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Research Article

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Modeling of Hybrid Piezoelectrodynamic Generators

Abstract: Vibrational energy harvesters are used for collecting electrical power from mechanical vibrations and supplying low power applications. Combining a piezoelectric generator with an electrodynamic generator creates a hybrid piezoelectrodynamic generator. In this paper, a model for this hybrid generator is proposed. The model allows the accurate simulation of the mechanical and electrical part of the generator. It also makes it possible to analyze the waveforms and phase relationships of the two electrical outputs of the hybrid generator and thus simulate power management circuits and their feedback on the generator.

Keywords: energy harvesting, modeling, electrodynamic generator, piezoelectric generator, hybrid

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Introduction

Vibrational energy harvesting transducers are used to convert vibrations of the ambient into electrical energy. Different types of transducers can be employed: piezoelectric, electrodynamic, and electrostatic generators are the ones that had significant attention in the last years.

The output power scales directly with the volume of the energy harvesters. Thus, increasing the power density of these generators is essential, as the volume of these devices is proportional to their price.

Challa, Prasad, and Fisher (2009) reported that a coupled piezoelectric–electromagnetic energy harvester can increase the power output density compared to a standalone piezoelectric or electrodynamic generator.

Another way to increase the overall output power of an energy harvesting system is by improving the power management circuits. In the case of a vibrational energy harvesting system, usually an AC/DC converter is employed to rectify the AC output voltage of the generator. Using active AC/DC circuits can increase the output power of the harvesting generator and that way increase the overall power density of the harvesting system, see Mateu et al. (2011) and Dayal, Dwari, and Parsa (2010).

Developing efficient active circuits requires the possibility to simulate the AC/DC converter in combination with the transducer. This article proposes an electromechanical model for the hybrid piezoelectrodynamic generator. This model allows the simulation of a combined electrodynamic and piezoelectric generator with the attached load electronics. This way, electric effects on the mechanical system can be simulated in a comprehensive way to help to develop power management circuits and the control strategies for maximizing the output power.

The structure of the paper is as follows. Section "Generator setup" describes the generator setup from which the equivalent circuit model is derived. Section "Modeling the piezoelectrodynamic generator" shows the model and a means of simulating the generator with correct phase dependencies. Section "Impedance calculation" describes the analytical calculation of the generator impedances. Section "Generator waveforms" shows voltage waveforms of the proposed simulation model and measurements of a piezoelectrodynamic generator. Finally, section "Conclusions" draws the main conclusions achieved in the paper.

Generator setup

The piezoelectrodynamic generator consists of a cantilever beam with the piezoelectric element attached to it. Usually, piezoelectric generators have a tip mass to change their resonance frequency. Now the tip mass consists of a permanent magnet. Placing a coil around this magnet introduces a second conversion principle, creating a vibration generator, which utilizes the piezoelectric and the electrodynamic effect.

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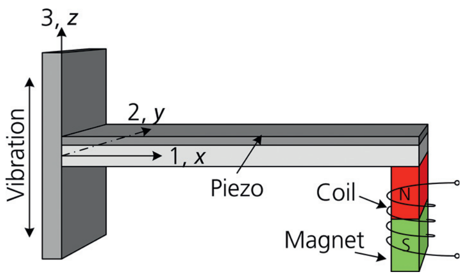

Figure 1 Setup of the piezoelectrodynamic generator

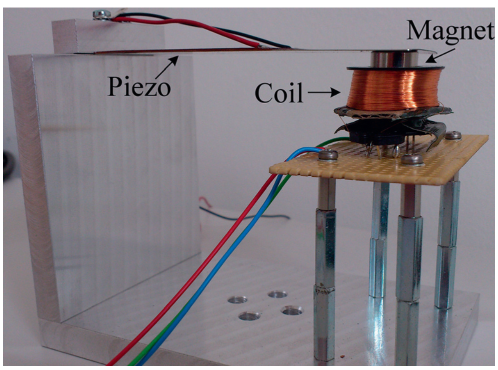

Figure 2 Piezoelectrodynamic generator prototype

The piezoelectric material is a modified lead-zirconate-titanate (PZT) compound by Pi Ceramic (DuraAct A12) with an active area of 61×35 mm^2. It is glued to a 97×30 mm^2 spring steel beam. The tip mass is a neodymium magnet (NdFeB) that weighs 10.3 g. The coil with the terminals for the electrodynamic generator part has 3,000 windings resulting in an electrical resistance of about 300 Ω and an inductance of 60 mH. It is made of 0.15 mm enameled copper wire.

Modeling the piezoelectrodynamic generator

A mechanical oscillation can be modeled by a mass, spring, and damper. Figure 3 includes the piezoelectric and electrodynamic part of the generator to depict both conversion principles.

The mechanical part of the electromechanical equivalent circuit in Figure 4(left) has been proposed by Brufeau-Penella and Puig-Vidal (2008) for a piezoelectric transducer. In this domain, a voltage corresponds to a force and a current to a velocity. I_{In} describes the base velocity of the harvester, where v_m corresponds to the tip

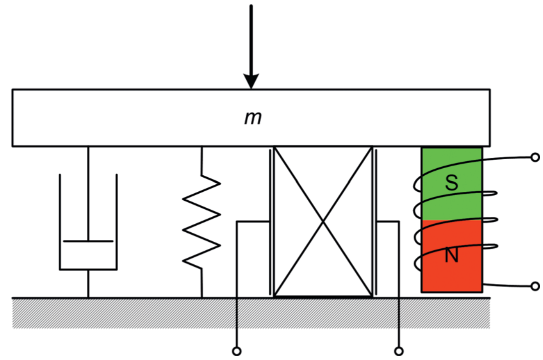

Figure 3 Mass–spring–damper system with piezo crystal and magnet with coil

velocity. Inductance ($L_{1,2m}$), capacitance (C_m), and resistance (R_m) represent mass, spring constant, and mechanical damping, respectively. The coupling coefficients are depicted as a transformer ratio (N_{PEG}) for the piezoelectric transducer and a gyration resistance (N_{EDG}) for the electrodynamic transducer in Figure 4.

Figure 4 (right) shows the electrical domain where the piezoelectric transducer is modeled with a capacitor (C_P). The electrodynamic transducer consists of an inductance (L_e) with a series resistance (R_e) due to the resistance of the windings. Load resistances ($R_{L,PEG}$, $R_{L,EDG}$) are connected to both outputs, and their corresponding output current (i_{PEG}, i_{EDG}) and voltage (U_{PEG}, U_{EDG}) are presented in the figure.

The right side of Figure 4 shows the electrical representation of the piezoelectric and electrodynamic generator. Through the transformer and gyrator, both have a feedback to the mechanical side, and damping from both electric outputs can be simulated.

The current v_m on the mechanical side represents the tip velocity of the generator. For the piezoelectric generator in open circuit, the maximum output voltage is reached, when maximum mechanical stress is applied to the piezoelectric element which means a maximum of force. This is the case at the turning point of the generator, where the speed and thus the current v_m is zero. With this physical correlation, it can be seen that the

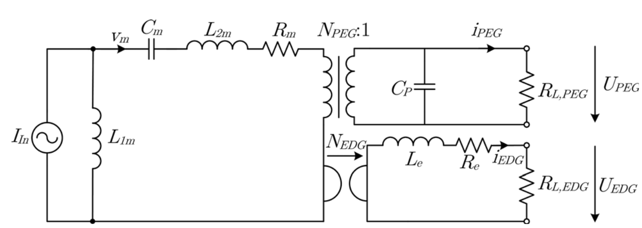

Figure 4 Equivalent circuit for piezoelectrodynamic generators

piezoelectric transducer has to be represented by a velocity–current and force–voltage relationship. This relationship can be achieved by a transformer element in the electromechanical equivalent circuit.

For the electrodynamic generator in open circuit, the maximum output voltage is reached when the change of the magnetic field through the coil is maximal. This is the case in between the turning points of the generator vibration, where the speed of the tip and thus the current v_m is maximal. This means the electrodynamic generator has to be represented by a velocity–voltage and force–current relationship.

The coupling element used to achieve this relationship is called a gyrator. A gyrator is a device which couples the current on one side to the voltage on the other side and vice versa. Cheng, Wang, and Arnold (2007) proposed this kind of coupling for the simulation of a standard electrodynamic transducer.

As the physical representation of v_m is the velocity of the magnet moving through the coil, this change of the magnetic field is represented as an induced voltage in the electrical domain of the generator.

The current flowing through the coil when a load is attached generates a force against the moving magnet. This force is represented by a voltage in the mechanical domain of the generator.

Figure 5 shows a possible representation for circuit simulators where the ideal transformer and the ideal gyrator are replaced by controlled voltage or current sources as follows.

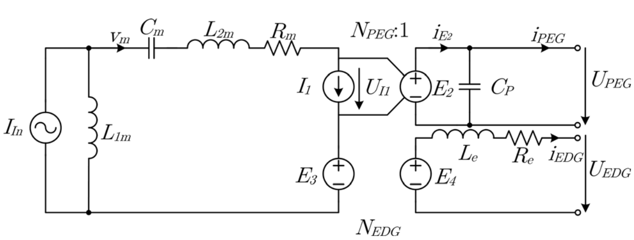

Figure 5 Simulation model for piezoelectrodynamic generators

The coupling between the mechanical domain and the electrical domain of the piezoelectric generator is simulated by an ideal transformer. I_1 is a current-controlled current source, where $I_1 = v_m = N_{PEG} \cdot i_{E2}$, and E_2, a voltage-controlled voltage source, where $E_2 = N_{PEG} \cdot U_{I1}$. This is a standard representation of an ideal transformer.

To accurately model the electromechanical physics of the system, the coupling between the mechanical domain and the electrical domain of the electrodynamic generator is represented by different controlled sources. Both E_3 and E_4 are current-controlled voltage sources, where $E_3 = N_{EDG} \cdot i_{EDG}$ and $E_4 = N_{EDG} \cdot v_m$.

Impedance calculation

The characteristic measurements of the piezoelectrodynamic generator can be performed with an impedance analyzer. By sweeping the frequency of an AC voltage and measuring the current, the Thévenin equivalent impedance of the piezoelectric and the electrodynamic part of the generator have been measured.

Using standard circuit analysis, the Thévenin equivalent impedances can be calculated by using the circuit in Figure 4. To calculate the Thévenin equivalent impedance of the piezoelectric part of the generator, the electrical part of the electrodynamic generator is transformed to the mechanical side of the model. Then, the combined impedance on the mechanical side is transformed to the electrical side of the piezoelectric generator leading to an overall Thévenin equivalent impedance in eq. [1].

$$Z_{PEG} = \frac{1}{\frac{N_{PEG}^2}{j\cdot\omega\cdot(L_{1m}+L_{2m})+\frac{1}{j\cdot\omega\cdot C_m}+R_m+\frac{N_{EDG}^2}{\left(j\cdot\omega\cdot L_e+R_e+R_{L,EDG}\right)}} + j\cdot\omega\cdot C_P} \quad [1]$$

The transformation with the gyrator is different from the well-known transformer equations. The transformation is always done by multiplying the inverse impedance with the square of the gyration resistance N_{EDG}.

The impedance of the electrodynamic generator can be calculated the same way, leading to the overall Thévenin equivalent impedance in eq. [2].

All variables can be found in Figure 4 and correspond to mechanical or electrical parts of the model. ω is the angular frequency used for the simulation.

$$Z_{EDG} = \frac{N_{EDG}^2}{\left(j\cdot\omega\cdot(L_{1m}+L_{2m})+\frac{1}{j\cdot\omega\cdot C_m}+R_m+\frac{N_{PEG}^2}{j\cdot\omega\cdot C_P+\frac{1}{R_{L,PEG}}}\right)} + R_e + j\cdot\omega\cdot L_e \quad [2]$$

These impedances were also measured at different load conditions. Figures 6 and 7 show the magnitude and phase of the impedance of the electrodynamic generator with the piezoelectric generator left in open circuit.

Figures 8 and 9 show the magnitude and phase of the impedance of the piezoelectric generator, with the electrodynamic generator short circuited.

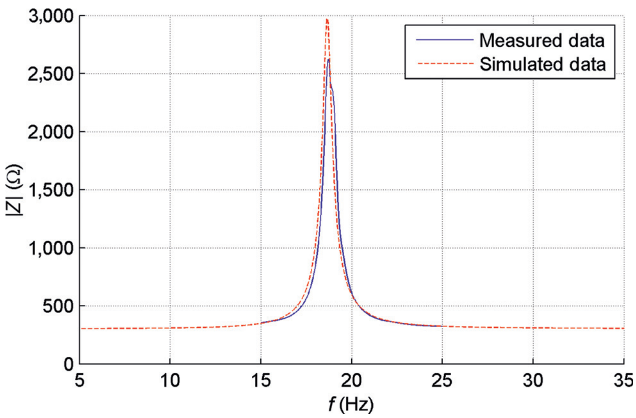

Figure 6 Modulus of the EDG impedance, PEG in open circuit

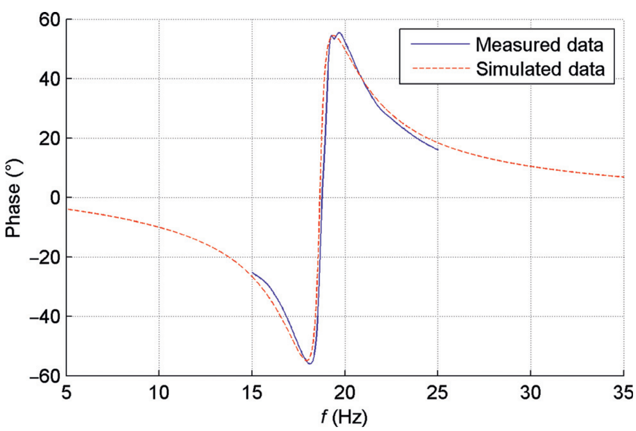

Figure 7 Phase of the EDG impedance, PEG in open circuit

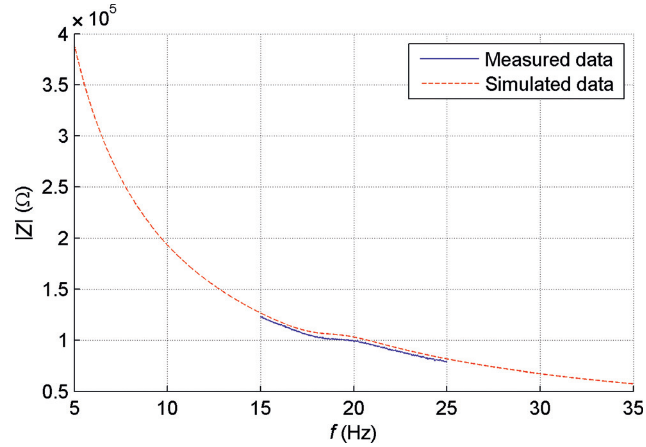

Figure 8 Modulus of the PEG impedance, EDG in short circuit

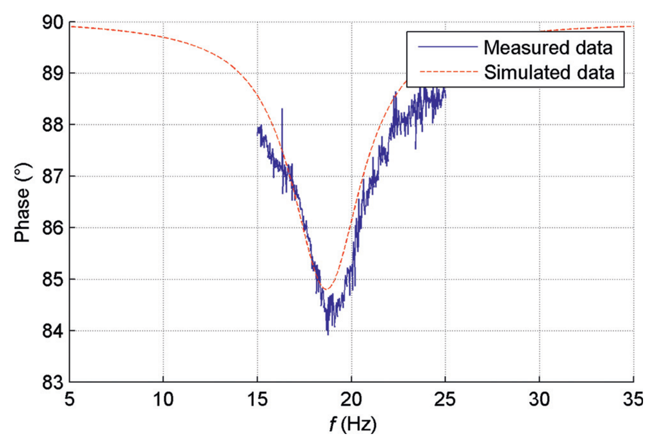

Figure 9 Phase of the PEG impedance, EDG in short circuit

Measuring and simulating either impedance in open circuit leads to the same result as if the generator would only use one of the two conversion principles. To determine if the feedback of the other conversion principle is modeled correctly, the measurement and simulation with a short circuited output was done.

The analytical result and the measured data agree, as can be seen in Figures 6–9, showing the accuracy of the proposed model. To simulate the impedance or voltage curves, the parameters of the circuit elements in Figure 4 are needed. The elements on the electrical side can be directly measured with an LCR meter and are listed in Table 1. The elements on the mechanical side of the model can be obtained by curve fitting the analytical impedance expression to the measured data. The values used for the simulation can be seen in Table 1. The first calculation for the curve fitting of the circuit parameters was performed with the formulas reported by Brufeau-Penella and Puig-Vidal (2008) using the physical dimensions and material coefficients of the generator.

Table 1 Circuit parameters

L_{1m}	L_{2m}	C_m	R_m	
6.37 mH	−311 μH	12.0 mF	17.6 mΩ	
N_{PEG}	N_{EDG}	C_P	L_e	R_e
384 μ	6.86	80.0 nF	60.0 mH	302 Ω

Generator waveforms

This section describes the waveforms of the generator voltages for different load conditions. In Figure 10, the 90° phase shift between the open circuit voltage of the piezoelectric and the electrodynamic generator can be seen. All waveforms have been normalized so that the peak voltage of the piezoelectric generator is one. The time axis shows two full periods.

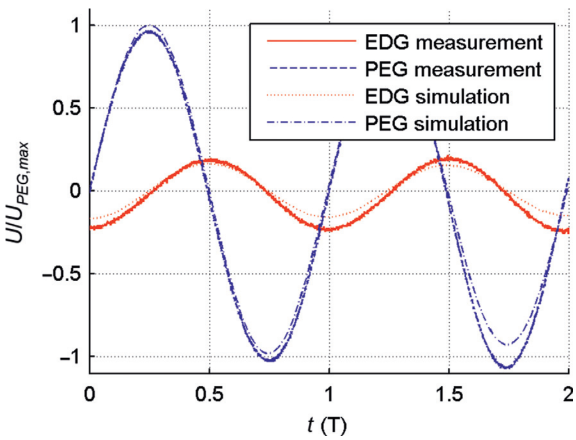

Figure 10 Open circuit voltages of the piezoelectric generator (PEG) and electrodynamic generator (EDG)

In Figures 11–13, voltages for different load conditions are measured and simulated. The difference between the measured and simulated waveforms is due to a not completely symmetric measurement, maximum and minimum do not have the exact same amplitude. Also the attached load resistance is not optimal. A load resistance is considered optimal when the maximum power is extracted from the generator. When the transducers are not in open circuit, the load on the piezoelectric generator is 120 kΩ and the load on the electrodynamic generator is 2 kΩ.

The phase shift between the generators is zero (or 180°, depending on the connection of the voltage probe) when a significantly high load is attached to the piezoelectric generator (Figures 12 and 13). A load attached to the electrodynamic generator does not change the phase shift.

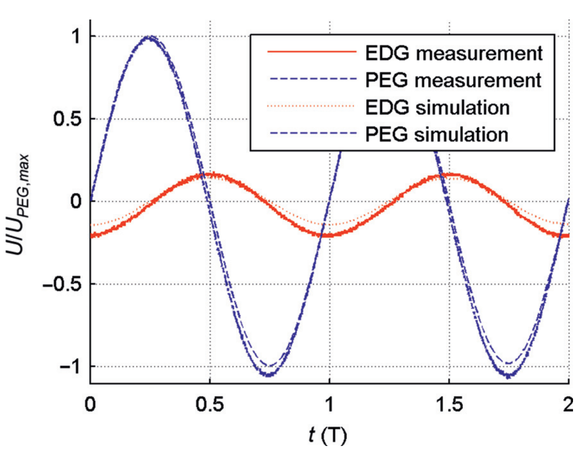

Figure 11 PEG in open circuit, EDG loaded with 2 kΩ

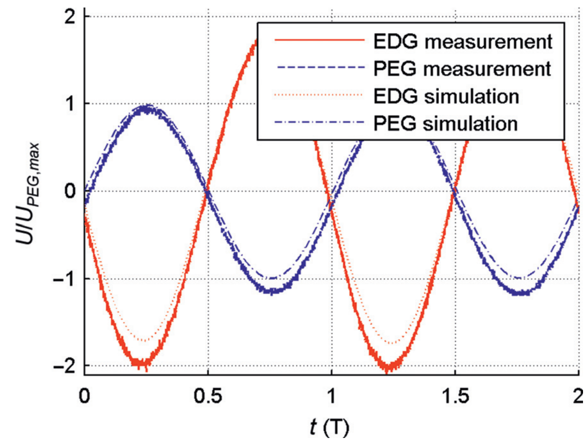

Figure 12 PEG loaded with 120 kΩ, EDG in open circuit

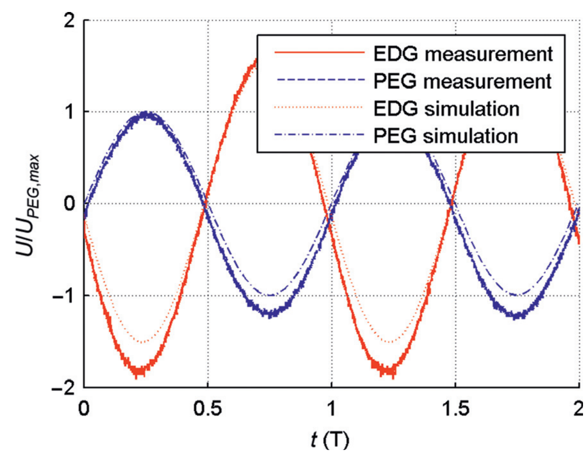

Figure 13 PEG loaded with 120 kΩ, EDG loaded with 2 kΩ

The models that provide these waveforms are essential for developing sophisticated power management circuits. Controlling these circuits and generating the needed control signals requires knowledge about the behavior of the generator voltages.

Conclusions

An electromechanical model of a hybrid piezoelectrodynamic generator has been presented. The model has been created to accurately simulate the behavior of both electrical outputs of the hybrid generator.

The analysis of the impedances of the generator was done and verified by comparing the results with the measured impedance curves. It is possible to characterize the generator just by measuring the impedances and identifying the values of the circuit elements by comparing the measured curves to the analytic expressions.

The correct electrical simulation of the phase and magnitude relationships between the piezoelectric and electrodynamic voltage output under different load conditions has been shown and compared to measurements.

With this model of the hybrid generator, it is possible to simulate non-linear power management circuits and analyze their effect on the mechanical system. This way, more sophisticated circuits can be developed and non-linear effects of switching converters can be taken into account.

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