Nathan Sharpes*, Abdessattar Abdelkefi and Shashank Priya

Comparative Analysis of One-Dimensional and Two-Dimensional Cantilever Piezoelectric Energy Harvesters

Abstract: A long-standing encumbrance in the design of low-frequency energy harvesters has been the need of substantial beam length and/or large tip mass values to reach the low resonance frequencies where significant energy can be harvested from the ambient vibration sources. This need of large length and tip mass may result in a device that is too large to be practical. The zigzag (meandering) beam structure has emerged as a solution to this problem. In this letter, we provide comparative analysis between the classical one-dimensional cantilever bimorph and the two-dimensional zigzag unimorph piezoelectric energy harvesters. The results demonstrate that depending upon the excitation frequency, the zigzag harvester is significantly better in terms of magnitude of natural frequency, harvested power, and power density, compared to the cantilever configuration. The dimensions were chosen for each design such that the zigzag structure would have 25.4×25.4 mm^2 area, and the cantilever would have the same surface area. The zigzag prototype of 25.4×25.4 mm^2 area was capable of generating 65 μW/cm^3 at 32 Hz when subjected to 0.1 G base acceleration.

Keywords: piezoelectric energy harvesting, vibration energy harvesting, zigzag beam shape, 2D beam shape, low frequency, high power density

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***Corresponding author: Nathan Sharpes,** Department of Mechanical Engineering, Center for Energy Harvesting Materials and Systems (CEHMS), Virginia Tech, Blacksburg, VA 24060, USA, E-mail: nsharpes@vt.edu

Abdessattar Abdelkefi, Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, NM 88003, USA, E-mail: abdu@nmsu.edu

Shashank Priya, Bio-Inspired Materials and Devices Laboratory (BMDL), Virginia Tech, Blacksburg, VA, USA, E-mail: spriya@vt.edu

Vibration-based energy harvesters have recently received significant interest in order to operate low-power electronics and replace small batteries that require expensive and time consuming maintenance. Several investigations have been performed to design high power density energy harvesters from unused mechanical vibrations (Abdelkefi and Ghommem 2013; Anton and Sodano 2007; Cook-Chennault et al. 2008; Karami and Inman 2012b; Priya 2007; Tang and Yang 2012). These designs vary from simple beam and beam-mass systems (Abdelkefi and Barsallo 2014; Erturk et al. 2009; Masana and Daqaq 2011; Stanton et al. 2009; Tang and Yang 2012) to more complex structures including bending-torsion, zigzag, and spiral systems (Abdelkefi et al. 2011; Karami and Inman 2011, 2012a). The objective of these diverse harvesters is to enable the harvesting of energy at specific frequencies from wasted mechanical energy in the environment through various transduction mechanisms. Examples include electrostatic (Anton and Sodano 2007), electromagnetic (Arnold 2007), and piezoelectric (Anton and Sodano 2007; Erturk et al. 2009; Stanton et al. 2009; Tang and Yang 2012). The piezoelectric option has gained prominence at small-scale, especially when the dimensions are in the range of few cubic millimeters.

A piezoelectric cantilever beam, subjected to harmonic or random vibrations, is the most common concept used in the literature. For these types of energy harvesters, resonant responses are obtained when the excitation frequency matches the natural frequency of the harvester. These resonant motions result in maximum deflection of the structure, straining the piezoelectric material and producing electrical charge, which can be channeled as an alternating current across an electrical load resistance. Different strategies and techniques have been applied to design efficient, low-frequency, piezoelectric energy harvesters, such as considering different shape geometries (Abdelkefi et al. 2011; Apo et al. 2014; Ben Ayed et al. 2014; Benasciutti et al. 2010; Berdy et al. 2012; Karami and Inman 2011, 2012a), developing bistable configurations (Daqaq 2011; Mann and Sims 2009), and including magnetic coupling (Abdelkefi and Barsallo 2014; Tang and Yang 2012). As for shape geometries, simple and complex systems have been proposed in order to design energy harvesters that can operate effectively at an excitation frequency that matches their resonant frequency. In general, passive tunable harvesters have been designed by using cantilever, spiral, or zigzag configurations. However, a comparative analysis in terms of the efficiency of these simple and complex energy harvesting designs is missing. In this study, we focus on the performance comparison between the classical piezoelectric energy harvesters (beam-mass system) and the complex piezoelectric energy harvesters (zigzag system). Experimental measurements are performed for these two types of piezoelectric energy harvesters with the same footprint (surface area) to quantify the power density and tunability. We first establish the designs for each harvester based on their most common configuration: the bimorph configuration for the cantilever harvester and the unimorph configuration for the zigzag piezoelectric energy harvester. The dimensions were chosen for each design, as shown in the schematics in Figure 1, such that the zigzag would have $25.4 \times 25.4\ \mathrm{mm}^2$ area and the cantilever would have approximately the same surface area. In this way, we can make conclusions about the performance of both these structures and identify the regions where one has advantages over the other.

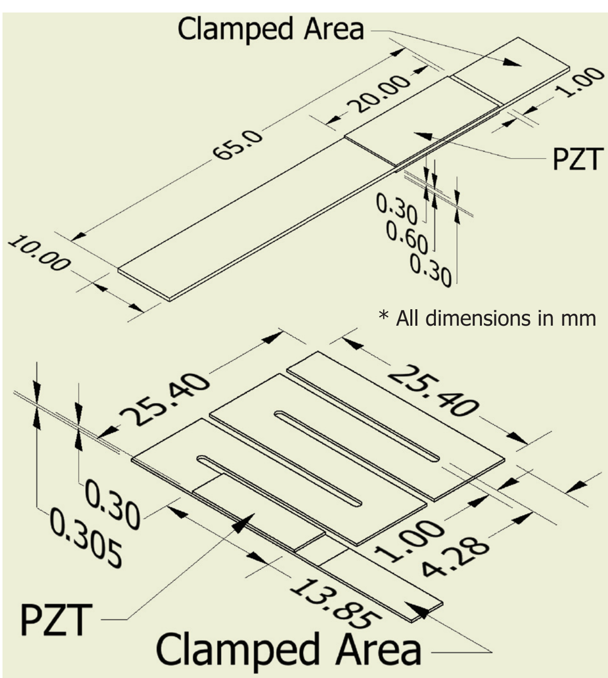

Figure 1 (Top) Schematic of a bimorph piezoelectric cantilever energy harvester (PZT on aluminum substrate), and (bottom) schematic of unimorph piezoelectric zigzag energy harvester (PZT on steel substrate)

The same piezoelectric material was used in both configurations to allow for reasonable comparisons to be made between the output power density and overall magnitude of the power. The piezoelectric material utilized was APC 850 PZT with $-d_{31}$ of 175 pC/N and $-g_{31}$ of 12.4×10^{-3}Vm/N. The energy harvesting prototypes were constructed by first cutting the beam substrate shapes from larger sheets of material (aluminum and steel for the cantilever and zigzag configurations, respectively) via CNC machining. Different substrates were chosen for this comparison so that the frequency response characteristics of the two harvesters might be brought closer together, allowing for reasonable comparisons to be made. Since the zigzag structure is naturally more compliant than the classical cantilever, steel was chosen as substrate making it stiffer. Thus, the two harvesters would have overlapping frequency regions when paired with appropriate tip mass. The piezoelectric elements were then bonded to the substrate at the areas of highest strain concentration in order to harvest the greatest amount of power without significantly increasing the stiffness and consequently the resonance frequency of the system. The bonding was accomplished using Loctite 120 HP epoxy and allowing a cure of no less than 24 h at room temperature before testing. The fabricated prototypes are shown in measurement setups in Figure 2. The constructed prototypes have reflective stickers (of negligible mass) placed in a pattern in the case of the zigzag configuration and at the beam tip in the case of the cantilever configuration to allow the use of a laser Doppler vibrometer to monitor the velocity at those points. We should note that the point labels were overlaid on the picture for illustration purposes and were not present on the physical stickers, thus not altering their reflectivity. The experimental setup and specific measurement equipment are displayed in Figure 3.

The measurement system was controlled via desktop computer interfaced with the SigLab. The SigLab produces a specified excitation voltage signal (sinusoidal wave in this case) at low current, which was then passed through the HP power amplifier, increasing the current of the signal, to power the TJ-2 shaker. The acceleration of the base of the energy harvesters (clamp) and velocity of the various points on the beam (Figure 2) were read by the SigLab from the PCB accelerometer and PDV100 vibrometer, respectively. The voltage output from the piezoelectric material was also recorded (across varying load resistances) by the SigLab. All measurements were done atop a vibration isolation pillar, which, by virtue of its

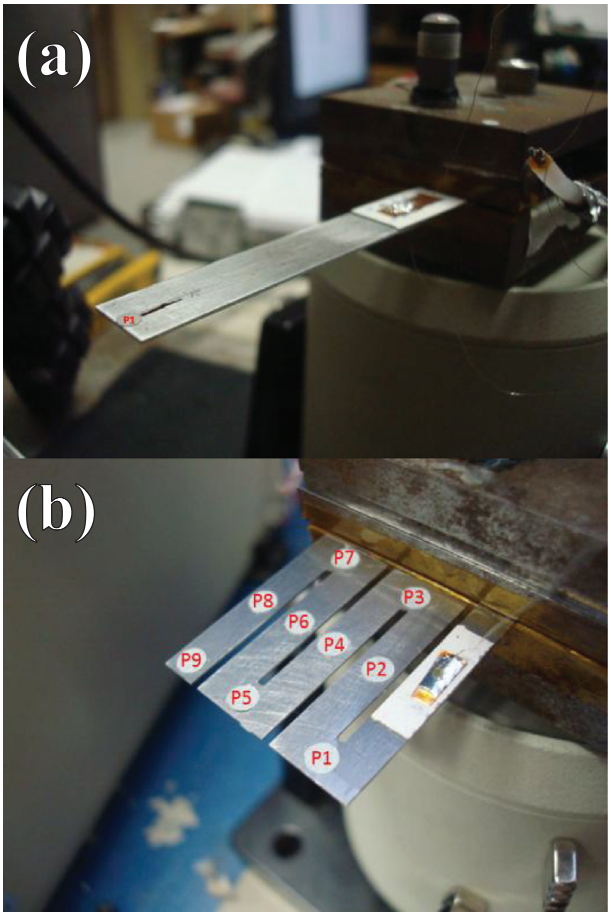

Figure 2 Constructed (a) cantilever bimorph and (b) zigzag prototypes mounted in shaker clamp with labeled points of interest

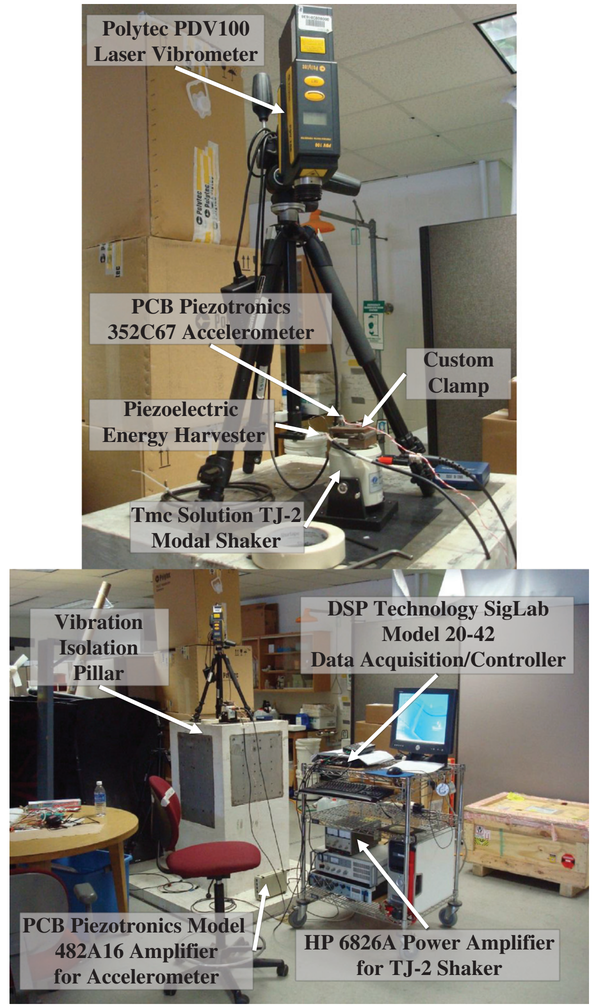

Figure 3 Experimental setup for cantilever and zigzag piezoelectric energy harvester tests, with key components labeled

mass and location, was largely uninfluenced by the vibration of the surroundings (i.e. building, floor, persons walking, machinery, etc.). All cables used were BNC type, to provide shielding from any external electromagnetic noise sources. As can be seen in Figure 2, the BNC cable was not directly attached to the piezoelectric elements in each harvester rather it was attached via copper tape (~51 μm thickness) and thin (44 gauge) wire. This approach was implemented for several reasons. Thin wire was used so that the attachment of wire(s) will not add any significant mass or stiffness to the piezoelectric energy harvesters and thus not alter their dynamics. The wires were first soldered to the copper tape and then adhered to the piezoelectric surface. In this way, one can avoid exposing the piezoceramic to high local temperatures associated with the direct application of molten solder. The poling directions and wire attachments used are shown in Figure 4.

We begin our analysis by examining the frequency response dynamics of both the systems. This is

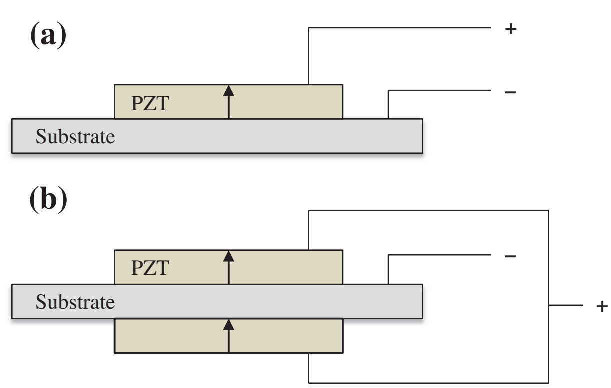

Figure 4 Poling direction and wire connection diagram for (a) zigzag unimorph harvester and (b) cantilever bimorph harvester with piezoelectric elements were wired in parallel

accomplished by applying a uniform white noise vibration input to each system and examining the Fourier transform of the cross-correlation between the base

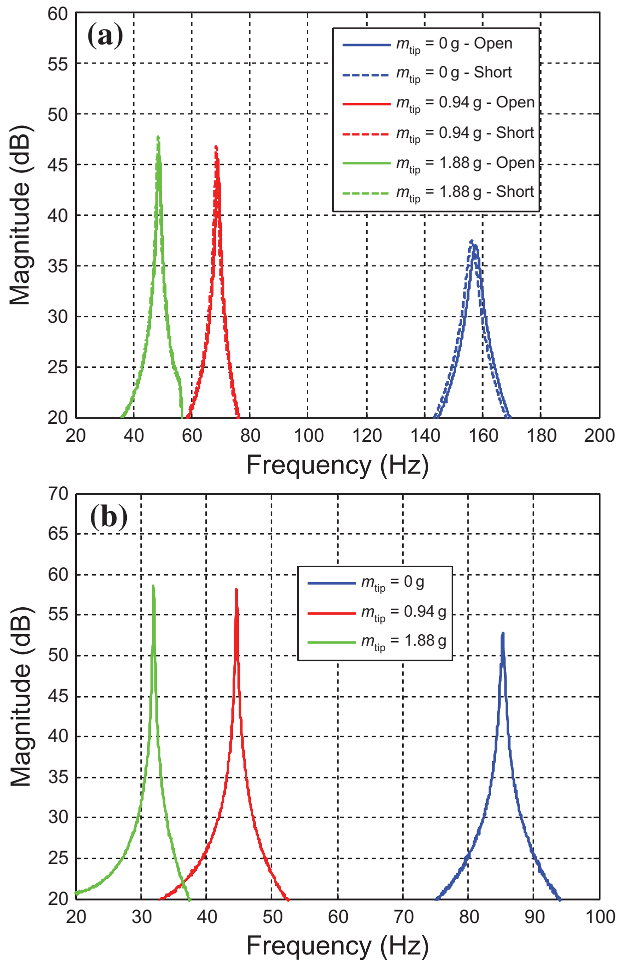

Figure 5 Frequency response of (a) cantilever and (b) zigzag piezoelectric energy harvester for varying tip mass and electrical connection. There is no measureable difference between open- and short-circuit response for the zigzag harvester, thus only the open-circuit response is shown

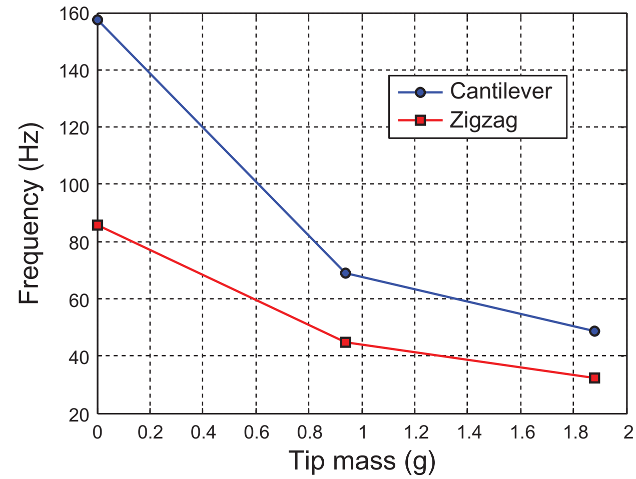

Figure 6 Variation of harvester's fundamental frequency as a function of tip mass for cantilever and zigzag systems

acceleration and beam tip velocity. In this way, all frequencies are subjected to the same spectral energy, and the curves in Figure 5 can be regarded as the transfer functions of their respective systems. These measurements are taken for connecting the piezoelectric elements in both short-circuit and open-circuit configurations. One would expect, as is evident in Figure 5, that connecting the piezoelectric in short-circuit configuration allows the most charge to flow, effectively reducing the natural frequency. For the zigzag case, a smaller volume of piezoelectric material is utilized than in the cantilever bimorph case and thus no measureable difference in frequency response between open- and short-circuit configurations is observed. For this reason, only the open-circuit cases for the zigzag configuration are plotted.

In addition, tip masses of 0 g, 0.94 g, and 1.88 g are employed to examine their effects on the natural frequency of the harvesters. It should be noted that increasing the tip mass for both the systems results in a decrease in the natural frequency and an increase in the amplitude of harvester's vibration. A plot of the variation of the fundamental frequency as a function of the tip mass is presented in Figure 6 where we note that for all tip mass cases the zigzag configuration has a lower fundamental frequency. The difference between the two systems is exceedingly evident at 0 g tip mass, when the harvester's shape was mostly responsible for its frequency response characteristics.

A comparison between the deflections of the two configurations at their respective first natural frequencies is performed for the case of zero tip mass. We note that all further tests are conducted at 0.1 G base acceleration. In Figure 7, it is observed that the deflections of the two systems at their respective natural frequencies are similar, with the cantilever system exhibiting a greater deflection at resonance. The displacement plots in Figure 7 show the relative displacement of the respective positions labeled in Figure 2 and are realized by integrating the laser vibrometer velocity data at each point and averaging over several (at least 10) periods. The base displacement, which is integrated from the base acceleration, then is subtracted leaving only the relative deflection of the respective points. Finite Element Analysis illustrations of the first natural frequency for each shape are also shown in Figure 7 in order to clarify the shape of each of the beams when excited at their respective fundamental frequencies.

The effects of the electrical load resistance on the performance of the harvesters are performed next, as shown in Figure 8. Here, the electrical load resistance is

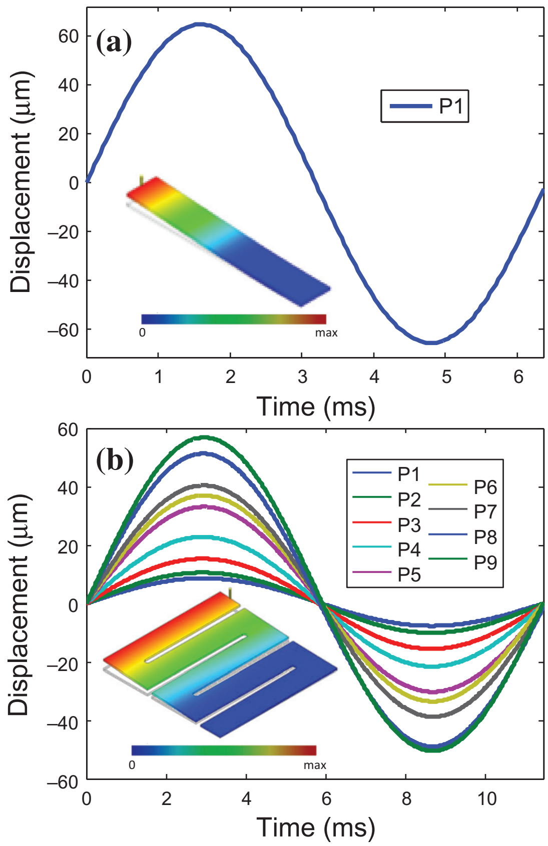

Figure 7 Deflections of the point(s) shown in Figure 2 at the first bending frequency of the (a) cantilever and (b) zigzag piezoelectric energy harvesters at a base acceleration of 0.1 G. Cantilever frequency is ~156 Hz, with max deflection ~66 μm, and zigzag frequency is ~86 Hz, with a maximum deflection of ~57 μm. Insets show FEA results for fundamental mode shape for clarity

varied while exciting the system at a fixed frequency and for a given tip mass. The frequency was varied across a range near that of the measured fundamental frequency of each system. It is shown that a similar power output is obtained from both systems, despite their different form factors. In addition, an increase in the tip mass is accompanied by an increase in the harvested power for both systems. However, the more tip mass is increased, the less effective it becomes in lowering the fundamental frequency and increasing the harvested power.

From Figure 8, it is clear that the dynamics of the system changes for different electrical load resistances. In fact, we note that there is a shift in the peak frequency to the right when increasing the load resistance. At higher load resistances, current is reduced, and higher values of the generated voltage are obtained. Furthermore, we note that varying the electrical load resistance impacts the amplitude of the average harvested power indicating that some optimum electrical load resistance must exist, whereby the most power can be derived from the piezoelectric element(s). To determine this optimal electrical load resistance, we fix the excitation frequency to either the short- or open-circuit fundamental frequency for the cantilever configuration or simply the open-circuit case for the zigzag configuration and vary the electrical load resistance, as shown in Figure 9. Figure 9 illustrates that the optimum electrical load resistance for the cantilever system is in the range of 50–100 kΩ, depending on the tip mass value and in the 500–700 kΩ range for the zigzag system. The high optimum load resistance of the zigzag configuration is why there is no observable difference between open- and short-circuit conditions, as the optimum condition is already very near open circuit. The difference in optimal load resistance values between the cantilever and zigzag results from the size of the piezoelectric layer(s) and the considered connection between these layers. These two main factors determine, along with material properties, the capacitance of each harvester. The optimal load resistance can be approximated by the complex impedance of the piezoelectric, or $1/(\omega C)$, where C is the capacitance of the piezoelectric layer(s) and ω is the excitation frequency in rad/s. Here, it can be seen how a larger capacitance for the cantilever piezoelectric energy harvester with parallel connection and its higher natural frequency for different tip masses result in a smaller optimum load resistance values for all considered cases.

With the information gathered, a final comparison of power density or power per unit volume is presented in Table 1. The volume of the cantilever harvester is 0.51 cm^3, and the volume of the zigzag harvester is 0.195 cm^3. Table 1 summarizes the first bending frequencies and power densities of the cantilever bimorph and zigzag piezoelectric vibration energy harvesters for different tip masses. Clearly, depending on the available excitation frequency, the zigzag harvester is always significantly better in terms of low natural frequency and power density.

In summary, two piezoelectric energy harvesters (bimorph cantilever and unimorph zigzag shape) were developed (surface <7 cm^2). The results showed that tuned energy harvesters can be designed to harvest energy at low-frequency excitation (<100 Hz) and levels of harvested power were on the order of micro-Watts from different configurations with an input acceleration of 0.1 G. Depending upon the available excitation frequency, the tip mass and load resistance can be changed to harvest maximal amounts of power. It was demonstrated that the zigzag piezoelectric energy harvester can be a means of increasing power density for a given harvester area (footprint).

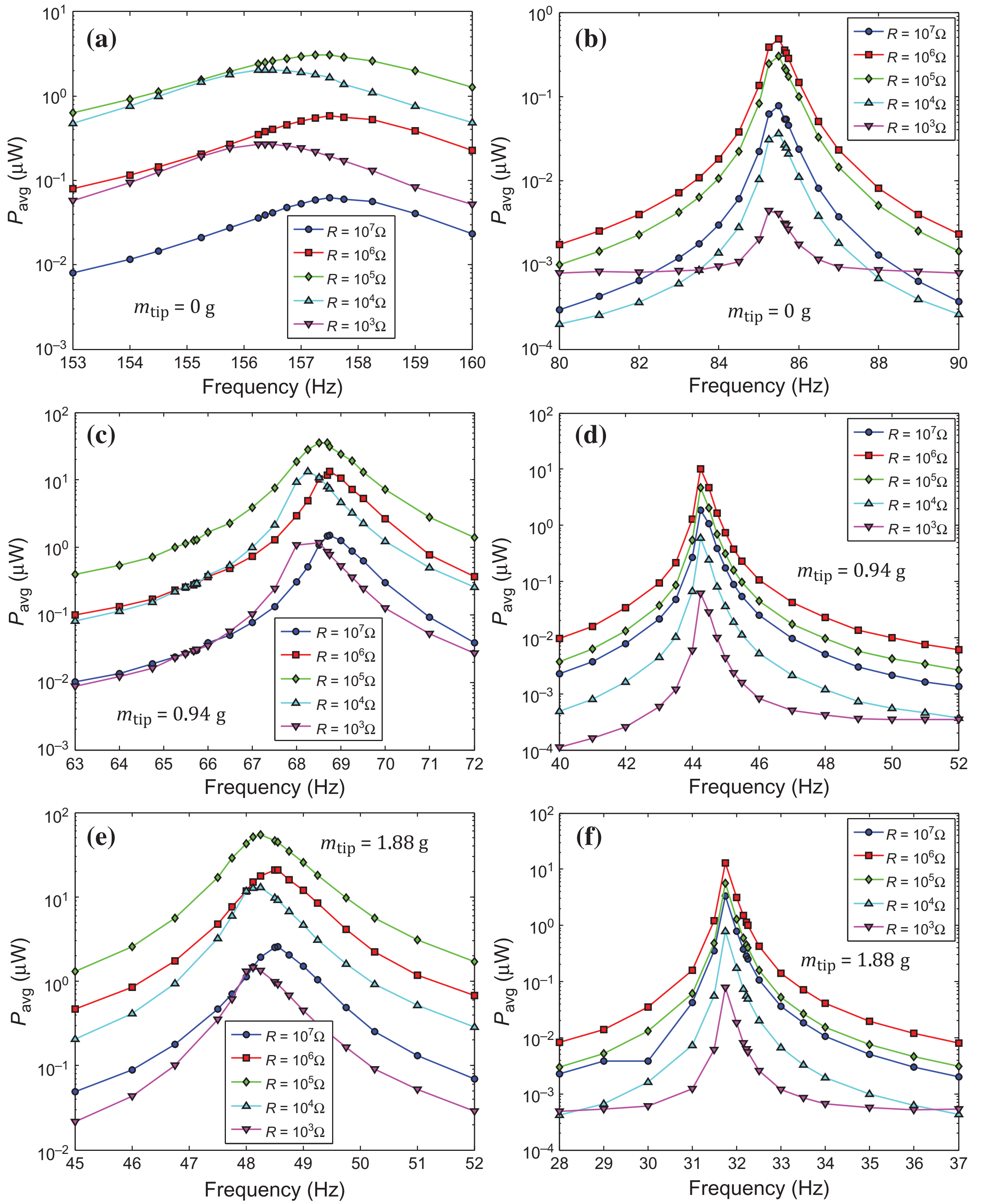

Figure 8 Average power output as a function of frequency, electrical load resistance, and tip mass for (a), (c), and (e) cantilever and (b), (d), and (f) zigzag harvesters, at a base acceleration of 0.1 G

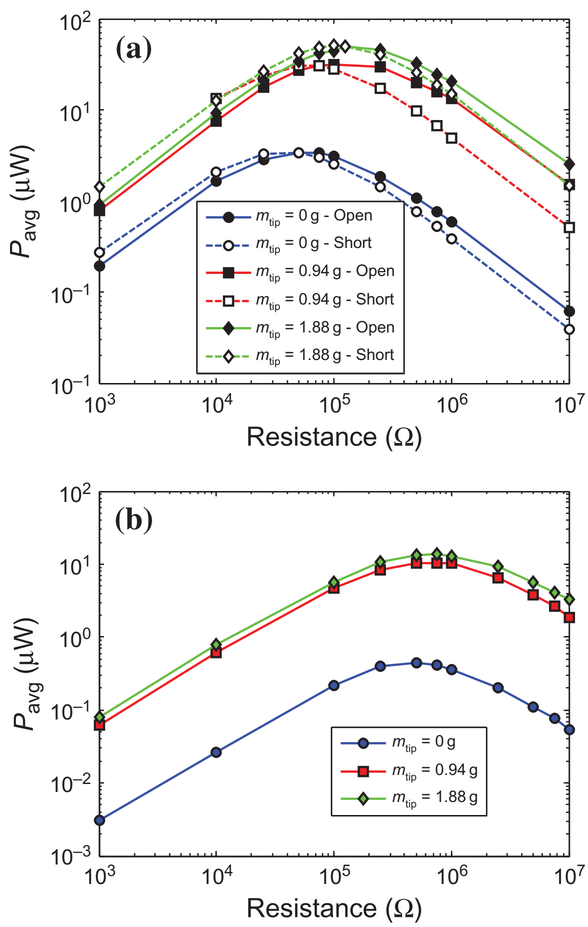

Figure 9 Average harvested power as a function of the electrical load resistance and tip mass at a base acceleration of 0.1 G, where for (a) the excitation frequency of the cantilever harvester is equal to the short- or open-circuit frequency and for (b) the excitation frequency of the zigzag harvester is equal to the open-circuit frequency. These frequencies were found from the Frequency Response Functions in Figure 5

Table 1 Summary of 1D cantilever bimorph and 2D zigzag unimorph piezoelectric energy harvesters power density as a function of excitation frequency and tip mass at 0.1 G base acceleration

Shape	Tip mass (g)	Natural frequency (Hz)	Power density (μW/cm^3)
Cantilever	0.00	157.50	0.73
	0.94	69.00	6.67
	1.88	48.75	10.60
Zigzag	0.00	85.78	2.13
	0.94	44.84	31.31
	1.88	32.34	65.08

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