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A Study on Bandwidth and Performance Limitations of Array Vibration Harvester Configurations

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Abstract: Many researchers introduced an array of generators for broadband energy harvesting. The array has been studied in comparison to a single element from this array, but never compared to a single reference harvester with same volume as the whole array. This paper presents a theoretical study of evaluating the performance of the array harvester in comparison to the reference harvester. Power from the reference harvester as well as from the array is analytically calculated. The array is compared to the reference harvester when loaded by their optimal resistances which provide maximum power capability. The comparison is divided into two sections: firstly when the elements of the array are tuned to resonate at matching frequencies and secondly when they are tuned to non-matching resonance frequencies. The comparisons lead to two significant limits of the working bandwidth of the array: the lower and the upper limit. Between the two limits, the power produced from the array is less than the reference harvester, but with a small additional bandwidth. Below the lower limit, the array has no advantage over the reference harvester. Above the upper limit, output power of the array is inconsistent. Hence, design guidelines for the array are provided.

Keywords: array of generators, broadband energy harvesting, multi-frequencies, vibration energy harvesting

1 Introduction

Vibration energy harvesting has received great interest during the last decades. Vibration exists in almost all industrial systems, and thus, provides an available source for energy harvesting. Since vibration in the real environment varies along a wide spectrum of frequencies, increasing bandwidth of the vibration harvester becomes a major objective of research. Many strategies were proposed for broadband energy harvesting as reviewed by Zhu, Tudor, and Beeby (2009) and Twiefel and Westermann (2013). Each strategy has its own advantages, disadvantages and limitations of applicability. Therefore, every generator is tailored to fit a specific application. One of the introduced strategies for a wider bandwidth is an array of generators.

Shahruz (2006a) and Shahruz (2006b) introduced a design procedure of an array of cantilevered beams. A mathematical model was derived to obtain the transfer function between transversal displacement at the tip and the input acceleration. The range of tuning frequencies for the array was defined based on the assumption that the transfer function for all the beams in the array had the same value. However, this design procedure did not consider the piezoelectric effect as well as the influence of the load resistance. Al-Ashtari et al. (2013) investigated an array of three cantilevered piezoelectric bimorphs with tuned frequencies. Output power of the array was obtained in parallel and in series connections. The array provided higher power and wider bandwidth compared to a single element from the array. Liu et al. (2011) proposed a new design of a low resonant frequency piezoelectric MEMS harvester. It consisted of a proof mass and a supporting beam integrated with parallel-arrayed piezoelectric elements on its side. At different input accelerations, power of the array was obtained for a different number of piezoelectric elements. The array showed wider bandwidth compared to a single piezoelectric element.

All the studies of an array of elements have compared output power and bandwidth of the whole array to a single element from this array. No previous study has investigated the array in comparison to a single harvester with same volume as the array and that operates under the same boundary conditions (called the reference harvester in this paper), see Figure 1. This study carries out the comparison in order to theoretically find the range of frequencies within which the array provides wider bandwidth than the reference harvester.

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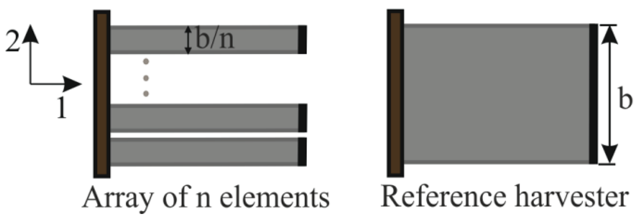


Figure 1: A sketch of an array and a reference harvester.

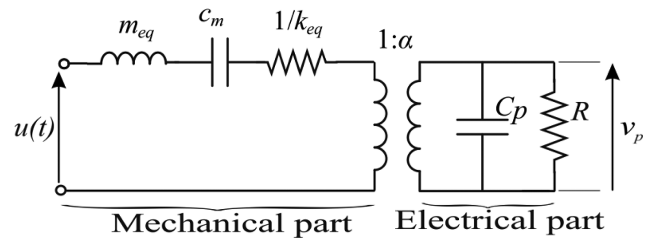


Figure 3: A sketch of the electrical circuit representing the SDOF system.

2 Analytical Modeling

All elements of the array under investigation as well as the reference harvester are in the form of the bimorph piezoelectric cantilever shown in Figure 2. The non-piezoelectric layer has a thickness of h_s, a length of l_s, a width of b_s, a modulus of elasticity of E_s and a density of ρ_s. Each piezoelectric layer has a thickness of h_p, a length of l_p, a width of b_p, a modulus of elasticity of E_p and a density of ρ_p. The beam holds a tip mass of m_t (which is normalized to the mass of the bimorph piezoelectric beam by the ratio β). The two piezoelectric layers are in parallel. An input harmonic excitation of a displacement *u(t)* is provided.

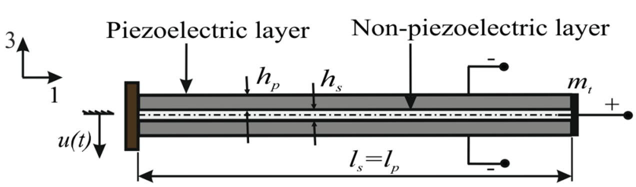


Figure 2: A sketch of bimorph cantilevered energy harvester.

The model introduced by Aboulfotoh and Twiefel (2016) is followed. The modeling procedure utilized the Rayleigh-Ritz method for the discretization of the cantilevered energy harvester into a single-degree-of-freedom (SDOF) system. The transversal displacement *w(x,t)* of any position x along the beam is expressed by:

$$w(x,t) = \phi_1(x/l)w_1(t) \tag{1}$$

where $w_1(t)$ is the generalized modal coordinate normalized on the displacement at the tip and $\phi_1(x/l)$ represents the first mode shape function of a cantilever with a tip mass. Since the second eigenfrequency is much higher than the first eigenfrequency for the cantilever, it is assumed that the first mode shape is sufficient to describe the deflection shape.

The SDOF system is represented by the electrical circuit shown in Figure 3. The governing equations of the system are expressed by eqs (2) and (3) where m_{eq} is the equivalent mass, c_m is the mechanical damping coefficient, k_{eq} is the equivalent stiffness, α is the electromechanical coupling factor, C_p is the capacitance of the two piezoelectric layers in parallel, F_{eq} is the equivalent driving force, R is the applied load resistance, I_p is the current passing through the load resistance and v_p is the output voltage. The output power p is expressed by eq. (6).

$$m_{eq}\ddot{w}(t) + c_m\dot{w}(t) + k_{eq}w(t) + \alpha v_p(t) = F_{eq} \tag{2}$$

$$\alpha\dot{w}(t) - C_p\dot{v}_p(t) = I_p(t) \tag{3}$$

$$v_p(s) = \frac{\alpha w(s)s}{\left(\frac{1}{R} + C_p s\right)} \tag{4}$$

$$I_p(t) = \frac{v_p}{R} \tag{5}$$

$$p = \left|\frac{v_p^2}{R}\right| = \left|I_p^2 R\right| \tag{6}$$

where Ω is the excitation frequency and j is the complex number, ($s = \Omega j$).

Table 1 displays the equations of the equivalent parameters where ζ is the mechanical damping ratio, s_{11}^E is the elastic compliance under constant electric field (according to direction of deflection, E_p equals $1/s_{11}^E$), d_{31} is the piezoelectric charge constant and ε_{33}^T is the permittivity of the piezoelectric material at constant stress.

3 Parallel and Series Connections between Elements in the Array

Elements of the array are connected together either in series or in parallel. Al-Ashtari et al. (2013) applied the superposition theorem to obtain output power of an array for both types of connection and verified it experimentally. The superposition theorem is utilized to determine a variable (voltage or current) in a circuit of multi-input sources. Contribution of each source to the variable is found independently where other voltage sources are

Table 1: Equations of equivalent parameters.

Parameter	Equation
Equivalent mass	$m_{eq} = (0.2357 + \beta)(\rho_s b_s h_s l_s + 2\rho_p b_p h_p l_p)$
Equivalent stiffness	$k_{eq} = \frac{1}{4}\frac{E_s b_s h_s{}^3}{l_s{}^3} + \frac{1}{4}\frac{E_p b_p h_p}{l_p{}^3}\left(8h_p{}^2 + 12hph_s + 6h_s{}^2\right)$
Damping coeffecient	$c_m = 2\zeta m_{eq}\omega_n$
Equivalent driving force	$F_{eq} = (0.375 + \beta)(\rho_s b_s h_s l_s + 2\rho_p b_p h_p l_p)\ddot{u} = m_f \ddot{u}$
Capacitance in parallel	$C_p = \frac{2b_p l_p}{h_p}(\varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E})$
Electromechanical coupling factor	$\alpha = \frac{3}{2}\frac{d_{31} b_p}{s_{11}^E l_p}(h_p + h_s)$
Resonance frequency	$\omega_n = \frac{1}{2l_p}\sqrt{\frac{E_p h_p\left(8h_p{}^2 + 12h_p h_s + 6h_s{}^2\right) + E_s h_s{}^3}{(\rho_s h_s + 2\rho_p h_p)(0.2357 + \beta)}}$
Anti-resonance frequency	$\omega_{anti} = \sqrt{\omega_n^2 + \frac{\alpha^2}{m_{eq} C_p}}$
Optimal load resistance at resonance frequency	$R_{op\_res} = \frac{2\zeta\omega_n m_{eq}}{\sqrt{4\zeta^2\omega_n^4 m_{eq}^2 C_p^2 + \alpha^4}}$
Optimal load resistance at anti-resonance	$R_{op\_anti} = \frac{\alpha^2 + \sqrt{\alpha^4 - (16C_p^2\zeta^2 m_{eq}^2\omega_{anti}^4)}}{4\zeta m_{eq} C_p^2 \omega_{anti}^3}$

replaced by a short circuit and other current sources are replaced by an open circuit. Then, by summing up all the contributions, the total value of the required variable is obtained, as explained by Sadiku and Alexander (2009).

The same method of using the superposition theorem is followed in this paper. The excitation force applied to the whole array is with a single frequency. All elements in the array are excited by the same force. The excitation force is considered in this case as the multi-input sources to the circuit. When calculating the contribution of each element in the array to a variable, either voltage or current, only the input excitation force to this element is considered, other input sources are replaced by a short circuit. The total impedance applied to the considered element is calculated as the internal impedances of all the other elements in the array added to the load resistance according to type of connection. The internal impedance of each piezoelectric element is obtained as expressed by eq. (7). It is noted that the absolute value of the internal impedance of a piezoelectric element at resonance frequency gives the same value as the optimal load resistance at resonance frequency (see Table 1). This coincides with the concept of the maximum power transfer theorem.

$$Z_k(s) = \frac{m_{eq\_k}s^2 + c_{m\_k}s + k_{eq\_k}}{\alpha_k^2 s + m_{eq\_k}C_{p\_k}s^3 + c_{m\_k}C_{p\_k}s^2 + k_{eq\_k}C_{p\_k}s} \quad (7)$$

where k is a counting number.

If the piezoelectric element i is considered as the input source, the total impedance when connected in series is obtained by adding the load resistance to the impedances of the other elements, see eq. (8). When connected in parallel, the total impedance is obtained as expressed by eq.(9).

$$Z_{i\_total\_s}(s) = \left[\sum_{k=1}^{n} Z_k(s)\right] - Z_i(s) + R \quad (8)$$

$$Z_{i\_total\_p}(s) = \frac{1}{\left[\sum_{k=1}^{n}\frac{1}{Z_k(s)}\right] - \frac{1}{Z_i(s)} + \frac{1}{R}} \quad (9)$$

where n is the number of the elements in the array.

For the array in series connection, contribution of each element is calculated independently to the current passing through the load resistance. Then, the total current is calculated by summing up all these contributions. The real power is estimated by the absolute value of multiplying the square of the total current by the load resistance. In case of parallel connection, the contribution of each element is calculated independently to the voltage drop across the load resistance, and then summing up all these contributions to obtain the total voltage. The real power is calculated from the absolute value of the square of the total voltage divided by the load resistance, see eq. (6).

4 Array of n Elements Compared to a Reference Harvester

Output power and bandwidth of the array are analyzed in comparison to those of a reference harvester in order to find the frequency band within which the array has additional benefit over the reference harvester. Hence, a guideline for designing the array is provided. All the analysis considers the following conditions for the elements of the array as well as the reference harvester: a constant-velocity input excitation of 7.5 mm/s, a low damping ratio of 0.02 and the material properties shown in Table 2. The mass ratio β is assumed to be zero for all elements in the array as well as the reference harvester. Since the equivalent stiffness and the equivalent mass are linear functions of the width, the equation of the resonance frequency is independent of the width, see Table 1. Hence, changing the width has no influence on the resonance frequency as long as the mass ratio is kept constant.

Table 2: Material properties.

	Piezoelectric	Non-piezoelectric
Mechanical data	$s_{33}^{E} = 20.6\text{e-}12\ \text{m}^2/\text{N}$, $s_{11}^{E} = 14.2\text{e-}12\ \text{m}^2/\text{N}$, $\rho_p = 8\text{e}03\ \text{kg/m}^3$	$E_s = 1.2\text{e}11\ \text{N/m}^2$, $\rho_s = 1.8\text{e}03\ \text{kg/m}^3$
Piezoelectric data	$d_{31} = 315\text{e-}12\ \text{C/N}$, $\varepsilon_{33}^{T} = 3.9825\text{e-}08\ \text{F/m}$	

The reference harvester always has the following dimensions: a length of 45 mm, a width of 21 mm, a thickness of the non-piezoelectric layer of 0.28 mm and a thickness of each piezoelectric layer of 0.25 mm. The dimensions of the elements in the array will be given according to the type of comparison that is divided into two sections. Firstly, the array has n number of identical elements tuned so that their first eigenfrequency has the same value (from now on referred to as: array of elements of matching frequencies). All the elements in the array in this section have the same material and dimensions as the reference harvester, but with a width of 1/n of its width. Therefore, they all have the same first eigenfrequency. Secondly, the elements of the array are tuned so that their first eigenfrequency does not have the same value (from now on referred to as: array of elements of non-matching frequencies). The elements of the array in this section will have different lengths, but the average length of all of them is kept equal to the length of the reference harvester to confirm the same volume condition. Each element in the array of non-matching frequencies will have a width of 1/n of that of the reference harvester. The thickness of the non-piezoelectric layer and of each piezoelectric layer will be as same as those of the reference harvester.

4.1 Section A: Array of Matching Frequencies

The elements of the array of matching frequencies are identical. Hence, their internal impedances are equal and the load resistance applied to the array is equally distributed between them. Thus, in series connection, the optimal load resistance of an array of matching frequencies is obtained by summing up individual optimal resistances of all elements or the absolute value of their internal impedances. The optimal load resistance of an array in series is given by eq. (10). In case of parallel connection, the optimal load resistance of the array is obtained by dividing the individual optimal resistance of element i by number of elements, as given by eq. (11).

$$R_{op\_s\_res\_match} = nR_{op\_i} = n\left|\bar{Z}_{i\_res}(s)\right| \tag{10}$$

$$R_{op\_p\_res\_match} = \frac{R_{op\_i}}{n} = \frac{\left|\bar{Z}_{i\_res}(s)\right|}{n} \tag{11}$$

where R_{op_i} is the individual optimal resistance of element i and Z_{i_res} is the average of the internal impedances of all elements.

Figure 4(a) and 4(b) display the output power of two 3-element arrays of matching frequencies, in series and in parallel, respectively. Both are loaded by their optimal resistances. The optimal resistance in series connection is much higher than in parallel connection. An array of matching frequencies either in series or in parallel has two power peaks: at resonance and at anti-resonance frequencies. The two arrays provide the same output power when loaded by their optimal resistances. Series and parallel connections are not interchangeable, but they are energy equivalent.

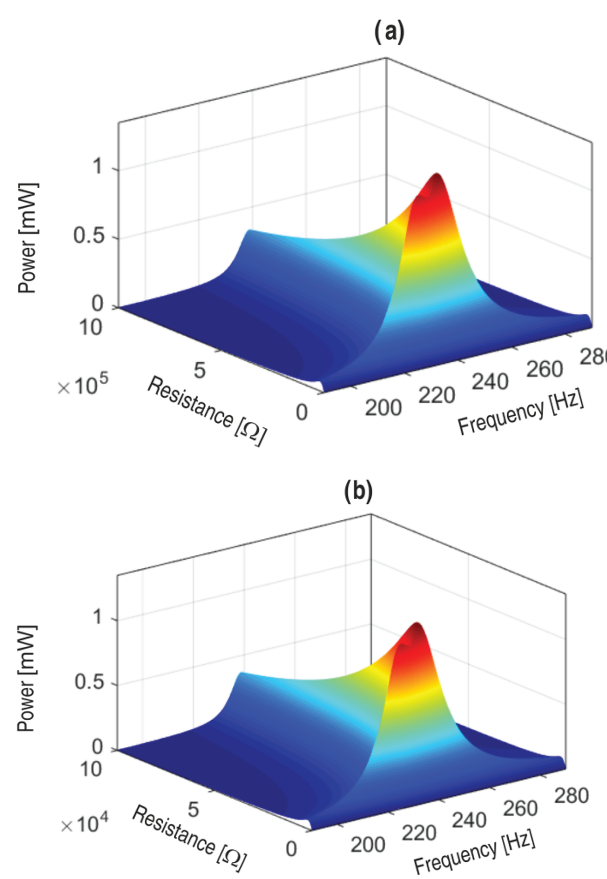


Figure 4: A 3-element array of matching frequencies (a) in series (b) in parallel.

Figure 5(a) displays the output power of the reference harvester and of the two previous 3-element arrays in comparison with the applied load resistance. Excitation

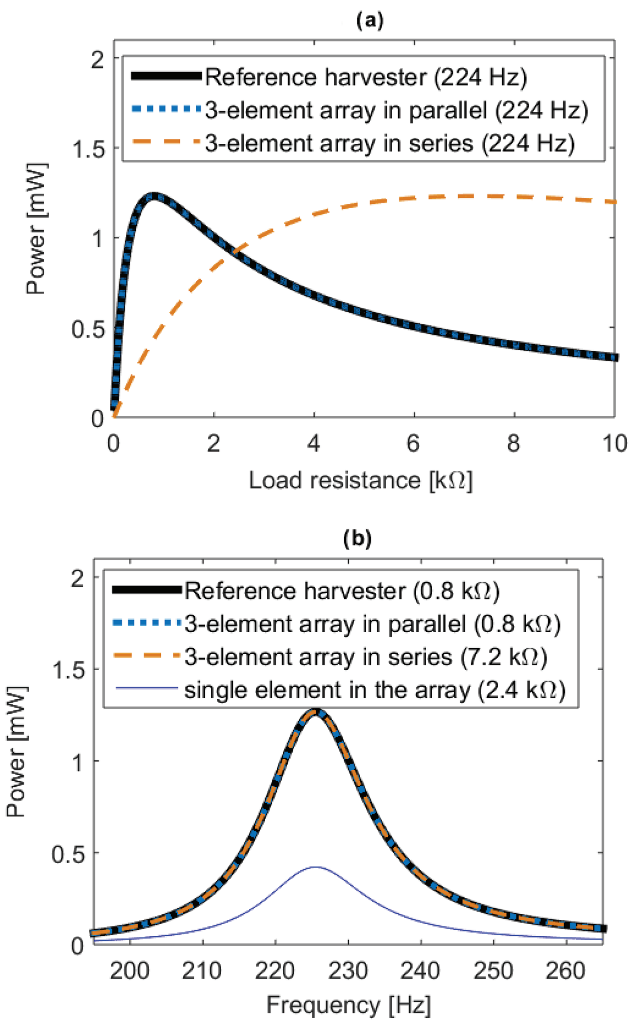


Figure 5: Power of a reference harvester and of a 3-element array of matching frequencies in comparison with (a) Load resistance (b) Excitation frequency.

frequency is 224 Hz, which equals the resonance frequency of the reference harvester and of all elements in the array. Figure 5(b) displays the output power of the reference harvester and of the two 3-element arrays in comparison with the excitation frequency. It is concluded that the output power of the array either in series or in parallel is equal to the output power of the reference harvester when all are loaded by their optimal resistances.

At different applied load resistances, the output power of the reference harvester and of n-element arrays is displayed, see Figure 6. Under the same volume, the reference harvester when loaded by a resistance of R provides the same output power and bandwidth as an array of n elements of matching frequencies in series connection and loaded by a resistance of n^2R or in parallel connection and loaded by a resistance of R.

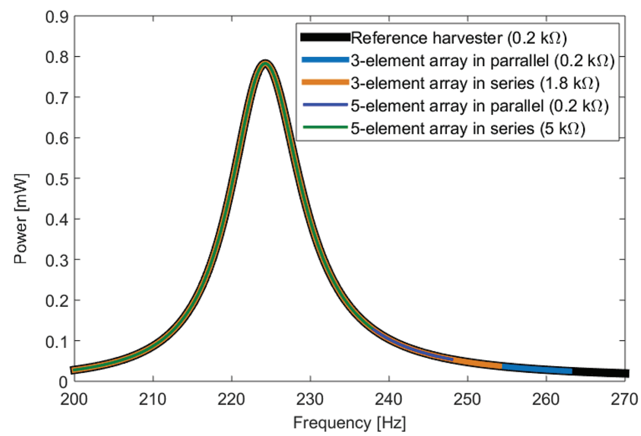


Figure 6: Power of a reference harvester loaded by R, compared to n-element arrays of matching frequencies in parallel (loaded by R) and in series (loaded by n^2R).

It is to be concluded from this section that since an array of matching frequencies has no additional benefit over the reference harvester, and taking into account the complexity of the array in mechanical setup and electrical connections, the array of matching frequencies is not recommended as a design for broadband energy harvesting.

4.2 Section B: Array of Non-Matching Frequencies

In this section, the performance of an array of elements of non-matching frequencies is studied. Output power of two 3-element arrays of non-matching frequencies in series and in parallel is displayed, see Figure 7(a) and 7(b), respectively. The peak value of output power of an array in series occurs at the lowest first eigenfrequency of the three elements, while in parallel it occurs at the highest first eigenfrequency of the three elements.

An array of non-matching frequencies produces variant bandwidths by changing the difference between the first eigenfrequencies of its elements Δf_n, see Figure 8. It is shown that the larger the difference, the wider the bandwidth, but the lower the output power. In the case of small differences, the bandwidth of the array is small and approximately equals the bandwidth of the reference harvester. In the case of large differences, the array produces inconsistent power. Hence, the available band of non-matching frequencies for an array is located between two limits: the lower limit B_{min} and the upper limit B_{max}. If the band of the non-matching frequencies is smaller than B_{min}, the array has no additional benefit over the

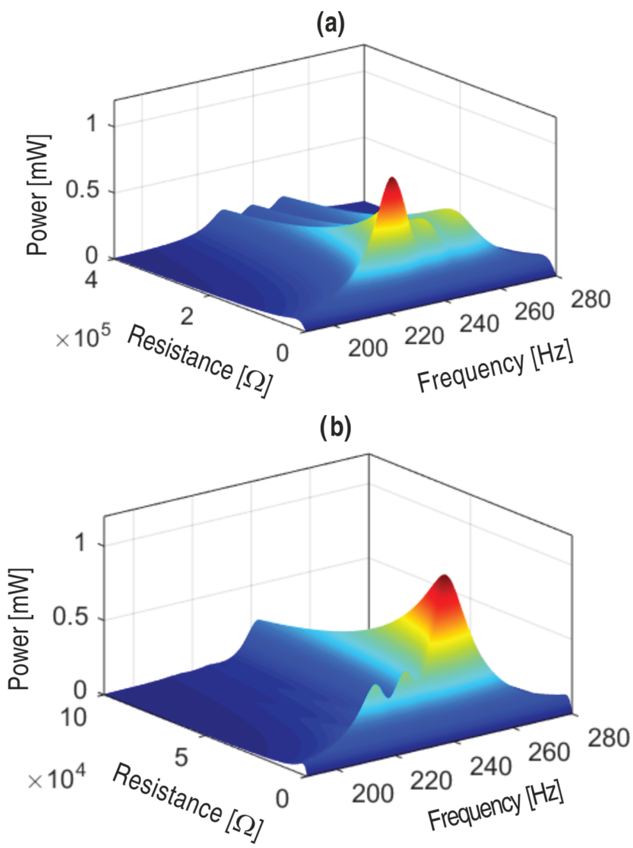


Figure 7: A 3-element array of non-matching frequencies (a) in series (b) in parallel.

reference harvester. If the band of the non-matching frequencies is larger than B_{max}, the power streamline has large drops.

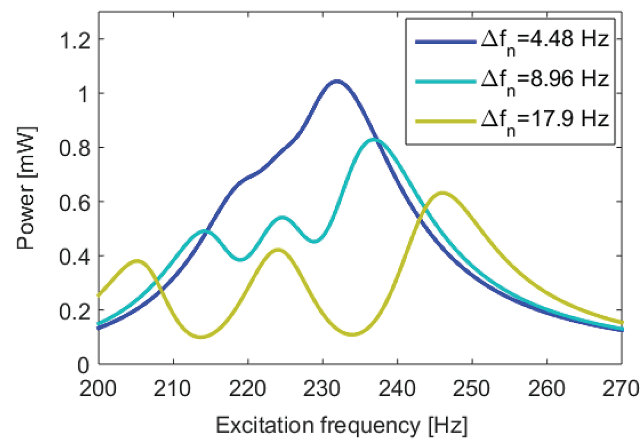


Figure 8: Array of non-matching frequencies in parallel (various Δf_n).

Equation (12) expresses the minimum band of frequencies (B_{min}). Figure 9 displays how this equation is obtained. The band of frequencies of the array has a middle frequency of ω_{mid}. Resonance frequency of the reference harvester is also ω_{mid}. Since the differences between mismatching frequencies are small, all elements of the array are assumed to have approximately the same bandwidth of $2\zeta\omega_{mid}$ which equals the bandwidth of the reference harvester. This bandwidth is divided by the number of elements in the array to get the difference between first eigenfrequencies of each two sequential elements as $\Delta\omega_{min} = 2\zeta\omega_{mid}/n$ or $\Delta f_{min} = (1/2\pi)(2\zeta\omega_{mid}/n)$. After that, this difference is multiplied by $(n-1)$ to obtain the minimum band of frequencies (B_{min}).

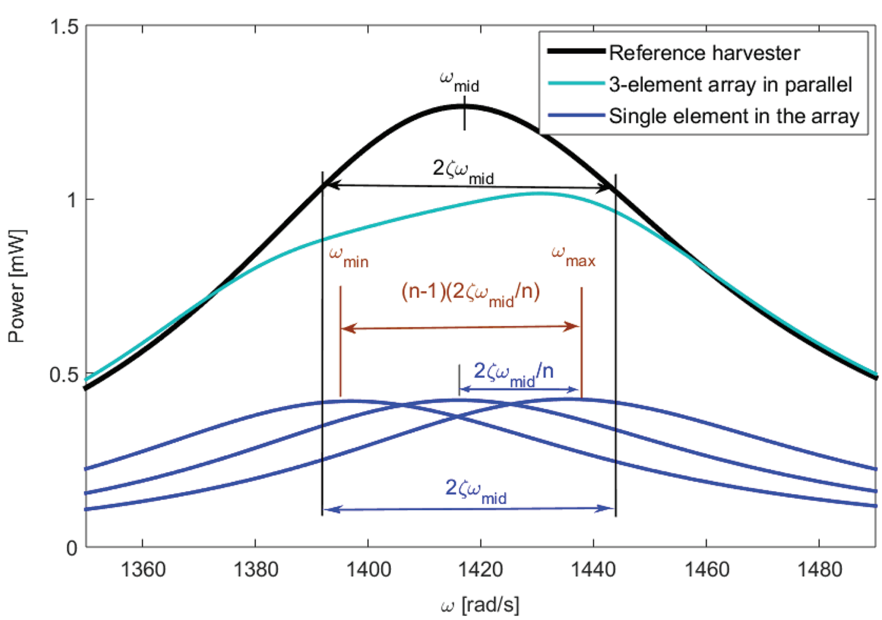


Figure 9: Finding the minimum band of non-matching frequencies B_{min}.

$$B_{\min} \approx \frac{1}{2\pi}\frac{n-1}{n}(2\zeta\omega_{\text{mid}}) \quad (12)$$

The maximum range of non-matching frequencies is found under the condition that the output power of the array should be consistent. Equation (13) expresses the maximum band of frequencies (B_{max}). Figure 10 explains how this equation is obtained. The half-power bandwidth ($2\zeta\omega_{mid}$) is assumed to be the maximum difference between first eigenfrequencies of each two sequential elements ($\Delta\omega_{max}$), hence, $\Delta f_{max} = (1/2\pi)(2\zeta\omega_{mid})$. The maximum band of non-matching frequencies is approximately the half-power bandwidth multiplied by (n–1).

$$B_{\max} \approx \frac{1}{2\pi}(n-1)(2\zeta\omega_{mid}) \quad (13)$$

In the following two sub-sections, the minimum and maximum bands of frequencies in parallel and in series connections are displayed in comparison with the reference harvester. For an array of elements of non-matching frequencies, the dimensions, materials or boundary conditions are varied. Thus, the internal impedances of the elements are not equal. When the array is excited by a frequency matching the first eigenfrequency of element i, only this element is at resonance, while all other elements are not and their internal impedances are added to the load resistance according to the type of connection. In series connection, internal impedances of (n–1) elements are algebraically summed up together and the resulting total impedance loaded to the element i is high. Thus, the elements of an array of non-matching frequencies in series connection are recommended to be tuned to their anti-resonances. In parallel connection, the resulting total impedance loaded to element i is low. Thus, it is recommended for an array of elements of non-matching frequencies in parallel connection to be tuned to their resonance frequencies.

4.2.1 Array of Non-Matching Frequencies in Parallel (At Resonance)

The reference harvester loaded by R is compared to n-element arrays of the minimum band in parallel and loaded by R, see Figure 11. The arrays produce approximately the same bandwidth as the reference harvester, but lower power. Thus, an array of the minimum band in parallel is not recommended as a broadband technique.

Figure 12 displays the reference harvester in comparison with n-element arrays of the maximum band in parallel. The bandwidth of the array is referred to as the band of frequencies that produce the maximum possible consistent output power. The array of the maximum band produces less power than the reference harvester but its bandwidth is relatively larger than the band of frequencies of the reference harvester that produce same power level. Thus, an array with maximum band in parallel may be an option for wider bandwidth if the application requirements are within this very low level of power.

4.2.2 Array of Non-Matching Frequencies in Series (At Anti-Resonance)

The reference harvester loaded by R is compared to n-element arrays of the minimum band in series and loaded by n^2R, see Figure 13. An array of elements of the minimum band in series has no additional benefit over the reference harvester.

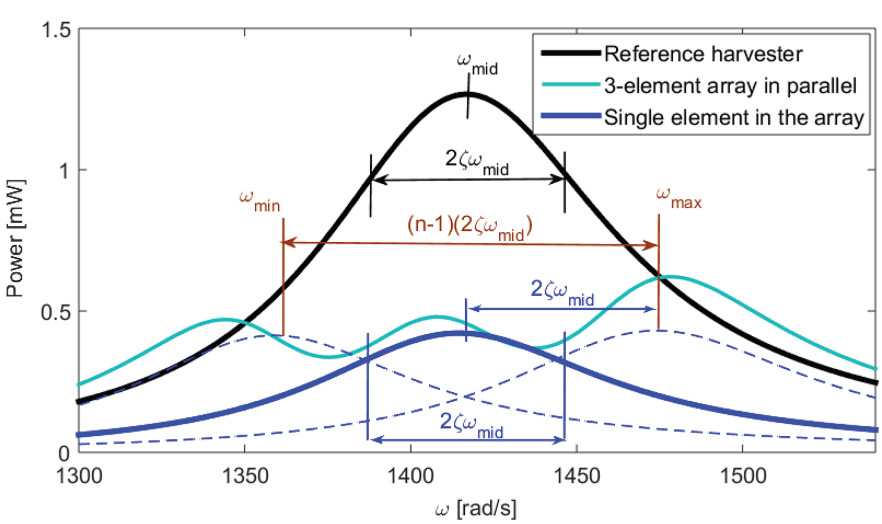


Figure 10: Finding the maximum band of non-matching frequencies B_{max}.

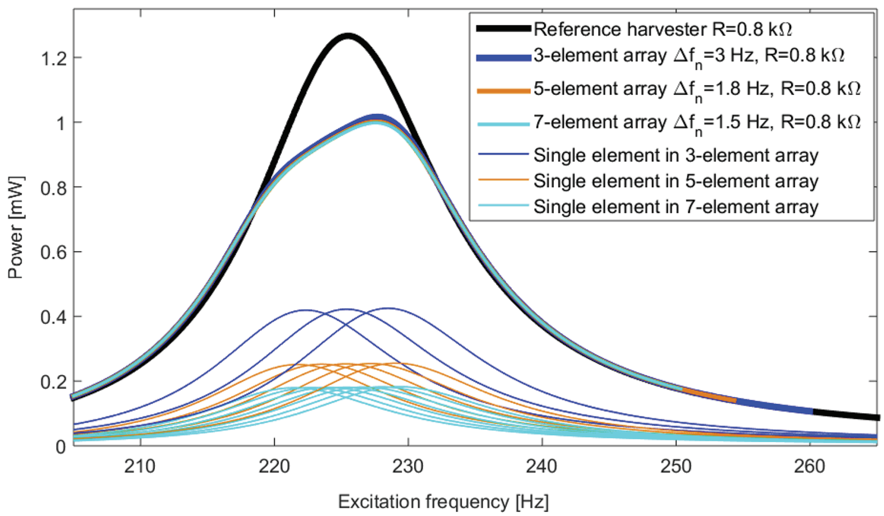


Figure 11: The reference harvester compared to n-element arrays of the minimum band (in parallel).

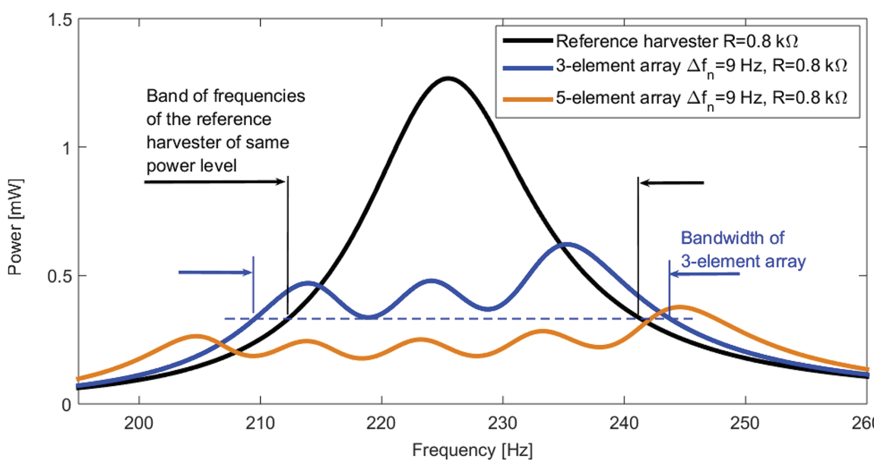


Figure 12: The reference harvester compared to n-element arrays of the maximum band (in parallel).

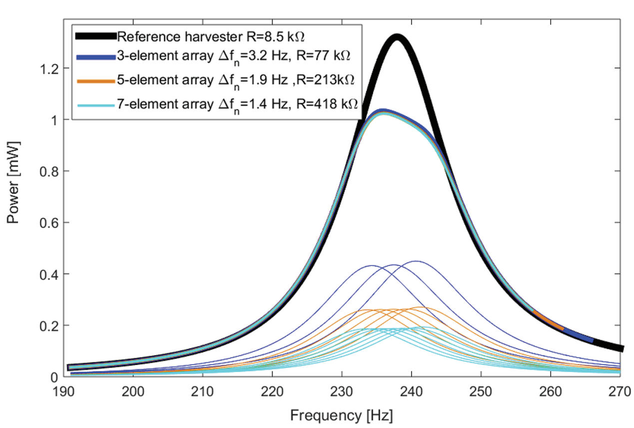


Figure 13: The reference harvester compared to n-element arrays of the minimum band (in series).

The reference harvester is compared to n-element arrays of the maximum band in series, see Figure 14. The array of the maximum band in series shows relatively larger bandwidth than the reference harvester but lower power.

5 Guidelines to Design the Array

The two eqs (12) and (13) provide guidelines to design the array. If the band of frequencies is around or less than B_{min}, the array is not the good selection because a single harvester of the same volume as the array provides approximately the same bandwidth and higher power. If the band of frequencies is around B_{max} the array provides an option for a broadband vibration harvester, but with very low level of power.

The two equations are obtained from the analysis of the bandwidth of the array in comparison with the reference harvester and are mainly based on:

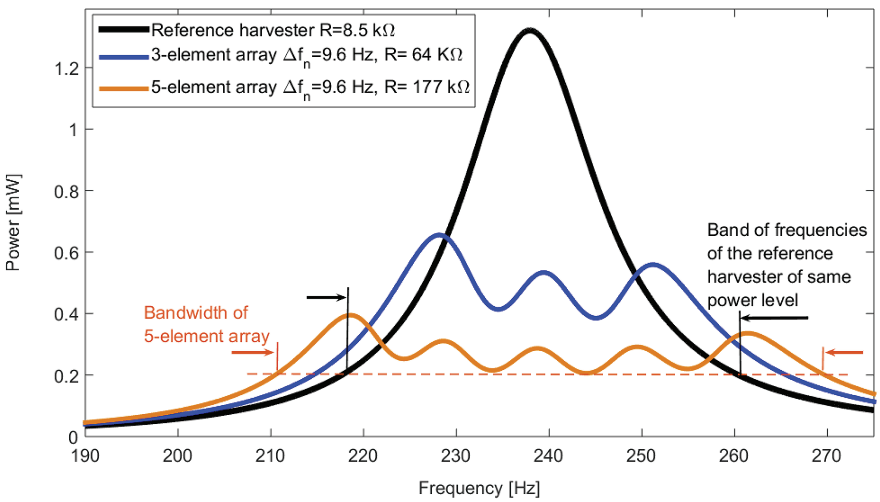
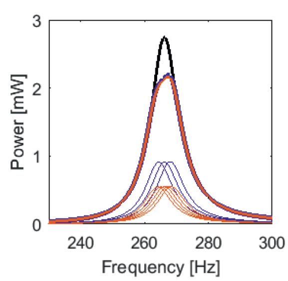
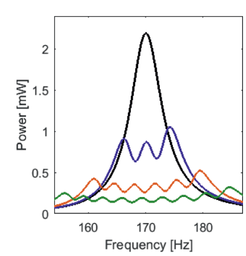
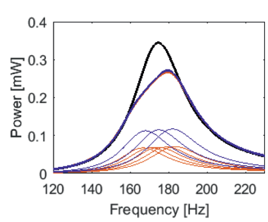
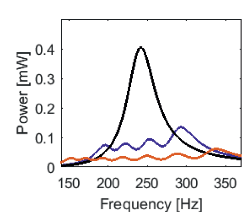


Figure 14: The reference harvester compared to n-element arrays of the maximum band (in series).

Table 3: Arrays in parallel (drawn in colored) of different frequency bands in comparison with reference harvester (drawn in black).

	B_{min}			B_{max}		
ζ	ω_{mid}, f_{mid}	Array compared to reference harvester	R	ω_{mid}, f_{mid}	Array compared to reference harvester	R
0.01	1,671 rad/s, 266 Hz		379 Ω	1,068 rad/s 170 Hz		474 Ω
0.06	1,100 rad/s, 175 Hz		2.2 kΩ	1,520 rad/s 242 Hz		1.8 kΩ

a. The bandwidth of the reference harvester (a function of the damping ratio and of its first eigenfrequency) and the bandwidth of any element in the array
b. The maximum output power of an element of an n-element array produces 1/n of the maximum output power of the reference harvester.

Hence, the equations are general in evaluating the outcome of the array in comparison with the reference harvester, but under the same volume, the same operating conditions, the same damping coefficient and the optimal applied load resistance from eqs (10) and (11). Different examples of the array compared to the reference harvester are included in Table 3.

6 Conclusion and Future Work

Performance of an array of elements is evaluated in comparison to a single reference harvester with same volume as the whole array. The comparison is carried out under the same operating conditions. The output AC power is analytically calculated for the array in series and in parallel. The comparison is divided into two sections: first, when the first eigenfrequencies of all the elements in the array have the same value (called in this paper: an array of matching frequencies), then when they don't have the same value (called in this paper: an array of non-matching frequencies). It is found that an array of matching frequencies either in series or in parallel provides the same power and the same bandwidth as the reference harvester when loaded by their optimal resistances. Hence, an array of matching frequencies is not recommended as a broadband energy harvester. For an array of non-matching frequencies, it is found that within low differences between the first eigenfrequencies, the array provides same bandwidth as the reference harvester, but lower power. The minimum band of frequencies below which the array has no advantage over the reference harvester is found. Within high differences between the first eigenfrequencies, the output power of the array is not consistent. The maximum band of frequencies above which the array provides inconsistent power is found. Between the minimum band and the maximum band, the array provides a relatively additional bandwidth over the reference harvester, but lower power. Thus, it can be an option as a broadband energy harvester if the application requirements are within very low power level. The minimum band and the maximum band of frequencies provide important guidelines for designing the array of generators.

In this paper, the array is evaluated based on its AC output power. Since most of energy harvesting applications require DC power, our future work will include the rectification of the array output power and compare it to the reference harvester.

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