Research Article

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The efficiency of linear electromagnetic vibration-based energy harvester at resistive, capacitive and inductive loads

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Abstract: Energy harvesters and almost all energy generation devices receive the motivation for design from their efficiency and efficiency play an important role in the feasibility and practicability of the design. In this paper, we investigate the efficiency of electromagnetic vibration-based energy harvesters at various electrical loads. In our problem the efficiency depends on excitation frequency, coil and load parameters as well as electromagnetic coupling coefficient. The author first proves that the input power that the harvester receives from its environment at constant base acceleration and constant excitation frequency is always equal to the power that consumes in electrical and mechanical dampers, then the author defines the resonance frequency and plot three efficiency diagrams i.e. plot of efficiency versus (excitation) frequency, plot of maximum efficiency at a constant frequency versus load and in the end plot of the efficiency versus output power at varying load capacitance and resistance. The author observes that maximum efficiency not only does not occur at resonance (i.e. at maximum power) but also is very low (less than 1e–10%) for typical parameters at resonance. Also the maximum efficiency for typical optimum parameters is around 17.45%.

Keywords: efficiency at maximum power; electromagnetic energy harvester; maximum efficiency; resonance definition.

Introduction

Efficiency is a fundamental parameter used to compare all kinds of energy harvesters with various sizes and designs. Usually the main goal of an energy harvesting system is to extract the maximum power from the environment. Almost all the authors have focused their efforts on maximizing the extracted electrical power for linear electromagnetic vibration-based energy harvesters (EVEH) and only a few of them have focused on maximizing efficiency (Almoneef and Ramahi 2015; Ashraf et al. 2013; Bright 2001; Blad and Tolou 2019; Roundy 2005; Smits and Cooney 1991; Wang et al. 1999; Wang and Cross 1998; Zhang et al. 2019). Among these a few, most of them focus on resistive load only. In this paper, we consider inductive and capacitive loads as well. The optimal resistor formula and the numeric optimal capacitor and inductor values that maximize the efficiency have been determined. Here efficiency is defined as the ratio of the electrical power extracted from the load resistor (output power) to the time rate of vibration energy of the environment (input power). Efficiency is considered one variable function of load resistance in resistive load, and is two variables for capacitive and inductive load, respectively. The efficiency is plotted versus excitation frequency at constant optimum coil and load parameters as well as constant electromagnetic coupling coefficient. Then the efficiency and output power is plotted versus load and the author observe that the maximum efficiency dose not occur at resonance frequency in other words load that maximize output power differs with the load that maximize efficiency. The efficiency versus variable load at constant parameters as well as constant resonance frequency is plotted and also efficiency versus excitation frequency at optimum loads is plotted. It should be noted that according the definition of resonance the resistive as well as inductive load dose not lead to a real or true resonance frequency and optimum case at Micro and Nano dimensions except for rare cases (this will be explained later in this paper).

Basic principles of EVEH

In this section we define resonance frequency and calculate the power consumed in the load resistor for three above mentioned loads i.e. resistive, capacitive and inductive loads.

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Definition of resonance

The definition of resonance differ from the other mechanical problems because here we seek condition that will maximize the electrical damping term in our problem and this is in contrast to other problems that the damping is a minimum and amplitude should have a maximum value. For resonance (the optimum frequency), the most important following conditions should be satisfied simultaneously:[1]

$$\frac{\partial c_e}{\partial \omega} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial \omega} = 0 \tag{2}$$

$$\frac{\partial E}{\partial t} = 0 \rightarrow -\omega c'_e + m\omega^2 - k_s = 0 \tag{3}$$

$$\frac{\partial P_{R_L}}{\partial R_L} = 0 \tag{4}$$

$$\frac{\partial P_{R_L}}{\partial C_L} = 0 \tag{5}$$

The result of condition 1 should be absolute maximum point and the result of condition 2 should be local minimum point. The third condition is partial derivative of total stored energy versus time that should be zero i.e. sum of all energy in the capacitors and inductors of electrical equivalent system is constant. P_{R_L} is output power in the load resistor and the last two equation are derived from Thévénin equivalent of the circuit of the system. The result of these conditions must be absolute maximum point. Q is the net reactive power that consumed or generated in the whole of the system. The Q can be derived easily from equivalent circuit of system (the mean (average) net reactive power that consumed or generated in the all capacitors and inductors of the equivalent circuit of the system as a whole) (Dezhara 2022). The first equation tells us that the electrical energy that enter in the electrical part should be maximum. In the third equation c'_e is called energy injection lock coefficient or simply lock coefficient, it will be explained later in this section. This equation comes from real part of transfer function denumerator which is equaled to the zero which is called energy boundary condition between mechanical part and electrical part it also called resonance condition and holds for all three kind of load that was mentioned in this paper. The maximum value of c_e is equals to mechanical damping coefficient and this is another condition that can be added in the above-mentioned resonance conditions. Note that in the process of analyzing the problem the above derivatives calculated symbolically and equaled to zero. These nonlinear equations as well as two equations from Thévénin circuit (optimum capacitive load i.e. equations of 18, 19)[2] can be solved simultaneously to give the optimum values of electrical parameter plus optimum frequency (resonance frequency). Note that in general this frequency differs from mechanical resonance frequency.

Equivalent circuit of EVEH

Based on the electrical similarity of mechanical systems (Cammarano et al. 2010) we can say:

$$\text{mass} \Rightarrow \text{capacitor, spring} \Rightarrow \text{inductor}$$

$$\text{damper} \Rightarrow \text{resistor}$$

$$m \rightarrow C, \frac{1}{k_s} \rightarrow L, \frac{1}{C_m} \rightarrow R$$

A diagram of the EVEH circuit is shown in Figure 1. F_{EM} is the electrical damping force and $m\omega^2 Y(\omega)$ is the excitation current source. The load impedance is Z_L. R_c and L_c are coil resistance and inductance respectively. m and k_s and C_m are the mass of the moving magnet and spring stiffness constant and mechanical damping coefficient respectively. We seek the Thévénin equivalence of circuits from the end terminal of the load Z_L. First, we calculate the voltage and impedance of Thévénin. This voltage is the voltage at the terminal of the current source.

$$X' = m\omega - \frac{k_s}{\omega} \tag{6}$$

where X' is the admittance of capacitor and inductor at mechanical side of Figure 2. Based on the Figure 3 we can also calculate short circuit current.

$$V_{th} = V_{o.c} = \frac{Km\omega^2 Y(\omega)}{C_m + jX'} \tag{7}$$

$$I_{s.c} = \frac{KV_1}{R_c + j\omega L_c} \tag{8}$$

$$V_1\left(C_m + jX' + \frac{K}{R_c + j\omega L_c}\right) = m\omega^2 Y(\omega) \tag{9}$$

1 If you are interested in designing other parameters such as electromagnetic coupling coefficient you can add to the conditions bellow the derivative of input power to electrical domain with respect to K and equal this derivative to zero.

2 Note that based on the resonance definition just capacitive load optimum frequency lead to a real number in almost all cases at Micro/Nano dimensions and this is explained more later in this paper.

$$V_1 = \frac{m\omega^2 Y(\omega)}{C_m + jX' + \frac{K}{R_c + j\omega L_c}} \quad (10)$$

If we substitute equation (10) into (8):

$$I_{s.c} = \frac{mK\omega^2 Y(\omega)}{K + (R_c + j\omega L_c)(C_m + jX')} \quad (11)$$

$$Z_{th} = \frac{V_{o.c}}{I_{s.c}} = \frac{K + (R_c + j\omega L_c)(C_m + jX')}{(C_m + jX')} \quad (12)$$

After simplifying we have:

$$R_{th} = R_c + \frac{K^2 C_m}{C_m^2 + X'^2} \quad (13)$$

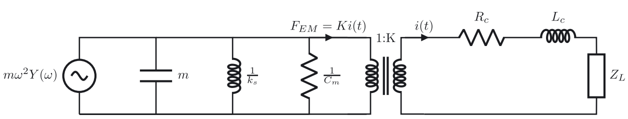

Figure 1: The circuit model of EVEH.

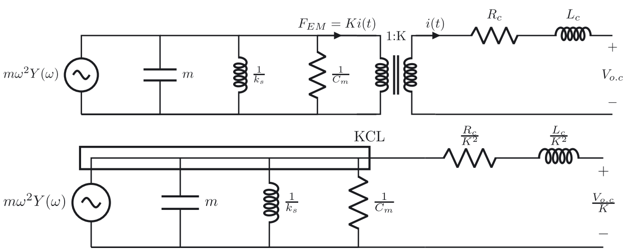

Figure 2: Thévénin circuit for open circuit voltage.

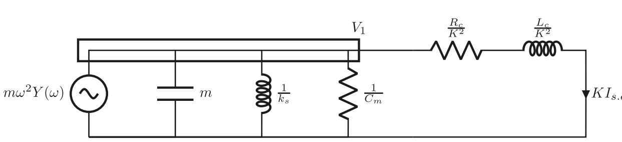

Figure 3: Thévénin circuit of short circuit current.

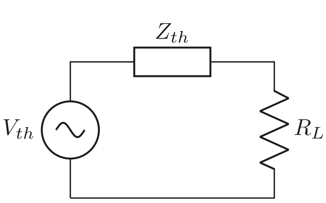

Figure 4: Thévénin equivalent circuit for resistive load.

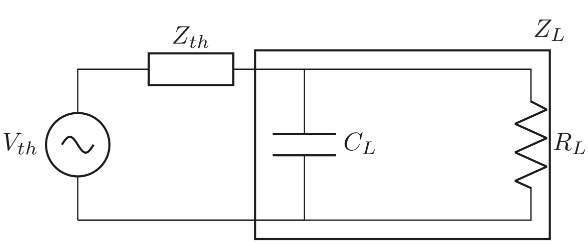

Figure 5: Thévénin equivalent circuit for capacitive load.

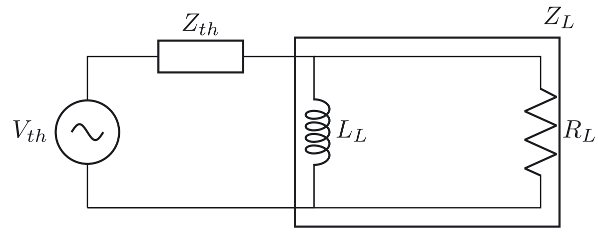

Figure 6: Thévénin equivalent circuit for inductive load.

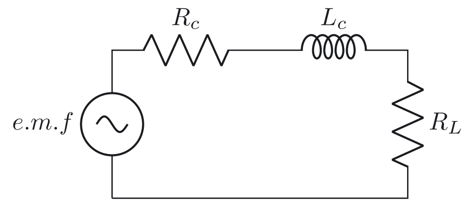

Figure 7: Electrical side of EVEH for resistive load.

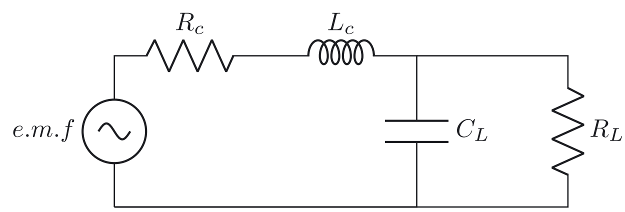

Figure 8: Electrical side of EVEH for capacitive load.

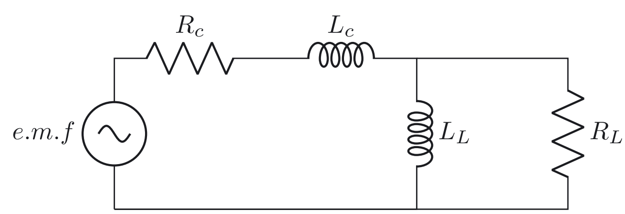

Figure 9: Electrical side of VEH for inductive load.

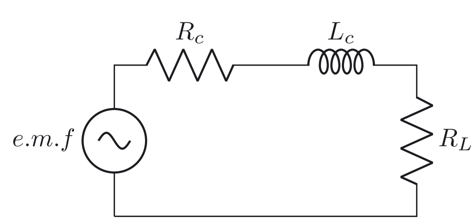

Figure 10: Electrical side of EVEH for resistive load

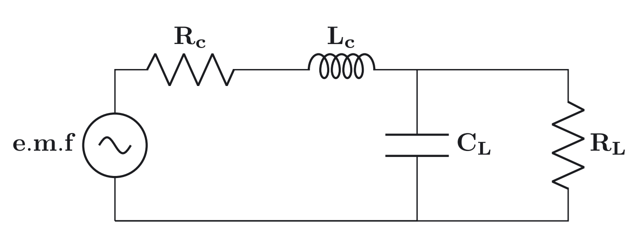

Figure 11: EVEH's electrical side for capacitive loads.

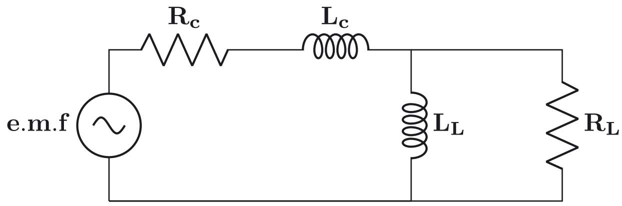

Figure 12: The electrical side of the EVEH for inductive loads.

$$X_{th} = X_c - \frac{X'}{C_m^2 + X'^2} \tag{14}$$

The condition of maximum power transfer

If the load impedance is equal to the conjugate of Thévénin impedance, then maximum power will be delivered from the source to the load (Cammarano et al. 2010). Note that this is true just for resistive load and in the case of capacitive and inductive loads the derivative of real power consumed in the load resistor in terms of load resistor and capacitor should be equaled to zero for obtaining the optimum load and rendering reactive part of Thévénin reactance to zero.

Resistive load

Based on the Figure 4 we have:

$$R_L = R_{th}, \quad X_{th} = 0 \tag{15}$$

$$R_L = R_c + \frac{K^2 C_m}{C_m^2 + X'^2} \tag{16}$$

Capacitive load

Based on the Figure 5, if you calculate the power consumed in the load resistor and equal the derivative of this power with respect to R_L and C_L to zero, the formulas for optimum capacitor and resistor will be achieved. these formulas are as follows:

$$R_L = \frac{|Z_{th}|}{\sqrt{\omega^2 C_L^2 |Z_{th}|^2 - 2C_L \omega X_{th} + 1}} \tag{17}$$

$$C_L = \frac{X_{th}}{\omega |Z_{th}|^2} \tag{18}$$

Putting the relation of (18) into (17), the result is:

$$R_L = \frac{|Z_{th}|^2}{R_{th}} \tag{19}$$

These equations are the optimum load formula for capacitive loads. Note that the optimum resistor is not equal to the Thévénin resistor.

Inductive load

Based on the Figure 6 and similar to the capacitive load we have:

$$R_L = \frac{-\omega L_L |Z_{th}|}{\sqrt{|Z_{th}|^2 + L_L^2 \omega^2 + 2L_L \omega X_{th}}} \tag{20}$$

$$L_L = -\frac{|Z_{th}|^2}{\omega X_{th}} \tag{21}$$

putting the relation of (21) into (20), the result is:

$$R_L = \frac{|Z_{th}|^2}{R_{th}} \tag{22}$$

These equations are the optimum load formulas for inductive load. Note that the optimum resistor is exactly the same as the case of capacitive load resistor.

The input power duo to vibration of environment

In this subsection the input power to the EVEH will be calculated and the author shows that this power is equal to power that consumes in the electrical and mechanical dampers. Consider the equivalent circuit of EVEH (Figure 1), the input power is equal to the power that is generated in the current source of the circuit.

$$P(\omega) = \frac{1}{2} V_s I_s^* = \frac{1}{2} V_s \left(m\omega^2 Y^*(\omega) \right) \tag{23}$$

where V_s and I_s are abbreviation of source voltage and current respectively. If we write the node equation for the mechanical side of the equivalent circuit, we have:

$$\frac{V_s - 0}{\frac{1}{j\omega m}} + \frac{V_s - 0}{\frac{j\omega}{k_s}} + \frac{V_s - 0}{\frac{1}{C_m}} + KI(\omega) = m\omega^2 Y(\omega) = I_s \tag{24}$$

Based on the differential equation governing the problem in frequency domain we have:

$$V_s = \frac{(m\omega^2 Y(\omega) - KI) Z(\omega)}{jm\omega + \frac{k_s}{j\omega} + C_m} = \frac{(-m\omega^2 + C_m j\omega + k_s) Z(\omega)}{C_m + j\left(m\omega - \frac{k_s}{\omega} \right)} \tag{25}$$

$$P_{in} = \frac{1}{2} \left(\frac{-m\omega^2 + k_s + j\omega C_m}{C_m + j\left(m\omega - \frac{k_s}{\omega} \right)} \right) m\omega^2 Y^*(\omega) Z(\omega) \tag{26}$$

Now if the resonance condition (relation 3) is applied to the transfer function (Dezhara 2022) we can simplify the above equation.

$$\frac{Z(\omega)}{Y(\omega)} = \frac{-m\omega^2}{-m\omega^2 + k_s + (C_m + c_e) j\omega + c_e' \omega} = \frac{-m\omega^2}{(C_m + c_e) j\omega} \tag{27}$$

If we put the equation (27) into equation (26) we have:

$$P_{in} = \frac{1}{2}\left(\frac{-m\omega^2 + k_s + C_m j\omega}{C_m + j\left(m\omega - \frac{k_s}{\omega}\right)}\right)(C_m + c_e)j\omega Z(\omega)Z^*(\omega) \quad (28)$$

After simplifying equation (28):

$$P_{in} = \frac{1}{2}(C_m + c_e)\left|\dot{Z}\right|^2 \quad (29)$$

The relation (29) tell us that the input power at resonance is equals to power that consumed in the mechanical and electrical dampers.

The specified load in various modes

Here the author determines the modes of load that should be applied in various condition in Table 1. It should be noted that the first case i.e. when excitation frequency is less that mechanical resonance frequency and Thévénin reactance (equation (14)) is positive, is the most popular mode and other cases are very rare at least in micro dimension. Based on the Thévénin reactance, when the excitation is less than mechanical resonance frequency, the Thévénin reactance has never a negative value and is always positive. So the case of $\omega < \omega_{\mathrm{Mech}}$ and $X_{th} < 0$ dose not mean.

Table 1: The loads that should be applied in various conditions.

Frequency	Thévénin reactance	Applied load
$\omega < \omega_{\mathrm{Mech}}$	$X_{th} > 0$	$R - C$
$\omega > \omega_{\mathrm{Mech}}$	$X_{th} > 0$	$R - C$
$\omega > \omega_{\mathrm{Mech}}$	$X_{th} < 0$	$R - L$

Resistive load calculations

In the Figure 7, if we designate the load voltage V_L then the Kirchhoff voltage low (KVL) around the loop is as follows:

$$e.m.f = L_c\dot{I}(t) + R_c I(t) + R_L I(t) \quad (30)$$

$$L_c j\omega\frac{V_L(\omega)}{R_L} + R_c\frac{V_L(\omega)}{R_L} + V_L(\omega) = e.m.f = K\dot{z} = Kj\omega Z(\omega) \quad (31)$$

$$V_L(\omega) = \frac{j\omega K Z(\omega)}{1 + \frac{R_c}{R_L} + \frac{j\omega L_c}{R_L}} \quad (32)$$

$$I(\omega) = \frac{V_L(\omega)}{R_L} = \frac{j\omega K Z(\omega)}{R_c + R_L + j\omega L_c} \quad (33)$$

where $I(\omega)$ is a coil current.

RMS Power for resistive load

The power is the time derivative work of the electric damping force F_{EM}:

$$P(t) = -\frac{d}{dt}\int F_{EM}dz = -K\frac{d}{dt}\int I(t)dz \quad (34)$$

$$P(\omega) = -j\omega K\int I(\omega)dZ(\omega) \quad (35)$$

We can calculate the power based on the generated electrical current. Inputting equation (33) into equation (35):

$$P(\omega) = -\frac{j\omega K}{R_L}\int V_L(\omega)dZ(\omega) \quad (36)$$

By putting equations (32) into (36), we can derive the power formula:

$$P(\omega) = -\frac{j\omega K}{R_L}\frac{j\omega K}{1 + \frac{R_c}{R_L} + \frac{j\omega L_c}{R_L}}\int Z(\omega)dZ(\omega) \quad (37)$$

$$P(\omega) = \frac{1}{2}\frac{K^2\omega^2}{R_L + R_c + j\omega L_c}Z(\omega)Z^*(\omega) \quad (38)$$

Based on the simplified equation above:

$$P(\omega) = \frac{1}{2}\frac{K^2\omega^2 Z(\omega)Z^*(\omega)}{(R_L + R_c)^2 + (\omega L_c)^2}(R_L + R_c - j\omega L_c) \quad (39)$$

$$P = \frac{1}{2}C_e\left|\dot{Z}\right|^2 \quad (40)$$

$$C_e = c_e - jc_e' \quad (41)$$

$$c_e = \frac{K^2(R_L + R_c)}{(R_L + R_c)^2 + (\omega L_c)^2} \quad (42)$$

$$c_e' = \frac{K^2\omega L_c}{(R_L + R_c)^2 + (\omega L_c)^2} \quad (43)$$

The real part of equation (41) corresponds to the electrical damping, which is a factor that affects the mechanical damping coefficient. The imaginary part of electrical coefficient i.e. imaginary part of equation (41) is called lock coefficient (Dezhara 2022). Reactive power may also be generated by the passive components of the circuit such as mass and spring. The reactive energy always flows from the electrical part, i.e. load capacitor and coil inductor, to the mechanical part, i.e. mass and spring. It's easy to conclude that the real part of equation (39) is energy generated in a coil and a load resistors, the imaginary part refers to the reactive energy generated in the coil's inductor. The resistor R_L consumes the following amount of power:

$$P_{R_L} = \frac{1}{2}\frac{K\omega^2 R_L}{(R_L + R_c)^2 + (\omega L_c)^2} Z(\omega)Z^*(\omega) \tag{44}$$

$$P_{R_c} = \frac{1}{2}\frac{K\omega^2 R_c}{(R_L + R_c)^2 + (\omega L_c)^2} Z(\omega)Z^*(\omega) \tag{45}$$

Capacitive load calculations

Based on the Figure 8, we can write the circuit equation of energy harvester with capacitive load as follows:

$$I = I_{C_L} + I_{R_L} = C_L\frac{dV_L}{dt} + \frac{V_L}{R_L} \tag{46}$$

where V_L is load voltage and C_L is the capacitance of load capacitor. If we put the equation of (46) into frequency domain we will have:

$$I(\omega) = j\omega C V_L(\omega) + \frac{V_L(\omega)}{R_L} \tag{47}$$

Now we should someway obtain the $V_L(\omega)$ and consequently $I(\omega)$ in terms of $Z(\omega)$.

$$L_c\dot{I}(t) + R_c I(t) + V_L(t) = e.m.f = K\dot{z} \tag{48}$$

$$j\omega L_c I(\omega) + R_c I(\omega) + V_L(\omega) = j\omega K Z(\omega) \tag{49}$$

If we put equation (47) into equation (49) and solve for $V_L(\omega)$ we will have:

$$\begin{aligned} &L_c j\omega\frac{V_L(\omega)}{R_L} - L_c C_L\omega^2 V_L(\omega) + \\ &R_c\frac{V_L(\omega)}{R_L} + V_L(\omega) + j\omega R_c C_L V_L(\omega) = j\omega K Z(\omega) \end{aligned} \tag{50}$$

$$V_L(\omega) = \frac{j\omega K}{1 + \frac{j\omega L_c}{R_L} + \frac{R_c}{R_L} + j\omega R_c C_L - L_c C_L\omega^2} Z(\omega) \tag{51}$$

If we put equation (51) into equation (47) we will have:

$$I(\omega) = \frac{-K\omega^2 C_L R_L + jK\omega}{(R_c + (1 - L_c C_L\omega^2)R_L) + (L_c + R_L C_L R_c)j\omega} Z(\omega) \tag{52}$$

RMS Power for capacitive load

Knowing the coil current we now can derive power formula versus frequency based on the definition of power (equation (34)):

$$P(\omega) = \frac{1}{2}\frac{\left(jK\omega^3 C_L R_L + K^2\omega^2\right)Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L) + (L_c + R_L C_L R_c)j\omega} \tag{53}$$

In order to distinguish real power generated in R_c and R_L from reactive power generated in L_c and C_L, we first calculate the real part of equation (53):

$$\begin{aligned} &\mathrm{real}(P(\omega)) = \\ &\frac{1}{2}\frac{\left(K^2\omega^2(R_c + R_L) + K^2\omega^4 R_L^2 C_L^2\right)Z(\omega)Z^*(\omega)}{(R_c + (1 - C_L L_c\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \end{aligned} \tag{54}$$

The real power is the sum of power that is generated in the coil and in the load. We can distinguish between these two powers, i.e. P_{R_L} and P_{R_c}, since the generated electrical current is known i.e. $I(\omega)$ (the electrical current that pass the resistor of coil) thus the real power consumed in the resistor of coil is as follows:

$$P_{R_c} = \frac{1}{2} I(\omega) I^*(\omega) R_c \tag{55}$$

$$P_{R_c} = \frac{1}{2}\frac{\left(K^2\omega^4 C_L^2 R_L^2 + K^2\omega^2\right)R_c Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \tag{56}$$

Subtracting equation (56) from equation (54) we find the power consumed by the resistor of the load (i.e. P_{R_L} for the capacitive load):

$$P_{R_L} = \frac{1}{2}\frac{K^2\omega^2 R_L Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \tag{57}$$

The frequency-response plot is the plot of P_{R_L} versus excitation frequency ω at the constant base acceleration. According to equation (53), the imaginary part of the equation is:

$$\begin{aligned} &\mathrm{imag}(P(\omega)) = \\ &\frac{1}{2}\Bigg(\frac{\left(-K^2\omega^3(L_c + R_L C_L R_c)\right)Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \\ &+\frac{\left(K^2\omega^3 C_L R_L(R_c + (1 - L_c C_L\omega^2)R_L)\right)Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2}\Bigg) \end{aligned} \tag{58}$$

we would like to conform equation (53) into standard form:

$$P = \frac{1}{2}C_e\left|\dot{Z}\right|^2 \tag{59}$$

The electrical damping coefficient of EVEH can be calculated by simplifying the equation of (54).

$$c_e = \frac{K^2 C_L\omega^2(R_c C_L) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L}\right)}{\left(1 + \frac{R_c}{R_L} - L_c C_L\omega^2\right)^2 + \left(R_c C_L\omega + \frac{L_c\omega}{R_L}\right)^2} \tag{60}$$

The imaginary part of complex damping based on simplification of equation of (58) is:

$$c'_e = \frac{K^2 C_L \omega (1 - L_c C_L \omega^2) - \frac{K^2\omega}{R_L}\left(\frac{L_c}{R_L}\right)}{\left(1 + \frac{R_c}{R_L} - L_c C_L \omega^2\right)^2 + \left(R_c C_L \omega + \frac{L_c \omega}{R_L}\right)^2} \tag{61}$$

Inductive load calculations

Based on the Figure 9 we have:

$$I = I_{L_L} + I_{R_L} = \frac{1}{L_L}\int V_L(t)dt + \frac{V_L}{R_L} \tag{62}$$

When we put the equation (62) into frequency domain, we get the following:

$$I(\omega) = \frac{V_L(\omega)}{j\omega L_L} + \frac{V_L(\omega)}{R_L} \tag{63}$$

L_L is the load inductance and I_L is the inductor current, and Here are the circuit equations:

$$L_c \dot{I}(t) + R_c I(t) + V_L(t) = e.m.f = K\dot{z} \tag{64}$$

$$j\omega L_c I(\omega) + R_c I(\omega) + V_L(\omega) = j\omega K Z(\omega) \tag{65}$$

Equation (63) into equation (65):

$$j\omega L_c \frac{V_L(\omega)}{R_L} + \frac{L_c}{L_L} V_L(\omega) + R_c \frac{V_L(\omega)}{R_L} + V_L(\omega) - \frac{R_c j}{L_L \omega} V_L(\omega) = j\omega K Z(\omega) \tag{66}$$

We solve for $V_L(\omega)$ from equation (66) and put it in equation (63).

$$V_L(\omega) = \frac{j\omega K Z(\omega)}{\frac{j\omega L_c}{R_L} + \frac{L_c}{L_L} + \frac{R_c}{R_L} + 1 - \frac{R_c j}{L_L \omega}} \tag{67}$$

$$I(\omega) = \frac{\left(\frac{1}{j\omega L_L} + \frac{1}{R_L}\right) j\omega K Z(\omega)}{\frac{j\omega L_c}{R_L} + \frac{L_c}{L_L} + \frac{R_c}{R_L} + 1 - \frac{R_c j}{L_L \omega}} \tag{68}$$

RMS Power for inductive load

$$P(\omega) = \frac{1}{2}\left(\frac{jK\omega}{\frac{-L_c L_L \omega^2}{R_L} + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right) j\omega + R_c} + \frac{jK\omega}{\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right) j\omega + \frac{L_c R_L}{L_L} + R_c + R_L}\right) Z(\omega) Z^*(\omega) \tag{69}$$

$$P = \frac{1}{2} C_e \left|\dot{Z}\right|^2 \tag{70}$$

Here is the real part of the equation:

$$\mathrm{real}(P(\omega)) = \frac{1}{2}\left(\frac{K^2\omega^2\left(R_c - \frac{L_c L_L \omega^2}{R_L}\right)}{\left(R_c - \frac{L_c L_L \omega^2}{R_L}\right)^2 + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)^2} + \frac{K^2\omega^2\left(\frac{L_c R_L}{L_L} + R_c + R_L\right)}{\left(\frac{L_c R_L}{L_L} R_c + R_L\right)^2 + \left(\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right)\omega\right)^2}\right) Z(\omega) Z^*(\omega) \tag{71}$$

$$c_e = \frac{\frac{K^2}{L_L \omega}\left(\frac{R_c}{L_L \omega} - \frac{L_c \omega}{R_L}\right) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)}{\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)^2 + \left(\frac{R_c}{L_L \omega} - \frac{L_c \omega}{R_L}\right)^2} \tag{72}$$

$$P_{R_c} = \frac{1}{2} I(\omega) I^*(\omega) R_c \tag{73}$$

$$P_{R_c} = \frac{1}{2}\left(\frac{K^2\omega^2 R_c}{\left(R_c - \frac{L_c L_L \omega^2}{R_L}\right)^2 + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)^2} + \frac{K^2\omega^2 R_c}{\left(\frac{L_c R_L}{L_L} R_c + R_L\right)^2 + \left(\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right)\omega\right)^2}\right) Z(\omega) Z^*(\omega) \tag{74}$$

As a result of subtracting equation (74) from equation (71):

$$P_{R_L} = \frac{1}{2}\left(\frac{-K^2\omega^4 \frac{L_c L_L}{R_L}}{\left(R_c - \frac{L_c L_L \omega^2}{R_L}\right)^2 + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)^2} + \frac{K^2\omega^2 R_L\left(1 + \frac{L_c}{L_L}\right)}{\left(\frac{L_c R_L}{L_L} R_c + R_L\right)^2 + \left(\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right)\omega\right)^2}\right) Z(\omega) Z^*(\omega) \tag{75}$$

$$\mathrm{imag}(P(\omega)) = -\frac{1}{2}\left(\frac{K^3\omega^3\left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)}{\left(R_c - \frac{L_c L_L \omega^2}{R_L}\right)^2 + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)^2} + \frac{K^3\omega^3\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right)}{\left(\frac{L_c R_L}{L_L} R_c + R_L\right)^2 + \left(\left(L_c - \frac{R_c R_L}{L_L \omega^2}\right)\omega\right)^2}\right) Z(\omega) Z^*(\omega) \tag{76}$$

$$c_e' = \frac{\frac{K^2}{R_L}\left(\frac{L_c\omega}{R_L}\right) + \frac{K^2}{L_L\omega}\left(1 + \frac{L_c}{L_L}\right)}{\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)^2 + \left(\frac{R_c}{L_L\omega} - \frac{L_c\omega}{R_L}\right)^2} \tag{77}$$

Efficiency

In this section the formulas for efficiency at resistive and capacitive and inductive load are derived. It should be noted that in the case of capacitive as well as inductive loads the efficiency is usually function of two variables the resistance and capacitance or inductance.

$$P_{in} = \frac{d}{dt}(E + U + W_d + W_e) = 0 + \frac{d}{dt}(W_d + W_e) \tag{78}$$

where:

(1) E: kinetic energy of moving magnet (mass)
(2) U: potential energy of spring
(3) W_d: dissipated energy in mechanical damper
(4) W_e: dissipated energy by electrical damping mechanism.

Resistive load

Here the typical values of efficiency is investigated note that these values are not true values.As noted previously, the time rate of the sum of kinetic and potential energies is zero as the vibration of the EVEH is forced and the sum of potential energy of the spring and kinetic energy of mass is constant, resulting in a zero time derivative of the energy. For resistive loads, based on the Figure 10 the output power consumed in the load resistor is:

$$P_{R_L} = \frac{1}{2}\frac{K\omega^2 R_L}{(R_L + R_c)^2 + (\omega L_c)^2} Z(\omega)Z^*(\omega) \tag{79}$$

from the above equation:

$$P_{\text{out}} = \frac{1}{2}\frac{K\omega^2 R_L}{(R_L + R_c)^2 + (\omega L_c)^2} Z_0^2 \tag{80}$$

The Z_0 represents the relative displacement of a moving magnet. and K is the electromagnetic coupling factor. The instantaneous input power is calculated as follows:

$$P_{\text{in}} = \int c\dot{z}d\dot{z} = \frac{1}{2}c|\dot{z}|^2 \tag{81}$$

$$c = C_m + c_e \tag{82}$$

$$c_e = \frac{K^2(R_L + R_c)}{(R_L + R_c)^2 + (\omega L_c)^2} \tag{83}$$

C_m and c_e are the mechanical damping and electrical damping coefficients respectively. Putting equations (83) and (82) into equation (81):

$$P_{\text{in}} = \frac{1}{2}\left(C_m + \frac{K^2(R_L + R_c)}{(R_L + R_c)^2 + (\omega L_c)^2}\right)\omega^2 Z_0^2 \tag{84}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{K^2 R_L}{C_m\left((R_L + R_c)^2 + (\omega L_c)^2\right) + K^2(R_L + R_c)} \tag{85}$$

It is shown in equation (85) that efficiency varies with load resistor i.e. R_L and that the maximum efficiency occurs when the derivative of efficiency with respect to load resistance R_L is zero:

$$\frac{\partial\eta}{\partial R_L} = 0 \to R_L = \sqrt{R_c^2 + \frac{K^2 R_c}{C_m} + (\omega L_c)^2} \tag{86}$$

As a result, this resistor will maximize the efficiency of the resistive load EVEH, note that this resistance is not equal to the resistance that maximizes the power extracted by EVEH.

Capacitive load

The output power consumed in the resistor of Figure 11 is:

$$P_{R_L} = \frac{1}{2}\frac{K^2\omega^2 R_L Z(\omega)Z^*(\omega)}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \tag{87}$$

according to above equation:

$$P_{\text{out}} = \frac{1}{2}\frac{K^2\omega^2 R_L Z_0^2}{(R_c + (1 - L_c C_L\omega^2)R_L)^2 + (L_c\omega + R_L C_L R_c\omega)^2} \tag{88}$$

According to equation (4), the input power is:

$$c_e = \frac{K^2 C_L\omega^2\left(\frac{L_c}{R_L} + R_c C_L\right) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L} - L_c C_L\omega^2\right)}{\left(1 + \frac{R_c}{R_L} - L_c C_L\omega^2\right)^2 + \left(R_c C_L\omega + \frac{L_c\omega}{R_L}\right)^2} \tag{89}$$

ω is the excitation frequency of the environment.

$$P_{\text{in}} = \frac{1}{2}(C_m + c_e)\omega^2 Z_0^2 \tag{90}$$

$$\eta = \frac{\frac{K^2R_L}{\left(R_c + \left(1 - L_cC_L\omega^2\right)R_L\right)^2 + (L_c\omega + R_LC_LR_c\omega)^2}}{C_m + \frac{K^2C_L\omega^2\left(\frac{L_c}{R_L} + R_cC_L\right) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L} - L_cC_L\omega^2\right)}{\left(1 + \frac{R_c}{R_L} - L_cC_L\omega^2\right)^2 + \left(R_cC_L\omega + \frac{L_c\omega}{R_L}\right)^2}} \tag{91}$$

MATLAB software can maximize this function which is a two-variable function of R_L and C_L.

Inductive load

The out power that consumes in resistor of Figure 12 is (Dezhara 2022):

$$P_{R_L} = \frac{1}{2}\left(\frac{-K^2\omega^4\frac{L_cL_L}{R_L}}{\left(R_c - \frac{L_cL_L\omega^2}{R_L}\right)^2 + \left(L_c + L_L\left(1 + \frac{R_c}{R_L}\right)\right)^2} + \frac{K^2\omega^2R_L\left(1 + \frac{L_c}{L_L}\right)}{\left(\frac{L_cR_L}{L_L}R_c + R_L\right)^2 + \left(\left(L_c - \frac{R_cR_L}{L_L\omega^2}\right)\omega\right)^2}\right)Z(\omega)Z^*(\omega) \tag{92}$$

$$c_e = \frac{\frac{K^2}{L_L}\left(\frac{R_c}{L_L\omega^2} - \frac{L_c}{R_L}\right) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)}{\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)^2 + \left(\frac{R_c}{L_L\omega} - \frac{L_c\omega}{R_L}\right)^2} \tag{93}$$

$$P_{\text{in}} = \frac{1}{2}(C_m + c_e)\omega^2Z_0^2 \tag{94}$$

After simplifying of P_{R_L} or P_{out}:

$$\eta = \frac{\frac{\frac{K^2}{R_L}}{\left(\frac{R_c}{L_L\omega} - \frac{L_c\omega}{R_L}\right)^2 + \left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)^2}}{C_m + \frac{\frac{K^2}{L_L\omega}\left(\frac{R_c}{L_L\omega} - \frac{L_c\omega}{R_L}\right) + \frac{K^2}{R_L}\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)}{\left(1 + \frac{R_c}{R_L} + \frac{L_c}{L_L}\right)^2 + \left(\frac{R_c}{L_L\omega} - \frac{L_c\omega}{R_L}\right)^2}} \tag{95}$$

MATLAB can also maximize the above equation, which is a two variable function of R_L and L_L.

Numerical example

In this section for resistive load case we just plot the efficiency function based on the typical data with respect to load because in these case the resonance that we have defined in Section 2.1 dose not lead to a real answer (the case of $\omega > \omega_{\text{Mech}}$ and $X_{th} < 0$ is rare in micro and nano dimension because the mechanical resonance frequency posses very high values). In the case of capacitive load and when Thévénin reactance is positive which is the most popular case in engineering practice at micro and nano dimension, three plot of efficiency versus angular excitation frequency at constant load and efficiency versus load at constant angular excitation frequency as well as maximum efficiency versus load are analyzed. According to the Table 2 typical data for resistive load are:

Table 2: Typical numerical data for resistive load.

1	L_c	Coil inductance	0.2	mH
2	R_c	Coil resistance	3	Ω
3	K	Electromagnetic coupling coefficient	5	$\frac{\text{Wb}}{\text{m}}$
4	ω	Excitation frequency	100	$\frac{\text{rad}}{\text{s}}$
5	C_m	Mechanical damping coefficient	0.05	$\frac{\text{N·s}}{\text{m}}$

Resistive load

Using the Figures 13 and 14 we can get these results:

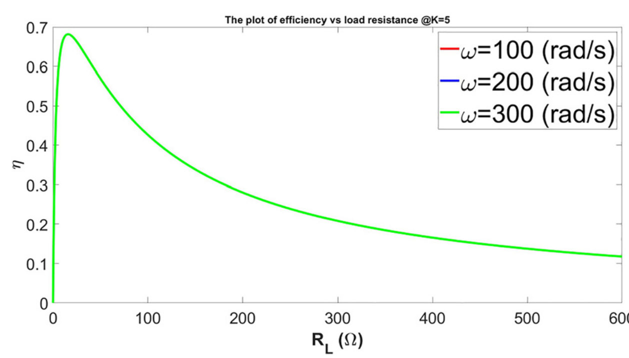

Figure 13: Efficiency plot at constant K, $L_c = 0.2\,\text{mH}$.

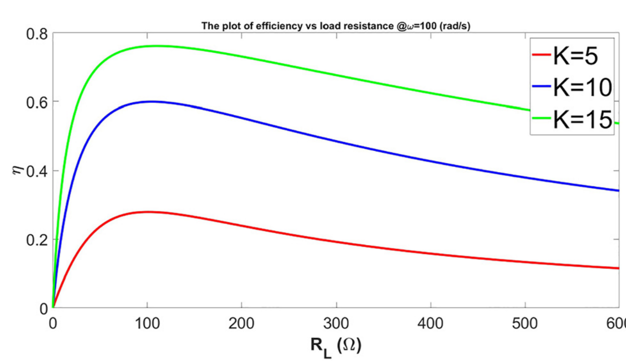

Figure 14: Efficiency plot at constant ω, $L_c = 0.2\,\text{mH}$.

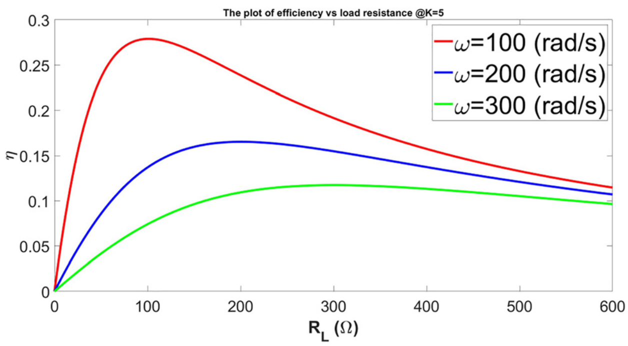

Figure 15: Efficiency plot at constant K, $L_c = 1\,\text{H}$.

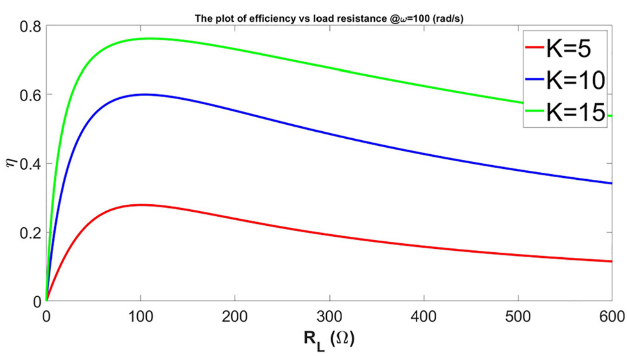

Figure 16: Efficiency plot at constant ω, $L_c = 1\,\text{H}$.

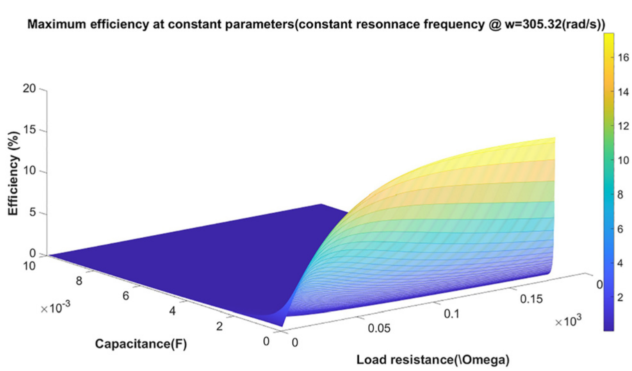

Figure 17: Maximum efficiency versus load.

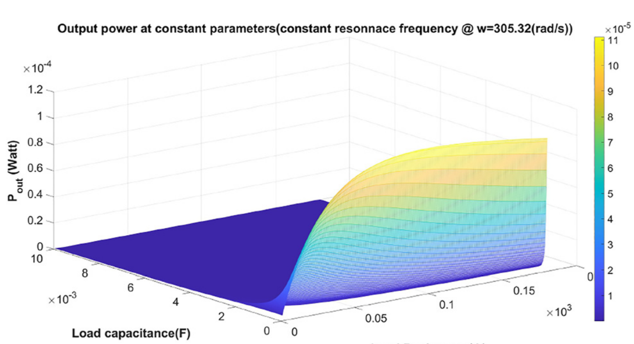

Figure 18: Maximum power versus load.

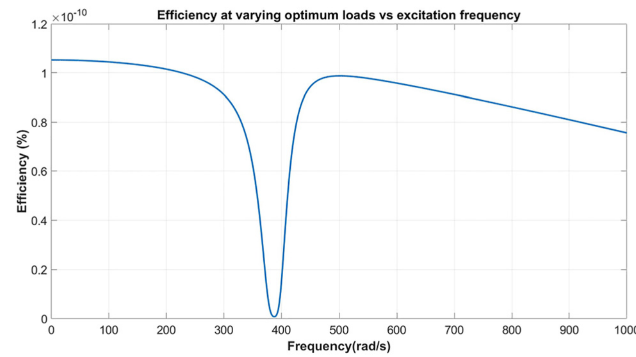

Figure 19: Efficiency versus excitation frequency for capacitive load.

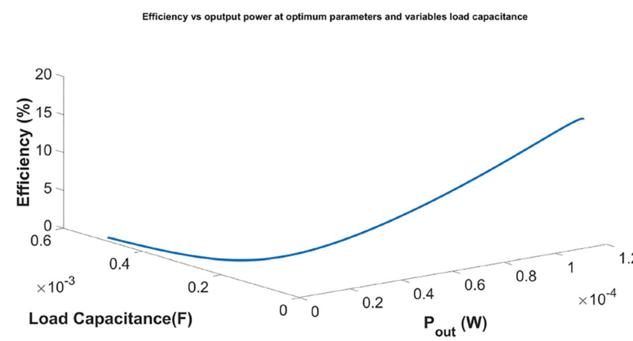

Figure 20: Efficiency at constant resistance but varying load capacitance versus output power.

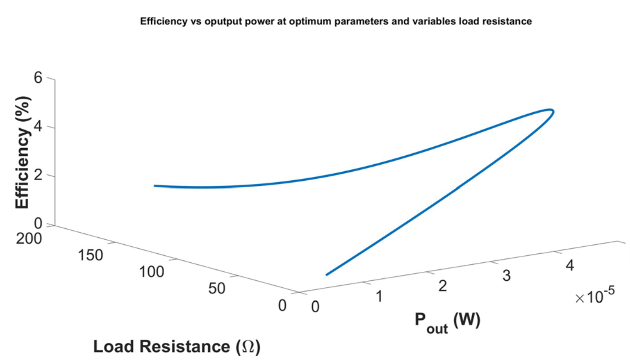

Figure 21: Efficiency at constant capacitance but varying load resistance versus output power.

Note that according to Figure 13 for $L_c = 0.2\text{mH}$ and constant K, all diagrams will coincide since the coil inductance is negligible at these frequencies. At higher values of L_c as in Figures 15 and 16 the term ωL_c becomes significant even at not very high frequencies and cannot be discarded.

Capacitive load

Generally in the case of capacitive load (at high mechanical resonance frequency which is the case for micro or nano

Table 3: Optimum numerical data for capacitive load.

ω	Resonance frequency	305.32	$\frac{\text{rad}}{\text{s}}$
L_c	Coil inductance	30.85	mH
R_c	Coil resistance	99.43	Ω
R_L	Load resistance	103	Ω
C_L	Load capacitance	3.1	μF
K	Electromagnetic coupling coefficient	5.6	$\frac{\text{Wb}}{\text{m}}$
ω_n	Mechanical resonance frequency	387.9	$\frac{\text{rad}}{\text{s}}$
C_m	Mechanical damping coefficient	0.31	$\frac{\text{N·s}}{\text{m}}$
A_b	Base acceleration	1.02 g	$\frac{\text{m}}{\text{s}^2}$

dimension) the definition of resonance leads to a numeric answer i.e. optimum load and coil parameters and optimum resonance frequency at assumed constant mechanical parameters.[3]

Note that the data of Table 3 is derived based on the definition of resonance and constant mechanical parameters of $k_s = 3220\ \frac{\text{N}}{\text{m}}$, $m = 21.4$ g, $C_m = 0.31\ \frac{\text{N·s}}{\text{m}}$.

Based on the MATLAB calculations the maximum efficiency occurs at $R_L = 140.8\,\Omega$, $C_L = 3.1\ \mu\text{F}$ in Figure 17 and the value of maximum efficiency is $\eta_{\max} = 17.45\%$.

Based on the MATLAB calculations the maximum power occurs at $R_L = 107.84\,\Omega$, $C_L = 1\,\mu\text{F}$ in Figure 18 and the value of maximum power is $P_{\max} = 110\,\mu\text{W}$. It should be noted that based on the example above the load that maximize power is different with the load that maximize efficiency i.e. maximum efficiency does not occurs at maximum power.

Note that based on the Figure 19 the value of efficiency at resonance $\left(\omega = 305.32\left(\frac{\text{rad}}{\text{s}}\right)\right)$ is less that $1e-10\%$. Also note that at mechanical resonance frequency, the value of efficiency is approximately zero. It should be noted that the efficiency at resonance which is $1e-10$ is different with maximum efficiency which is 17.45%.

Note that based on the projection of plots of Figure 20 and Figure 21 efficiency has approximately linear relationship with the output power up to some limits which is less than maximum extracted power i.e. 110 μW. Also note that power at maximum efficiency is between 40 and 50 μW and differ from maximum power.

3 Note that if you consider the mechanical parameters as a optimum variable and not some constants, there should be a relation in mechanic that relates mechanical damping with mass and stiffness but this is beyond the scope of this paper. So here the three mechanical parameters of stiffness, mass, and mechanical damping are assumed constant.

Conclusions

Based on the resonance definition and numerical results we conclude that the resonance frequency for resistive and inductive loads does not lead to a real answers. Nevertheless we plot the efficiency versus resistive load for typical numeric parameters. We also conclude that the loads that maximize the efficiency differ from the loads that maximize the power, in other words efficiency at maximum power differs from the maximum efficiency. And at mechanical resonance frequency, the efficiency will decrease considerably. Our example shows that the typical maximum efficiency of EVEH for optimum parameters is around 17.45% and this is a low value. Note that with increasing output power the efficiency is also increase but not up to resonance i.e. at some power that considerably less that maximum extracted power.

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