

Research Article

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Roof Geometry in Building Design

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Abstract: The paper shows the usefulness of the roof geometry in determining of building outline. It was pointed out how analysis of the roof skeleton shape avoids many design errors. Omitting the criterion for the shape of the roof skeleton in the case of multi-criteria optimization can pose a lot of difficulties when designing a building. The problem is discussed in the case study.

Keywords: roof geometry, building outline, straight skeleton, shape of roof skeleton

1 Introduction

Roof geometry [1–5] is a theory describing the class of well-known objects, so-called straight skeletons [6–11]. The orthographic projection of the roof, *i.e.* the skeleton of the roof in the literature since 1995 has been called the straight skeleton [6]. In descriptive geometry, this object has been known for a long time [12, 13]. It is uniquely defined by the base polygon. The skeleton of a roof in recent years has been the object of interest of many authors dealing with computational geometry [6, 7, 14]. The concept of roof skeleton allows to determine the span of a building [15]. The roof skeleton structure, as it turns out, can be used to describe embankments [16]. There are many publications in which the issue of roofs has been treated very widely [17–23]. The authors in this paper pose the problem of the importance of the roof design process slightly differently.

Roof geometry can be useful for preliminary analysis of some building structural features. Especially when the building's outline has a complicated shape. This applies to the initial design stage. According to the authors, the shape of the building's outline should be adopted follow-

ing an analysis of the roof shape. This means that designing a house architect should start by designing the shape of the roof. Three examples are considered in the paper (Figure 1, Figure 2, Figure 3, Table 1).

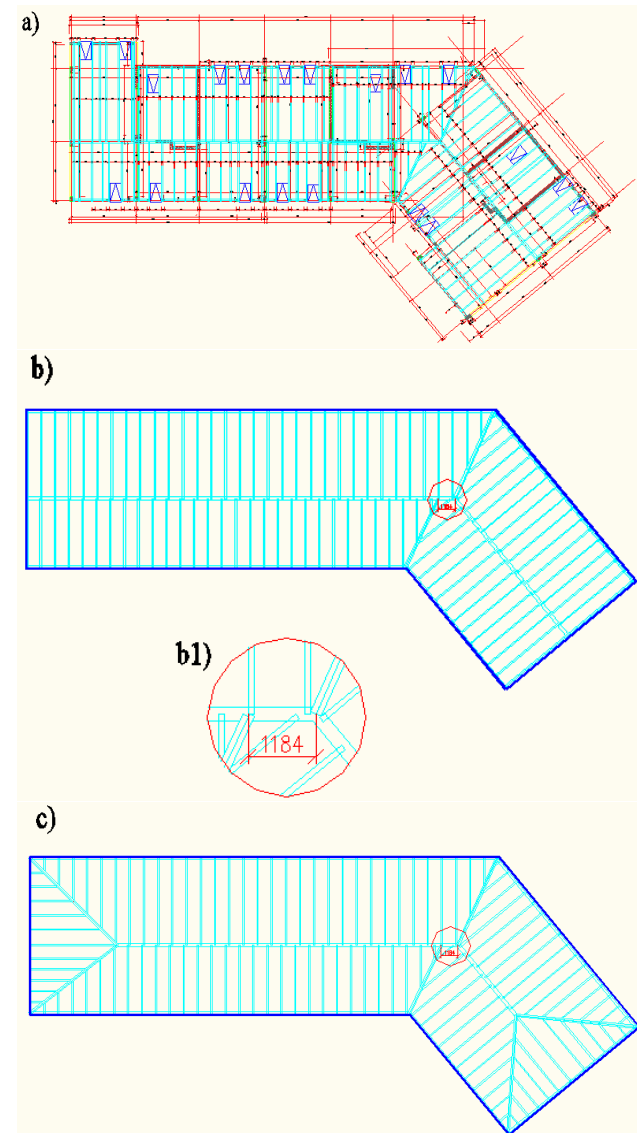


Figure 1: An unsuccessful attempt to design a roof truss on an ill-considered contour design: b) short (1180 mm) section of the ridge purlin, oblique to the straight line of the eaves, "forcing" a fragment of the saddle surface and various slope angles in the roof design; c) a multi-slope roof with slopes in the gable parts of the building with the same disadvantage

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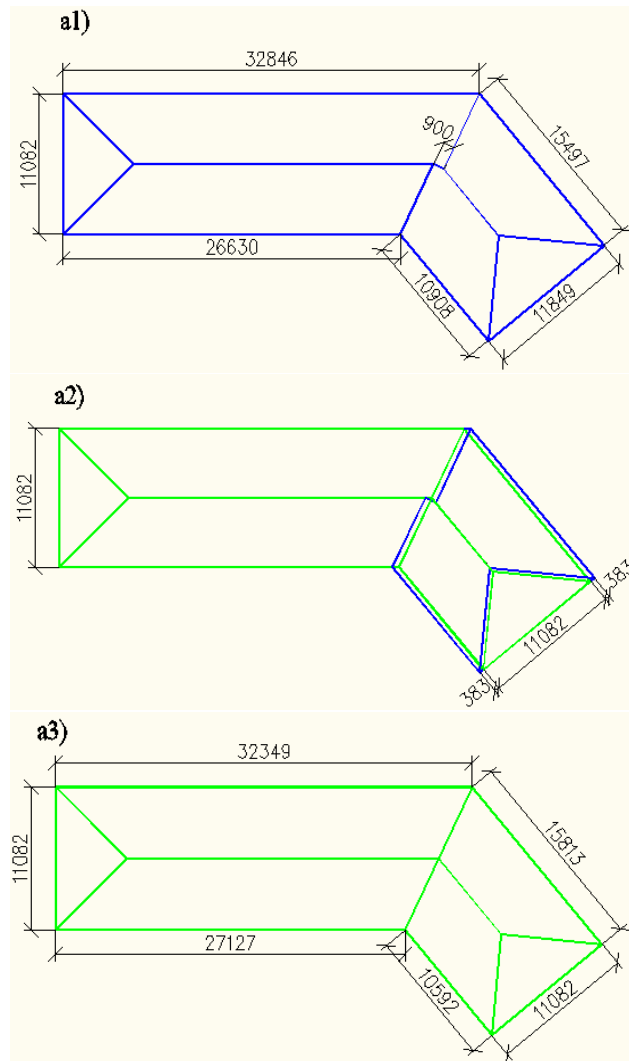


Figure 2: Correction of the building's outline: a1) roof skeleton with given building outline - short (900 mm) section of the roof ridge hindering the design of the board or ridge purlin; a2) correction of building contour - moving two lines of the building plan by only 383 mm; a3) locally symmetrical plan of the building after correction

2 Examples of using roof geometry - case study

Example 1. An initiated student project was analyzed (Figure 1a). In the example, the roof shape was not checked before designing (Figure 1a). In particular, the symmetry of the building's outline was not checked. This fact became the reason for further mistakes made when designing the roof truss.

Figure 1a shows a projection of the designed roof truss. An error was made in determining the building outline - no preliminary analysis was carried out. As a result, an asymmetrical roof was obtained, generating a short ridge edge

(Figure 1b, 1c). Then a second mistake was made in designing the roof ridge on the longer part of the building (Figure 1b1). It turned out that part of the roof surface would not be flat. The edges: ridges and eaves will then be skew. A fragment of the saddle surface will be created. To start designing the building correctly, it was worth proceeding as follows. Assuming that the roof slope should be the same, a locally symmetrical system should have been adopted (Figure 2a3). Would be avoided: The following would be avoided: 1) the need to implement a short, non-horizontal (1180 mm) section of the roof ridge (Figure 1a1) or 2) the construction of a curvilinear fragment of the roof slope in the form of a saddle surface (Figure 1b, c). The correction of the building's dimensions made it possible to obtain a locally symmetrical building with a loss of approx. 4 m² of area (Figure 2a1: Area=1943973357.8854 [mm²], Perimeter = 217623.3146 [mm]; Figure 2a3: Area = 1903476713.4383 [mm²], Perimeter = 216089.6351 [mm]). The area loss of approx. 4 m² can be compensated by increasing the dimensions from approx. 400 mm to 10592 by 11992 [mm] and from 15813 to 16213 [mm]. Although the deviations discussed will not have a significant impact on the roof truss technique, precision in design taking into account such details will certainly be a manifestation of the designer's diligence and reliability.

Example 2. Another example concerns a designed building (Figure 3). Solving the roof over a building so designed, a too short (35 cm) section of the roof ridge is obtained (Figure 3a1). In this situation, the solution most often adopted as in the figure Figure 3a2. Then the roof consists of four quadrangular and one triangular hipped roof end. Then in the left part of the building one of the hipped roof ends is not a parallelogram, and the other is not an isosceles trapezoid. Other hipped roof ends are not 45° polygons. It all disturbs the roof aesthetics. In order to improve the symmetry is sufficient to raise the upper contour line of 50 cm (Figure 3a3).

Example 3. In the monograph [24] the authors presented the results of many years of research on the multi-criteria optimization of energy-saving buildings. They adopted three criteria:

- minimum construction costs, including material costs and erection of the building,
- minimum seasonal energy demand for heating,
- minimum emissions of solid and gaseous pollutants arising in the process of heat generation in the sources considered. The study tried to take into account all significant restrictions so that the result-

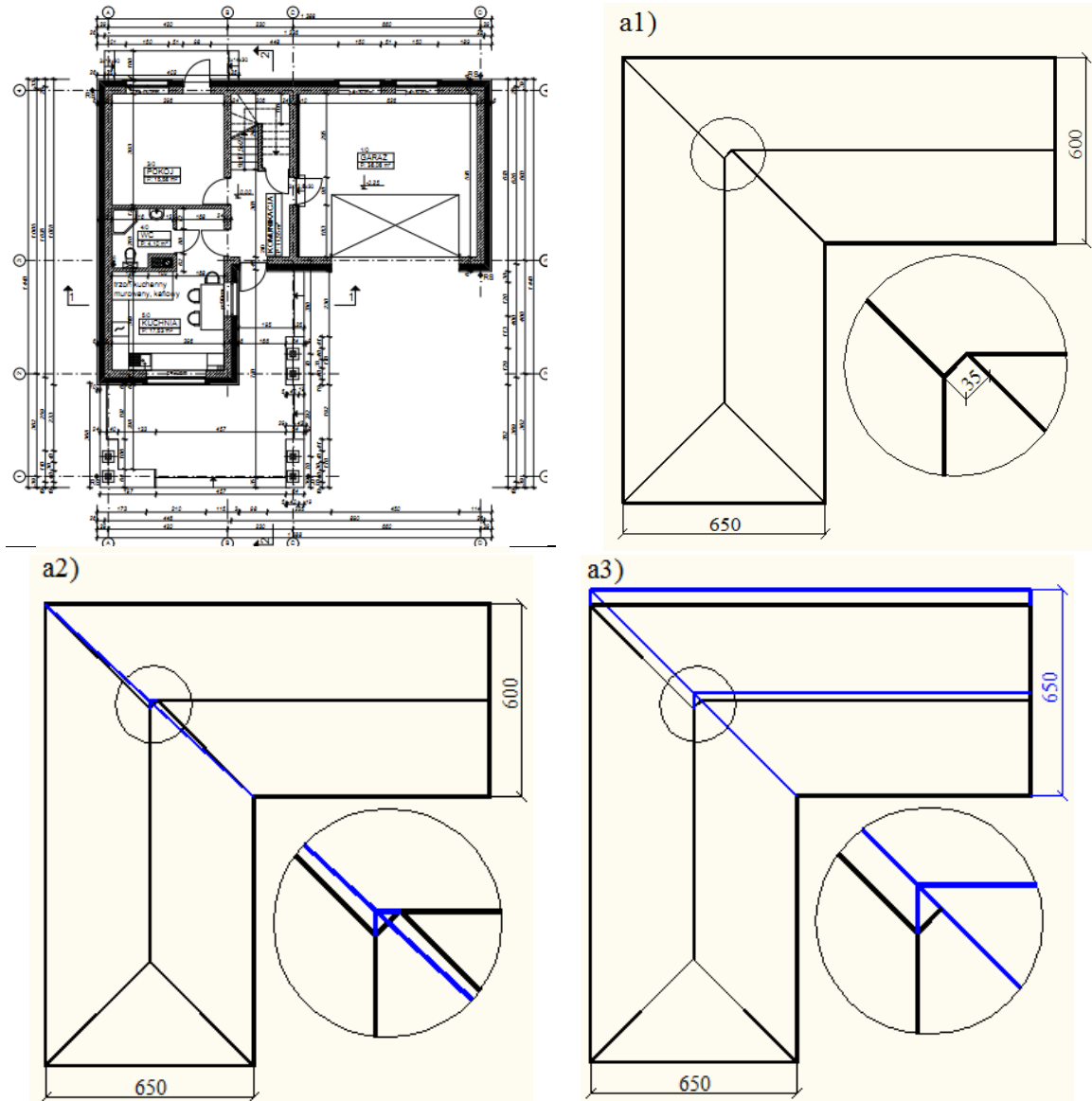


Figure 3: Correction of the outline of the designed building: a1) roof skeleton with given building outline - a short (35 cm) section of the roof ridge hindering the design of the board or ridge purlin; a2) roof solution for the designed outline; a3) correction of building outline - 50 cm shift of the building plan line and the locally symmetrical building plan obtained in this way

ing solutions could be useful in further work on the building design.

Two groups of decision variables were adopted:

- variables determining the shape and dimensions of the building and its external partitions (wall lengths, number of storeys, angles of wall inclination to the north-south direction, thermal resistance of walls, windows, roof and floor, ratios of window surface to wall surface),
- variables defining heating devices and the way they are used (power of individual heating devices, num-

ber of heating devices with a given power, annual shares of individual devices in heat generation).

In the monograph [24] building outlines were described using polar coordinates l_i (distance), ϑ_i (azimuth).

The monograph does not take into account the structural features of the building, such as the roof truss span (1) and the length of the smallest edge of the roof skeleton (e_{min}). Hence, it was interesting to analyze the roof of buildings assuming a multi-hipped shape with the same slope. By the way, the compactness of (2) buildings in the proposed solutions was examined. The value of indicator (2) was then compared with indicator (3).

Table 1: List of building shapes obtained in the optimization process [24], wall length and azimuth, roof skeleton and edge length, building span

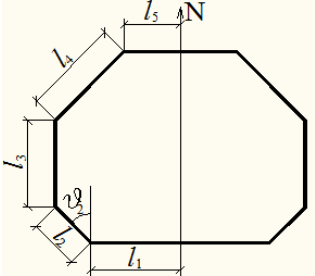
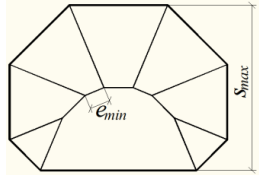
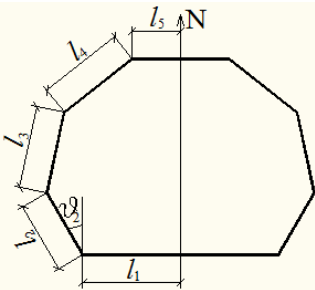
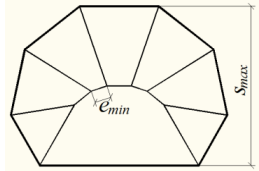
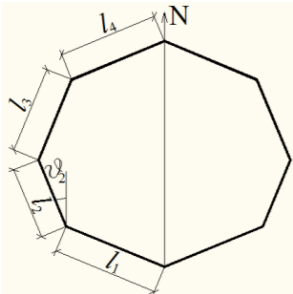
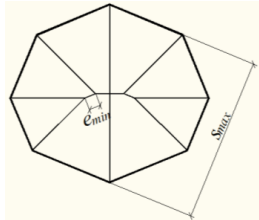
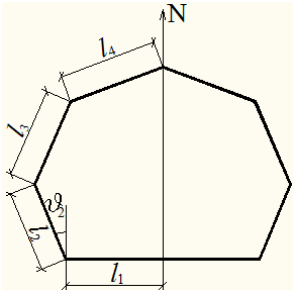
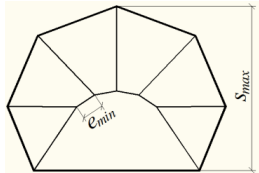
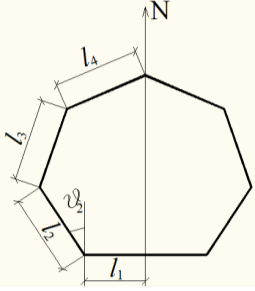
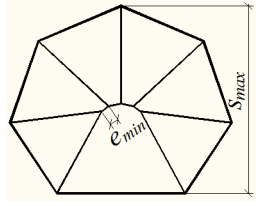
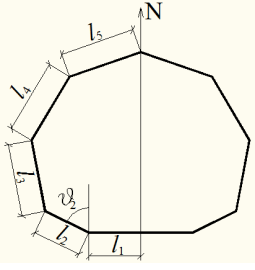
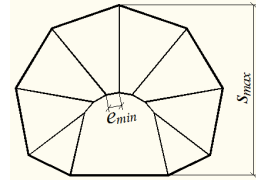
	optimal geometric solution			dimensions: length l_i [m] i azimuths ϑ_i [°]	roof skeleton according to the building's outline	
	RC_{cir}	Area A [m ²]	Perimeter P [m]		e_{min} [m]	S_{max} [m]
1		1.0488	111.07	39.18	$h = 9.00$ $l_1 = 4.61$ $\vartheta_1 = -90$ $l_2 = 2.59$ $\vartheta_2 = -45$ $l_3 = 4.48$ $\vartheta_3 = 0$ $l_4 = 5.01$ $\vartheta_4 = 45$ $l_5 = 2.90$ $\vartheta_5 = 90$	 1.21 9.85
2		1.0487	110.99	39.17	$h = 9.00$ $l_1 = 4.97$ $\vartheta_1 = -90$ $l_2 = 3.62$ $\vartheta_2 = -30$ $l_3 = 4.17$ $\vartheta_3 = 12$ $l_4 = 4.36$ $\vartheta_4 = 52$ $l_5 = 2.47$ $\vartheta_5 = 90$	 1.03 9.90
3		1.0311	111.43	38.58	$h = 9.00$ $l_1 = 5.61$ $\vartheta_1 = -67.5$ $l_2 = 3.81$ $\vartheta_2 = -22.5$ $l_3 = 4.58$ $\vartheta_3 = 22.5$ $l_4 = 5.28$ $\vartheta_4 = 67.5$	 0.77 11.01
4		1.0574	111.09	39.51	$h = 9.00$ $l_1 = 5.22$ $\vartheta_1 = -90$ $l_2 = 4.38$ $\vartheta_2 = -22.2$ $l_3 = 4.58$ $\vartheta_3 = 23.2$ $l_4 = 5.31$ $\vartheta_4 = 69.4$	 1.36 10.38

Table 1: ... continued

	optimal geometric solution			dimensions: length l_i [m] i azimuths ϑ_i [°]	roof skeleton according to the building's outline	
	RC_{cir}	Area A [m ²]	Perimeter P [m]		e_{min} [m]	S_{max} [m]
5				$h = 9.00$ $l_1 = 3.32$ $\vartheta_1 = -90$ $l_2 = 4.40$ $\vartheta_2 = -33.36$ $l_3 = 4.49$ $\vartheta_3 = 19.00$ $l_4 = 4.62$ $\vartheta_4 = 67.81$		
	1.0402	83.66	33.73		0.46	9.75
6				$h = 9.00$ $l_1 = 3.10$ $\vartheta_1 = -90$ $l_2 = 2.93$ $\vartheta_2 = -64.33$ $l_3 = 4.34$ $\vartheta_3 = -10.66$ $l_4 = 4.43$ $\vartheta_4 = 30.90$ $l_5 = 4.52$ $\vartheta_5 = 70.80$		
	1.0343	111.06	38.64		0.84	10.82

In this work, the geometric shape of the polygons of the bases of building models obtained as a result of multi-criteria optimization using indicators was examined:

- span of the polygon [15], i.e. doubled the length of the largest height of the hipped roof end, i.e.

$$S_{max} = 2 \cdot \max_{ij} \{s_{ij}\}, \quad (1)$$

where i denote the number of polygon C_i and j denote the number of vertex of polygon C_i of generalized polygon $P(C_1, C_2, \dots, C_k)$ (cf. [1]),

- the length of the smallest ridge of the roof skeleton e_{min} ,
- indicator of compactness of a region relative to the circle (the inverted indicator [18] to POP ratio [25])

$$RC_{cir} = \frac{P}{2\sqrt{\pi A}}, \quad (2)$$

where P – perimeter of region, A – area of region of the base of building model,

- indicator of compactness of the square in relation to the circle

$$RC_{cir} = \frac{2}{\sqrt{\pi}} = 1,1284. \quad (3)$$

The circumference of a square is almost 13% larger than the circumference of a circle with the same surface area.

The optimization results obtained in the monograph [24] for octagon, heptagon and nine-sided can be interpreted in the context of the concept of building span and length of ridge sections. In the second and third columns of Table 1 the shape of the geometric roof skeleton is given, in the second column – the compactness of the base polygon relative to a circle with the same surface, in the third column – the base and perimeter of the base, in the fourth column – the length of the shortest ridge of the roof skeleton and in the fifth column – roof truss span.

A similar analysis can be made for a building with a contour in the fifth row of Table 1. Maybe it is worth eliminating small, 0.46 [m] sections of roof ridge edges and thus simplify the topology of the roof skeleton, accepting (perhaps) small losses as a result of increasing the value of the objective function [24]. The shape of the roof is therefore an important parameter for the shape of the building, it is a derivative of the contour, if we plan the building arbitrarily; but it can become a determinant stimulating the

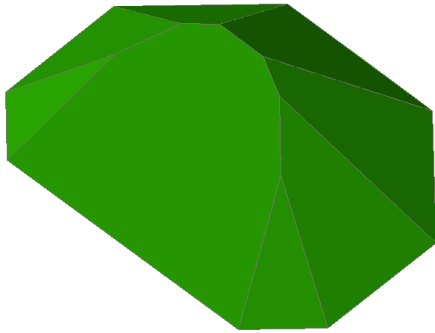


Figure 4: Axonometric view of the roof model of the building No. 1

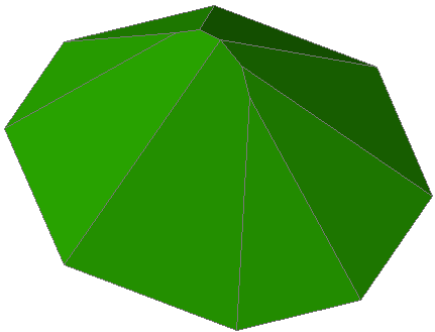


Figure 5: Axonometric view of the roof model of the building No. 3

development of the project when we start the process of designing the building geometry with the analysis of the shape of the roof skeleton.

Note that the values in the second column of Table 1 are definitely more favorable than the values calculated relative to the square. No wonder, because the optimal solution obtained is close to the shape of the circle. The idea of obtaining such a solution guided the authors of the work [24]. The solution is worse than ideal (for the wheel) only by about 5% (in the second row the value 1.0488) and by about 3% (in the third row the value 1.0311).

3 Conclusions

- 1: The building outline, including arcades and covered terraces, determines the shape of the roof. The shape is unique, the slope of the hipped roof ends are the same.
- 2: There is convenient software that allows preliminary simulation of the roof skeleton (e.g. AutoCAD environment). Checking the shape of the roof on the proposed outline of the designed building is a simple operation.

- 3: Omission of the initial analysis of the building's outline shape can cause many difficulties in the correct design of the building.
- 4: When using multi-criteria methods, the shape of the roof skeleton and the parameters resulting from it should be among the criteria.

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