Research Article

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Contractor's bid pricing strategy: a model with correlation among competitors' prices

Abstract: The approach used by construction companies to determine bid prices is an element of their strategy used to win jobs in competitive tenders. Such strategies build upon an analysis of the contactor's potential and capabilities (am I able to deliver? am I eligible to participate in the tender?), and the analysis of the economic environment, including the expected behavior of competitors. The tender strategy sets out both the guidelines and the procedure in deciding whether or not to bid as well as the rules for determining the price. The price, on the one hand, should be high enough to cover expected direct and indirect costs as well as risk-adjusted profit. On the other hand, it needs to be low enough to be considered most attractive (typically: the lowest) among the prices offered by the competitors. The paper focuses on the price definition component of the bidding strategy. It provides a brief overview of the existing methods that support bidding decisions by comparing their demand for input and limitations in practical applications and presents a simulation-based method supporting the determination of the profit ratio. This probabilistic method assumes the existence of a positive correlation between the prices offered by the competitors. Its application is illustrated by means of a numerical example. The outcomes of the simulation prompt the amount of the profit margin that maximizes the expected value of the contractor's profit.

Keywords: competitive bidding strategy, bid price, markup, correlation among bids; probability of winning

1 Introduction

The procurement of construction works is expected to ensure that competitive bids are obtained and that the one that best meets the investor's requirements is selected from among them. Bids may be evaluated according to a variety criteria, but the key one is typically the total price.

The low bid (LB) model treats the bid with the lowest price as the most beneficial to the client. This approach is believed to have its merits especially in the case of works procured in the traditional design-bid-build way, where the contractor is expected to deliver works defined by precise drawings and prescriptive specifications. If the client wishes to enforce completion time and conditions of payment, the circumstances leave little space for differentiating the bids according to criteria other than the price. Besides, clients from the public sector tend to use LB for momentary benefits of transparency, as selection based solely on the measurable criterion of price is unequivocal. However, the practice of the lowest price tendering is observed to result in the low quality of work, claims, disputes, and time overruns, especially in the times of economic downturn when the orders are scarce [1]. It is also reported to promote bid rigging [2, 3]. Thus, the clients' practice of "buying cheap" necessitates introducing strict controls to mitigate risks (*e.g.* to eliminate unreasonably low bids). It also results in increased cost, as concisely summarized by Constructing Excellence [4].

The average bid method and its variations, where the winner is determined by comparing the submitted bids to their average, was believed to be the answer to the key deficiency of LB, so the risk of selecting a bid of an unrealistically low price due to either underestimated cost or the bidder's anticipation to win more money through claims and disputes [5]. This approach does not compromise the ease of use of a single criterion-based selection and, at least in theory, works well, though does not automatically exclude collusion [1, 6].

The best value or most economically advantageous tender model relies upon the optimum combination of the whole-life cost and quality to meet the user's requirements. It enables the client to take account of criteria that reflect qualitative, technical, and environmental aspects of the bid as well as the price. Nevertheless, it involves much more effort of the client in selecting the set of criteria and

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their relative importance to reach the desired effect [3, 7–9].

Regardless of the procurement strategy adopted by the clients, the price for the construction works belongs to the most important criteria of bid assessment. Current public procurement regulations in Poland (as for 2019) may serve as an example. In 2014 and 2016, following EU directives, Polish regulations switched from the lowest price to multicriteria bid selection. The price stays an obligatory criterion (it may be replaced by the whole life cost if justified by circumstances), but it can be used as a sole criterion or as a dominating criterion (with a weight greater than 60%) if specifications precisely define quality standards. Other criteria are to be related to the characteristics of the subject of the contract and not to the characteristics of the contractor; the latter can be verified at the preselection stage or the conditions of participation in the tendering procedure. The report on effects of changes in public procurement law [10] focuses solely on the number, the type and weights of the criteria in use, and not on the actual impact of using more bid assessment criteria on the efficiency of public spending. However, the overall picture (at least in tenders for construction works) is as follows: the lowest price is what the clients want the most. From the point of view of a bidder, it is therefore important to set the price at a level that ensures that the tender is won.

The paper investigates into models that could be used to support bidding decisions of contractors operating in local markets of specialty works who win their orders mostly through open tendering and bid against a relatively predictable set of local competitors. Contractors who operate in such market niches are, on the one hand, well informed on their competitors' bidding practices. On the other hand, the price competition may be fierce, and the chance of reducing costs is not high: the competitors often rely on the same suppliers, draw from the same local pool of labor resources and apply the same construction methods. Therefore, the costs of such contractors become similar, and the markup becomes the key factor affecting the price and the chance of winning the job.

Construction companies inevitably strive to set the size of the markup at a level ensuring the maximum value of the expected profit in the long term. They also adjust its planned amount on an ongoing basis, according to the market situation, their current project portfolios, and attractiveness of a particular job. In the times of economic downturn, when orders become scarce, they are likely to undertake less profitable jobs and bid low to stay in the market. Since the same factors affect decisions of all competitors in the local market (as operating under similar conditions and participating in the same tender proce-

dures), there are grounds to assume that their decisions on pricing are correlated. Therefore, while constructing a bidding decision support model intended for local market competitors, the authors attempt to account for a correlation between the competitors' bid prices.

The remainder of the paper is structured as follows: Section 2 presents a concise literature review on mathematical models that support decisions in competitive bidding. Section 3 discusses the assumptions of the proposed simulation model and the methodology for determining the markup-related probability of winning the tender. Discussion on the impact of the scale of correlation between competitors' bid values on the chances to win and the recommended markup is presented by means of a numerical example in Section 4. The concluding section points to directions for further research.

2 Competitive bidding models in construction

The body of literature on multifaceted aspects of auction theory is enormous. Therefore, the authors narrow down their scope to the models intended to support construction contractor in deciding on best price in single-stage sealed bids tendering, with the assumption that the price is the sum of estimated cost and the markup. The "best price" in this context is to maximize the expected monetary value of the job to the contractor [11, 12]. Hosny and Elhakeem [13] divide these models into three categories: probability theory and statistical models, multicriteria utility, and artificial intelligence, though it does not cover all techniques to capture the problem and provide a solution.

The probability theory-based models aim at defining probabilities of winning in competition. These probabilities are typically estimated on the basis of experimental distributions of the competitor's past bidding behavior. Friedman [11] provided one of the first such models, defining the expected profit as a product of the contractor's markup and the probability of winning under a set of simplifying assumptions. These assumptions were as follows: there exists a pattern in each competitor's bidding defined by a particular probability density function, the competitor's patterns are independent (thus the probability of winning in the lowest bid auction is a product of probabilities to beat each of the competitors); the competitors should be known in advance, though if the identity and number of competitors is unknown – as in real life – some more assumptions on the distribution on the number of bidders were to be made, and the "unknown" competitors' pattern

of bidding was advised to be described by averaging patterns of competitors observed in the past). A contrasting model was introduced by Gates in 1967 [14], who provided a different formula for assessing the probability of winning at a specified markup (proved later by Skitmore *et al.* [15] to hold only if the bidding patterns follow Weibull distribution). Carr [16] put more attention to the possible differences in the cost estimates between the tender participants – this model (rather unrealistically) assumed that the competitor's cost estimates (and not only bid values) distributions can be estimated for all historic projects, and that the distributions of cost estimates and bid values are independent.

The above mentioned models reached much attention from other researchers and have been extensively modified. Solutions were proposed to be found either analytically or by means of simulations. For instance, King and Mercer's [17] model was equivalent to Friedman's but it accounted for the effect of an inaccurate cost estimate on the contractor's expected profit. Hosny and Elhakeem's simulation-based tool [13] allowed the user to select specific projects for eliciting bidding behaviors of competitors instead of using data on all historic projects available to better account for characteristics of the project in question. Bahman-Bijari [18] and Yuan [19, 20] provided analyses based on similar assumptions that those in Friedman's model, but accounted for a correlation between the competitor's bids.

As the competitors' bidding pattern may be difficult to capture by the statistical models, many factors are likely to affect bidding decision in a particular case, and the objectives of bidding is not necessarily to maximize profits [12], some researchers propose tools that use fuzzy input to infer on the most suitable markup size, but base on a predefined range of the margin [*e.g.* 21, 22]. Models based on statistical regression and neural networks were also tried to account for more factors that characterize particular projects and are likely to affect both cost and bid values; neural networks are generally claimed to be an effective tool in the search for the optimal markup [23–25].

3 Proposed bidding model

3.1 Assumptions

The model presented in this chapter stems from Friedman's model [11] with assumptions modified by adding correlation between the competitors' bids.

Let us take the point of view of a bidder A_0, who intends to bid against n competitors $A_1, A_2, \ldots, A_n$ in a particular tendering procedure. The bid price b_i of each competitor A_i is established on the basis of the sum of individually estimated costs c_i and profit m_i:

$$b_i = c_i + m_i, \tag{1}$$

The costs comprise the direct costs of works c_i^d (materials c_i^M (including their buying costs), labor c_i^L and plant c_i^P), and indirect costs c_i^{ind}. Let us assume that these can be calculated as a "true commercial cost" adjusted for risks of delivering the work to the client.

The profit is often expressed as a percentage of costs. In Poland it is usual to present it as a percentage of the sum of labor, plant and indirect costs, but not materials [26, 27]. Such percentage is used in market reports prepared by Polish construction price book publishers. Adhering to this convention:

$$m_i = w_i^m \left(c_i^L + c_i^P + c_i^{ind} \right) = w_i^m \left(c_i - c_i^M \right) \tag{2}$$

where w_i^m – the markup percentage calculated by the contractor A_i, further referred to as markup.

Therefore, the markup can be expressed as:

$$w_i^m = \frac{b_i - c_i^M}{c_i - c_i^M} - 1 \tag{3}$$

In the considered tender, the contractor A_0 naturally does not know the cost estimates c_i, bid prices b_i nor markups m_i of their competitors. Therefore, let us treat them as random variables further referred to as C_i, B_i and W_i^m.

Let us assume that the distribution of each competitor's markup W_i^m can be estimated on the basis of historic records of the competitor's bid prices noted down by the contractor A_0 at each public bid opening in the past. It is thus assumed that there exists some pattern in the bidding behavior of the competitors that can be inferred from these records.

As construction prices change naturally over time, each project involves different costs, and the cost estimates of the competitors stay unknown to contractor A_0, the historic markups of the competitors can only be estimated and expressed with reference to costs calculated by the contractor A_0 for the respective tenders in a "standardized" form:

$$w_{i,k}^m = \frac{b_{i,k} - c_{0,k}^M}{c_{0,k} - c_{0,k}^M} - 1, \quad i = 1, 2, \ldots, n, \tag{4}$$
$$k = 1, 2, \ldots, s,$$

where:
$w_{i,k}^m$ – estimated standardized markup of competitor A_i at

tender k,

$b_{i,k}$ – bid price offered by competitor A_i at tender k, as announced at bid opening,

$c_{0,k}$ – total cost of works calculated by contractor A_0 to define their bid price for tender k,

$c_{0,k}^M$ – material cost calculated by contractor A_0 to define their bid price for tender k,

s – the number of historic tenders providing insight into the competitor's bidding patterns.

The optimal markup $w_0^{m,\star}$ would correspond to the highest expected value of profit:

$$\max E(V) = P\left(win | w_0^{m,\star}, n\right)\left(c_0 - c_0^M\right) w_0^{m,\star} \qquad (5)$$

where the probability of the contractor's A_0 winning the current job with the markup of w_0^m equals the probability of winning against all n competitors who decided to participate in the tender:

$$P\left(win | w_0^m, n\right) \qquad (6)$$
$$= P\left(W_1^m > w_o^m \cap W_2^m > w_o^m \cap \ldots \cap W_n^m > w_o^m\right)$$

3.2 Procedure for determining the optimum markup

Once a database of estimated markups $w_{i,k}^m$ of the competitors' historic tenders is prepared, it can be used in the following steps:

1. Establish bidding patterns of the competitors from the sample recorded in the database: define distribution type and parameters of the random values of their standardized markups W_i^m;
2. Establish correlation coefficients among W_i^m.
3. Generate (simulate) correlated random values according patterns established in 1. and 2. to serve as bidding profiles of the competitors and the basis for inference on the relationship between the markup value and the probability of winning and the expected value of profit.
4. On the basis of the above mentioned dependencies, determine of the optimum value of the markup, corresponding to the maximum expected profit.

3.3 Correlation between the contractors' bid prices

The correlation between random variables means interdependence or association between them. This relationship may be functional or statistical. A functional relationship is characterized by the fact that each value of one independent variable corresponds to only one unambiguously defined value of a dependent variable. A statistical relationship is the fact that specific values of one variable correspond to precisely defined average values of the other variable. Correlation does not mean that there is a cause and effect relationship between the variables in question. The co-existence of two variables may result from the influence of another variable on both of them.

A contractor's decision on the markup size is affected by many factors: perceived risk, project complexity, type of work, current workload (affecting the desire to get a new order), an expected rate of return, location, etc. [28]. Considering that some factors are characteristic for the market and the project, they affect all competitors in a particular tender. Though the scale of this impact is subjectively assessed by each competitor, it is likely that there exists a positive correlation between markups in the same tender: if one of the bidders decides to use a higher markup as justified by the circumstances, other competitors, basing on the same input, would probably also bid higher. A similar relationship occurs in the case of the cost: the competitors are likely to resort to the same/similar methods and materials because they are at least partially enforced by the specification, and as they may have access to the same pool of subcontractors and suppliers.

There exist many measures of the strength of the relationship between random variables. The authors resorted to the most popular one, the Pearson coefficient of linear correlation, applicable to variables that are normally distributed; it assumes values between –1 (the total negative linear correlation), through 0 (no linear correlation), up to 1 (the total positive linear correlation). A number of analyses of tender results presented in the literature prompt that markups W_i^m may follow a normal distribution [18–20, 29].

The Pearson coefficient of correlation between random variables X and Y is estimated on the basis of relationships discovered in a sample of observations:

$$\rho_{XY} = \frac{1}{s_X \cdot s_Y} \sum_{i=1}^{m} (x_i - \overline{x})(y_i - \overline{y}),$$

where: (x_i, y_i) – observed values of variables X and Y, m – the number of observed pairs, $\overline{x}$, $\overline{y}$ – the sample means, s_X, s_Y – the standard deviations of observations.

The coefficient calculated on the basis of the sample is an estimator of the correlation coefficient of the population. Its reliability depends on the sample size.

In practical cases, the sample of usable tender results may occur small: even if the number of tenders was significant, not all competitors participated in each of them.

If the data are missing due to the fact that a particular competitor did not participate in a particular tender, the case-wise elimination technique should not be used as it considerably limits the sample size and, as a result, increases the estimation error and reduces the significance level of the tests. Pairwise elimination is a more efficient approach - cases are removed when there are no values for the pair of variables for correlation. An alternative approach is to substitute missing data [29]:

- with the means of the whole sample or a subset of the sample,
- by pattern matching,
- by expectation-maximization algorithm.

With small samples, to enhance the reliability of Pearson coefficient estimation, the Bayesian inference may be adopted [31]; this approach was used *e.g.* by Yuan [19, 20] in his correlated competitive bidding model being an extension to the Friedman's model.

Ultimately, if the input is insufficient, the correlation coefficient may be subjectively assumed. Touran [32] recommended using expert opinions of correlation between two random variables expressed in an ordinal scale (weak, moderate, strong) that are arbitrarily assigned values of Pearson coefficient of, respectively, 0.15, 0.45 and 0.8. The same approach was used by Chau [33] to assess relationships between cost of processes. Cho [34] employed the concept of concordance probability by Gokhale and Press [34] to estimating correlation coefficient between normally distributed random values of process durations; the concordance probability is a monotone increasing function of correlation coefficient whose parameters are assessed by experts according to the fraction of expected cases where a random variable Y takes values greater than its expected value, given that random variable X is greater than its expected value.

3.4 Generating correlated random variables

Two distinct approaches have been proposed to generate correlated random variables. The first approach is applicable in cases where the joint multivariate distribution of the random variables is fully specified. The other approach is applied if the joint distribution is not fully specified, but the marginal distributions and correlations between the variables are known.

A survey on methods and applications of multivariate simulation can be found in Johnson [36]. In particular, methods developed for one-dimension random variables, *i.e. acceptance-rejection*, *composition* and *convolu-*

tion, are applied to generating multivariate variables [37]. In the case of a multivariate normal distribution with n marginal variables X_i of a normal distribution $N(\mu_i, \sigma_i)$ with the mean μ_i, $i = 1, 2, \ldots, n$, the standard deviation of σ_i, $i = 1, 2, \ldots, n$, and the covariance matrix of $\Sigma = \left[cov\left(X_i, X_j\right)\right] = \left[\rho_{X_i X_j}\sigma_{X_i}\sigma_{X_j}\right]$, a following transformation may be applied: $\Sigma = \mathbf{C}\mathbf{C}^{\mathbf{T}}$, where $\mathbf{C}$ is a Cholesky factor of Σ [37]. The covariance matrix ought to be semipositive definite, otherwise the elements of $\mathbf{C}$ matrix are imaginary. If Touran subjective correlation coefficients [32] are intended to be used and the covariance matrix is not positive definite, it is advised that the estimated correlation coefficients are incrementally reduced by 1% until the values of C matrix become real. A full description of Cholesky's transformation algorithm can be found in the literature on numerical methods. The process of generating correlated normally distributed random values X_i is as follows:

1. Generate a sample of standardized normal values Z_i ~'N(0, 1): Z=$[z_1, z_2, \ldots, z_n]^T$.
2. On its basis, calculate $X_i = [x_1, x_2, \ldots, x_n]^T = \boldsymbol{\mu} + \mathbf{CZ}$.

If the correlated random values to be generated are to be of a distribution other than normal, the NORmal-To Anything (NORTA) transformation can be applied [38, 39]. With this transformation, the samples of random values can be obtained from a partially specified distribution by transforming the elements of a sample from a multivariate standard normal distribution according to the desired marginal distribution, where the correlations of the elements of the deriving normally distributed random vector are set to generate the desired correlations in the transformed random vector.

4 Numerical example

The input for the example is purely notional, though the range of input values (distribution parameters of estimated markups) are suggested by the survey of Mielec *et al.* [29] conducted on a local market of external thermal insulation contractors.

A local contractor A_0 competes usually against two local competitors A_i ($i=\{1,2\}$). To discover patterns in their bidding, the contractor keeps records of competitor's bid values and estimates their markups W_i^m relative to the contractor's own cost estimates. The distribution of estimated markups of Competitor 1 was found to be N(0.102;0.014), and for Competitor 2, N(0.098;0.012). Let us assume that, due to the small size of the contractor's database and incomplete data (the competitors did not always participate
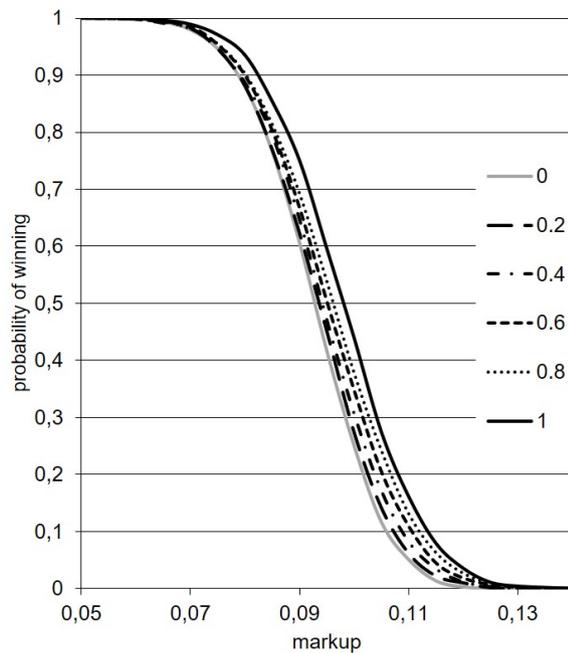
Figure 1: Simulation results: probability of winning against bid markup at different levels of correlation between the competitor's bids (Pearson correlation coefficient $\rho = \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$)
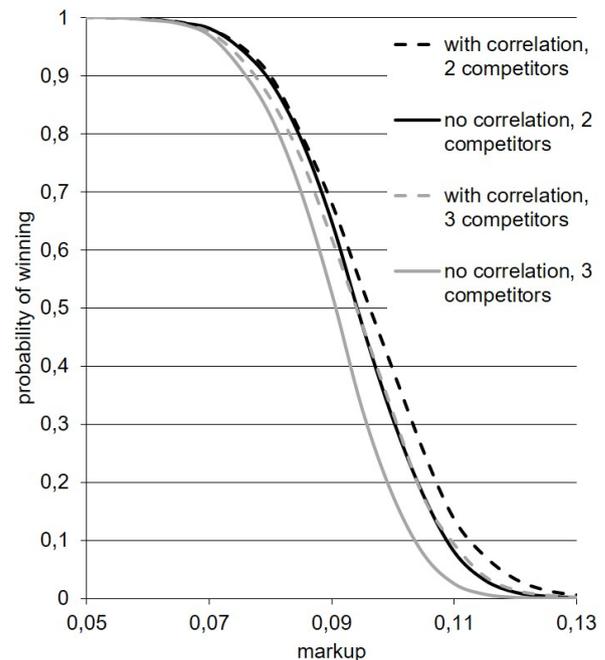

Figure 2: Simulation results: the expected profit to cost ratio $\frac{E(V)}{c_0 - c_0^M}$ against bid markup at different levels of correlation between the competitor's bids (Pearson correlation coefficient 0, 0.2, 0.4, 0.6, 0.8, 1.0)

Figure 3: Simulation results: probability of winning against markup for different number of competitors and at different correlation level (description in the text)
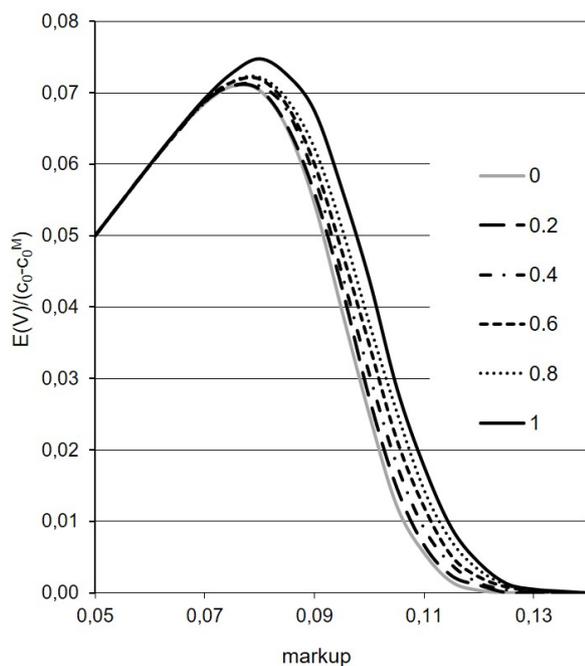
in all tenders), the correlation coefficients cannot be reliably estimated.

Therefore, simulations are going to be repeated for a number of arbitrarily assumed Pearson correlation coefficient values only to assess the scale of impact of correlation on the optimum markup.

The results of simulations are shown in Figures 1 and 2 presenting, respectively, the probability of the contractor's A_0 winning versus the markup and the expected profit to cost ratio versus the markup.

If there exist a high correlation between the competitor's bids ($\rho > 0.6$) then the optimal markup level, corresponding to the highest expected profit index, is about 8%. Lower correlation shifts the maximum expected profit index to the left, and the optimum markup is prompted to be about 7.5%. The probability of winning a tender with a markup close to the optimum differs significantly according to correlation: it is clearly greater in the case that the competitors' bids are uncorrelated. Correlation-related differences in expected profit are also noticeable. Therefore, if correlation was observed between the competitor's bids, a contractor using the proposed method of analysis to support their bidding decision would be prompted to submit a bid with a higher markup.

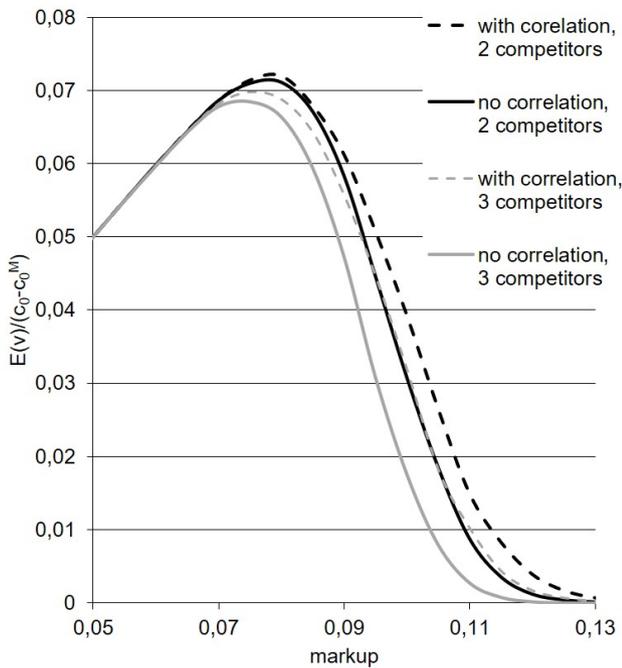Figures 3 and 4 illustrate the impact of the number of competitors on the probability ow winning and ex-

Figure 4: Simulation results: expected profit to cost ratio $\frac{E(V)}{c_0 - c_0^M}$ against markup for different number of competitors and at different correlation levels (description in the text)

pected profits by comparing simulation results for a theoretical case of two ($i=\{1,2\}$) and three ($i=\{1,2,3\}$) competitors whose bids W_i^m follow normal distributions of identical parameters: N(0,102;0,014). For the sake of brevity, only two options of correlation level were considered: no correlation ($\rho = 0$) and $\rho = 0.5$ between all pairs of competitors. As expected, the probability of winning drops with a growing number of competitors. The optimal markup (corresponding to the highest expected value of profit) is prompted to be 7.5% for three other bidders, and 8% for two bidders. An important observation is that, with the greater number of competitors, there are greater differences in the probability of winning a tender with a given markup in the case of no correlation and considerable correlation between the competitors' bids.

5 Conclusions

The presented approach – the modified Friedman's model with correlations – may be difficult to apply in practice if there is little historical data from past tenders. As illustrated by the example, the impact of correlation on the probability of winning a tender and the expected profit is considerable. Therefore, relying on subjective expert esti-

mates of correlation may generate large errors and, if overestimated, significantly reduce the chances of winning a tender. Moreover, the number of competitors, which is in practice unpredictable, has a profound impact on the estimates. Actual complexity of the problem makes simple models based on a handful of input unreliable. It is thus advisable to develop methods enabling the planner to account for a variety of factors affecting the bid price, as well as their uncertainty.

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