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Designing Thermally Actuated Bimorph as Energy Harvester

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Abstract: Microelectromechanical systems (MEMS) are micrometre-size systems that are capable of transforming electrical signals to mechanical signals. In this paper, the use of MEMS devices as a method for harvesting waste heat is explored. A background for the technology and methods is discussed and a method proposed for a working device based upon a thermally heated bimorph actuated by waste radiation alone is investigated. A chopper interrupts the radiant energy inducing an oscillation in the bimorph temperature and thus creates a mechanical motion which can be taken advantage of using piezoelectric materials. Difficulties in harvesting radiant energy with this method are highlighted and suggestions for non-passive energy generation are made based on the outlined principles.

Keywords: MEMS, energy, thermal radiation, bimorph, piezoelectricity, cantilever

Background

Microelectromechanical system (MEMS) devices are used in a wide variety of applications performing basic signal transduction operations as sensors and actuators. They are usually constructed using lithographic processes derived from very-large scale integration (VLSI) technology. Because these devices must react to mechanical signals, many of them use construction topologies that require physical motion. Energy harvesting, energy conservation, and efficiency are the universal applications of MEMS devices that make them ideal for any expeditionary platform, such as ships, aircraft, and automobiles. Energy harvesting provides an unending power source (few milliwatts), which is adequate to power these MEMS devices having remote area applications. While the words power and energy have vastly different definitions, the terms “power harvesting” and “energy harvesting” are used interchangeably throughout much of the literature to describe the same process of extracting

electrical energy from ambient sources. Even though most of the energy coupling materials currently available have been around for decades; their use for the specific purpose of power harvesting has not been thoroughly examined until recently, when the power requirements of many electronic devices have reduced drastically. Piezoelectric type energy harvesters are the most promising and efficient vibration energy harvesters that are widely used.

In the past few years, much attention has been paid to the power sources of small wireless electronic devices, due to the rapid development of remote microsensors and microelectronic devices in various applications. Earlier studies on the structures of various piezoelectric devices include a piezoelectric bimorph harvester which can scavenge power from low-level ambient sources by an electrically induced damping term (Roundy et al. 2005). In their analysis on micro-piezoelectric generators, Lu, Lee, and Lim (2004) have studied the conversion of mechanical energy into electrical energy using a circuit represented by a resistor linked with a rectifier. Yang et al. (2005) and Jiang et al. (2005) studied the mechanical-to-electrical energy conversion of a piezoelectric element driven mechanically. Furthermore, researchers have already been able to design a harvesting structure with a very low natural frequency, suitable for scavenging energy from a low-frequency ambient vibration (Hu, Hu, and Yang 2006). It is a well-known fact that vibrations come in varying frequency. In that case, the piezoelectric structures of power harvesters should be designed with adjustable natural frequencies so that strong interaction can happen between the time scales corresponding to the bimorph natural frequency and the external driving frequency.

Very recent studies on this field include a new design of guided four beam cantilever type piezoelectric energy harvester by Saxena, Sharma, and Pant (2017). In their work, maximum stress was calculated for a certain input acceleration on piezoelectric layer and on guided beams. It was found that this stress is well below the fracture stress for silicon. In another work by Porter and Berfield (2014), a bi-stable switching energy harvester made from a buckled steel structure mounted with uniaxially poled piezoelectric polyvinylidene fluoride and 3D printed

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polylactic acid components was constructed and tested. The energy harvester was tested with the centre beam compressed to different degrees of buckling as well as in its unloaded state. Their conclusion is that larger applied compression levels (0.25 mm) are well beyond the minimum required to induce a bi-stable buckled profile and are detrimental to the device performance at different accelerations. Guido et al. (2016) reported on the development of a flexible energy harvester based on piezoelectric Aluminium Nitride (AlN) thin film in order to generate electrical energy from human motion at very low frequencies. They found that the “piezoelectric skin” undergoing folding-unfolding exhibits fast snapping transitions due to buckling effect. As a result, this increases the mechanical stress of the piezoelectric structure and improves the generated output voltage.

A micro-piezoelectric cantilever array device consisting of five piezoelectric cantilever beams and a single proof mass for vibration energy harvesting was designed, microfabricated, and characterized by Zhao et al. (2015). They found that the power loss in the rectifier when the beams are arranged in parallel is 5 times more than when they are in series. This demonstrates that series connection for array harvester can effectively reduce power loss in the rectifier. Many review papers have written exceptional reviews on performance of energy harvesting devices (Selvan and Mohamed Ali 2016), state-of-the-art and challenges of piezoelectric energy harvesting (Toprak and Tigli 2014), piezoelectric thin films as transducers and energy harvesting devices (Khan et al. 2016) and mechanics of flexible and stretchable piezoelectric for energy harvesting (Chen et al. 2015).

Introduction

Harvesting heat energy on micro-scale is of relevance for harvesting and generating power. Small changes in temperature can be transformed into micron level deflection, which could lead to more efficient automobiles and power generation cycles in the future by reducing the amount of waste heat. It is a well-known limit in the study of thermodynamics that the limitation of energy extraction from a hot reservoir at temperature T_h to a cold reservoir at temperature T_c is given by the Carnot efficiency η as shown in eq. (1). In practice, the Carnot Cycle is impossible but can produce small increase in efficiency of heat engines which would lead to a dramatic reduction of energy usage while energy harvesting can have profound impact on powering wireless devices.

$$\eta = 1 - \frac{T_c}{T_h} \quad (1)$$

Bimorph structures which are under-reported by the literature could prove to be an interesting response to the Carnot Cycle on the MEMS scale. A bimorph is a cantilever system used for actuation or sensing that consist of a beam with two active layers, called the electrodes, and a passive layer, usually made up of piezoelectric material. Consider a bimorph which derives micron scale deflection from a change of a few degrees Kelvin coupled with this low thermal mass and ability to rapidly heat and cool. Such a cantilever can be made to operate by heating from a hot reservoir and discharging its heat to a cold reservoir upon deformation. Such a design could lead to not only harvesting of energy, but could be used for active energy generation in a MEMS scale heat engine. Radiant energy is one particular form of energy that may provide a useful insight into the development of this class of MEMS devices.

Radiant Energy

Energy is found in different forms, such as light, heat, sound, and motion. There are many forms of energy, but they can all be put into two categories: kinetic and potential. Radiant energy is electromagnetic energy that travels in transverse waves. Radiant energy includes visible light, x-rays, gamma rays, radio waves, etc. Light is one type of radiant energy. Solar energy is an example of radiant energy. The quantity of radiant energy may be calculated by integrating radiant flux (or power) with respect to time and, like all forms of energy, its SI unit is Joule. Radiant energy is generally thought of as radiation emitted by a source into the surrounding environment. It propagates in the form of electromagnetic waves, or traveling subatomic, atomic or molecular particles. Specific forms of radiant energy include electron space discharge, visible light, and other wave types. Radiant energy is exhibited in the spontaneous nuclear disintegration with emission of particulate or electromagnetic radiations.

Harvesting Energy Using Photoelectric Effect

The emission spectra for a blackbody falls under a spectral emission defined by Planck distribution, where the

emitted radiation varies continuously with wavelength. At any wavelength, the magnitude of the emitted radiation increases with temperature (Incorpria 2007). The blackbody radiation emission from the sun is highest in the visible light spectrum of 0.4 to 0.7 μm corresponding to a photon energy of 3.1 and 2.07 eV, where increasing wavelengths result in lower photon energies. Solar collectors made of semiconductors with bandgaps in this light regime in order to take advantage of the photoelectric effect are emerging. However, photons with energy levels below that of the semiconductor bandgap such as those in the infrared regime cannot be absorbed by traditional methods.

Harvesting Energy Using Pyroelectric Materials

Pyroelectric materials have been commonly used for applications of IR sensing by generating a measurable voltage, current, or charge caused by a change in temperature. However, they are not ideal for energy harvesting purposes because they absorb radiation for a narrow band and are difficult to grow (Lee, Guo, and Bhalla 1998). On the contrary, a harvesting method using bimorphs can potentially lead to a higher absorption regime and cheaper technology.

Bimorph

The typical application of a bimorph is for actuation, where a bimorph actuator has a number of advantages:

- Easier control of the structure and effective conversion of mechanical strain into electrical voltage and vice versa
- Simple structure containing only piezoelectric film, electrodes, and very flexible layer (does not introduce any significant stiffness in the diaphragm, nor residual stress)
- Larger sensitivity and signal/noise ratio and higher fundamental resonant frequency with inherently higher figure of merit than a unimorph device
- Conformal deposition allowing various diaphragm shapes

The potential difficulties of working with this type of bimorph devices are low energy conversion efficiency (around 7 times less the energy conversion efficiency of

the multilayer piezoelectric actuator) and higher power consumption. In addition, they have unstable displacement, which increases shift and creep phenomena and low response speed.

While bimorphs for energy harvesting are not common in the literature, the cantilever structure is a common place for harvesting applications such as vibration harvesting. Most often, the bimorph is designed in a cantilever configuration with a tip mass added to increase strain and to lower the natural frequency of the vibrating beam. One particular configuration of the bimorph cantilever uses a thin layer of metal sandwiched between two or more layers of piezoelectric material as shown in Figure 1.

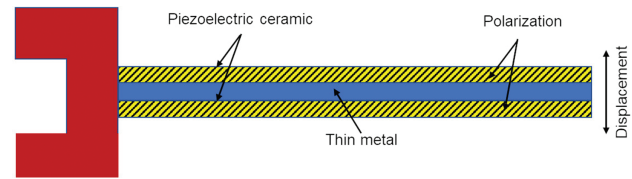


Figure 1: Schematic of a bimorph structure.

The exact size of the mass attached to the tip can also be specified so that the bimorph operates within the range of an ambient driving frequency base excitation. Figure 2 is a graphical representation of the simplest model for such a device. The first natural frequency of a slender cantilever beam with a concentrated end mass as given by Blevins (1979) is

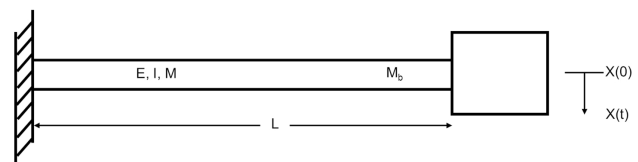


Figure 2: Cantilever beam with added tip mass.

$$\omega_n = \sqrt{\frac{3EI}{L^3(M_b + 0.24M)}} \quad (2)$$

Here, E is the modulus of elasticity, I is the moment of inertia, L is the length of the beam, M is the fixed mass of the beam, and M_b is the concentrated end mass. In reality, when an electromagnetic shaker is used to simulate the excitation and for many of the harvesting applications, the left end of the beam cannot be

modelled as fixed but rather as another very large vibrating mass. Improved models were developed by Erturk and Inman (2007) to better model the left side of the beam. However, to simplify the equations in this section, the left end is assumed to have a fixed boundary condition.

A piezoelectric bimorph is more complicated than this simple cantilever beam model in that it consists of a metal shim layer sandwiched between two piezoelectric layers. Therefore, an equivalent Young's modulus and moment of inertia must be calculated in order to determine the first natural frequency. In case of thermally activated bimorph cantilevers, dissimilar structural materials with different coefficients of thermal expansion (CTE) are sandwiched together. When the top layer has greater CTE than the bottom layer, bimorphs flatten when heated and curl when cooled. In addition, due to intrinsic stresses generated in the structural layers during the fabrication process, bimorphs are initially deflected from the substrate surface. Intrinsic stresses apply during a bimorph actuation can be modelled using Stoney's theorem in eq. (3), which gives the stress at the material interface as,

$$\sigma_f = (\alpha_f - \alpha_s)\Delta T E_f \quad (3)$$

Here, σ_f is the induced stress at the interface, α_f and α_s are the coefficients of thermal expansion (CTE) of the film and the substrate respectively, ΔT is the change in temperature, and E_f is the Young's modulus of the film. Beam analysis with one end fixed and assuming a hyperbolic shape of the beam, the shape is depicted in eq. (4). Further detail on the derivation of this equation can be seen in Appendix A.

$$u(x) = \frac{-3(\alpha_f - \alpha_s)\Delta T E_f t x^2}{E_s h^2} \quad (4)$$

Here, the quantities α_f , α_s , ΔT , and E_f have been discussed above. Aside, t is the thickness of the film, h is the thickness of the substrate, x is the distance along the length of the beam from its fixed end, and E_s is the Young's modulus of the substrate. Using eq. (4), it is possible to develop a displacement for the beam under very small increase in temperature of the order of few degrees Kelvin if the materials and bimorph dimensions are carefully selected.

Working Principal of Radiation Activated Bimorph Cantilever

Radiation activated bimorph functions as a thermally actuated bimorph where, instead of using joule heating, this method uses radiation energy to heat the bimorph and induce a mechanical deformation of the cantilever as seen in Figure 3. In this particular example, a mechanical chopper must be utilized to interrupt the flow of radiation and to allow the cantilever to cool. This is illustrated in Figure 3(d). The top view of the device with the bimorph array substrate and mechanical chopper can be seen in Figure 4.

Analysis and Behaviour of the Bimorph Cantilever

Analysing the bimorph using lumped capacitance method and assuming a low Biot number the energy balance for the system is,

$$mC_p \frac{dT}{dt} + hA(T - T_o) = \eta\varphi \quad (5)$$

Here, m is the mass of the cantilever beam, C_p is the heat capacity at constant pressure of the beam, h is the heat transfer coefficient, A is the cross-sectional area of the beam, T is the surface temperature of the beam at time t , T_o is the temperature of the surrounding, and $\frac{dT}{dt}$ is the rate of change of beam's temperature with time. The first term on the left-hand side in eq. (5) represents stored energy, whereas the second term represents energy loss due to convection. The term $\eta\varphi$ represents the amount of absorbed incident radiation energy by the bimorph. This analysis is based upon the assumption that the heating can be modelled using lumped capacitance method and that the entire cantilever is uniformly heated by the radiation. Biot number gives the ratio of the heat transfer resistances in the bulk of the material to its surface. At very high repetition rates, only the top layer of the bimorph will be heated resulting in a faster response than what is modelled here. It is obvious that the cantilever will fall under a steady state condition if allowed to be exposed to a continuous stream of radiation. For harvesting energy, a chopper must be used. If we assume the use of some form of a chopper, we can convert eq. (5) into a transfer function as discussed in Appendix B. For our system we assume the

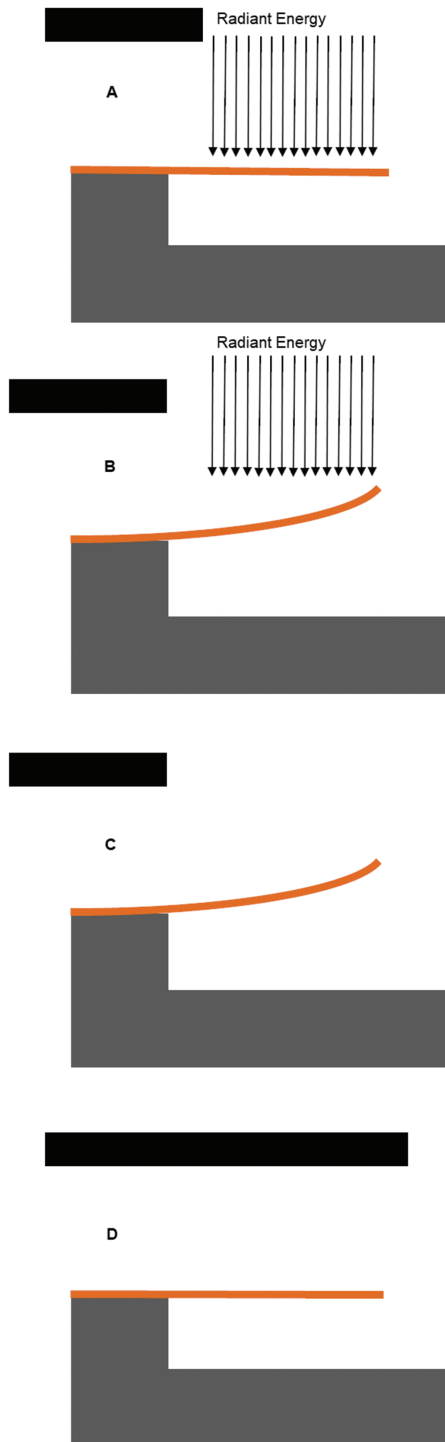


Figure 3: Radiation harvesting bimorph actuation while heating (A), deformation (B), blocked radiation (C), and cooling (D).

properties as illustrated in Table 1 with a convection coefficient of $25 \text{ W/m}^2\text{-K}$, which is the upper end of the estimated range for free convection cooling lower values resulting in a higher response per energy input.

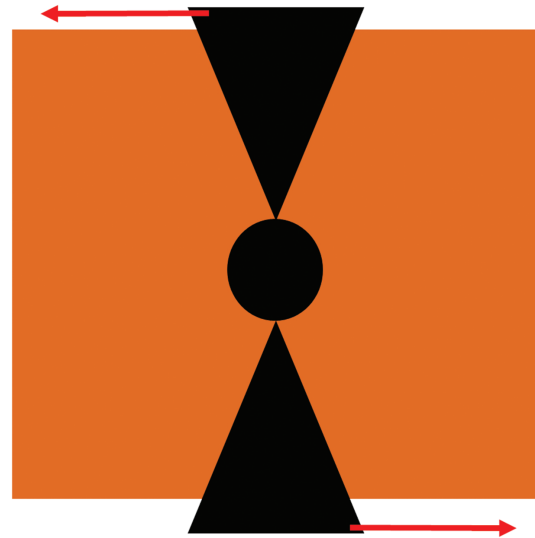


Figure 4: Bimorph array (orange) and a mechanical radiation chopper (black).

To understand the frequency response of a linear time-invariant system, Bode diagram is generally used. The Bode diagram in Figure 5 indicates that the desired range of input frequency can be as high as 775 rad/s before the input energy will have no effect on the bimorph. At very low frequency the amplitude is approximately 6 dB. Additional analysis of the minimum temperature T_0 indicates that this temperature is not affected until a much higher frequency range is achieved.

Solving the Blevins' equation, it is possible to determine the appropriate mass to put at the end of the cantilever to achieve a resonant frequency in lower range of the heating Bode diagram. In doing so, it is assumed that the cantilever deformation is in phase with the temperature change. Then, by selecting a chopped radiation frequency of 100 rad/s where the temperature to radiation response is approximately 6 dB, the required concentrated mass can be determined as shown in Appendix C. This approximation indicates that the mass at the end of the cantilever must be equal to 185 nanograms for a chopped radiation frequency of 100 rad/s . A mesoscale chopper can be used on the principal of a toy radiometer, which will rotate due to currents induced by the heated surfaces of the blade.

Fabrication of Bimorph Actuators

Bimorphs used for actuation have been discussed in literature and a polyimide-gold bimorph is outlined in the

Table 1: Bimorph design specifications.

Bimorph Cantilever Properties	
Length	100 μm
Width	50 μm
Substrate Thickness	10 μm
Film Thickness	1 μm
Substrate Material	Silicon
Film Material	AlN

schematic in Figure 6. The detailed steps of fabrication have been discussed in a paper by Al Aioubi et al. (2004), where they used a *N*-doped silicon wafer coated with a 0.5 μm thick layer of silicon dioxide in a plasma enhanced chemical vapour deposition (PECVD) system. This was followed by the deposition of a thin layer of gold using sputter deposition onto a Cr adhesion layer (Figure 6(a)). Polyimide was next spin coated and cured at 120 °C. For the microheater fabrication, a 0.2 μm thin layer of chromium was deposited on top of the polyimide and patterned (Figure 6(b)). An additional polyimide layer was spin-coated on top of the patterned chromium and then the cantilever structures were defined using a

combination of photolithography for the polyimide and wet chemical etching for gold (Figure 6(c) and 6(d)). Release of the bimorph was achieved by under etch of the Cr adhesion layer beneath the Au coating.

Contrary to the polyimide type bimorph, materials with higher thermal conductivity must be used for a radiation actuated bimorph. Here, the author suggests the use of a silicon substrate using an AlN thin film. The design proposed here is for a radiation actuated bimorph. The fabrication steps include silicon on insulator (SOI) technology, which is illustrated in Figure 7. This is similar to the method proposed by Yu et al. (2006) to fabricate short and thin AFM cantilevers.

Alternative Approach

The method proposed here demonstrates a technique of harvesting radiation energy from an external source using a passively controlled radiation chopper. However, this micro-scale device could be a source of large energy loss, inefficiency, and inability to control the cycle and would require a complex control system to harvest the energy

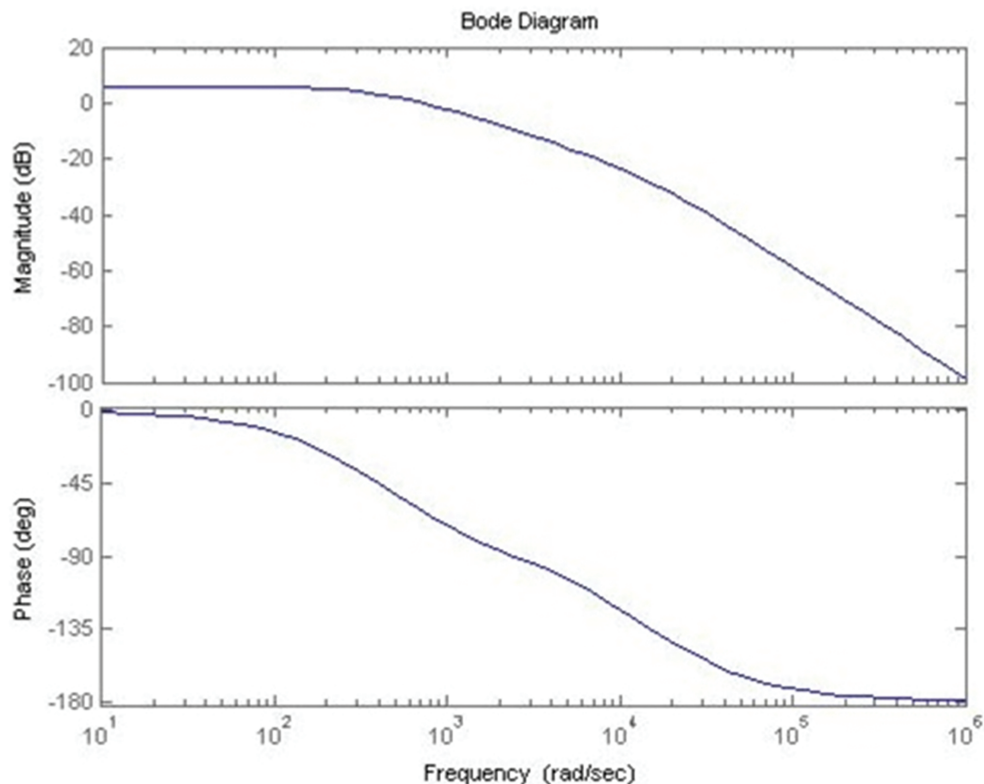


Figure 5: Bode diagram of the response temperature of the cantilever with respect to the energy input for convection cooling coefficient of 25 $\text{W}/\text{m}^2\text{-K}$.

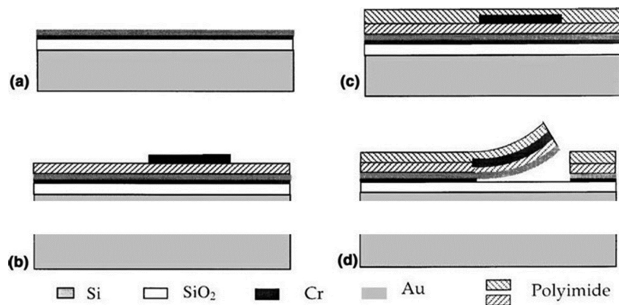


Figure 6: Polyimide-gold cantilever fabrication process sequence (Al Aioubi et al. 2004).

Step 1: Starting with an SOI wafer, pattern the back oxide and bulk etch the backside.



Step 2: Pattern the front side oxide and pattern the silicon.



Step 3: Remove the front side oxide and reduce the thickness of the silicon to the desired thickness.



Step 4: Atomic layer deposition (ALD) and pattern to create the AlN layer and repeat to form the PZT Layer. Finally deposit metal contacts for energy harvesting.



Step 5: Pattern the backside oxide, and the device is now complete.



Figure 7: Steps involving bimorph fabrication using silicon on insulator (SOI) technology.

from the cantilever due to its non-synchronous behaviour. An alternative approach to this, beyond proving the initial concept, might include an active generation of energy such as a MEMS engine, which would operate an array of bimorphs on a large-scale combustion manifold. The combustion manifold would be a source of both radiant and convective heat, whereby the bimorphs would deform and touch a porous silicon substrate filled with a flammable fuel such as methanol. The methanol will then be vaporized at 64.7°C and enter the combustion manifold and ignited by a control loop that can monitor temperature. The entire process is depicted in Figure 8.

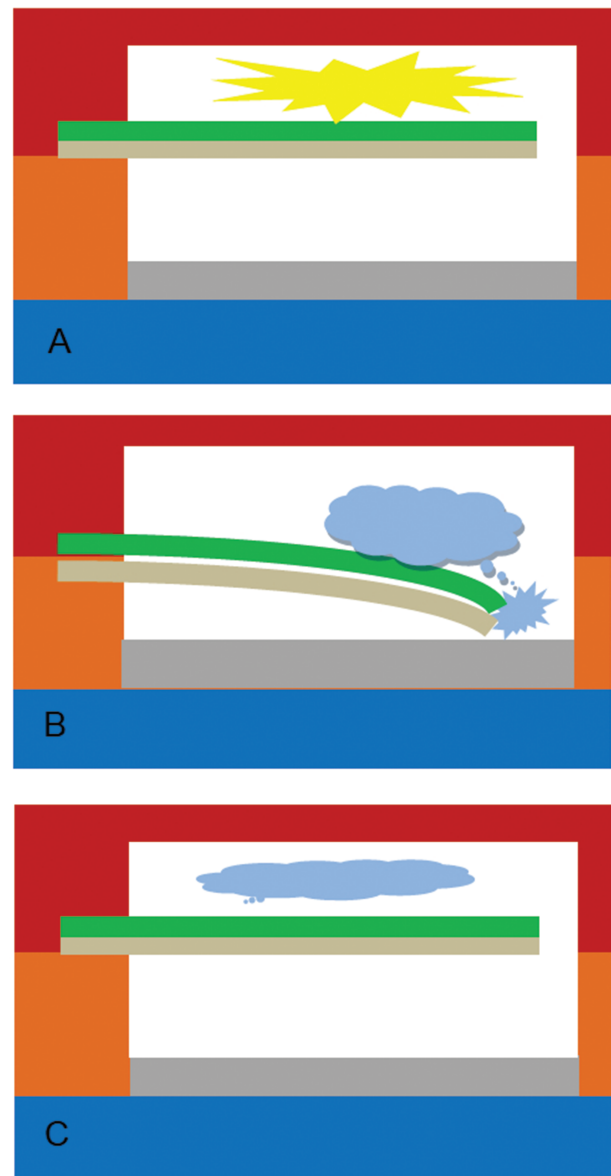


Figure 8: Methanol powdered heat engine using the concept of a bimorph actuator showing the combustion (A), expansion and interaction with methanol (B), and cooled cantilever with methanol cloud ready for combustion (C).

Conclusion

Thermally actuated bimorphs have the potential to be realized as an efficient method for both passive harvesting and active energy generation. Astute applications of the bimorph and the exploitation of the fact that they can generate high deformations with small temperature inputs can lead to devices that can rapidly build up and discharge heat to their environments. Devices based upon radiation harvesting require large scale chopper to create an intermittent input radiation and do not offer a large degree of control over the behaviour of the system. Alternative systems that are actuated by some form of controllable input such as a combustion cycle must be considered when attempting to develop a MEMS device based upon a heat cycle.

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Appendix A: Deformation of the Bimorph Cantilever Using Stoney's Theorem

Bending in a bimorph is caused by the stresses induced by the thermal expansion mis-match of the materials.

$$\varepsilon = (\alpha_f - \alpha_s)\Delta T$$

Using Stoney's formula for thin film stress we resort to

$$\sigma_f = (\alpha_f - \alpha_s)\Delta TE_f$$

Referring to Figure 9, for the case of a cantilever, the moment of the beam is

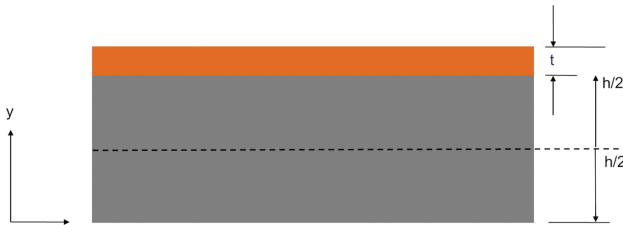


Figure 9: Moment analysis of a beam which is composed of a thin film of thickness t on a substrate of thickness h .

$$M_t = \int_{h/2}^{\frac{h}{2}+t} \sigma y dy$$

$$M_t = \int_{h/2}^{\frac{h}{2}+t} (\alpha_f - \alpha_s)\Delta TE_f y dy$$

$$M_t = \left[(\alpha_f - \alpha_s)\Delta TE_f \frac{1}{2} y^2 \right]_{\frac{h}{2}}^{\frac{h}{2}+t}$$

$$M_t = (\alpha_f - \alpha_s)\Delta TE_f \left[\left(\frac{h}{2} + t \right)^2 - \left(\frac{h}{2} \right)^2 \right]$$

$$M_t = (\alpha_f - \alpha_s)\Delta TE_f \frac{h}{2} t$$

For a beam,

$$EI \frac{d^4 u}{dx^4} + \frac{d^2 M_t}{dx^2} = 0$$

Boundary conditions are given by,

$$\begin{aligned} \text{At } x = 0, \\ u = 0 \text{ and } \frac{du}{dx} = 0 \\ \text{At } x = L, \end{aligned}$$

$$EI \frac{d^3 u}{dx^3} + \frac{dM_t}{dx} = 0$$

$$EI \frac{d^2 u}{dx^2} + M_t = 0$$

We assume a parabolic type shape to the beam bending.

$$u(x) = a_1 x^2$$

$$\frac{d^2 u}{dx^2} = 2a_1$$

$$2a_1 E_s I = -M_t$$

$$a_1 = \frac{-12}{2E_s h^3} (\alpha_f - \alpha_s) E_f \Delta T t \frac{h}{2}$$

$$a_1 = \frac{-3(\alpha_f - \alpha_s)E_f \Delta T t}{E_s h^2}$$

$$u(x) = \frac{-3(\alpha_f - \alpha_s)E_f \Delta T t x^2}{E_s h^2}$$

Appendix B: Bimorph Beam Heating

The assumptions made here are

1. Lumped capacitance method is used.
2. The substrate is used as the bulk material ignoring the film thermal characteristics.
3. The coefficient of convection is on the range of 2–25 W/m²-K.

Because the beam is very thin, we make the assumption that the Biot number is very high. Thus, we can use lumped capacitance to solve the problem.

$$mC_p \frac{dT}{dt} + hA(T - T_o) = \eta\varphi$$

$$\frac{dT}{dt} + \frac{hA}{mC_p}(T - T_o) = \frac{\eta\varphi}{mC_p}$$

Let T_o be a constant and from Laplace's equation for heat flow,

$$\frac{dT}{dt} = \alpha \frac{\partial^2 T}{\partial t^2}$$

Using method of separation of variables,

$$\frac{dT}{dt} = -\lambda\alpha T$$

Therefore,

$$-\lambda\alpha T + \frac{hA}{mC_p}(T - T_o) = \frac{\eta\Phi}{mC_p}$$

Let, $-\lambda\alpha = s$. (Here, α is the thermal diffusivity and λ is a constant.)

$$Ts + \frac{hA}{mC_p}(T - T_o) = \frac{\eta\Phi}{mC_p}$$

$$T \left[s + \frac{hA}{mC_p} \left(1 - \frac{T_o}{T} \right) \right] = \frac{\eta\Phi}{mC_p}$$

$$\frac{T}{\eta\Phi} = \frac{\frac{1}{mC_p}}{s + \frac{hA}{mC_p} \left(1 - \frac{T_o}{T} \right)}$$

For the cooling case, we use the same transfer function but let the radiant energy be equal to zero.

$$Ts + \frac{hA}{mC_p}(T - T_o) = 0$$

$$\frac{T}{T_o} = \frac{\frac{hA}{mC_p}}{s + \frac{hA}{mC_p}}$$

Appendix C: Calculation of the concentrated mass using the assumed beam parameters of Table 1

$$\omega_n = \sqrt{\frac{3EI}{L^3(M_b + 0.24M)}}$$

$$(\omega_n)^2(M_b + 0.24M) = \frac{3EI}{L^3}$$

$$(\omega_n)^2(M_b + 0.24M) = \frac{3EI}{L^3}$$

$$M_b = \frac{3EI}{L^3(\omega_n)^2} - 0.24M$$

$$M_b = \frac{3E\left(\frac{1}{12}bh^3\right)}{L^3(\omega_n)^2} - 0.24\rho b h L$$

$$M_b = \frac{3\left(160 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)\left(\frac{1}{12}(50E - 6m)(10E - 6m)^3\right)}{(100E - 6m)^3(100 \text{ rad/s})^2} \Big| 1000 \text{g/kg} \Big|$$

$$- 0.24(1.24 \text{g/cm}^3)(50E - 4 \text{cm})(10E - 4 \text{cm})(100E - 4 \text{cm})$$

$$M_b = 1.85E - 7 \text{g} = 185 \text{ng}$$

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