Regular Article

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Control logic algorithm to create gaps for mixed traffic: A comprehensive evaluation

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Abstract: Over the last decade, the increase in the number of vehicles has affected traffic performance, causing traffic congestion. However, intersections, where different flows intersect, are among the primary causes of traffic congestion besides bottlenecks. Bottlenecking in the minor stream is mainly due to the extended queueing, specifically due to minimal gaps in the mainline stream as the intersection's high priority exists with the major stream. This research aims to control connected and automated vehicles (CAVs) to help generate additional usable gaps for the minor road vehicles to enter the intersection without interrupting the mainline traffic flow. A probability function is developed to estimate the probability of CAVs creating additional usable gaps. The proposed logic is simulated in unsignalized and semi-actuated signalized intersections, and a field investigation is conducted. Simulation results show that the minor road delays and queue length are minimized without causing a significant delay to the mainline. Results show that major road interruptions are reduced at a semi-actuated signal control scenario when CAVs' penetration increases. It can be observed that deploying CAVs in the road network with the proposed method can positively impact traffic efficiency, where the intersection's performance and safety are improved.

Keywords: connected and automated vehicles, unsignalized intersections, intersection priority, headway distribution

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1 Introduction

Over the last few decades, the rapid population growth and the attendant increase in vehicle numbers have caused widespread travel problems, with traffic congestion forecast to increase by 60% by 2030 [1,2]. Those problems lead to inefficient use of the transportation network, forcing drivers to spend more time commuting and increasing the overall fuel consumption. Among the major bottlenecks that cause traffic congestion are road intersections with different flow conflicts.

Most intersections in urban areas are signalized to regulate the movement of traffic flows. The unsignalized intersections are typically found in suburban areas where low and high traffic flows meet. There is a higher chance of collision at an unsignalized intersection than signalized intersection due to the driver's indecision or wrong decision to enter the intersection from the unsignalized approach. A report by the National Highway Traffic Safety Administration (NHTSA) compiled in 2011 indicated that 40% of car collisions in the US happen at intersections, and 60% of them are related to unsignalized intersections [3].

Accidents occur at unsignalized intersections because of lack of traffic control devices and often due to geometric condition constraints or inappropriate speed control [4,5]. With such limitations, drivers from the minor road often face the challenge of selecting a proper vehicular gap to enter the intersection, as any mistake could lead to a safety hazard, in addition to a negative effect on the efficient intersection operations.

The safety and performance of unsignalized intersections are largely influenced by the driver's gap acceptance behavior. Gap acceptance behavior represents the choice to accept or reject a gap of a specific size [6]. A critical gap is the most common metric of driver's gap acceptance behavior. Different scholars have provided several definitions of the critical gap, but it is considered as the time interval between two succeeding major road vehicles, required to be maintained by the driver of a minor road vehicle for taking the desired turn safely [7].

In many applications, the critical gap is measured by the smallest gap drivers accept on intersections [8].

At major–minor intersections, traffic control for vehicles on the major road generally prioritizes the minor road. Driver behavior from the minor road vehicle is highly affected by the number of available gaps on the major road. With light traffic on the major road, larger gaps may be found in the traffic stream; on the other hand, heavy traffic on the major road is more likely to have smaller and less safe gaps for the minor road drivers to choose from. The vehicle delay for the minor road vehicle influences the critical gap selection. A longer waiting time experienced by the driver while waiting can result in losing patience to accept a shorter critical gap [9].

Moreover, additional factors may also influence the gap selection behavior of the driver, including age group, time of the day, and trip purpose, but the most observed factors are the presence of a queue behind the driver, the number of gaps rejected, as well as the wait time. The presence of a queue behind the driver is likely to make the driver feel pressured and thus, accept a smaller gap; when more gaps are rejected with an extended waiting time, the minor road drivers may become impatient and choose to accept smaller gaps [10,11].

Connected and automated vehicles (CAVs) use advanced wireless technologies like Cellular Vehicle-to-Everything (C-V2X) or Dedicated Short-Range Communication (DSRC). These wireless technologies would create a link between the vehicle and the network or among vehicles to share information such as speed [12], location, traffic control, etc. Such data communication capabilities and the supporting infrastructure may help improve traffic operation and reduce traffic flow interruption, thereby enhancing safety, fuel consumption, and emission [13,14].

When combined with the wireless communication systems, the automated driving and data processing technology empower CAVs to be a potential solution to many field problems that are difficult to solve or require excessive resources to deal with by using the traditional engineering method. It is also well understood that, before CAVs become dominant, frequently used technology, a mixed form of vehicles, and human-driven vehicles (HDVs) along with CAVs, will be driving simultaneously on the road, so it is important to investigate CAVs at intersections considering mixed traffic condition with different CAVs' penetration.

In the past decades, emerging technologies that could assist or automatically control the driving process of intelligent vehicles have drawn great interest from researchers and engineers. With the development of Vehicle to Vehicle (V2V) and Vehicle to Infrastructure (V2I) communication technologies, intelligent vehicles could make driving informed decisions based on multi-source data, such as the speed/location of surrounding vehicles and the signal plans of surrounding intersections. The headway distribution of vehicles streaming on a mainline road towards an intersection can significantly affect the number of usable gaps that a minor road enters the intersection.

This research aims to utilize CAVs to help generate additional adequate gaps for the minor road vehicles to enter the intersection without interrupting the mainline traffic flow. The methodology works when vehicle arrivals on the main road permit the implementation of the control strategy to improve intersection efficiency in mixed traffic conditions. This study consists of three main contributions, which are: formulating a probability model to identify the headways that are less than the critical headway that can be used by minor road vehicles when CAVs reduce speed. Additionally, this research developed a framework that guides CAVs in creating gaps at such intersections while considering safety and efficiency under mixed traffic conditions. The framework was simulated in a microscopic environment to validate and evaluate the efficiency and safety of the intersection before and after the method. A field investigation was conducted at two different locations to study the feasibility of the proposed algorithm.

2 Literature review

CAV technologies offer potentially transformative traffic impacts, including significant mobility, safety, and environmental benefits. Numerous algorithms have been developed to improve intersection throughput, and most of them used simulation to evaluate the proposed methods [1]. Simulation assumptions limit the validity of evaluation results, and the lack of conducting field experiments in real traffic conditions exacerbates this problem of inaccuracy, leading to improper model evaluation. This problem has resulted in a wide range of differences in effectiveness among studies examining the same CAV applications. Since it is too early to achieve high penetration of CAVs and AVs on roads, it is practical and essential to investigate the mixed flow of HDVs and intelligent vehicles at currently used intersections [15,16].

Many CAV application studies on unsignalized or signalized intersections have been reported, and most of them are conducted under the assumption of a 100% CAV environment to explore potential benefits of the

CAVs; only a few studies have focused on unsignalized intersections or minor roads with semi-actuated signal control in mixed traffic conditions [1]. Zhong et al. studied a priority unsignalized intersection management to ensure vehicle crossing, while considering the efficiency and safety of both the approaches [17]. Similarly, a gap-based eco-driving speed control algorithm for unsignalized intersections was studied [18] by considering the realistic traffic conditions. The proposed algorithm incorporates gaps, initial speed, and vehicular position as control variables while optimizing the acceleration/deceleration to cross the intersection. Although the proposed method was found to improve efficiency while considering safety, the study considers only fully automated vehicles at the intersection.

Many studies have presented various distributions to represent the headways for traffic with only conventional vehicles, but not many exist for pure CAVs or a mixed traffic stream. For single-lane traffic, there are various sequence combinations of conventional vehicles and CAVs, such as conventional vehicles following conventional vehicles, conventional vehicles following CAVs, CAVs following CAVs, and CAVs following conventional vehicles. With an intention to develop an algorithm that can help minor road vehicles to have more usable gaps to enter the intersection based on the technology of CAVs, this article is focused on the probability distribution of the headways to counter for the number of possible headways where CAVs can create the usable gaps for the minor road vehicles.

Autonomous Intersection Management, or AIMS, was first put forth in the works of Dresner and Stone, which was followed by several researchers in similar concepts, most of which used the First-come, first-serve (FCFS) model [19]. Several methods have been used to develop AIMS; including mixed-integer linear programming [20], linear programming [21], and mixed-integer non-linear programming [22]. The purpose of formulating AIMS in different models was to ensure optimization [23] while also serving as a control framework [24]. Likewise, there have been models that focus on the issues related to dynamic optimization [25] and models of predictive control framework [26] of AIMS.

Various AIM models illustrated different findings. Some models discussed the vehicle arrival time at the departing time or conflicting points alongside the calculation of intersection exit time [27]. While on the other hand, some models focused on the vehicle number which was allowed to move [22]. Nevertheless, all these models are intended to maximize the overall throughput of AIM [28] while also limiting the travel period [29] as well as reducing the fuel consumption [30] to diminish the potential threats, if any [24].

The purpose behind the development of AIM is to ensure that all the vehicles follow the controller by compromising the connected automated or autonomous vehicle (AVs/CAVs). However, limited studies have focused on the interaction of AIM with human vehicles (HVs) that are either partially or completely un-equipped with V2V and V2I communication facilities, along with the ones with no autonomous driving module [18]. Most of the researchers studying the interaction use traffic lights as the medium of communication with HVs, while others only consider automated vehicles and autonomous vehicles without considering the mixed traffic.

The application of CAV on unsignalized intersections has been explored in many studies. Among these studies, a major proportion of this research is conducted on the assumption of a 100% CAV environment to explore the benefits of the CAVs. However, only a few studies focus on the unsignalized intersections in mixed traffic conditions [31]. Chen et al. [32], in their study, explored the prioritization of unsignalized intersection management to ensure vehicle crossing along with considering the safety and efficiency of the approaches. It is suggested that vehicles on the major road enjoy priority while also creating gaps for those on the minor road. In this regard, a gap-based approach equipped with an eco-driving speed control algorithm has been studied for unsignalized intersections to take into consideration the realistic traffic conditions. The purpose of the algorithm is to include information related to gaps, vehicle position, and initial speed as control variables, while suggesting ways to optimize and develop acceleration and deceleration profiles and it helps the vehicle at the stop line. Though the study incorporated only fully automated vehicles at the intersection, the study has been found to have positively improved the efficiency of AIM while factoring in the element of safety.

All the above studies have treated the traffic coming to the intersection at an equal level of importance. More studies are needed at a major–minor intersection with priority set for the major road under mixed traffic conditions. A few studies conducted a feasibility study to test the capability of the control algorithm in real traffic scenarios. This research aims to utilize CAVs to help generate additional adequate gaps for the minor road vehicles to enter the intersection without interrupting the mainline traffic flow. The methodology works when vehicle arrivals on the main road permit the implementation of the control strategy to improve intersection efficiency in mixed traffic conditions.

The rest of the article is organized as follows: First, the number of headways that CAVs can create under mixed traffic conditions using the expected probability headway distribution is calculated. Then, the framework that guides CAVs in creating gaps is proposed, including the formulation, communication, and simulation of the proposed method in two different cases. Next the simulation results are conducted to validate the effectiveness of the proposed method. Also, a field test is then conducted to validate the proposed algorithm in real traffic conditions, and finally, the conclusion of this work and some future works to improve this study are presented.

3 Methodology

The methodology in this article is presented in sections, and consists of three main sections: (i) developing a headway distribution of the main road-stream to predict the possibility of gaps that CAVs can help to create; (ii) developing a framework (control logic) that guides CAVs to create additional gaps on the mainline for the minor road vehicles to enter the intersection safely; (iii) evaluating the proposed algorithm by an experiment of case studies for two types of intersections.

3.1 Gap acceptance of unsignalized intersections

Gap acceptance at an intersection of a major road and a minor road depends on the choice of the minor road vehicle to accept an available headway between approaching vehicles. However, the amount of delay by the minor road vehicle before merging into the major roads depends on the frequency and distribution of headways equal to or greater than the critical headway selected as the minimum acceptable headway to enter. At a major–minor intersection most seen in suburban areas, the upstream signals often do not affect vehicle arrivals at intersections. Numerous studies have found that such vehicle arrivals follow the Poisson distribution in light to medium traffic flows [33,34].

3.1.1 Headway distribution on major road

Headway is the time interval between two successive vehicle arrivals at a point. A waiting vehicle on the minor road approach will merge into the major road only if a proper headway t is available on the major road. The Poisson distribution describes the probability of having a certain number of vehicles arriving at a given time interval, and it is calculated as follow:

$$p(X = x) = \frac{e^{-m} m^x}{x!}, \quad 0 \leq x \leq \infty, \tag{1}$$

where m is the average arrival rate, and x is the number of vehicles. Since m is dependent on the duration of t, it can be written as vt, where v is the number of vehicles expressed in a unit time.

Since the probability of having no vehicles arriving during time t represents the probability of having the next headway at t or longer, the headway distribution for those at least the size of the critical headway, t_{cr}, for merging into the main road can be estimated by modifying equation (1). For vehicles to merge into the major road from a minor approach, headways at or larger than t_{cr} are required, which and all headways, can be obtained through an adjusted probability density function, as:

$$\bar{P}_0(t \geq t_{cr}) = \int_{t_{cr}}^{\infty} \frac{2\sqrt{v}}{\sqrt{\pi}} e^{-v(t_{cr}^2)} dt, \tag{2}$$

where t_{cr} represents the critical headway in seconds, and v is the traffic volume in veh/s.

Solving equation (2) using the Gaussian error function method, the cumulative probability that headways in the mainline traffic are greater than or equal to the critical headway t_{cr} would be:

$$\bar{P}_0 = 1 - \mathrm{erf}(\sqrt{v}\ t_{cr}). \tag{3}$$

Vehicles' arrival at a major–minor road intersection on the major road is affected by traffic volume and influenced by the local conditions, with interarrival times not less than the minimum headway, t_{min}, which is the shortest car-following time interval or spacing expressed in time. The expected cumulative probability of all headways less than the minimum headway is determined as follow:

$$P_{min}(t \geq t_{min}) = \int_{0}^{t_{min}} \frac{2\sqrt{v}}{\sqrt{\pi}} e^{-v(t_{min}^2)} dt, \tag{4}$$

where t_{min} is the minimum headway in seconds.

Solving equation (4) to get:

$$\bar{P}_{min} = \mathrm{erf}(\sqrt{v}\ t_{cr}). \tag{5}$$

The critical headway is larger than the minimum headway. The purpose of this research is to help create additional gap time between arriving vehicles on the major road by using CAVs that reduce the approaching speed to the preceding vehicles so that some of the gaps less than the critical gap may be extended and become usable. For safety concerns, the slowdown by a CAV is limited to the maximum allowable reduction in speed in case of small vehicle spacing behind the CAV. Based on field data in this research, the size of the additional gaps that CAVs can create is in a certain range, definable at the specific testing sites with sight distance and data communication limits. More discussion on the related aspects is given in the later sections of the article.

For simplicity in developing the prototype model and control logic, we focused our discussion on the single-lane situation where the entering vehicle from the minor road only needs to watch the traffic on one lane from each direction. To help estimate the scope of application of the proposed study, the probability that CAVs can create additional gap times in the major road traffic stream, or $\bar{P}_{\mathrm{CAV}}$, is estimated as the difference between the headways less than the critical headway and those greater than the minimum headway:

$$\bar{P}_{\mathrm{CAV}} = (1 - \bar{P}_0) - \bar{P}_{\mathrm{min}}. \quad (6)$$

Since CAVs are mixed with regular vehicles, the total number of additional headways, N_{cav}, that CAVs can possibly create depends on the number of CAVs, q_{cav}, on the major road and can be determined as:

$$N_{\mathrm{cav}} = (\bar{P}_{\mathrm{CAV}})^{*} q_{\mathrm{cav}}. \quad (7)$$

The opportunities for CAVs to help create additional usable gaps for the minor road vehicles are influenced by the traffic volume and market penetration rate of CAVs on the major road. Using equations (2)–(7), Figure 1 shows how N_{cav} for right-turning vehicles on the minor road changes at different major road volumes and CAV penetrations.

The results show that N_{cav} increases with the increase in CAVs' penetration and the major road volume. However, when the major road volume increases, the number of gaps decreases. This is understandable because with the increase in the major road volume, the headway between two successive vehicles becomes smaller, making it more difficult to create a headway greater than the critical gap.

Equations (2)–(7) have considered only one direction of the mainline traffic, so they can only be applied to the right-turning vehicles from the minor road. More commonly, a two-way traffic condition should be considered since there are left-turn vehicles from the minor road as well. Therefore, the number of gaps that can be created in both directions for the vehicles turning left relies on the conditional probability in the desired gap range as follows:

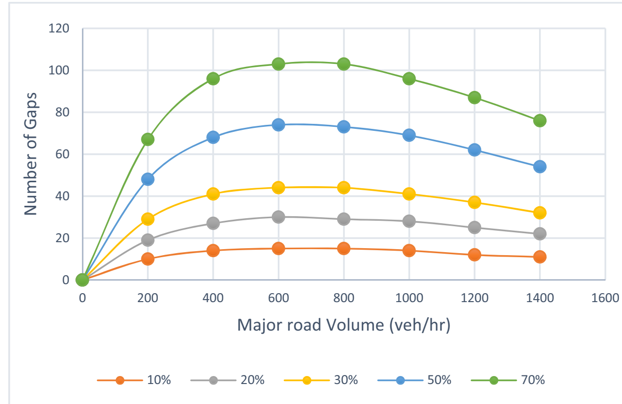

Figure 1: Number of usable gaps CAVs may possibly help create for right turns.

$$\bar{P}_{\mathrm{CAV(L)}} = \bar{P}_{\mathrm{CAV(E)}}{}^{*}\, \bar{P}_{\mathrm{CAV(W)}}, \quad (8)$$

where $\bar{P}_{\mathrm{CAV(E)}}$ is the probability of headways that an eastbound CAV can help create and $\bar{P}_{\mathrm{CAV(W)}}$ is that for the westbound CAV. The number of headways that CAVs can utilize to create additional usable gaps for the minor road vehicles turning left is shown in Figure 2, at different levels of CAV penetration and major road volume.

Figure 2 shows that, since the left-turn vehicles would need to find a safe gap in both directions on the major road, the probability of finding such a gap is less than vehicles turning right. Similarly, the number of gaps creatable by CAVs for the minor road left turns increases with the increase in the major road volume and the CAVs penetration, and as the major road volume continues to increase, the number of such gaps decreases as more vehicles on the major road result in smaller gaps.

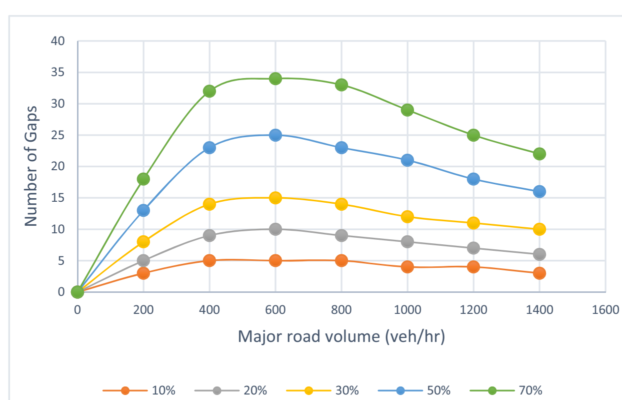

Figure 2: Number of gaps CAVs can possibly help create for left turns.

3.2 Control logic

It has been shown that CAVs can help create additional usable gaps for the minor road vehicles to enter the major road. This section discusses the development of an operational algorithm for generating such gaps and the supporting vehicle control strategies. The objective is to develop a systematic framework and implementation plan that uses CAVs for gap creation so that the application can be analyzed in simulation and its feasibility tested in the field. The control algorithm considers safety as the priority, and it intends to minimize interruptions to the mainline traffic flow due to vehicles from the minor road.

3.2.1 Intersection layout

T-intersections are commonly seen in areas when a minor road joins a major road, where vehicles from the minor road merge into the mainline traffic stream by making a left or right turn. The minor road approach is controlled by either an actuated signal or a stop sign where the major road traffic is given the right of way most of the time, as shown in Figure 3. CAVs are operated only on the major road, and the vehicles on the minor road are all HVs; the case of CAVs on the minor road is not considered here since it is easier for communication and vehicle control.

The main assumptions of the modeling method are as follows:

1. The intersection has been instrumented with a roadside unit (RSU) for data communication with all CAVs.
2. The geometric and environmental conditions do not hinder the decision-making and operation of the minor road vehicles.
3. CAVs can detect the leading and following vehicles and determine the spacings at different speeds of those vehicles.

3.2.2 The front gap

The proposed control system works in a mixed traffic environment since it considers both CAVs and HVs. When a CAV is within the communication range with the RSU, and a vehicle is detected in the minor street, the CAV detects the vehicle ahead of it and calculates whether the front gap when it moves to the intersection is greater than the critical gap. If the gap is not more than the critical gap, as explained in the previous section, the CAV will decide how much speed reduction is needed to help maintain a distance beyond the established critical gap if other conditions permit.

In the scenario shown in Figure 4, the CAV is following an HV and separated by an existing gap time, T_1, which is measured as:

$$T_1 = \frac{L_{AV}}{v} - \frac{L_{HV}}{v}, \tag{9}$$

where v is the approaching speed in ft/s, L_{AV} is the distance of the CAV from the intersection, and L_{OV} is the distance of the HV to the intersection. Since the distances are measured near the intersection, speed variation before reaching the intersection is not considered.

The minor road vehicle looking for a safe gap to enter the intersection requires a gap equal to or greater than the critical gap, T_0. For a minor street vehicle to enter the intersection safely, T_1 must be greater than T_0. In the scenario where T_1 is less than T_0, and approaching CAV will be looked at for possible speed reduction to add some extra gap time, Δt_c, in the front vehicle, as shown in Figure 5:

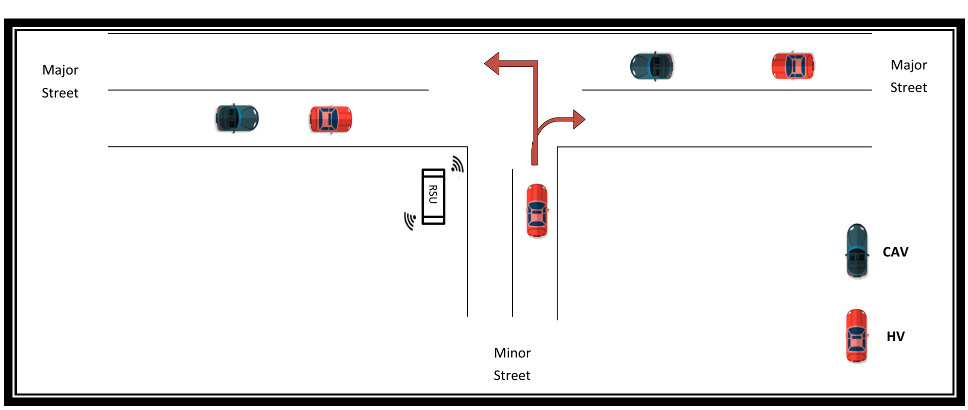

Figure 3: Layout of the intersection studied.

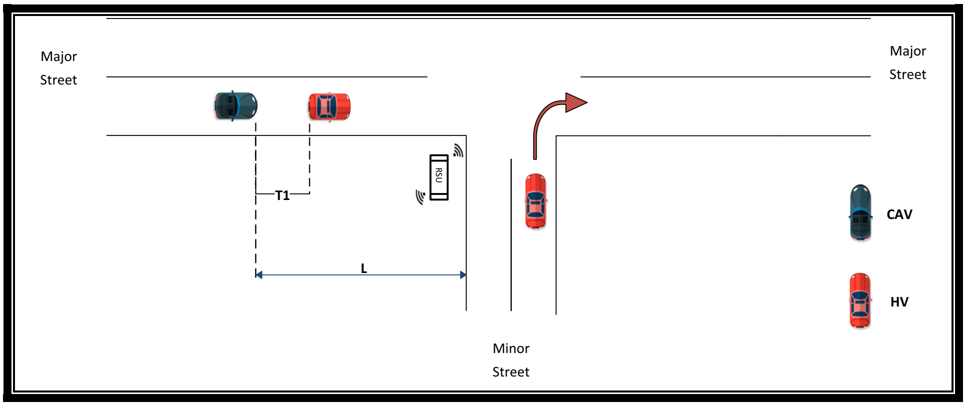

Figure 4: Front gap between CAV and the leading vehicle, T_1.

$$\Delta t_c = \frac{L_{\mathrm{AV}}}{v_c} - \frac{L_{\mathrm{AV}}}{v}, \quad (10)$$

where v_c is the speed of CAV after reduction.

The extra gap time added to T_1 must have a combined value greater than T_0. As a result, the following equation is obtained:

$$\frac{L_{\mathrm{AV}}}{v} - \frac{L_{\mathrm{HV}}}{v} + \Delta t_c \geq T_0. \quad (11)$$

Substituting Δt_c using equation (10), then:

$$\frac{L_{\mathrm{AV}}}{v} - \frac{L_{\mathrm{HV}}}{v} + \frac{L_{\mathrm{AV}}}{v_c} - \frac{L_{\mathrm{AV}}}{v} \geq T_0. \quad (12)$$

This results in:

$$\frac{L_{\mathrm{AV}}}{v_c} - \frac{L_{\mathrm{HV}}}{v} \geq T_0. \quad (13)$$

If the level of speed reduction by the CAV is denoted as β , then

$$v_c = \beta v, \quad (14)$$

and

$$\frac{L_{\mathrm{AV}}}{\beta v} - \frac{L_{\mathrm{HV}}}{v} \geq T_0. \quad (15)$$

The time delay for communication between the CAV and the RSU through DSRC or C-V2X can be neglected compared with the time needed for vehicle speed adjustment [33]. However, the time for the CAV to reach the desired speed during gap creation, or the transition time, $\Delta t_{\mathrm{trans}}$, must be taken into consideration. This transition time is measured from when CAV receives the speed control instruction till it reaches the required speed. This extra time must be compensated during gap creation; thus, equation (15) is modified as

$$\frac{L_{\mathrm{AV}}}{\beta v} - \frac{L_{\mathrm{HV}}}{v} \geq T_0 + \Delta t_{\mathrm{trans}}. \quad (16)$$

The equation for the gap that CAV creates is then calculated at a specific reduction in speed β to achieve the gap needed for the minor road vehicle to enter the intersection, as follows:

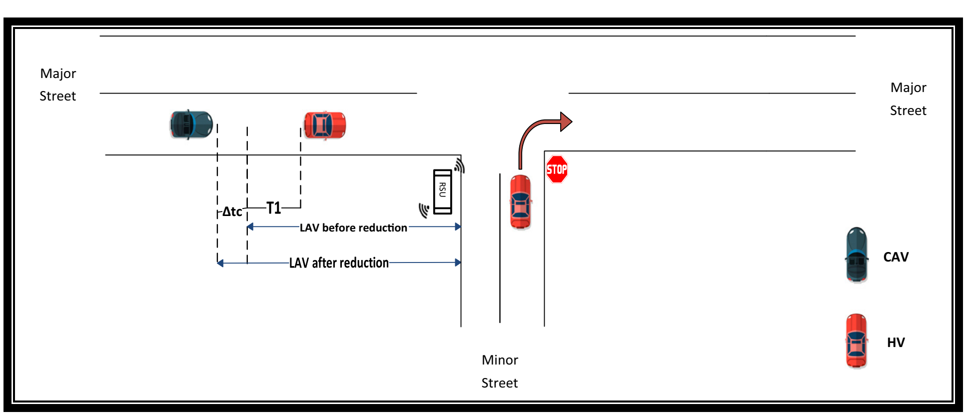

Figure 5: Extra time added to increase the gap time, Δt_c.

$$\frac{L_{\mathrm{AV}}}{\beta} - L_{\mathrm{HV}} - (v \times \Delta t_{\mathrm{trans}}) \geq (v \times T_0). \tag{17}$$

3.2.3 The back gap

Equations (9)–(17) have considered only the case of a vehicle ahead of the CAV. However, there may also be a vehicle behind the CAV at the same time. This back gap, C_{back}, as well as the speed of the following vehicle imposes a major constraint on the potential speed reduction by the CAV. As shown in Figure 5, before CAV reduces speed to expand the front gap, it checks the spacing and speed of the following vehicle and calculates to ensure that the following vehicle would not be severely affected by the speed reduction of the CAV. For safety, the back gap should be at least greater than the safe car-following distance (CFD) for consideration of reducing the CAV's speed, that is

$$C_{\mathrm{back}} \geq \mathrm{CFD}, \tag{18}$$

where

$$\mathrm{CFD} = v_o\, t_r + \frac{v_o^2 - v_f^2}{30(f \pm G)}, \tag{19}$$

where V_o refers to the initial speed of the following vehicle (mph); v_f represents the post-reduction speed of CAV (mph); t_r is the reaction time (s); f is the friction coefficient, and G is the grade level of the street.

An additional term is further used to ease the impact of the back gap reduction. As shown in Figure 6, a threshold distance (clearance) between the CAV and the following vehicle is considered at different speed levels. This clearance distance between the two vehicles can be expressed using a time factor, Δt_c, which can be added to equation (18) to get:

$$C_{\mathrm{back}} - (\Delta t_c\, v_f) \geq \mathrm{CFD}. \tag{20}$$

The case of CAV with a front vehicle and a back vehicle represents a complex situation; the safety conditions established also apply if a CAV is following another CAV. In this case, the first calculation will start with the front gap to ensure a gap can be created before checking the safety behind the CAV reducing speed.

3.3 Experiential evaluation

To prove the concept and estimate the potential effectiveness of the proposed control method, a simulation was performed on the VISSIM platform. VISSIM is chosen because it has been widely used by the consulting industry, transportation authorities, and universities [14].

Two types of intersection control are considered to demonstrate how the proposed algorithm works, including a stop sign control and a semi-actuated signal control at a T-intersection. The three-legged intersection may include one or more lanes on the major road, and only one lane on the minor road. Vehicles from the minor road approach and merge into the major road from a designed left-turn or right-turn bay. The detector placed on the minor road-stream is connected with RSU to obtain the information when a minor road is detected and needed a gap.

3.3.1 Unsignalized intersection

In this case, as shown in Figure 3, the major road traffic has the right of way, and the minor road vehicle has to wait at the stop sign before merging. The minor road vehicle will enter only if there is a gap equal to or greater than the critical gap. For a vehicle waiting to join the major road by a right turn, the control system has two options to take, as explained below:

Option 1: Do nothing: This scenario is when a CAV is within the range of communication but do not help create a usable gap due to one or more of the following possible conditions:

1. The front gap time of the CAV from the leading HV is greater than the critical gap, so there is no need for gap extension, or:

$$T_1 \geq T_0$$

2. The front gap time of the CAV is less than the critical gap, but the CAV is not able to help because the CAV is too close to the leading HV and not enough time can be saved by slowing down, that is:

$$T_1 + \Delta t_c < T_0$$

3. The back gap behind the CAV from the following HV is less than the safe distance required. This scenario is when the following HV is very close to the CAV, and it is unsafe for the CAV to reduce speed, or:

$$C_{\mathrm{back}} - (\Delta t_c\, V) < \mathrm{CFD}$$

Option 2: Reduce speed: This scenario is when a CAV can reduce speed to help create a usable gap for the minor road vehicle after satisfying both of the following conditions:

1. The front gap time is less than the critical gap and CAV, and a CAV can help extend the gap not less than the critical gap by reducing speed, that is:

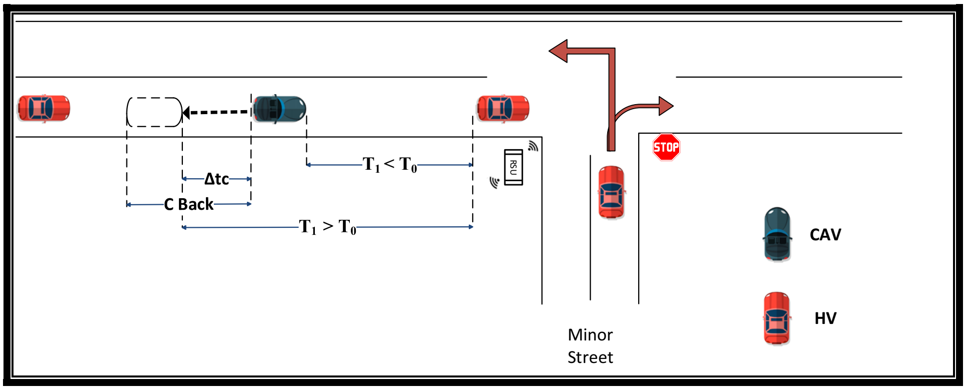

Figure 6: The back gap and front gap after speed reduction.

$$T_1 + \Delta t_c \geq T_0$$

2. The back gap of the CAV from the following HV is greater than the safe distance:

$$C_{\text{back}} - (\Delta t_c\ V) \geq \text{CFD}$$

The control logic of the above process is shown in Figure 7.

The above discussion has focused only on one direction of traffic on the major road when the minor road vehicles make a right turn. However, for a minor road vehicle to turn left, safety considerations must be given to both directions of traffic on the major road because of the increased potential conflicts. Figure 8 shows a minor road vehicle waiting for a gap to turn left and join the mainline traffic.

In this case, the system will check if the condition, $T_i < T_0$, exists and the CAV can slow down to create the needed gap value of Δt_c in each direction, so that $T_i + \Delta t_c \geq T_0$. In addition, the algorithm will check if the back gap conditions discussed above can be satisfied. If either the front or back gap in any direction does not meet the requirements, the algorithm will select Option 1 (do nothing). The added control logic is shown in Figure 9.

3.3.2 Semi-actuated signal control

Another application of the proposed control strategy is at a T-intersection under semi-actuated signal control. In most such intersections today, green light interruptions to the mainline traffic are used when an individual vehicle arrives and is detected at the minor road approach. Since semi-actuated control generally does not optimize intersection performance, this often causes multi-vehicle stoppings and significant delays to the mainline vehicles even though it could serve just one vehicle on the minor road.

Our research intends to improve the operation of this type of intersection in a similar way to the unsignalized

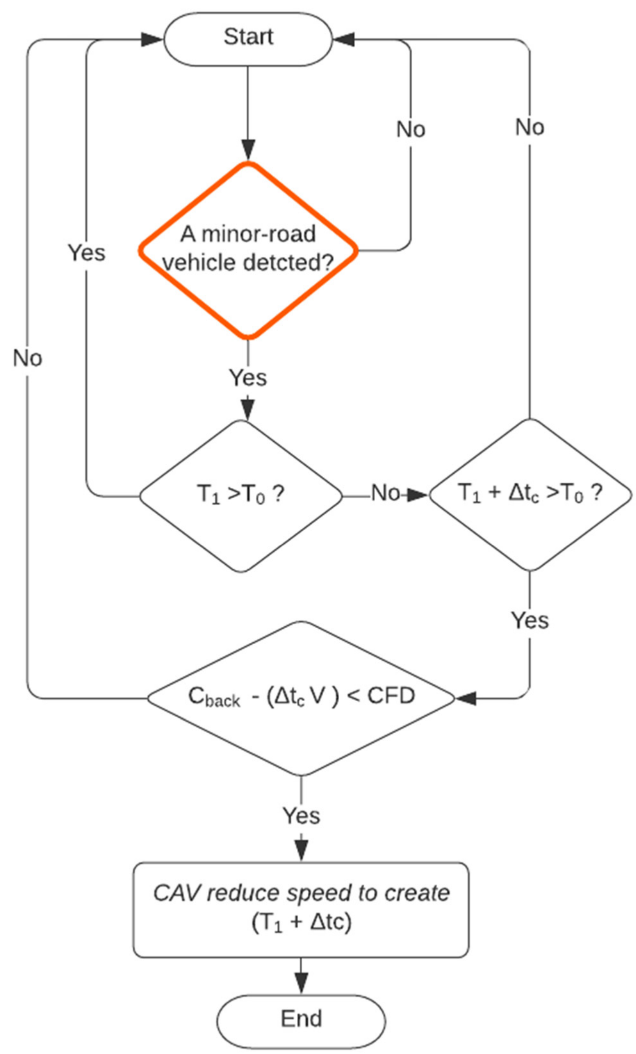

Figure 7: Control logic for a right-turning vehicle in mixed traffic flow.

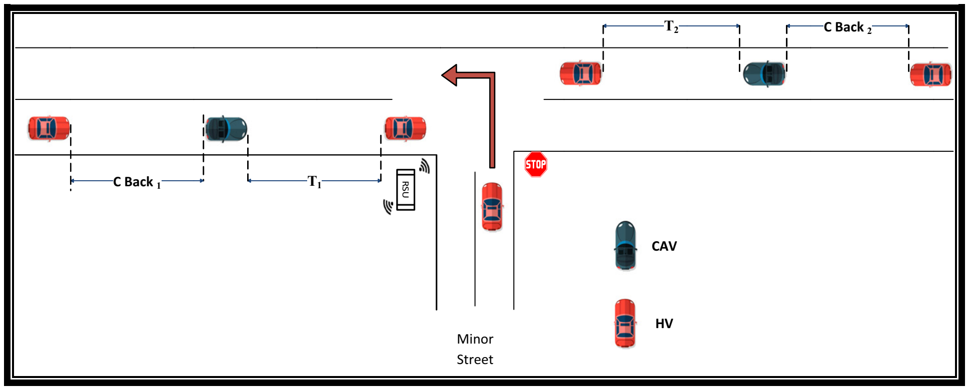

Figure 8: Road entry through a left turn.

intersections. Taking advantage of the modern detection and signal control technologies, the control method proposes to extend the green time on the major road to allow gap creation by CAVs without switching the green time to the minor road. Specifically, for application of the system, the major road signal is set in the green-rest (GR) mode while a flashing red (FR) is used for the minor road, where the minor road vehicle can still merge into the mainline flow if a proper gap is available. The FR operation is functionally similar to a stop sign, but its advantage lies in the ability of the signal system to change operation according to real-time detection. When a vehicle is detected waiting at the minor road approach, the detector "holds off" the detector status change information by employing the detection delay function, while at the same time, the gap searching and creation procedure using CAVs is executed. The minor road will be able to join the mainline flow if a usable gap is successfully created; otherwise, the detector status change is reported, and the "hold" is released to allow the green signal switch to the minor road, avoiding excessive delay to the minor road waiting for the vehicle.

As the modified algorithm temporarily operates a FR for the minor road when a vehicle is waiting to enter the intersection, the CAVs in the mainline flow can execute the gap creation. If the minor approach vehicle cannot

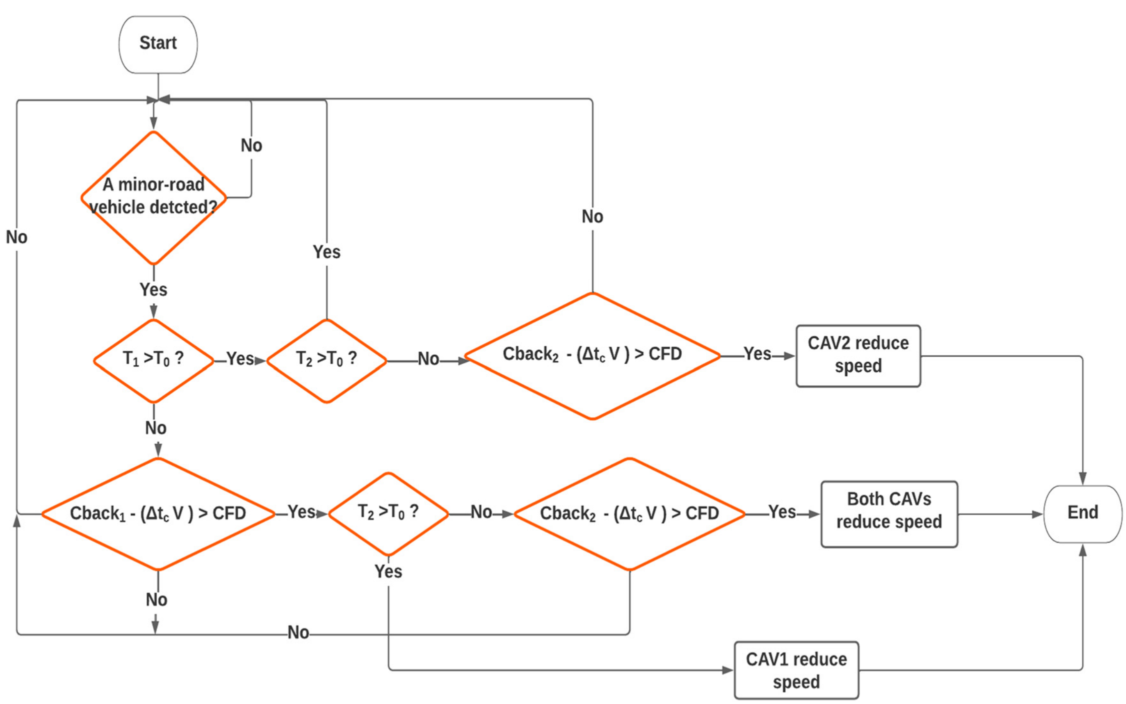

Figure 9: Control logic for a left-turn vehicle in mixed traffic flow.

find a proper gap to use if there are no CAVs in the mainline traffic, the traffic signal resumes its semi-actuated operation. The control logic for the algorithm to work at a semi-actuated signal is shown in Figure 10. The effectiveness of the control algorithm can be measured by comparing how often the mainline signal is interrupted with and without the help of CAVs.

3.3.3 Simulation results

Several key parameters are determined before the simulation study as shown below. While the critical gap values recommended by HCM may be used for general traffic conditions, it is understood that a more accurate estimation of the critical gap is from field observations at a specific location, especially if other relevant model parameters are also developed from the local data. This research has conducted a field study and analyzed gap selection conditions to obtain the critical gap for the use in the simulation. Similarly, the clearance distance (Δt_c) is obtained by looking at the minimum clearance between stopped vehicles and the transition time (Δt_{trans}) is calculated by applying the normal deceleration rate to the speed difference. Additionally, the gap utilization factor is developed by finding the number of gaps accepted by the minor road vehicles out of the total number of usable gaps based on field observations. Finally, the level of speed reduction (β) is determined by tracking the speed changes when individual vehicles from the major road reduce speed to turn into the minor road at the selected field location.

Due to the stochastic nature of traffic arrivals, a minimum of ten simulation runs were performed for each volume and CAV rate combination with different random speed numbers to ensure that the values reported incorporated the stochastic changes in the traffic flow. The test plan includes comparisons to evaluate intersection performance before and after implementing the CAV algorithm to create gaps. Thus, two types of vehicle compositions are specified. The first is when there are no CAVs on the major road, which is the benchmark for comparison, and the second includes a mixed traffic condition with CAVs and HVs.

The performance results of the proposed algorithm of both cases (unsignalized and semi-actuated) are presented and analyzed next. Different traffic demand levels are considered on the major and minor roads, along with different penetration rates of CAVs on the major road. The measures of effectiveness (MOEs) are used to evaluate the proposed method, including delay and capacity.

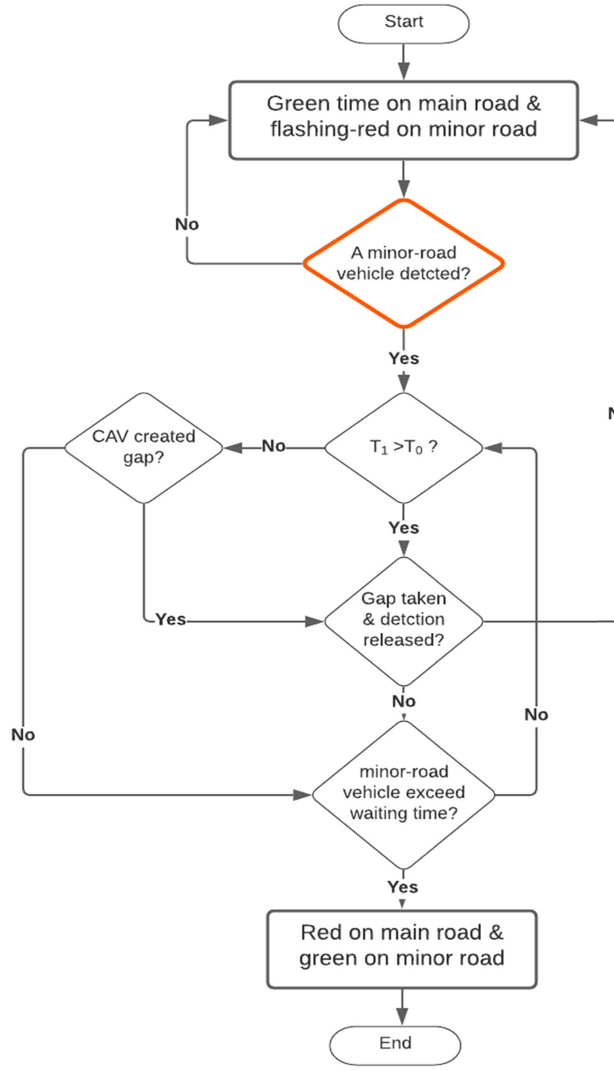

Figure 10: Control logic for semi-actuated signal control.

3.3.3.1 Unsignalized intersection

The stopped delay occurs when a vehicle is waiting for a safe gap at the minor road-stream. The average stopped delay is shown in Figure 11, where three different CAV percentages are presented as 30, 50, and 70%. Figure 11 corresponds to the minor road volumes as 100, 150, 200, and 250 veh/h, respectively.

As shown in Figure 8, when the CAV rate to the total volume of vehicles on the major road increases, the stopped delay on the minor road approach decreases to a certain level. This is primarily because the increase in CAVs can help create more safe gaps, resulting in less waiting time and improved stopped delay on the minor road compared with the no CAV situation on the major road. However, once the major road volume reaches a high level, the improvements start to decrease because more vehicles on the major road result in shorter headways between vehicles, and the smaller headways reduce the possibility for CAVs to create safe gaps for the minor road vehicles.

Gap creation by CAVs may cause additional delays on the major road. This impact is also studied by obtaining the average delay on the major road approaches before and after the proposed method, as:

$$\text{Change in averge delay} = \text{Delay with CAVs} - \text{Delay without CAVs}. \quad (21)$$

Table 1 shows a comparison between the delay added to the major road and the reduced delay on the minor road. The results show that the delay caused to the major road at low minor road volumes is very low while the reduction in the minor approach is 23–30%. When the added delay to the major approach reaches 11% at a higher level of traffic on the major and minor roads, the improvement in the minor road approaches increases to 62%. Thus, the proposed method at unsignalized intersections, in general, helps the minor-road approach vehicles without seriously affecting the major road operation.

3.3.3.2 Semi-actuated signal

During simulation, a minimum green and a maximum green time are used for the semi-actuated signal on the major road. The traffic flow is classified into three volume levels: low, medium, and high, for which the corresponding maximum green time for the major road is 50, 60, and 70 s and the minimum green is 20, 25, and 30 s,

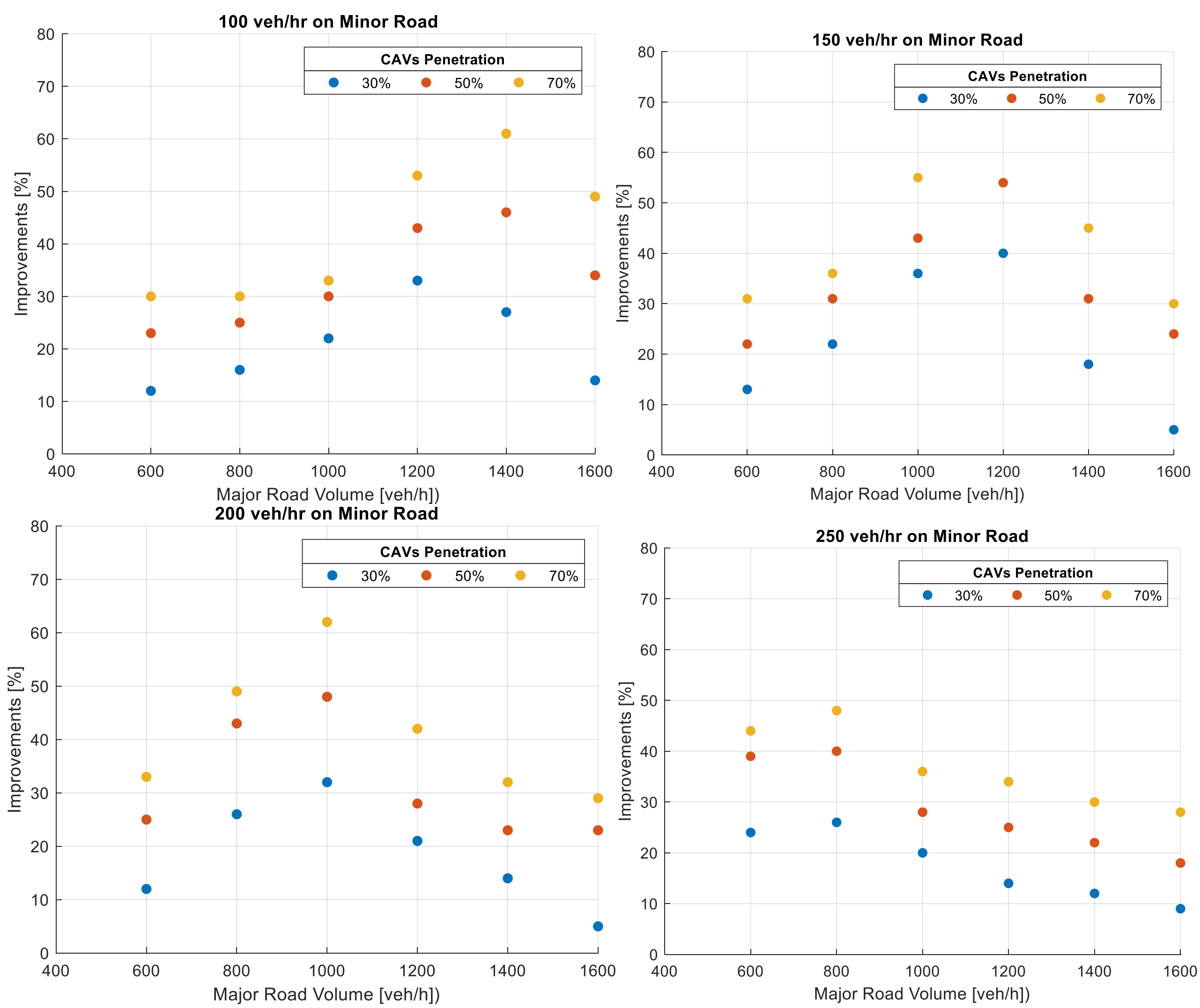

Figure 11: Improvement in delay for the minor road vehicles.

Table 1: Comparison of delays due to gap creation by CAVs

Minor road volume (vph)	Major road volume (vph)	CAV (%)	Delay added (major road) (%)	Improvements in delay (minor road) (%)
100	600	50	1	23
	1,000	50	2	30
150	800	30	3	15
	1,200	30	6	40
200	600	70	4	23
	1,000	70	11	62
250	800	30	6	22
	1,200	50	20	25

respectively. However, if no calls are placed by the minor road vehicle, the major road will stay on GR. On the other hand, for the minor road, the "holding" time (before reporting the detector status change) for low, medium, and high major road volumes is set as 15, 20, and 25 s, respectively.

The signal interruptions are counted when the minor road vehicles are not able to use a safe gap in the mixed flow. The signal switches stop the major road traffic and serve the minor road vehicles. Figures 12 and 13 illustrate the number of interrupted signals at different CAVs' penetration rates with different allowable maximum waiting times on the minor approach.

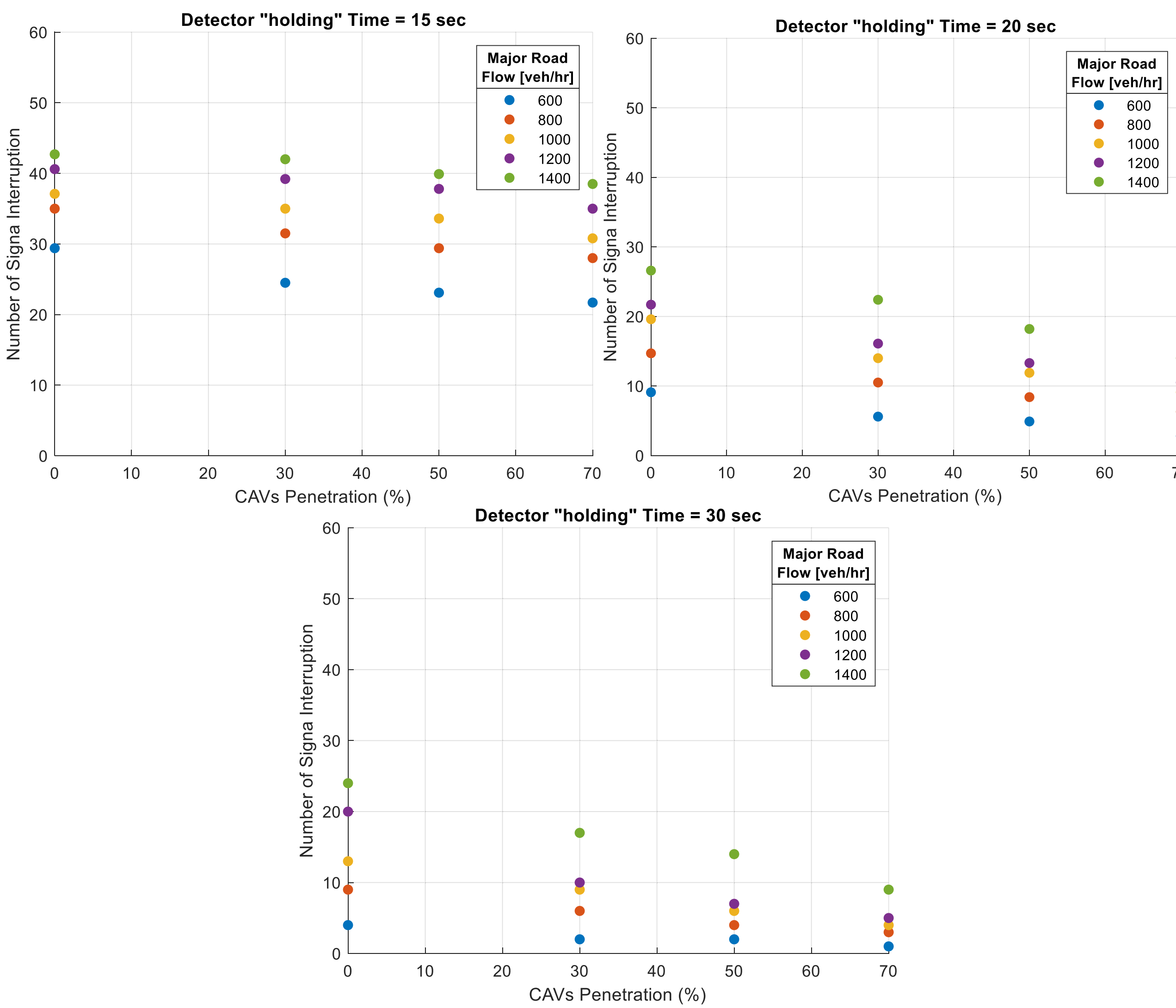

Figure 12: Signal interruptions on the mainline at 150 veh/h on the minor road.

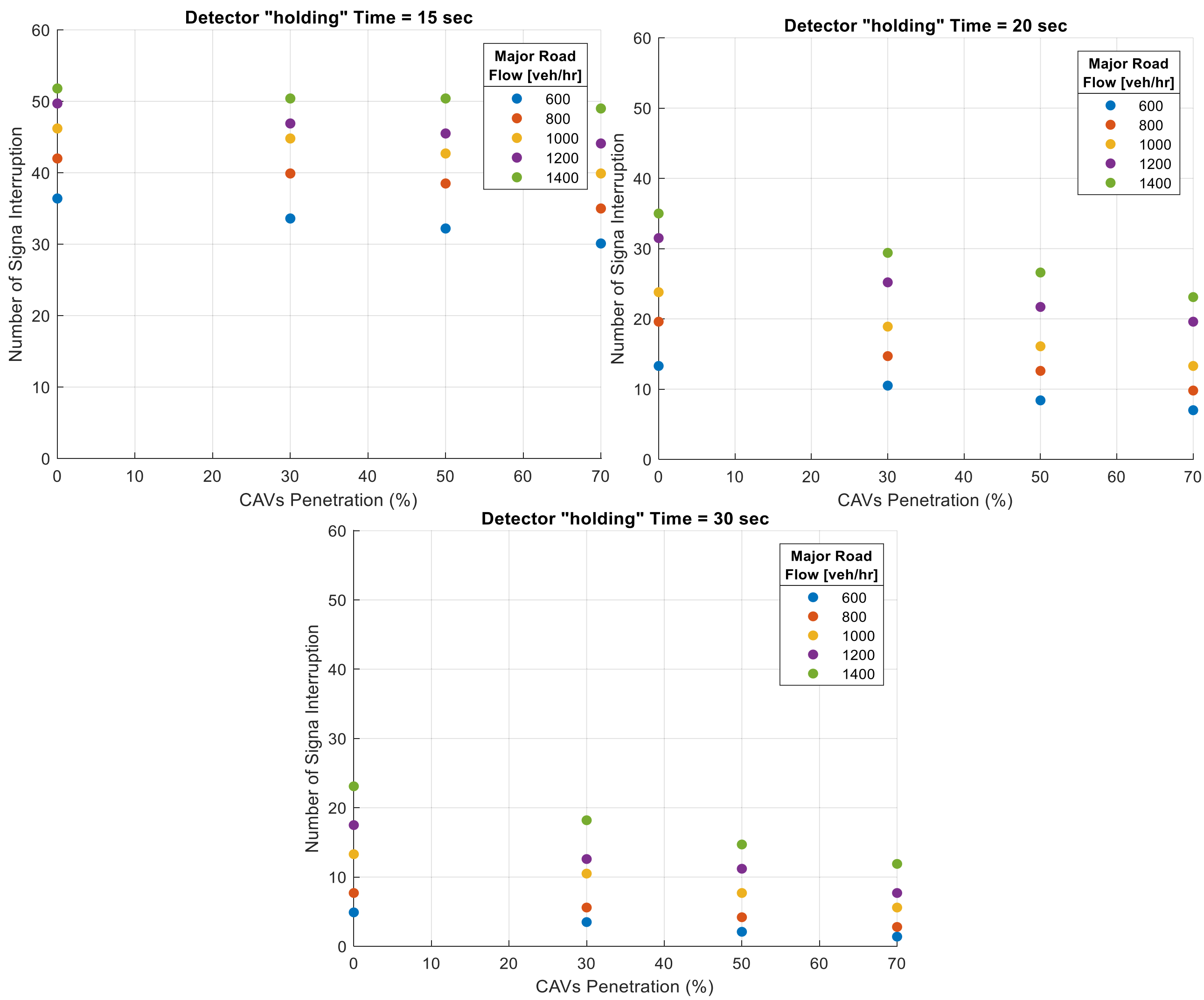

Figure 13: Signal interruptions on the mainline at 250 veh/h on the minor road.

It can be found that at a low volume on the major road, the number of mainline interruptions is less than when the major road volume gets higher. Also, an increase in the minor road volume leads to more signal interruptions. Further, with the increase in the CAV rate in the mainline traffic, the number of signal interruptions decreases, since more CAVs increase the probability of creating safe gaps for the minor road to enter the main road. This helps improve the waiting time at the minor road approach and thus reduces the need to switch the signal.

The improvement in the intersection operation is evaluated by measuring the control delay, a primary LOS indicator in the Highway Capacity Manual (HCM). Figure 14 presents the control delay at different levels of traffic volume on the major and minor roads along with different CAV rates. Figure 14 shows the results when the minor road volume is 100, 150, 200, and 250 veh/h, respectively.

It can be seen that at all levels of minor road volume, the control delay increases with the increase in the major road volume if no CAVs are used to help create usable gaps. However, when different levels of CAV penetrations are introduced, the control delay decreases as more CAVs become available. The results in Figure 13 also show that with the increase in the minor road volume, the control delay increase due to the increase in calls from the minor road detection; however, at a given minor road volume, the CAVs can help reduce the signal interruptions.

Since CAVs help to create additional usable gaps on the mainline traffic stream, the number of minor road vehicles that need signal accommodations would decrease, leading to a potential increase in intersection capacity, referred to as surplus capacity in this articl. This surplus capacity is analyzed by subtracting the number of gaps successfully created by CAVs and taken by the minor road vehicles, from the total demand on the minor road that would otherwise have required the signal service. Figure 14 shows the surplus capacity as a percentage increase to the intersection capacity with different traffic volumes on the major and minor roads.

Figure 15 presents the surplus capacity at different traffic volumes on the major and minor roads. The results show the surplus capacity increases as more CAVs are available in the mainline traffic stream. However, the capacity increase starts to decline when the mainline traffic volume continues to grow, indicating the difficulties for CAVs to find opportunities to create useable gaps. Figure 15 also shows that surplus capacity becomes much lower at a high minor road volume, reflecting that the remaining vehicles on the minor road still need signal service after other vehicles have taken advantage of the created gaps to merge into the main road.

4 Field test

This field test aims to study the feasibility of the control logic implementation in real traffic conditions. The investigation evaluates the execution of the framework in this research under the scenarios explained in the above section.

4.1 Site selection

The selection of the test site to conduct the experiments is based on the following criteria:

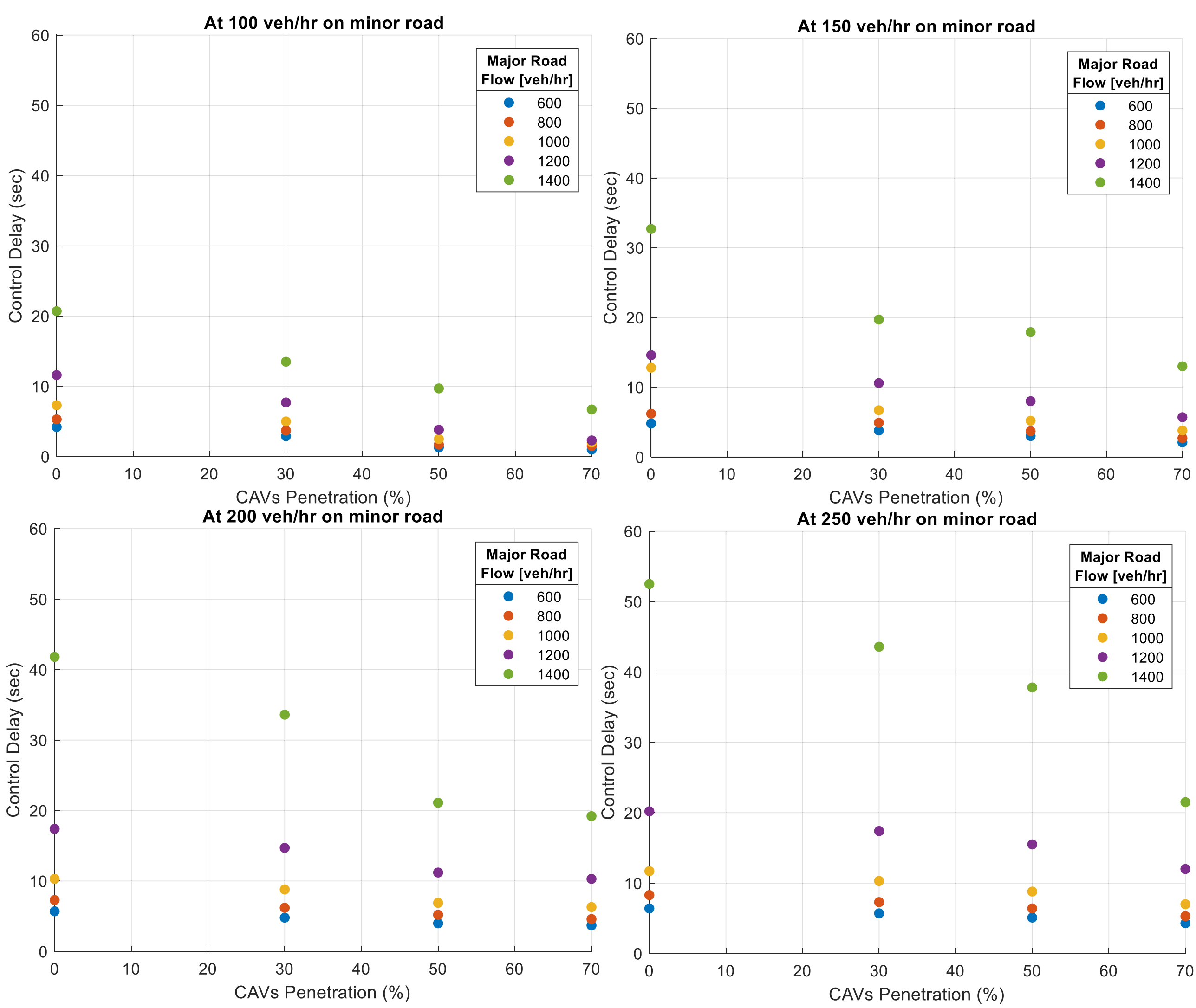

Figure 14: Improvement in control delay at different levels of CAV penetration.

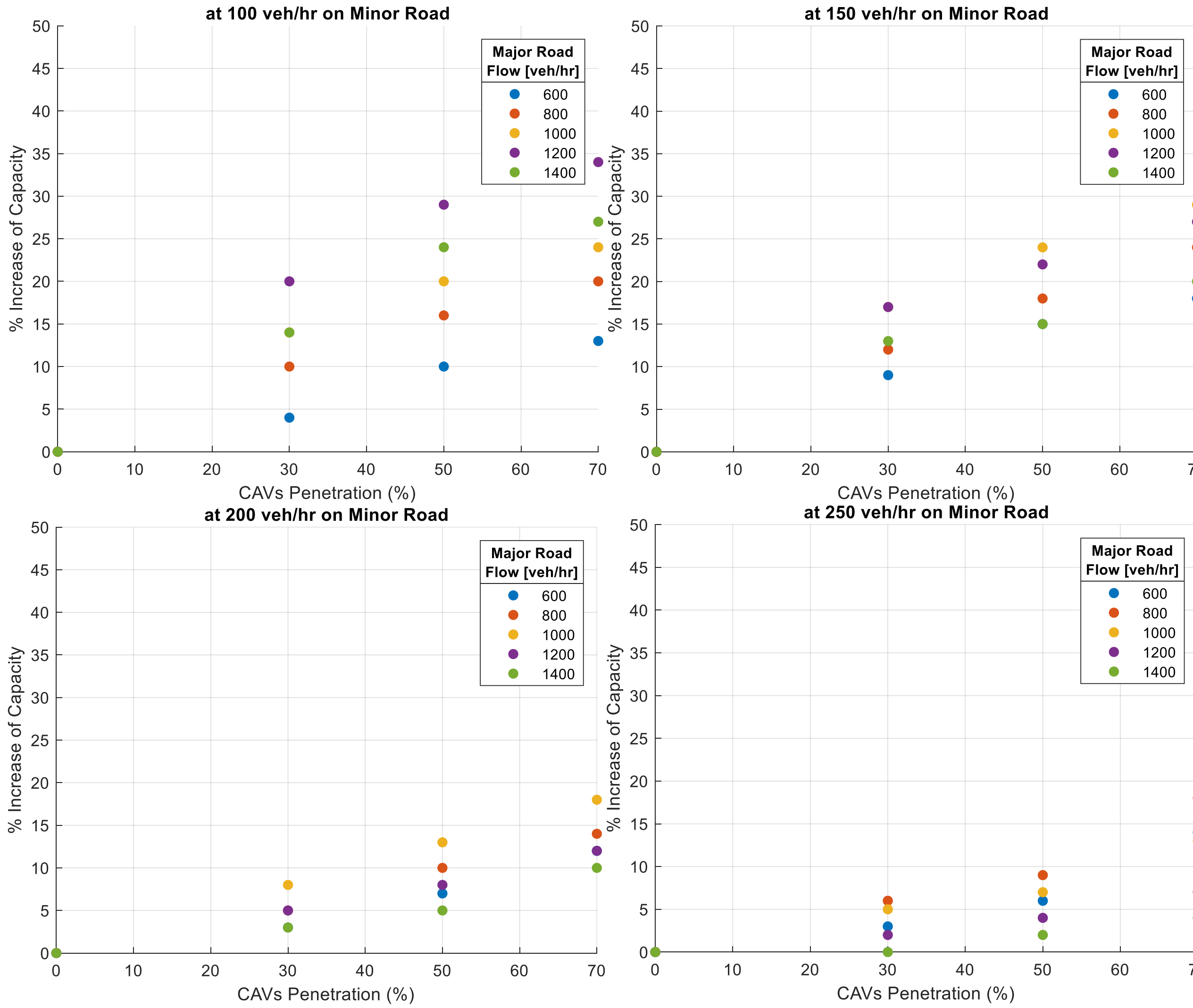

Figure 15: Surplus capacity at different levels of traffic volume.

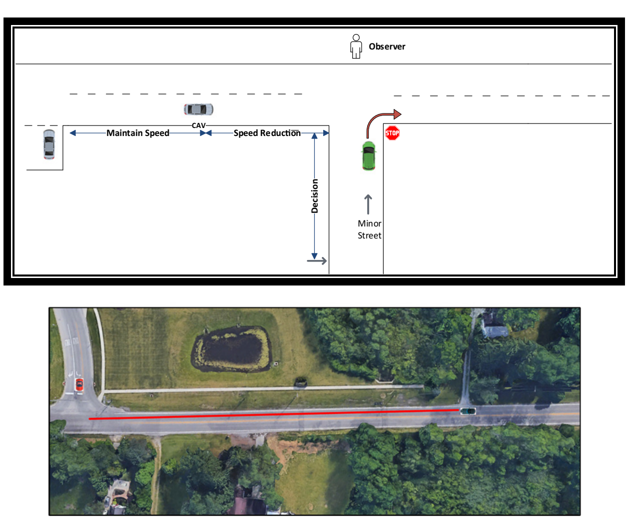

Figure 16: Setup for field test.

1. Good access to the site and easiness to markup reference points on the roadside.
2. Reasonable traffic volumes on both major and minor roads, so that good quality data for normal traffic conditions can be collected.
3. Good sight distances in both directions to ensure that the poor geometric conditions do not impose complications to the test.
4. Suitable safe spots to station the test vehicles on the major road without being too close to the intersection.

After visiting several T-intersections, two sites have been selected for the field test. The intersection of Wilbeth Road and Inman Street (minor road) in Akron, OH is signalized with a FR on Inman and a flashing yellow on Wilbeth as the default control plan. On the other hand, the intersection of Fishcreek Road and Sowul Boulevard

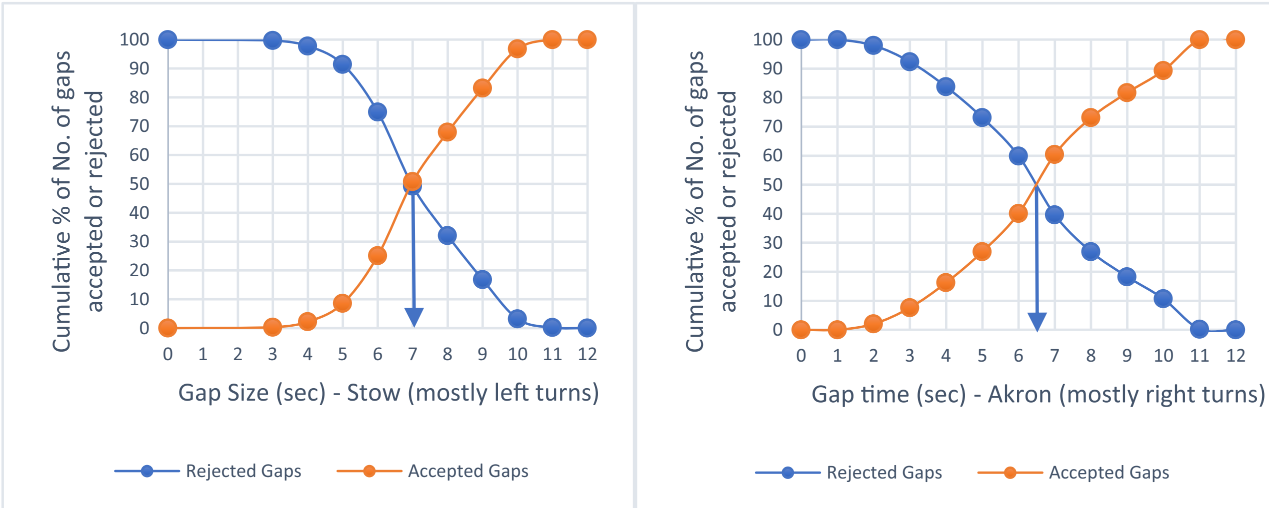

Figure 17: Critical gap estimate using Raff's method.

(minor) in Stow, OH is controlled by a stop sign on Sowul Boulevard.

4.2 Experiment design and execution

Since the CAV technologies for applications in this study are still being developed, we used our own vehicle as a CAV in each direction of the main road. Communications within the system were realized by voice commands over cellphones between the observer at the intersection and the test vehicles.

Each test vehicle included the driver, front gap watcher, and back gap watcher. The front gap watcher estimates the distance between the test vehicle and the preceding vehicle using temporary roadside marks. This person uses pre-established reference points from spacing requirements to determine if the test vehicle is too close to the front vehicle (cannot create a long enough gap) or if the vehicle has reached the speed reduction zone to carry out the procedure (shown by the red line in Figure 16).

The back gap watcher estimates the distance between the test vehicle and the following vehicle also by using the roadside marks. The observer at the intersection communicates with the driver regarding conditions at the intersection and issues a "NO" command if an unusual situation arises, such as a pedestrian or bicycle crossing the road. When a minor road vehicle is approaching the intersection, the observer at the intersection informs the test vehicles waiting upstream of the intersection to enter the major road from each direction. If the conditions reported all support a "YES" command, the test vehicles slow down to generate additional gap time. If the minor-road vehicle accepted the expanded gap to enter the main road, it would be counted as an accepted gap. On the other hand, if the minor-road driver did not select the gap, it would be considered a rejected gap.

To help verify the success rate in the gap created by our test vehicles, two methods were used to collect field data during the experiment, a manual data sheet and a high-quality video recorded using a Mini-DVI drone for data checking along with the manually entered data points. The tests were conducted at different times of the day and different days of the week to increase the generality of the data. Each test usually takes 1–2 h from setting up the site to finishing the data collection, as the video drone records the data simultaneously. After each field test, data from the video were extracted manually and compared with the manual sheets at the Transportation Lab of The University of Akron.

4.3 Critical gap

Before the test vehicles were deployed, gap acceptance data without the presence of CAVs were collected at the intersection to help establish the critical gap and other

Table 2: Data point collected on the field

Location	Minor road vehicle movement		Driver decision		Total
	Right	Left	Accept	Reject	
Akron	43	23	42	24	66
Stow	8	47	34	21	55
Total	51	70	76	45	121

local traffic parameters that were used in the simulation study described before. For example, the minor road vehicle's waiting time was also determined from the field data, which was used to suggest the detector status "holding" time for the semi-actuated signal control in the simulation.

The critical gap was estimated by following Raff's method [17]. The data includes a total of 512 gap selections, including 197 from the Akron site and 315 from the Stow site. An example of the cumulative accepted and rejected gap curves is illustrated in Figure 17, where it can be seen that the critical gap at the Akron location is approximately 6.5 s, whereas at the Stow location is nearly 7.0 s.

4.4 Results

In the entire field-testing effort, a total of 121 successfully executed cases have been recorded, roughly 60 for each intersection. As shown in Table 2, the results show that at the Akron test site, the acceptance rate by the minor road vehicles to the gaps created is close to 64%, indicating that most drivers are willing to accept the gaps created by the test vehicles. On the other hand, gaps that were rejected by the minor road drivers are 36%. This indicates that many drivers are very cautious regarding roadway entry from a minor road. The underlined numbers in parentheses represent the number of accepted cases for the right or left turns.

The data at the Stow site have shown a similar pattern in gap acceptance and rejection, as shown in Table 2. The results indicate an overall acceptance rate of 62%, and there is no obvious difference in the acceptance rate between the left turns (mainly at this site), and the right turns at the Akron site.

While the simulation of the proposed method shows improvement in the performance of the intersection studied, field testing is conducted, and the objective is to investigate the feasibility of implementing the proposed method in field conditions. By using our test vehicles to "function" as CAVs, the vehicle sensing and data exchange work was done by human assistants. While the data accuracy is not the focus in the process, this proof of concept test has shown that the proposed method can create gaps for the minor road vehicles to use through coordination and control of the test vehicles.

5 Conclusion

This article developed and evaluated a systemic framework that guides CAVs to create additional safe gaps in the mainline traffic stream so that the minor road vehicles may reduce delay while waiting to enter the main road and the interruptions to the continuous mainline flow. The article considers a mixed traffic environment, in which both CAVs and HVs share the same right-of-way. A probability function was developed to assess the number of usable gaps that CAVs may help create at different volume levels and CAV rates on the major road. Since the right turn movements from the minor road conflict with only one direction of the mainline traffic, the number of possible gaps to be created is greater than the left-turn movements, since the latter conflicts with the mainline traffic in both directions, with a reduction in the possibility of simultaneous assistance by CAVs.

A simulation study was performed on the VISSIM platform involving two types of intersection control to investigate the effectiveness of the proposed algorithm. The simulation results showed that the proposed method might effectively reduce delay for the minor road vehicles without causing a significant increase in delay on the major road at unsignalized intersections. When a semi-actuated signal control is considered, the results show that the creation of additional gap times by CAVs would substantially reduce the number of interruptions to the mainline traffic flow and reduce the intersection control delay. Field testing was also conducted, and the objective is to investigate the feasibility of implementing the proposed method in field conditions. This proof-of-concept test has shown that the proposed method can create gaps for the minor road vehicles to use through coordination and control of the test vehicles.

The ability of CAVs to improve traffic operation and safety is highly anticipated in the near future. Although many of the technical details are still being worked out by the auto and communications industries, it is time for the transportation profession to develop and evaluate the impact of the technologies once implemented. This article has only looked at a simple application of advanced sensing and data exchange abilities to reduce unnecessary interruptions to the mainline traffic flow. Future work in this direction by the researchers is underway. It involves a four-legged intersection with multiple lanes, where CAVs are also expected to help improve the operation efficiency in mixed traffic conditions.

Conflict of interest: Authors state no conflict of interest.

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