Research Article

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Improving Calculus Learning Using a Scientific Calculator

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Abstract: This article discusses the use of a scientific calculator in teaching calculus by using representations of mathematics notions in different sub-languages (analytical, graphical, symbolical, verbal, numerical and computer language). Our long-term experience shows that this may have a positive and significant effect on the enhancement of conceptual understanding of mathematical concepts and approaches. This transcends the basic computational uses, and implies a potential for real improvement in the learning success, cognitive motivation and problem solving skills of the student. We illustrate the steps we have taken towards doing this through some examples.

Keywords: scientific calculator; calculus teaching; different representations; conceptual understanding; cognitive motivation.

1 Introduction

Despite the near half-century of their widespread presence in schools, colleges and universities, educators have yet to agree on the best way to use electronic pocket calculators in teaching. For example, some claim that the use of electronic pocket calculators harms mathematics learning, while others are indifferent to their presence and are not fully aware of recent steps in their development. Indeed, many school and university teachers simply ignore the possible benefits of calculator use in teaching mathematics, or other fields of STEM (Kissane and Kemp, 2013). In academic studies, relatively few educators pay enough attention to the broad educational opportunities made possible with calculators. This includes not only the basic computational tools, but also their potential use in enhancing conceptual understanding of mathematical concepts and approaches, essential for developing research and critical thinking by the student.

This is very important nowadays, because academic institutions have been dealing with the failure of students in the first year of their studies in general and especially in mathematical courses (Lowe and Cook, 2003; Yorke and Longden, 2004). With technological advances and a large flow of students to STEM professions at universities and colleges, there is a constant decline in the mathematical basic knowledge of the novice academic students (Gueudet, 2013; Bosch, Fonseca & Gascon, 2004). Undergraduate students have difficulties in understanding definitions and various representation of mathematical concepts. It is because teaching and learning in high school is still based more on algorithmic exercises and memorization, rather than on deeper understanding of mathematical language and using its computer applications in solving complex and interesting problems. Using the scientific calculator may contribute to a reduction of this gap between enough developed algorithmic skills of novice engineering students and insufficiency of research, creative and critical mental skills, required in our current academic studies.

2 Pedagogical strategy

Our main pedagogical approach is based on the active use of different representations of mathematics notions and problems through diverse sub-languages (analytical, graphical, symbolical, verbal, numerical and computer language) of mathematics (Dagan et al., 2018, 2019b). For convenience, we will designate these basic forms of representations by means of the capital first letters of the appropriate words:

A – Analytical representation
V – Verbal representation

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G – Graphical representation
N – Numerical representation
C – Computer representation (Calculator depiction, in the context of this article)

For example, the problem: "Calculate 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10" formulated in numerical form and required numerical answer is NN representation of the problem in this classification. If we want to present it in computer language (in CASIO fx-991ES PLUS sub-language, in the context of this article) we must submit it in CN form: "Calculate $\sum_{x=1}^{10}(x)$".

Verbal representation (VN) of the same problem will be "Calculate the sum of natural numbers from 1 to 10". Graphical representation (GN) of the same problem will be "Calculate the area of the figure shown on figure 1".

Our hypothesis, based on long-term pedagogic research and teaching experience, is:

> *"The permanent and systematic use of different sub-language representations, and various transitions among them, in a calculus course, in addition to extensive use of scientific calculators and graphic applications, yields greater achievement and a higher level of understanding, while enhancing cognitive motivation and creative thinking of students".* Such approach is in accordance with the requirements of 21 century that needs profound changes in the teaching of mathematics and other STEM disciplines for engineering students.

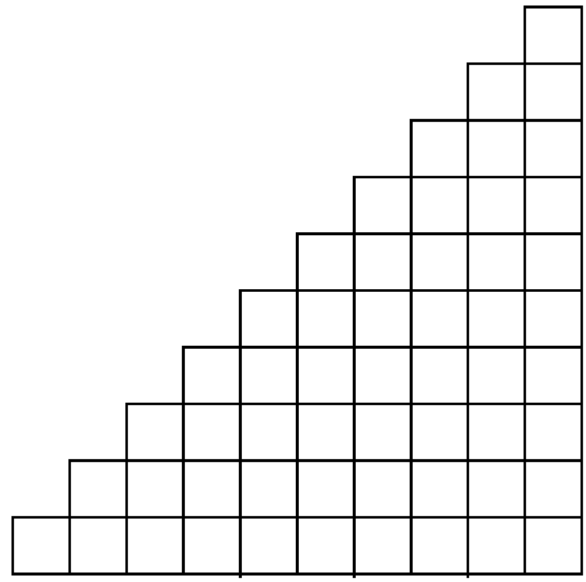

Figure 1: Graphical representation of the sum $\sum_{x=1}^{10}(X)$.

3 Using calculators in teaching calculus

In most study programs, the calculus course plays a central role. After many years of experience in teaching calculus, we have found that the calculator presents opportunities for significantly developing exploratory and critical thinking by the student. In this paper, we will describe some of our findings in this direction. Since most of our students have worked with the fx-99IES PLUS CASIO calculator, which allows for mid-term tests, final exams and high school exams, we have used this calculator in all our examples. As is the case for many advanced calculators, the lecturer will play a key role in utilizing the full power of this device that transcends its basic computational functions.

In a survey of lecturers in our department, several years ago, we found that most lecturers did not use this calculator in the teaching process and some of them were not even familiar with it and its potential for the teaching and learning of the calculus course (as well in others mathematical disciplines). To correct this situation, we organized special seminars dedicated to the use of the *fx*-99IES PLUS CASIO calculator in teaching of mathematics for engineering students. We treat this calculator as an interactive math dictionary and, as regards the calculus course, as a modern machine for evaluating elementary functions, and, more generally, as a convenient device for storing and processing numerical information. This approach has changed the attitudes of students and teachers regarding the use of the scientific calculator, not only in the calculus course, but also in other math courses and engineering disciplines. Note, that the skillful use of the calculator not only improves the student's exam results, but also enhances the understanding of the key ideas of calculus, as well as students' motivation and interest in studying them.

4 Classroom teaching examples

Consider an example involving simply the skillful use of the scientific calculator.

Students are asked to evaluate the following expression:

$$2.018^{2.019}+\frac{2.018}{2.018+2.019}-\frac{\sqrt{2.018\times 2.019}}{2.018^2+2.019^2}$$

Although all students had the *fx*-991ES PLUS CASIO calculator, they did this task in a direct "arithmetic" manner, step by step. This appears to be a routine calculation, requiring attention and numerous keystrokes, and no more than that. Only rarely did we observe a student who might be familiar with the useful algebraic options of the calculator, before the teacher's explanation. This explanation begins by inviting the student to enter the following algebraic expression in the calculator:

$$A^B + \frac{A}{A+B} - \frac{\sqrt{AB}}{A^2+B^2}$$

After that, we ask the student to press the key "CALC". What does the calculator do? It requests the value of A and then, after pressing the key "=", the value of B, and compute the numeric value of algebraic expression for the given values of A and B.

After that, we invite the students to evaluate the effectiveness of algebraic approach by asking the question: "How much faster will the calculation be via the algebraic expression, than by direct arithmetic calculation". At the beginning, most students do not understand how to evaluate this difference. In this case, there is data entry time. We ask students to estimate this time both theoretically and practically, comparing two data entry methods. To clarify the task we ask the question: "What is an elementary action with a calculator?" The correct answer is "pressing any calculator key". How many clicks will it take directly calculating the given arithmetic expression? The answer is 9 × 5 + 12 = 57 (45 clicks to enter 9 numbers and 12 clicks for arithmetic operations including "="). In addition, how many clicks are required to input an algebraic expression? The answer is 9 × 2 + 10 = 29 (18 clicks to enter 9 letters and 10 clicks for arithmetic operations. In fact, after that, you will need to press the key "CALC" and twice the key "=" and then to enter the numbers 2.018 and 2.019 and to press the key "=". There are ultimately 13 extra clicks. Therefore, computing in the algebraic way will require 42 clicks. One may wonder, what is the significant advantage of the algebraic way? It is important to understand the major difference in the reliability of these two methods. Firstly, we immediately see whether the algebraic expression is entered correctly, secondly, the possibility of error when entering two numbers is much less than when entering nine numbers, thirdly we can use this formula for any other numbers, fourthly, future engineers need to consider these issues for the development of research thinking.

We ask students to calculate the sum.

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + 25^3$$

We ask students to do this with their calculators and the first one, who correctly completes this task, will win this competition. Our experience shows that not many students are familiar with the summation operator $\sum$, which is available in this device. Therefore, in most cases, students trying calculate this sum directly, as is written in the expression, term-by-term. Such a way takes essential time and effort, and the hidden meaning of the task is not clear to students. After that, we have a good opportunity to show students the strength and convenience of the $\sum$ operator. By using $\sum$ the calculator gives an answer immediately $\sum_{x=1}^{25} X^3 = 105\,625$.

In addition, what about the sum $1^3 + 2^3 + 3^3 + 4^3 + \ldots + 1000^3$?

Now students see with surprise for the first time, that the calculator does not give an answer immediately. Moreover, what about the sum $\sum_{x=1}^{10000} X^3$? It takes some minutes to calculate this enormous sum! After that, we ask students to calculate $\sqrt{Ans}$ and instead of a cumbersome result, $\sum_{x=1}^{10000} X^3 = 2.500500025\times10^{15}$ they see the nice natural number 50005000. This is something that makes you think.

We invite students to find a formula for quick summation $\sum_{x=1}^{A} X^3$ for any natural number *A* (*AA* problem according to our classification). We enter the expression $\sum_{x=1}^{A} X^3$ into the calculator and using the "CALC" key start to sequentially calculate the sums and record the results obtained:

$$A=1:\ 1^3 = 1 \to \sqrt{Ans} = 1$$
$$A=2:\ 1^3+2^3 = 9 \to \sqrt{Ans} = 3$$
$$A=3:\ 1^3+2^3+3^3 = 36 \to \sqrt{Ans} = 6$$
$$A=4:\ 1^3+2^3+3^3+4^3 = 100 \to \sqrt{Ans} = 10$$
$$A=5:\ 1^3+2^3+3^3+4^3+5^3 = 225 \to \sqrt{Ans} = 15$$
$$A=6:\ 1^3+2^3+3^3+4^3+5^3+6^3 = 441 \to \sqrt{Ans} = 21$$

We see that for each *A* the sum is an exact square and we can only guess the squares of which numbers. It is easy to see that they are the squares of the sums of the bases of the cubed numbers:

$A=1:\ 1^3=1=1^2$
$A=2:\ 1^3+2^3=3^2=(1+2)^2$
$A=3:\ 1^3+2^3+3^3=6^2=(1+2+3)^2$
$A=4:\ 1^3+2^3+3^3+4^3=10^2=(1+2+3+4)^2$
$A=5:\ 1^3+2^3+3^3+4^3+5^3=15^2=(1+2+3+4+5)^2$
$A=6:\ 1^3+2^3+3^3+4^3+5^3+6^3=21^2=(1+2+3+4+5+6)^2$

Using the formula $1+2+3+4+\ldots+A=\dfrac{A(A+1)}{2}$, known from high school, we obtain

$$1^3+2^3+\ldots+\mathrm{A}^3=(1+2+\ldots+\mathrm{A})^2=$$
$$=\left(\frac{\mathrm{A}(\mathrm{A}+1)}{2}\right)^2=\frac{\mathrm{A}^2(\mathrm{A}+1)^2}{4}$$

Now we can even compete with the Sigma Operator (S) of the CASIO calculator and leave it far behind in calculating the sum:

$$\sum_{x=1}^{10\,000} X^3=(5\,000\times 10\,001)^2=(50\,005\,000)^2$$

Alternatively, even $\sum_{x=1}^{1000000} X^3$ and so on. Rigorous proof of this formula, obtained by analyzing particular results, is easy to get using the mathematical induction method.

5 Approximate calculation of sums using integrals

Let $f(x)$ be a decreasing and continuous function in the domain $x\geq 1$, then from figure 2 it is easy to understand that $\sum_{k=1}^{k=N} f(k)$ is equal to the sum of the areas of N rectangles depicted in figure 2.

The sum of the areas of these rectangles is more than the area under the curve $y=f(x)$ on the segment [1, N+1]. i.e. the following inequality holds:

$$\sum_{k=1}^{k=N} f(k) > \int_1^{N+1} f(x)dx$$

On the other hand, the sum of the excess areas of N rectangles above the curve $y=f(x)$ i.e. $\int_1^{N+1} f(x)dx$, is the sum of areas of N "curved triangles" in figure 3, in that all of them can be moved into the first rectangle (the highest of all) which has an area equal to $f(1)$.

Therefore, the following inequality holds:

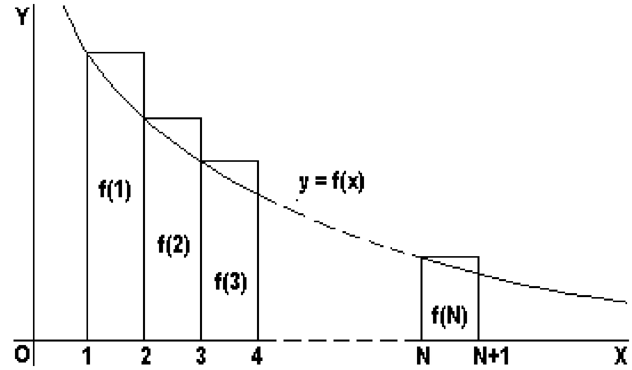

Figure 2: Graphical representation of the sum $\sum_{k=1}^{N} f(k)$.

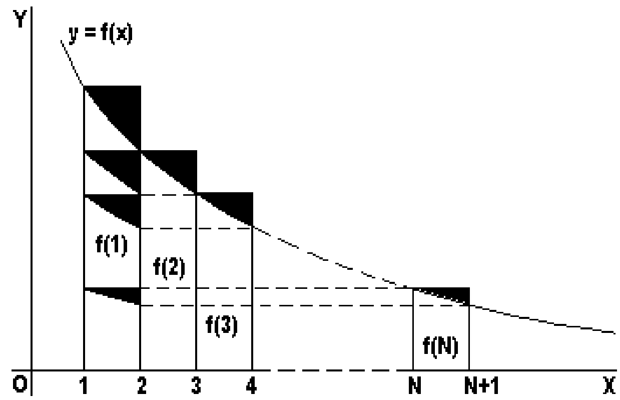

Figure 3: Graphical representation of the differences $\sum_{k=1}^{k=N} f(k) - \int_1^{N+1} f(x)dx$.

$$\sum_{k=1}^{k=N} f(k) - \int_1^{N+1} f(x)dx < f(1)$$

Thus, we arrive at a double inequality:

$$\int_1^{N+1} f(x)dx < f(1)+f(2)+f(3)+\ldots+$$
$$+f(N) < \int_1^{N+1} f(x)dx + f(1)$$

As example, for $f(x)=1/x$, we get:

$$\ln(N+1)=\int_1^{N+1}\frac{dx}{x} < 1+\frac{1}{2}+\frac{1}{3}+\ldots+$$
$$+\frac{1}{N} < \int_1^{N+1}\frac{dx}{x}+1=\ln(N+1)+1$$

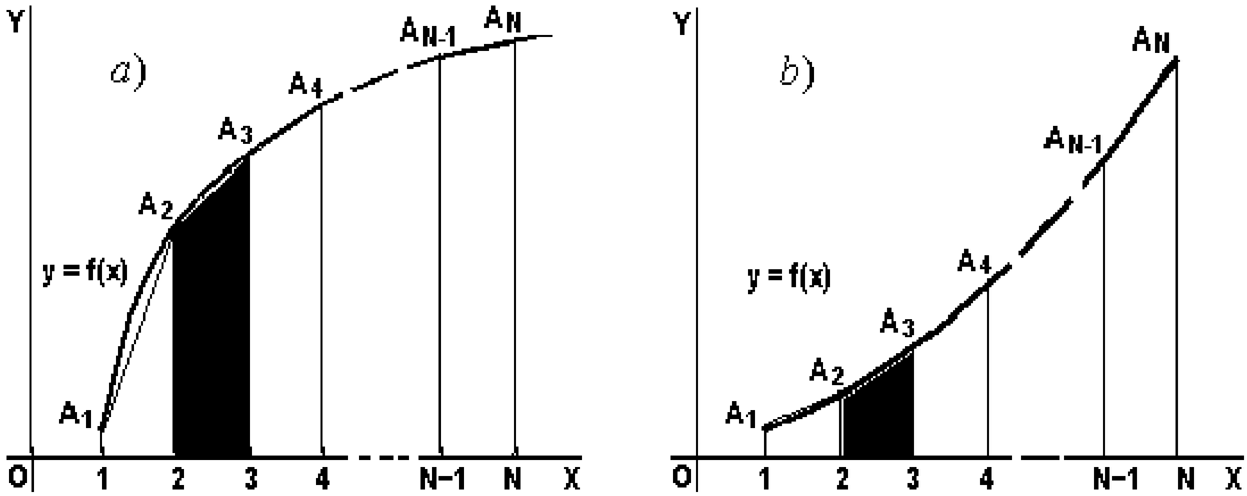

Figure 4: Approximation of the sum $\sum_{k=1}^{N} f(k)$ as sum of areas of N trapezoids.

Since for large values of N the differences between $\ln(N+1)$ and $\ln(N)$ is very small and tends to zero, we get very simple approximate formula for large values of N:

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N}\approx\ln(N)$$

Denote the direct calculation result, using Σ operator of CASIO, as S_N

$$S_N=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N}=\sum_{x=1}^{x=N}\frac{1}{X}$$

Denote approximate result as A_N, that in this case is $A_N = \ln N$.
We estimate the relative error of approximation by formula:

$$\delta_N(x)=\left|\frac{A_N-S_N}{S_N}\right|\cdot 100\%$$

For N=100 the direct calculation by CASIO calculator of the sum $\sum_{x=1}^{x=N} X^{-1}$ we get

$$S_{100}=5.1873;\ A_{100}=\ln 100\approx 4.6052 \qquad \delta_{100}=11\%$$
$$S_{1000}\approx 7.4855;\ A_{1000}=\ln 1000\approx 6.9078 \qquad \delta_{1000}=7\%$$
$$S_{10000}\approx 9.7876;\ A_{1000}=\ln 10000\approx 9.2103 \qquad \delta_{10000}=5.9\%$$
$$S_{20000}\approx 10.4807;\ A_{1000}=\ln 20000\approx 9.9035 \qquad \delta_{20000}=5.5\%$$

Note that it needs about a quarter of hour for CASIO using direct calculation for N=20000.

Geometric representation of the sum $f(1) + f(2) + f(3) + \ldots + f(N)$ as the appropriate number of rectangles is very easy, but we will get a better integral approximation if we think about the sum of the appropriate trapezoid areas. An integral approximation of the sum $f(1) + f(2) + f(3) + \ldots + f(N)$ will be better if we think about the sum of the appropriate trapezoid areas (Figure 4).

Sum of areas of N trapezoids depicted in the figure 4 equals to

$$\frac{f(1)+f(2)}{2}+\frac{f(2)+f(3)}{2}+\ldots+\frac{f(N-1)+f(N)}{2}=$$
$$f(1)+f(2)+\ldots+f(N)-\frac{f(1)+f(N)}{2}$$

This number is close to the integral of the function $f(x)$ on the segment $[1, N]$

1) $f(1)+f(2)+\ldots+f(N)-\frac{f(1)+f(N)}{2}\approx\int_1^N f(x)dx$

Therefore, we get the approximate formula:

2) $f(1)+f(2)+\ldots+f(N)\approx\int_1^N f(x)dx+\frac{f(1)+f(N)}{2}$

Note that for the function concave downward (Figure 4*a*)

3) $\sum_{k=1}^{k=N} f(k)<\int_1^N f(x)dx+\frac{f(1)+f(N)}{2}$

In addition, for the function concave upward (Figure 4*b*)

4) $\sum_{k=1}^{k=N} f(k)>\int_1^N f(x)dx+\frac{f(1)+f(N)}{2}$

As example for the sum, $\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots+\sqrt{N}$ we obtained double inequality:

Table 1: Outcomes of direct and approximate formulas and calculation time.

N	100	1000	10000	20000
$\sum_{k=1}^{N}\sqrt{k}$	671.46	21 097.45	666 716.46	1 885 688.59
Approximate Formula 5	671.5	21 097.50	666 716.50	1 885 688.63
Direct calculation time	4 sec.	34 sec.	330 sec.	615 sec.

Table 2: Outcomes of questionnaire 1.

Knowing the calculator	1	2	3	4
Number of teachers	14	10	6	5

5) $$\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots+\sqrt{N}\approx\int_{1}^{N}\sqrt{x}dx+\frac{1+\sqrt{N}}{2}\Rightarrow$$

$$\Rightarrow\sum_{k=1}^{k=N}\sqrt{k}\approx\frac{2}{3}N\sqrt{N}+\frac{\sqrt{N}}{2}-\frac{1}{6}$$

In Table 1, you can see the direct outcomes (by using operator S of CASSIO fx-991ES Plus calculator) and approximate outcomes (by using formula 5) calculation of this sum for some values of N, as well as the calculation time.

Comparison of approximate results (by using formula 5) with direct computation outcomes (by using operator S of CASSIO fx-991ES Plus calculator), shows the great accuracy of that approximate formula.

6 Research questions and assessment experiment

Our research questions were:
1. To what extent do lecturers know and use the calculator in teaching calculus?
2. To what extent do students know this calculator?
3. To what extent use of the calculator helps you in calculus learning (in the student's opinion)?

Concerning the first research question, Questionnaire 1 was completed by 35 teachers of the mathematical department of Shamoon College of Engineering, in the winter semester 2017.

Questionnaire 1. To what extent are you familiar with the CASSIO fx-991ES Plus calculator?
1. I don't use this calculator and I am not familiar with this devise.
2. I use this calculator for computation arithmetic operations only.
3. I am familiar with some basic options of this devise.
4. I am familiar with most of options of this devise and use them.

Questionnaire 2. To what extent do you use CASSIO fx-991ES Plus in teaching calculus?
1. I don't use calculator in the teaching process.
2. I use this calculator to check computation carried out during the lesson.
3. I show students algebraic possibilities of this devise, and how to use them.
4. I constantly demonstrate the great opportunities of this devise in calculus studies.

Concerning the second research question, Questionnaire 3 was completed by Building Department students of Shamoon College of Engineering at the beginning of the first semester of their academic studies (two groups, A and B, with 50 students each).

Questionnaire 3. To what extent are you familiar with the CASSIO fx-991ES Plus calculator?
1. I use this calculator for arithmetic computation only.
2. I am familiar with some basic modes of this devise.
3. I use the algebraic possibilities of this devise.
4. I am familiar with most of the options of this devise.

As an example of our assessment experiment of how advanced calculator knowledge helps in solving computation problems of calculus, we describe the same 15 minutes test, consisting of two questions, two groups of first year engineering students (50 students in each group) mentioned above. In one of the groups, (A), the teacher paid particular attention to the opportunities of the calculator in solving calculus computation problems, while in the other group, (B), the teacher paid no attention

Table 3: Outcomes of questionnaire 2.

Using the calculator in teaching of calculus	1	2	3	4
Number of teachers	20	7	6	2

Table 4: Questionnaire 3 (group A).

Knowing the calculator	1	2	3	4
Number of students	22	18	9	1

Table 5: Questionnaire 3 (group B).

Knowing the calculator	1	2	3	4
Number of students	24	21	5	0

Table 6: Questionnaire 4.

	1	2	3
Group A	0	18	32
Group B	31	12	7

to the skillful use of the calculator. The initial level was the same for these groups (see tables 4, 5).

Question 1. Find the slope of the tangent line to the graph of the function $f(x)$ at the point x=3

$$f(x) = 2^{3^{\sqrt{x}}}$$

Question 2. Calculate the integral:

$$\int_2^4 \left(\sqrt{x} + \frac{1}{x} - \frac{1}{x^2+1} \right) dx$$

These were standard questions for both groups and with all symbolic calculations students of each group succeeded equally but while in "calculator expert learned group" (A) all students did the numerical calculation correctly and quickly, the students in the other group (B) made a lot of errors and many of them did not have enough time to complete these numerical calculations. For Question 1, the symbolic calculations result is

$$f'(x) = 2^{3^{\sqrt{x}}} \ln 2 \cdot 3^{\sqrt{x}} \cdot \ln 3 \cdot \frac{1}{2\sqrt{x}}$$

The students of the "calculator expert" group (A) found the value of $f'(3)$ using the CASIO fx-991ES PLUS option $\frac{d}{dx}[\ ]_{x=[\ ]}$ and obtained $\frac{d}{dx}\left[2^{3^{\sqrt{x}}}\right]_{x=[3]} \approx 153.773$ and the students of the "calculator reject" group (B) calculated the value of expression:

$$2^{3^{\sqrt{3}}} \ln 2 \cdot 3^{\sqrt{3}} \cdot \ln 3 \cdot \frac{1}{2\sqrt{3}}$$

For the Question 2, the symbolic calculations result is:

$$\int_2^4 \left(\sqrt{x} + \frac{1}{x} - \frac{1}{x^2+1} \right) dx = \left(\frac{2}{3} x^{\frac{3}{2}} + \ln|x| - \arctan(x) \right)_2^4$$

The students of "calculator expert" group (A) found the numerical value of integral using the CASIO fx-991ES option $\int_{[\ ]}^{[\ ]}[\ ]dx$ and obtained

$$\int_{[2]}^{[4]} \left[\sqrt{x} + \frac{1}{x} - \frac{1}{x^2+1} \right] dx \approx 3.922$$

The students of "calculator reject" group (B) calculated two values and then their subtraction:

$$\left(\frac{2}{3} 4^{\frac{3}{2}} + \ln|4| - \arctan(4) \right) - \left(\frac{2}{3} 2^{\frac{3}{2}} + \ln|2| - \arctan(2) \right)$$

That takes more time (if the algebraic features of CASIO fx-991ES PLUS is not used) and so often leads to mistakes.

Concerning the third research question, questionnaire 4 was completed by the students of groups A and B, mentioned above, at the end of the first semester.

Questionnaire 4. To what extent does use of the calculator help you in calculus learning?

1. I use this calculator for arithmetic computation and no more.
2. It helps me to check the results and detect errors.
3. It helps me not only check errors but also to understand the notion of calculus.

7 Conclusions

Our long-term experiences of use of the scientific calculator in teaching calculus, and our assessment experiment results show, that permanent and systematic use of the scientific calculator yields greater achievement and a high level of understanding, while enhancing cognitive motivation and creative thinking of students. Note, that we always observed surprise and admiration of students, when they see unexpected possibilities of mathematical and engineering thinking in the process of using their scientific calculator. The above examples and numerous others, demonstrate to students the skillful use of the calculator when studying each topic and each concept of the Calculus course. Note that we also encourage engineering students to investigate the calculator as a computation machine. For example, we direct them to think about calculation speed of this device for those or other computation problems, and about the possible algorithms used by it in these calculations. In this way, the calculator has become an integral part of the studies of Calculus and other mathematics courses we teach.

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