

Research Article

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Ergodic capacity of correlated multiple-input–multiple-output channel with impact of transmitter impairments

<https://doi.org/10.1515/eng-2022-0338>
received March 15, 2022; accepted June 02, 2022

Abstract: The ergodic capacity of the ideal multiple-input and multiple-output (MIMO) channel grows unboundedly at the-high SNRs because of the assumption that the transceivers used in the system do not affect the signal that passes through them. In reality, the transceivers used in the MIMO system suffer from many impairments such as phase noise, quantization noise, and non-linear amplifier. These physical transceivers will affect the signal that passes through them. Thus, the influence of the physical transceivers cannot be neglected in the analysis of the MIMO channel capacity. Additionally, another essential thing that must be considered when analyzing the MIMO channel capacity is the correlation between the channel elements. Therefore, to get results close to the behavior of the practical results for the capacity of any MIMO system, this article proposes an algorithm that simulates the capacity of the correlated MIMO channel that suffers from Rayleigh flat fading with the presence of the physical transmitter. This work is done by considering that the effect of the physical transmitter at the n th transmitting antenna is a Gaussian distortion noise with a zero mean and variance proportional to the power of the signal transmitted through the same antenna. The simulation process for the proposed technique is manipulated by MATLAB software with 10,000 realizations of the channel. Also, the transmitter impairment thresholds employ in the simulation are 0.08 and 0.175. The simulation results showed that the channel capacity with the physical transmitter stopped

growing at a specific limit in the very high-SNRs, and it decreased at the medium and high SNRs when the correlation coefficient increased.

Keywords: transceiver impairments, transmitter distortion, ergodic capacity, correlated multiple-input–multiple-output channel, channel state information, physical transmitter, Kronecker model

1 Introduction

In the past few decades, most wireless communication systems contain one antenna for transmitting the signal and another for receiving it. That system was called the single-input and single-output system (SISO system). The problem in that system is represented by having one link between the transmitting antenna and the receiving antenna. Thus, the transmitted signal becomes more subject to the fading out. Therefore, other systems, such as single-input and multiple-output system (SIMO system) and multiple-input and single-output system (MISO system), have appeared recently to ensure that the transmitted signal reaches the receiver without fading out [1,2]. However, the demand to obtain a high data rate without increasing the transmitting power or bandwidth led to the emergence of what is known as a multiple-input and multiple-output (MIMO) system, which means having more than one antenna on both the transmitting and receiving sides. Some relevant works such as refs [3,4] show the actual need for the MIMO system.

Consequently, scientists turned toward designing the transceivers used in MIMO systems at the lowest possible cost, which led to a decrease in the quality of these devices. In other words, the transceivers used in the MIMO system suffer from many impairments such as phase noise, quantization noise, and non-linear amplifier. These impairments affect the signal that will pass over the transceivers used in the system, which degrades the MIMO channel performance [5]. Thus, the effect of the

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transceivers used in the MMO system cannot be neglected when analyzing the performance of the channel capacity.

In addition, another thing that influences the channel capacity of the MIMO systems, which cannot be neglected in the analysis of the MIMO channel, is the correlation between the channel elements. The correlation between the channel elements depends on the spacing between the antennas used in the system [6,7]. Therefore, many previous works analyze the ergodic capacity of the MIMO channel in different cases and different goals, some of which will have our focus later.

In the literature, the ergodic capacity of the MIMO systems is analyzed in many different ways. Telatar [8] assumed ideal transceivers and derived the basic formulas of the capacity for both the deterministic MIMO channel and random uncorrelated MIMO channel that suffers from Rayleigh flat fading. The researcher relied on the basic definition of channel capacity, which states that the capacity of any wireless channel is the maximum mutual information between the transmitter and receiver. His research results show the increase in channel capacity of the MIMO system compared to the SISO system if the channel state information is available at the receiver (CSIR). Alimi et al. [9] analyzed the ergodic capacity of the correlated MIMO channel in a frequency-flat Rayleigh environment in two different cases of channel state information (CSIT and CSIR), assuming that the transceivers used in the system are ideal. The research results showed that the capacity of the correlated MIMO channel decreases as the amount of correlation between the channel elements increases.

For different analyses, Bjornson et al. [10] described the behaviors of both the uncorrelated MIMO channel capacity experiencing Rayleigh flat fading and the finite-SNR multiplexing gain of the same channel in the presence of the physical transmitter with an impairment level of 0.05. Their research results showed that the capacity of the uncorrelated MIMO channel decreases as the level of impairments of the physical transmitter increases. Their work considered that the channel state information is available at the transmitter (i.e., CSIT). However, Studer et al. [11] demonstrated a new method for optimizing the ergodic capacity of the MIMO channel that suffers from Rayleigh flat fading in the presence of the physical transmitter, assuming that the channel components are independent of each other (i.e., an uncorrelated MIMO channel). They also assumed that channel information would be available in the receiver (i.e., CSIR). This method was implemented by considering that the impairment level of each RF chain on the transmitter side is proportional to the power of the signal transmitted through the antenna to which this RF chain belongs.

Furthermore, the researchers in ref. [12] discussed the situations of the MIMO channel capacity in the presence of the ideal transmitter and with the presence of the physical transmitter having an impairment level of 0.15, under the conditions of the Rayleigh flat fading, assuming that the channel information is only available in the receiver (i.e., CSIR). They assumed that the channel matrix elements are independent of each other (i.e., uncorrelated MIMO channel). Their research results showed that in the high SNR regime, the channel capacity behavior with the physical transmitter is utterly different from the channel capacity behavior with the ideal transmitter. In contrast, the authors in ref. [13] dealt with different levels of impairments of the transmitter to investigate the behavior of the uncorrelated MIMO channel capacity in a Rayleigh flat fading case.

Moreover, Singal et al. [14] analyzed the capacity of the uncorrelated MIMO-OFDM channel that suffers from Rayleigh flat fading, assuming that the transmitter used in the system is physical with an impairment level of 0.05. Also, they assume that the channel state information is available at the transmitter and receiver (i.e., CSIT). The simulation results of their work showed that the capacity of the uncorrelated MIMO-OFDM channel is stopped growing at a particular value in the high SNRs, which is called the capacity limit. Also, the same authors in ref. [15] investigated the energy efficiency of bulk and per-subcarrier antenna selection strategies in the MIMO-OFDM system with and without hardware limitations. The channel used in their work is the uncorrelated channel that suffers from Rayleigh flat fading. Their work required finding the ergodic capacity of the uncorrelated MIMO-OFDM channel with and without hardware defects. It has been noticed from their research results that as the value of these hardware deficiencies grows, the energy efficiency falls.

For different types of channels, Papazafeiropoulos et al. [16] analyzed the ergodic capacity of Rayleigh-product MIMO channels in the presence of the physical transmitter with an impairment level of 0.15. The channel of the MIMO system in the case of the Rayleigh product consists of two channels: the first channel between the transmitter and scatters and the second channel between the scatters and the receiver. The researchers assumed that the entries of the two channels are independent of each other (i.e., each channel is an uncorrelated MIMO channel). Additionally, they considered that the pieces of information of both channels are known to the receiver (i.e., CSIR). Their work results showed that at the high SNRs, the capacity of the Rayleigh-product channel in the case of the physical transmitter is stopped growing at a

particular value called the capacity limit. Papazafeiropoulos et al. [17] directed their focus to investigate the impact of residual hardware impairments on the ergodic capacity of dual-hop (DH) amplify-and-forward (AF) MIMO relay systems. The authors assumed that the channel from users to the relay and the channel from the relay to the base station are both uncorrelated MIMO channels. Also, they assumed that the level of impairments of the transceiver used in the system is changing between two values {0.1 or 0.25}. Their results showed that the ergodic capacity with transceiver impairments saturates after a certain SNR.

This article presents an algorithm focused on analyzing the effect of the transmitter impairment on the capacity of the MIMO system based on the correlated channel under the consideration of the Rayleigh flat fade. The most important practical applications of this work are cellular communications, advanced Wi-Fi networks, and 5 G massive systems.

To be more exact, in this article, we consider the case where the channel state information at the receiver (CSIR) is available. In contrast with the previously mentioned works [10–15], which analyze the uncorrelated MIMO channel with the presence of the physical transmitter, we try to analyze the correlated MIMO channel that undergoes the Rayleigh flat fade with the presence of the physical transmitter. Also, in this article, we compared the capacity of the uncorrelated MIMO channel with the capacity of the correlated MIMO channel in the presence of the physical transmitter. Finally, we show how the increase in correlation coefficient or the number of transmitting antennas affects the ergodic capacity of the MIMO channel in the presence of the physical transmitter.

The article is organized as follows. Section 2 introduces the MIMO system model and the problem formulation. Section 3 shows the ergodic capacity (average capacity) of the correlated MIMO channel with the presence of the physical transmitter. Section 4 focuses on the methodology of the proposed algorithm. Discussions and numerical results are provided in Section 5, while Section 6 concludes the article.

2 Correlated MIMO channel model with transmitter impairments

One of the most important things that affect the performance of the MIMO channel in practical applications is the defects of the transmitter used in the system, such as phase noise, quantization noise, and IQ imbalance. Numerous of these defects are avoided or compensated by implementing complex compensation operations in the system. However, some of these defects remain unaffected by the compensation operations. Thus, these residual defects will affect the quality of the signal that passes through the transmitter [18]. The residual defects in the transmitter are called the transmitter distortion, which in MIMO systems is modeled as Gaussian distortion noise with a zero mean and variance denoted by the symbol δ_t [19]. Therefore, the transmitted signal vector (S) in the physical MIMO systems will be affected by the surrounding environment (correlated channel matrix H_{cor}), the noise vector (Z), and the transmitter distortion vector (σ_t); Figure 1 illustrates the block diagram of the affine correlated MIMO channel model in the presence of the physical transmitter.

The above-mentioned channel model consists of N antennas to transmit the signals and M antennas to receive them. Also, the received signal vector (R) of the above model can be expressed by the following equation:

$$R = \sqrt{\text{SNR}} * H_{cor} * (S + \sigma_t) + Z, \quad (1)$$

where R is the complex-valued received signal vector with dimensional $(M \times 1)$ (i.e., $R \in \mathbb{C}^M$, where $\mathbb{C}$ is the complex numbers vector), SNR is the signal-to-noise ratio, $S \in \mathbb{C}^N$ is the transmitted signal vector with covariance matrix $Q_s = \mathbb{E}(SS^H)$, and Z is the zero-mean circular-symmetric complex Gaussian noise (i.e., $Z \sim \mathcal{CN}(0, I)$). The symbol σ_t is the distortion of the transmitter used in the MIMO system, and H_{cor} indicates that the Kronecker model represented the channel matrix to take the correlation between the channel elements into account, as shown in the following equation [20]:

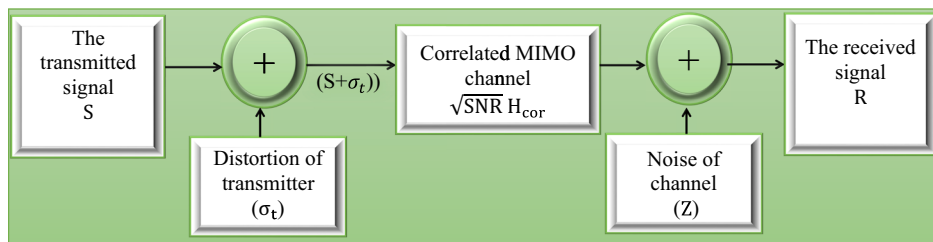


Figure 1: Block diagram of the affine correlated MIMO channel model in the presence of the physical transmitter.

$$H_{\text{cor}} = D_r^{1/2} * H_{i.i.d} * D_t^{1/2}, \quad (2)$$

where D_r is the correlation matrix on receiving side with dimensional $(M \times M)$, and D_t is the correlation matrix on the transmitting side with dimensional $(N \times N)$. Also, $H_{i.i.d}$ refers to the channel matrix with dimensions $(M \times N)$ whose entries are random complex variables, all subject to the Rayleigh distribution and independent of each other. Both matrices D_r and D_t can be found by the following equation [21–23]:

$$D_t = \begin{bmatrix} 1 & \alpha_t & \alpha_t^4 & \dots & \alpha_t^{(N-1)^2} \\ \alpha_t & 1 & \alpha_t & \dots & \vdots \\ \alpha_t^4 & \alpha_t & 1 & \dots & \alpha_t^4 \\ \vdots & \vdots & \vdots & \ddots & \alpha_t \\ \alpha_t^{(N-1)^2} & \dots & \alpha_t^4 & \alpha_t & 1 \end{bmatrix} \quad D_r = \begin{bmatrix} 1 & \alpha_r & \alpha_r^4 & \dots & \alpha_r^{(M-1)^2} \\ \alpha_r & 1 & \alpha_r & \dots & \vdots \\ \alpha_r^4 & \alpha_r & 1 & \dots & \alpha_r^4 \\ \vdots & \vdots & \vdots & \ddots & \alpha_r \\ \alpha_r^{(M-1)^2} & \dots & \alpha_r^4 & \alpha_r & 1 \end{bmatrix}, \quad (3)$$

where α_t indicates the signal correlation coefficient between two adjacent antenna elements on the transmitting side and α_r denotes the signal correlation coefficient between two adjacent antenna elements on the receiving side. In this article, for simplicity, we assume that $\alpha_t = \alpha_r = \alpha$. Also, the alpha value ranges from zero (an uncorrelated MIMO channel) to one (a highly correlated MIMO channel).

3 Ergodic capacity with transmitter impairments

The ergodic capacity of the uncorrelated MIMO channel in the presence of the physical transmitter can be found in the following equation [10]:

$$C_{\text{ER}} = \sup_{\text{trace}(Q_s)=1} \mathbb{E} \{ \log_2 (\det(I_M + (\text{SNR} H_{i.i.d} Q_s (H_{i.i.d})^H) + (\text{SNR} H_{i.i.d} \delta_t (H_{i.i.d})^H + I_M)^{-1})) \}, \quad (4)$$

where C_{ER} refers to the ergodic capacity of the uncorrelated MIMO channel in the presence of the physical transmitter, $\mathbb{E}(\cdot)$ refers to the expectation operator, $(H_{i.i.d})^H$ indicates to apply a Hermitian operation on the matrix $H_{i.i.d}$, and I_M refers to an identity matrix with size $(M \times M)$. The term $\text{trace}(Q_s) = 1$ refers to the power constraint of the system, and the symbol δ_t indicates the variance of transmitter distortion.

In this article, we want to find the ergodic capacity of the correlated MIMO channel in the presence of the physical transmitter. Therefore, we can rewrite equation (4) as follows:

$$C_{\text{ERC}} = \sup_{\text{trace}(Q_s)=1} \mathbb{E} \{ \log_2 (\det(I_M + (\text{SNR} H_{\text{cor}} Q_s (H_{\text{cor}})^H) + (\text{SNR} H_{\text{cor}} \delta_t (H_{\text{cor}})^H + I_M)^{-1})) \}, \quad (5)$$

where C_{ERC} is the ergodic capacity of the correlated MIMO channel with the presence of the physical transmitter.

In single-carrier MIMO systems, the effect of the physical transmitter at the n th transmitting antenna is modeled as a Gaussian distortion noise with zero mean and variance proportional to the signal's power transmitted through the same antenna. Therefore, the variance of the transmitter distortion can be given as the following equation [24–26]:

$$\delta_t = \kappa^2 \text{diag}(q_1, \dots, q_N), \quad (6)$$

where the elements $(q_1, \dots, q_N)$ are the diagonal elements of the matrix Q_s , and the parameter κ (kappa) is the level of impairments of the transmitter used in the system, which is given in the range of $[0.08, 0.175]$ in LTE systems [27].

In multi-carrier scenarios, there is a distortion leakage between the subcarriers. Thus, equation (6) cannot be applied to find the distortion of the transmitter. Therefore, for simplicity, the transmitter distortion will be considered to arrive at its average if the system is multi-carrier [10]. Thus, in the multi-carrier system, equation (6) can be rewritten as follows [10,26]:

$$\delta_t = \kappa^2 \frac{\sum_{j=1}^N q_j}{N} = \frac{\kappa^2}{N}, \quad (7)$$

where q_j is the power of the j th transmit antenna, the term $\sum_{j=1}^N q_j$ is the sum of the diagonal elements of the matrix Q_s (i.e., $\text{trace}(Q_s)$). Also, we know that $\text{trace}(Q_s) = 1$. Thus, the term $\sum_{j=1}^N q_j$ is equal one (i.e., $\sum_{j=1}^N q_j = 1$).

Moreover, this article will assume that the channel state information is available at the receiver (CSIR). Thus, equal power will be allocated to each transmitting antenna (i.e., $Q_s = \frac{I_N}{N}$), which means that δ_t will be equal to $\frac{\kappa^2}{N}$ regardless of the type of system used, whether a single carrier or multi-carrier. Hence, the ergodic capacity of the correlated MIMO channel shown in equation (5) becomes as follows:

$$C_{\text{ERC}} = \sup_{\text{trace}(Q_s)=1} \mathbb{E} \left\{ \log_2 \det \left(I_M + \frac{\text{SNR}}{N} H_{\text{cor}} (H_{\text{cor}})^H \left(\frac{\text{SNR} \kappa^2}{N} H_{\text{cor}} (H_{\text{cor}})^H + I_M \right)^{-1} \right) \right\}. \quad (8)$$

Suppose we want to find the ergodic capacity of the correlated MIMO channel with the presence of the ideal

transmitter. In this case, we put the parameter κ equal to zero in equation (8) and get on the following equation:

$$C_{\text{ERI}} = \sup_{\text{trace}(\mathbf{Q}_s)=1} \mathbb{E} \left(\log_2 \det \left(\frac{\text{SNR}}{N} \mathbf{H}_{\text{cor}} (\mathbf{H}_{\text{cor}})^H + \mathbf{I}_M \right) \right), \quad (9)$$

where C_{ERI} is the ergodic capacity of the ideal correlated MIMO channel. Equation (9) is also found in ref. [9].

4 Methodology of proposed algorithm

In order to obtain realistic results that show the performance of the channel capacity of a particular MIMO system before implementing it in practice, this article proposes an algorithm that analyzes the capacity of the MIMO channel that suffers from Rayleigh flat fading in the presence of the physical transmitter, taking into account the effect of the correlation between the elements of the channel. Before implementing the proposed algorithm, the following considerations must be taken into account:

1. The number of transmitting antennas in the system must be greater or equal to the number of receiving antennas.
2. The correlation coefficient must be greater or equal to zero and smaller than one (i.e., $0 \leq \alpha < 1$).

After taking into account the matters mentioned earlier, the algorithm is implemented by the MATLAB program, and the implementation steps are as follows:

Step 1: Entering all required inputs (number of transmitting antennas (N), number of receiving antennas (M), correlation coefficient (α), number of realizations of the channel (L), transmitter impairments level (κ), and the range of SNR in decibel).

Step 2: Converting the range of SNR from decibel to linear scale.

Step 3: Creating a zero matrix with size $(\text{length}(\text{SNR}) \times L)$ to save the value of ergodic capacity in the case of the ideal transmitter for each value of SNR at each realization of the channel.

Step 4: Creating zero matrices with size $(\text{length}(\text{SNR}) \times L)$. The number of zero matrices that must be generated equals the length of the impairment level vector (e.g., if the impairments level entering consists of two values, the number of the zero matrices is two matrices). Each matrix is devoted to saving the value of ergodic capacity in the case of the physical transmitter for each SNR value at each channel's realization.

Step 5: Finding the $\mathbf{D}_t$ and $\mathbf{D}_r$ matrices by using equation (3).

Step 6: Generating the channel matrix ($\mathbf{H}_{i,i,d}$) whose components are complex random variables subject to a Rayleigh distribution and independent of each other. The Generating of $\mathbf{H}_{i,i,d}$ matrix is done by using the following equation:

$$\mathbf{H}_{i,i,d} = \sqrt{(0.5)} * (\text{randn}(M, \max(N), L) + 1i * \text{randn}(M, \max(N), L)), \quad (10)$$

where L is the number of realizations.

Step 7: Creating the first loop to cover all the realizations of the channel. This loop includes finding the $\mathbf{H}_{\text{cor}}$ matrix at each realization of the channel. The finding of $\mathbf{H}_{\text{cor}}$ matrix is done by using equation (2).

Step 8: Creating the second loop inside the first loop to cover all the values of SNR at each realization of the channel. This loop includes computing the ergodic capacity of the correlated MIMO channel in the case of the ideal transmitter according to equation (9).

Step 9: Creating the third loop inside the second loop to cover each value of the impairment level if the impairment level is entered with more than one value. Within this loop, the correlated MIMO channel capacity is computed in the case of the physical transmitter by using equation (8).

Step 10: Drawing the ergodic capacity in each case as a function of SNR.

5 Results and discussion

Here, we will explain the results of applying the proposed algorithm to the MIMO channel in various cases. We will show how the distortion of the transmitter and correlation between the channel elements affect the ergodic capacity (average capacity) of the MIMO channel, assuming channel information is available at the receiver (CSIR). The number of realizations used in the simulation is 10,000 (i.e., $L = 10,000$), and the transmitter's impairments level values used in the simulation process are 0.08 and 0.175.

Initially, we assume the existence of a MIMO system with $N = 4$, $M = 4$, and varying SNR. Figure 2 shows the ergodic capacity of the channel of this wireless system in different cases that can be summarized as follows:

1. Ergodic capacity with the ideal transmitter ($\kappa = 0$) and different correlation coefficients $\alpha \in \{0, 0.4\}$ (i.e., the ergodic capacity of uncorrelated MIMO channel ($\alpha = 0$) and the ergodic capacity of correlated MIMO channel

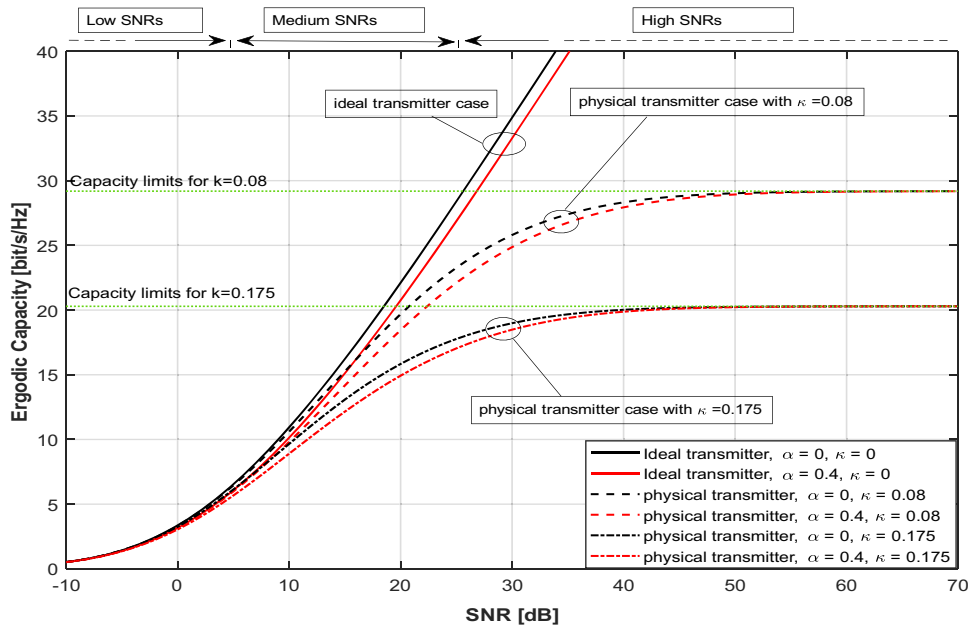


Figure 2: Ergodic capacity of the 4×4 MIMO channel with different levels of impairments $\kappa \in \{0, 0.08, 0.175\}$ and different correlation coefficients $\alpha \in \{0, 0.4\}$.

($\alpha = 0.4$)), both in the case of the presence of the ideal transmitter).

2. Ergodic capacity in the case of the physical transmitter with $\kappa \in \{0.08, 0.175\}$ and $\alpha \in \{0, 0.4\}$.

The earlier figure shows that the channel capacities with the physical transmitter behave in the same manner as the channel capacities with the ideal transmitter in the low and medium SNRs. While in the high SNRs, the channel capacities behavior in the case of the physical transmitter is entirely different from the channel capacities behavior in the case of the ideal transmitter. Also, Figure 2 shows that the channel capacities in the case of the physical transmitter are stopped growing at a specific value in the very high SNRs, which is called the capacity limit. On the other hand, it shows that the channel capacities in the case of the ideal transmitter grow infinitely in high SNRs.

In addition, at medium and high SNRs, Figure 2 shows that the channel capacity in the case of the ideal transmitter with $\alpha = 0$ is greater than the channel capacity in the case of the ideal transmitter with $\alpha = 0.4$. In other words, in the case of the ideal transmitter at the medium and high SNRs, there is a spacing between the channel capacity with $\alpha = 0$ and the channel capacity with $\alpha = 0.4$. At the same time, as illustrated in Figure 2, there is a spacing between the channel capacity with $\alpha = 0$ and the channel capacity with $\alpha = 0.4$ in the case of the physical

transmitter for any level of impairments $\kappa \in \{0.08, 0.175\}$. However, this spacing decreases until it disappears entirely in the very high SNRs. Thus, we can say that the capacity limit (the value at which capacity stops growing) for any level of impairments $\kappa \in \{0.08, 0.175\}$ in the case of a physical transmitter is constant for any value of alpha.

Moreover, Figure 2 shows that the capacity limit decreases as the transmitter impairments level (κ) increases. Thus, the ergodic capacity also decreases as the level of impairments increases. Table 1 shows the capacity limit values of the 4×4 MIMO channel over different impairment levels and different correlation coefficients.

Now, we suppose the same previously MIMO system, but with the varying correlation coefficient not equal to zero $\alpha \in \{0.4, 0.6, 0.8\}$; Figure 3 shows the effect of increasing the correlation coefficient on the capacity of the MIMO channel in the case of the physical transmitter.

It can be seen from Figure 3 that the channel capacity in the case of the physical transmitter with any level of

Table 1: Capacity limit values of the 4×4 MIMO channel with different impairment levels and different correlation coefficients

Level of impairment (κ)	Correlation coefficient (α)	Capacity limit (bit/second/hertz)
0.08	$\alpha = 0$ or $\alpha = 0.4$	29.19
0.175	$\alpha = 0$ or $\alpha = 0.4$	20.29

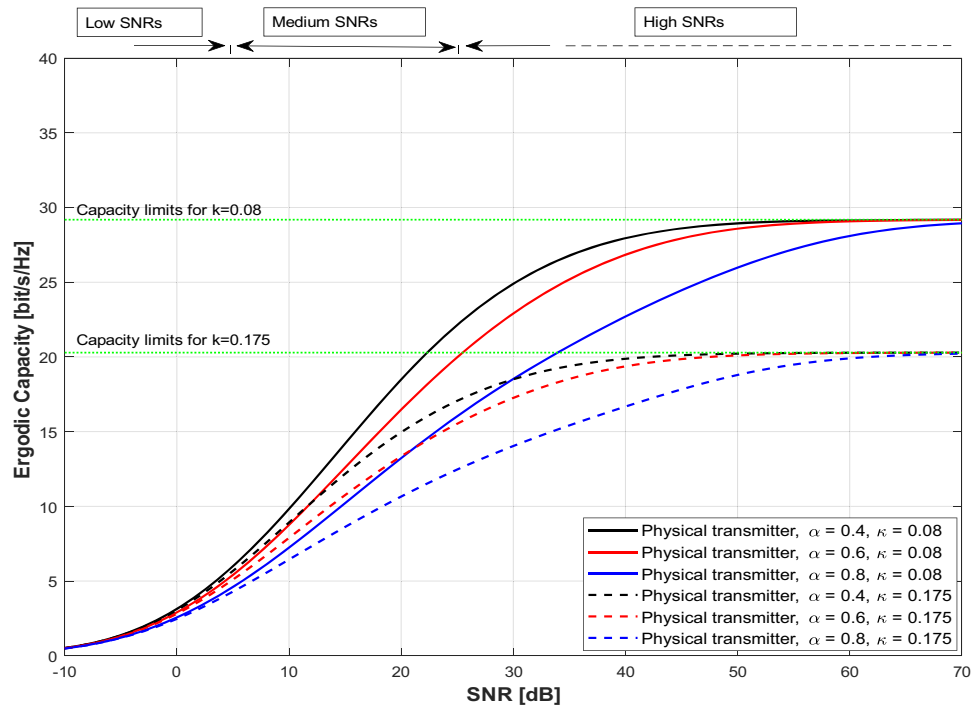


Figure 3: The effect of increasing the correlation coefficient on the 4×4 MIMO channel capacity in the case of the physical transmitter with different impairment levels $\kappa \in \{0.08, 0.175\}$.

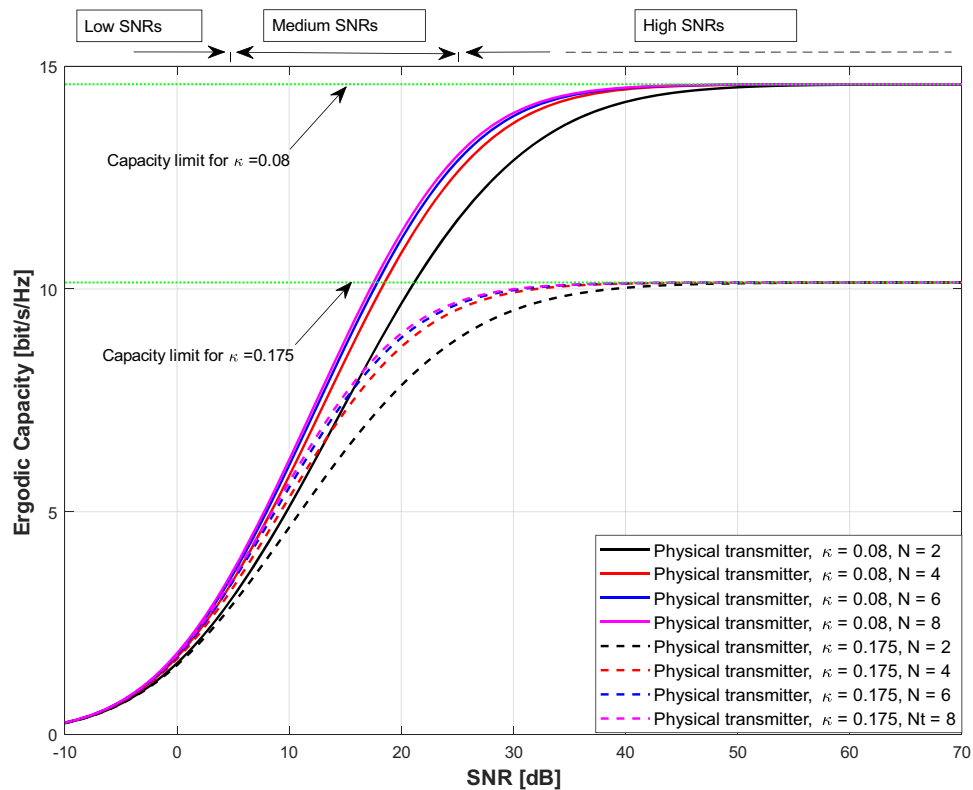


Figure 4: The ergodic capacity of the correlated MIMO channel in the case of physical transmitter with $M = 2, N \in \{2, 4, 6, 8\}$, $\kappa \in \{0.08, 0.175\}$, and $\alpha = 0.4$.

Table 2: Capacity limit values of the correlated MIMO channel with $M = 2$, $N \in \{2, 4, 6, 8\}$, $\kappa \in \{0.08, 0.175\}$, and $\alpha = 0.4$

Level of impairment	Number of transmitting antennas	Capacity limit (bit/second/hertz)
0.08	$N = 2, 4, 6, 8$	14.59
0.175	$N = 2, 4, 6, 8$	10.15

impairments $\kappa \in \{0.08, 0.175\}$ is affected by the increase in the correlation between the channel elements, where the higher the correlation coefficient, the channel capacity decreases more in the medium and high SNRs. However, the capacity limit (the limit at which the capacity stops growing in the very high SNRs) remains constant whatever the correlation coefficient value (i.e., the capacity limit is the same for any value of α), as shown in the above figure. Thus, we can say that the capacity limit value does not depend on the correlation coefficient value. In contrast, the increase in the impairment transmitter level (κ) leads to a decrease in the capacity limit, which decreases the channel capacity. Therefore, the level of transmitter degradation (κ) plays an essential role in channel capacity performance because it controls the ultimate limit of channel capacity.

Now, we will show how the ergodic capacity of the correlated MIMO channel in the presence of the physical transmitter will be affected if the number of transmitting antennas increases. We suppose a correlated MIMO channel with $M = 2$, $N \in \{2, 4, 6, 8\}$, and varying SNR. The level of impairment changes between two values $\kappa \in \{0.08, 0.175\}$, and the correlation coefficient is 0.4 (i.e., $\alpha = 0.4$). Figure 4 shows the ergodic capacity of this correlated MIMO channel.

It can be seen from Figure 4 that the correlated MIMO channel capacity increase with a certain amount when the number of transmitting antennas increases. This particular amount of the growth in the channel capacity decreases every time the number of transmitting antennas increases without increasing the number of receiving antennas, as shown in the above figure. In other words, the increase in the number of transmitting antennas to a large number without increasing the number of receiving antennas will not significantly increase the channel capacity.

In addition, from Figure 4, it can be noted that the capacity limit is not affected by the increase in the number of transmitting antennas when the number of receiving antennas remains constant. In other words, the capacity limit remains constant for any number of transmitting antennas when the number of receiving antennas remains constant. Table 2 shows the values of

the capacity limit of the MIMO channel with $M = 2$, $N \in \{2, 4, 6, 8\}$, $\kappa \in \{0.08, 0.175\}$, and $\alpha = 0.4$.

Moreover, the capacity limit of the MIMO channel with $M = 4$, $N = 4$, $\alpha = 0.4$, and $\kappa = 0.08$ is equal to 29.19 bit/second/hertz, as shown in Table 1, while the capacity limit for the same criteria, but with $M = 2$, is equal to 14.59 bit/second/hertz, as shown in Table 2. Therefore, we can say that the number of receiving antennas affects the capacity limit. In other words, the capacity limit is directly proportional to the number of receiving antennas.

6 Conclusion

This article clears that the transmitter distortion and correlation between the channel elements cannot be neglected in the MIMO channel capacity analysis, in which this article presents an algorithm to analyze the ergodic capacity of the correlated MIMO channel in the presence of the physical transmitter. The simulation results of this algorithm showed that the ergodic capacity of the MIMO channel is decreased more in the medium and high SNRs when increased the transmitter impairment level (κ), correlation coefficient (α), or both. Also, the simulation results showed that the ergodic capacity in the case of the physical transmitter would stop growing at a specific value in the very high SNRs, which is called the capacity limit.

Finally, the simulation results of this algorithm showed that the capacity limit is affected by the impairment level and the number of receiving antennas and do not affect by the correlation coefficient and the number of transmitting antennas.

Conflict of interest: Authors state no conflict of interest.

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