Research Article

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Calculators as Facilitators of Understanding Computational and Mathematical Contexts

https://doi.org/10.1515/edu-2019-0012
received April 17, 2019; accepted October 15, 2019.

Abstract: Learning is a complex phenomenon. Contemporary theories of education underline active participation of learners in their learning processes. One of the key arguments supporting this approach is the learner's simultaneous and unconscious development of their ability of "learning to learn". This ability belongs to the soft skills highly valued by employers today.

For Mathematics Education, it means that teachers have to go beyond making calculations and memorizing formulas. We have to teach the subject in its social context. When the students start understanding the relationship between real-life problems and the role of numbers and formulas for their solutions, their learning becomes a part of their tacit knowledge. Below we explain the theoretical background of our approach and provide examples of such activities.

Keywords: calculator; mathematics education; computational; mathematical context.

1 Introduction

The main role of education in a classroom is to prepare learners for their future life. Contemporary theories of education underline their active participation in the processes. One of the key arguments supporting this approach is the learner's simultaneous and unconscious development of their desire of "learning to learn" (Rábeková & Hvorecký, 2015). This competence is defined by Educational Council (2006) as "the ability to pursue and persist in learning, to organize one's own learning, including through effective management of time and information, both individually and in groups". The learning to learn competence belongs among today's most frequent requirements of employers (Pirrie & Thoutenhoofd, 2013). They expect their employees to be capable of adapting to their changing environment and learning new facts and procedures. For education it implies that instead of memorization of formulas and theorems they should be steered towards problem-solving and creativity. It does not mean that formulas and theorems should be excluded from the courses. On the contrary, they must be incorporated into the problems in a way which will clarify their meaning and importance i.e. their context. For example, Lovászová & Hvorecký (2002) show that one – virtually the same – problem can be solved using several mathematical methods depending on expectations of their results.

Unfortunately, the capability of finding an applicable method is often misunderstood. Learners are tempted to apply their existing knowledge and produce a result regardless of its "price" and meaning. It may end in generating a(ny) numerical outcome regardless of the logic behind it. In this context, (Greer, 1997) demonstrates his student's solution to the following problem: "There are 125 sheep and 5 dogs in a flock. How old is the shepherd?" The student used the following reasoning: "125+5=130... this is too big, and 125-5 is still too big... while ... $\frac{125}{5}$= 25 ... that works... I think that the shepherd is 25 years old." This example demonstrates an inconsistency between two principal types of knowledge defined by Knowledge Management: explicit and tacit (Kendal & Creen, 2007):

- *Explicit knowledge* refers to the part of person's knowledge which can be clearly demonstrated using facts, formulas, instructions, drawings, and similar. The pieces of explicit knowledge can be fixed and distributed using appropriate media. Formulas, laws of physics, recipes, user instructions and other exactly formulated recommendations, rules and regulations belong to this category.
- *Tacit knowledge* represents the part of our knowledge having an informal character. Its content and ways of application are much foggier. We gain it unnoticed through our day-to-day communication. It becomes "visible" only through our understanding of the

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relationships among pieces of explicit knowledge, by our ability to apply them, by synchronizing our behaviour to our community's expectation and its values, etc.

In the above example, the student likely wanted to accommodate his/her teacher by delivering "a reasonable figure". He/she was ready to interpret the input data in a wrong way in order to favour his/her socially-developed interpretation of the community's desire. His/her tacit knowledge likely guided his/her reasoning in the following way: "Whenever my teacher gives me a problem, using a calculation leads to its solution. I am supposed to calculate."

Similar risks grow with the application of calculators in classrooms. Using technology, experiments leading to similar "socially affected" solutions can be executed faster and the student is capable of doing more calculations within the same time period. The learner's self-conviction about finding the expected solution may grow, too. In our paper, we indicate methods of facilitating tacit knowledge in order to train students to become more critical of their own procedures for problem solving as well as of their results (for details see Hvorecký & Koreňová, 2018).

In the real world, this process runs as a series of trials and errors. Its equivalent in education is discovery learning. Regardless of whether children work individually or collaborate while solving problems, making trials, errors and corrections helps them to understand the complexity of learned phenomena, their relationships and properties. In this paper, we exemplify the approach using a series of examples.

First, we present this learning as a social activity which simultaneously develops both explicit and tacit knowledge – the elements and laws of the subject as well their meaning and relevance. Then, we show a set of problems that explain to the students that Mathematics is a standard human activity which is not free of errors, imperfections and confusions. Using advanced calculators, we demonstrate problems that may introduce them to the field of discovery learning by combining mathematics and information technology.

2 Learning Concepts

2.1 Discovery Learning

Learning is a very complex, multifaceted phenomenon (see Bianco & Ulm, 2010). Initiatives aiming at the development of discovery in mathematics education have to take the nature of learning into account. Let us have a quick look at some fundamental aspects of discovery learning (e.g. Ruthven, 2007; Haberlandt, 1997).

- Discovery learning is a constructive process. Knowledge and understanding cannot be simply transported from teachers to students. Cognitive psychology describes learning as a process of construction and modification of cognitive structures. From the view of neurosciences, learning is the construction of neural networks. During learning, connections between neurons develop and change (Prodromou & Lavicza, 2017).
- It is an individual process from one side because learning and its outcomes take place inside the mind of each learner. He/she creates his own knowledge and understanding by interpreting his/her personal perceptions on the basis of his/her prior individual knowledge and prior understanding. According to the constructivist approach (Karagiori & Symeonu, 2005), every learner must construct his/her own new knowledge in his/her specific way. His/her presence in the team does not mean that every learner gets the identical knowledge and skills.
- On the other hand, this type of learning is also a social process. The sociocultural environment has great impact on educational processes. Learning is strongly based on interpersonal cooperation and communication between students and teachers (Prodromou, Lavicza & Koren, 2015).
- This type of learning is always an active process. Cognitive activity means working with the content in mind, viewing it from different perspectives and incorporating it into a person's existing knowledge network. The use of models plays an important role. According to Hejný et al. (2006) suitable separate and generic models are needed for every kind of abstract knowledge.
- It is a self-organized process. The learner is at least partially responsible for the organization of his/her individual learning. The degree of responsibility may vary in the phases of planning, realizing or reflecting of learning processes. In the frame of constructionism (Papert & Harel, 1991), "learning-by-doing" is a typical method of knowledge gathering. Students are active and build new concepts, which help them to understand new notions according to their personal expectations and needs.
- It is a situated process. It is influenced by each and every particular situation. A meaningful context or a pleasant atmosphere can foster learning, while

fear can hamper it. According to Hejný et al. (2006) the role of the teacher is critical because he/she must organize the activities in the class and to invite students to construct their new knowledge according to their individual dispositions.

2.2 Inquiry Based Learning

Inquiry based learning supports the development of analytical skills, and on techniques for stimulating intrinsic motivation for learning mathematics and science. Here, the students are facing facts and their role is to understand their meaning by querying them. Context-based learning in which the relevance of science is highlighted and inquiry-based techniques are used to stimulate the formulation of ideas and their testing through direct experimentation have been shown to be effective in stimulating interest in science, but they need to be developed and implemented more widely (see http://scientix.eu).

By means of constructive, feasible problem-solving teaching techniques, it is possible to demonstrate mathematics as a science to students (Hejný et al., 2006). They should have an opportunity to discover and formulate the solutions to problems on their own. These abilities and skills are also essential for the research work in the field of mathematics, physics, geography and other fields of science. In this methodology, all stages of mathematization are applied: transferring problems into the language of mathematical symbols, solving them using calculations and formal manipulations and interpreting the solutions using the language of the source discipline.

On the other hand, according to this theory of education, a mutual transfer of knowledge and experiences between sciences and the content of teaching are very important. This transfer is more effective if accompanied by activities introducing students to science by closely linking to educational theory and practice. A collaboration of experts in educational theory with teachers and students will help achieve this objective. In the field of Science education, it is very important that the students' knowledge not be isolated. To know how to link knowledge from various thematic units as well as to use the inter-thematic relations, experts' support is necessary. This guided educational process should support their inductive and deductive reasoning as well as their ability for abstraction and specification (Hejný et al., 2006).

In this case one suitable idea for teaching mathematics and natural sciences will be the Georges Charpak's "Hands On" method which is special educational method for science and mathematics education at primary level. This method promotes experimentation and observation in class where teachers apply phases of separate and generic model(s). The phases split the process into understandable "chunks". Unfortunately, they are often absent in the teaching of mathematics and science at Slovak schools so the knowledge of the pupils is superficial (Csachová & Jurecková, 2018).

Teaching mathematics to pupils should stimulate intrinsic motivation for learning mathematics, supporting interdisciplinary relations between mathematics and science, facilitating their personal growth, responsibility and self-confidence in addition to developing their analytical skills.

This method has ten main principles (see Charpak, 1997):

1. Students in the class observe an object or phenomenon in the real, tangible world and experiment with it.
2. During their investigations, students develop their argumentation and reasoning, gathering and discussing their ideas and results, and they build their knowledge, as manual activity on its own is insufficient.
3. The activities that the teacher proposes to students are organized in sequences within teaching modules. They are related to the State Educational Curriculum and offer students a great deal of independence.
4. It is important to arrange the continuity of activities and teaching methods, which are ensured throughout the school program.
5. Each student keeps teaching materials, written and updated in his/her own words.
6. The main objective is a student's gradual adoption of scientific concepts and techniques, along with the strengthening of their oral and written skills.
7. It is an advantage, if families and/or neighborhoods take part in students' work done in class.
8. Local scientific partners, such as universities and engineering schools, support classwork by making their skills available.
9. It is an advantage, if teachers exchange their experiences in the use of the "Hands On" method (see https://www.fondation-lamap.org/en/ international).
10. Teachers can access teaching modules, ideas for activities, and answers to various questions from resource websites (e.g. www.fondation-lamap.org). They can also exchange ideas with colleagues, trainers and scientists via their collaborative labor. They support also collaborative work of their students in the classroom.

According to Kovács, Recio & Vélez (2018) using ICT tools develops innovative practices and research approaches of these tools from many new perspectives. The international character of the community of teachers who use tools provides many inspirations on how to use ICT tools in the inquiry based and discovery learning. It encourages also others to get involved in the creative work and apply their efforts in the development of learning materials and methods for this kind of teaching (see Kónya & Kovács, 2019).

2.3 Developing Tacit Knowledge

In this section, we present three examples of problems that can enhance the atmosphere in Mathematics classes in our desired direction.

Take a piece of rope and hold its ends in your hands. Move your hands closer and further from each other. Look at the rope and think about its shape. Which curve could it be?

If your guess is "the parabola", you are wrong. Do not get frustrated – many great mathematicians including Galileo Galilei made the same wrong guess. In reality, the curve's name is the catenary – derived from the Latin word for "chain". In 1669, many years after Galileo, a much less known Jungius proved it. The catenary is formed by a hanging flexible wire or chain supported at its ends and uniform gravitational force (Hvorecký, 2006).

This and similar errors in reasoning should be presented to students. They are supportive examples of inquiry-based learning. They present famous scientists as humans making mistakes. Students might be scared of starting their discovery processes due to their image of scientists as ideal, never-failing persons. Reverting this image and placing an equality symbol between them and science celebrities may enhance diminishing their mental barriers.

The students often do not comprehend formulas, their graphs and real objects. This example coming from (Hvorecky, 2005) demonstrates how the ties between them can be pointed to and facilitated.

In 1965, a huge arch was built in Saint Louis (Missouri, USA). It symbolizes an important period in the town history: for almost two centuries, St. Louis used to be the gateway to the United State's West. Its shape has the form of the inverted cosine hyperbolic. Its shape follows the catenary (now formally renamed as the cosine hyperbolic – cosh). Its exact formula is

$$y = 693.8597 - 68.7672 \cdot \cosh(0.0100333x) \quad (1)$$

On one hand, the example shows that the arch and the formula are virtually the same and can be used for a discussion about the meaning of such identity: "To what degree are they identical and what makes them different?" Within this frame students can be asked to complete two tasks:

(a) to draw the graph using their graphic calculators to visualize the similarity,

(b) to find the picture of Saint Louis Arch on the Internet.

They might be surprised by their relation or – if their dissimilarity is big – to investigate the reasons. For their teachers, a problems carries another lesson. Typical classroom problems use whole numbers as their parameters. Their role is to simplify and speed up learners' manual calculations. With calculators, there is no difference between calculating with 20 or 327 and the numbers like above 693.8597 and 0.0100333. This is the good news: solving similar problems with information technology does not require any artificial simplifications.

The visualization of Saint Louis Arch and its expression by a formula can serve as an ignition point for continuing problems:

- *What is the height of the arch?* In the discovery-oriented part of the solution, students have to come up with a conclusion that they are supposed to find the maximum of the function (1). Using their calculator, the next stage of the solution is fast and simple. As a result, teachers can spend more time on the initial stage which is critical to students' comprehension.
- *What is the distance between its branches measured on the ground?* Now, the roots of (1) has to be found. Again, learners should come up themselves with the idea. This will help them to understand a meaning of "root" and its importance. After catching the idea and using an intelligent calculator, getting the result is easy and quick.
- *There is an elevator in a branch of the Arch going to its top. How long is it?* This problem is an appropriate introduction to calculus. Using a graphic calculator, one can quickly demonstrate that splitting the basis of the arch to ten sections brings the elevator's length exact to centimeters. For educators, the growth of the number of arch sections is an example of phasing the problem leading to an increasingly accurate solution. At the same time, learners quickly begin to understand that more than ten sections leads to negligible improvements in the precision. They will learn more about the interpretation of the concept of precision in mathematics and technology.

2.4 Technology Advancement

Educators are often apprehensive about the quick development of technology. They are challenged by its new capabilities and frustrated by a necessity to modify their teaching routines. Similarly, parents dislike buying newer and newer devices as their child advances from class to class.

Calculator producers are aware of this animosity and started looking for solutions optimal for all partners. One such solution is offered by CASIO. Its CASIO Classwiz allows splitting solutions, "freezing" them and revoking their solution from their frozen stage using QR codes. As the QR codes can be copied, every learner can get his/her identical copy of the solution.

As a result, not all students need calculators. They can use their smartphones with an official application. It allows reading the QR code, storing it on the phone, presenting the solution and (to a limited degree) manipulating it. A problem can be solved with one device (non-graphic CASIO Classwiz) and then presented and analyzed on the smartphone. In our examples, students will be able to read the QR codes using their mobile phones, i.e. store them. To demonstrate a possible phasing of the process, the below subproblem is solved.

Example 1. Mitosis is a method of cell division based on a normal (Gaussian) probability distribution. In our example, the expected value of cell division is 1 hour with the variance of 5 minutes. What is the probability that a cell will divide in 45 (x) minutes?

The most advanced calculators are capable of presenting probability distributions. For the above reasons we present using CASIO Classwiz in two phases. In Phase 1, we select Distribution from the Classwiz main menu and then Normal PD. The calculator asks for input values, i.e. the requested parameters of the Gauss function. We type:

- The value $x = 45$,
- The variance $\sigma = 5$
- The expected value $\mu = 60$ minutes (i.e. 1 hour)

The calculated probability is 0.000886, i.e. very low. This process can be followed by students. Here we presume that it is done by the educator using the outcome of Phase 1 as his/her preparation for Phase 2. In this case, he/she creates the QR code with the Classwiz display. It replaces the previous displayed content (showing the parameters of the normal probability distribution) by its QR code. Both screenshots are presented in Figure 1.

Phase 2 starts with students reading this QR code using their smartphones. (The free CASIO EDU+ application must be installed in advance). After the successful reading

Figure 1: Normal distribution and its QR code.

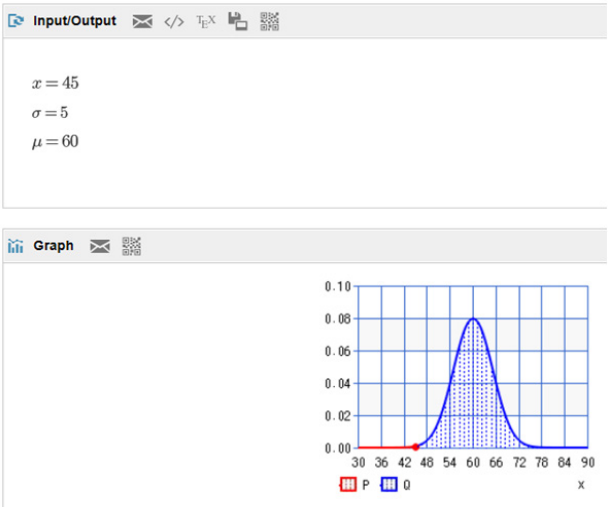

Figure 2: Transferred normal distribution to a smartphone.

of the QR code, their screens display the data and graph – see Figure 2.

The visualized normal distribution – the input of Phase 2 – has several positive aspects for education:

- The students' preparation time for getting to Phase 2 is virtually zero.
- The content of all their smartphones is identical.
- The teacher is now certain that there will be no confusion caused by different data and graphs. He/she can fully concentrate on the properties of the normal distribution expressed by the graph.
- Then the students can start experimenting and be certain that they can at any moment return to the initial stage defined by the QR code. This makes them feel at ease with their experiments. They are aware that their mistakes can be easily corrected by starting their calculations from this specified point.
- The teachers are also in a more comfortable position than when using for example computers in a network. Technological problems are easier to resolve, e.g. by replacing the calculator with another one. It is therefore suggested to have a few spare functioning calculators. As a result, the delays caused by technological difficulties are minimal.

3 Conclusions

Calculators can simplify the teachers' role in their Mathematics classrooms under a crucial condition: They are applied in accordance with the principles of active learning. It means that their function remains supportive and does not become the goal of learning. Activities performed with their use would therefore concentrate on analyzing the properties of the objects and intensifying the students' comprehension.

At the same time, technology can enrich education by introducing realistic problems. Most textbook problems use simplified parameters (small integers). These are remnants from the times when all calculations were done manually. Using more complex numbers would slow down the classroom processes. Now, the problems can use data which are closer to those used in science, technology and economics – as big and complex as demonstrated by the Saint Louis Arch.

Information Technology minimizes the time necessary for completion of operations and allows students to concentrate on their learning (see also Muller, Buteau, Klincsik, Perjési-Hámori & Sarvári, 2009). Naturally, the input to such problems should be preprocessed by the teacher otherwise it could result in an even bigger waste of time. The case of Classwiz shows a way to do this in a speedy and easy manner.

All this means that the future of Mathematics education can benefit from IT in classrooms presuming the educators' creativity and good will (Dostál & Kropáč, 2017; Verma, Stoffová & Illés, 2018).

Acknowledgements: This article is supported by grants KEGA 012UK-4/2018 "The Concept of Constructionism and Augmented Reality in the Field of the Natural and Technical Sciences of the Primary Education (CEPENSAR)", VEGA 1/0079/19 "Analysis of critical points in school mathematics and identification of factors influencing pupils' attitude to mathematics", and APVV-15-0378 (OPTIMAT) "Optimization of mathematics teaching materials based on analysis of the current needs and abilities of pupils of younger school age".

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