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Neutrosophic Inventory Management: A Cost-Effective Approach

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Neutrosophic Inventory Management: A Cost-Effective Approach

[1]Ankit Dubey, [2,*]Ranjan Kumar
[1,2]VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India
*Corresponding author: ranjank.nit52@gmail.com

Abstract: Classical Inventory models (IM) serve as quantitative tools for determining the optimal order quantities, timing of orders, and safety stock levels for specific inventory items or item groups. In 1965, Zadeh introduced the concept of fuzzy theory. Subsequently, Dubois and Parade(1988) introduced the fuzzy inventory model, which, however, exhibits limitations in effectively handling uncertainty, inaccuracies, and imprecise data. To handle more uncertainty, Smarandache introduced the concept of neutrosophic set theory in 1999. This research article applies the neutrosophic set theory to the realm of inventory management, with a specific focus on addressing the uncertainty linked to holding costs, shortage costs, and ordering costs by utilizing trapezoidal neutrosophic numbers. Additionally, we conduct a comparative analysis of our proposed model against existing models in this paper. Based on this comparative study, our findings assert the superior performance of our proposed model in relation to some of the existing models. In conclusion, we wrap up our research by presenting graphical, logical, and tabular comparisons with the existing methods.

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Introduction

Operations Research, often abbreviated as OR, is a multidisciplinary field at the intersection of mathematics, statistics, and decision science. Operations research is intricately connected to inventory management by providing a systematic approach to optimize key inventory decisions. Through mathematical models, simulations, and analysis, operations research enables businesses to determine optimal reorder points, order quantities, and inventory policies, accounting for factors like demand variability, lead times, and cost structures. This connection empowers organizations to minimize holding and ordering costs while maintaining desired service levels, enhancing overall supply chain efficiency and profitability through data-driven decision-making in the realm of inventory management.

The last few years have witnessed a growing body of research focusing on various dimensions of inventory control and management. A series of studies have delved into intricate aspects of this field. Das et al. (2020) (Das, Zidan, Manna, Shaikh, & Bhunia, 2020) concentrated on the integration of preservation technology within an inventory control system, specifically addressing price-dependent demand and partial backlogging. Mashud (2020) (Mashud, 2020) contributed by presenting an EOQ framework for a deteriorating IM that incorporates diverse demand types and complete backlogging. In the year 2021, Khan and Sarkar (Khan & Sarkar, 2021) explored the terrain of supply chain risk transfer, effectively merging pricing and inventory decisions while meticulously tackling shortages. Setiawan et al. (2021) (Setiawan, Lesmono, & Limansyah, 2021) offered valuable insights into managing exponential and quadratic demand within the context of Weibull deterioration, thereby addressing challenging inventory control scenarios. The subsequent year introduced the work of Sharma et al. (2021) (Sharma, Tyagi, Verma, & Kumar, 2022), who devised a model that takes into account demand-driven production while accommodating time and stock-related demand for deteriorating inventory items. Antic et al. (2022) (Antic, Djordjevic Milutinovic, & Lisec, 2022) contributed an adaptive IM tailored for pharmaceutical distribution, incorporating dynamic discreteness and accounting for both deterministic and stochastic demand. Duary et al. (2022) (Duary, et al., 2022) extended the scope by considering payment timing and inventory discounts in a model for deteriorating items, thoughtfully incorporating capacity constraints and partially backlogged shortages. The most recent addition, by Jani et al. (2023) (Jani, et al., 2023), offers a decision support system tailored for retailer's deterioration control, factoring in trade credit dynamics and the presence of shortages. Together, these studies form a comprehensive mosaic of inventory management research, contributing valuable perspectives to this intricate and dynamic field. Classical IM faces a multitude of challenges, including accurate demand forecasting to prevent stockouts or overstocking, optimizing reorder points and safety stock levels to balance costs and service levels, etc. In certain decision-making scenarios involving multiple complex factors, some of which may defy easy quantification, fuzzy logic offers a more adaptable and nuanced alternative to traditional inventory models. As a result, Lotfi A. Zadeh (Zadeh, 1965) introduced the fuzzy theory in 1965, recognizing its potential for enhancing decision-making processes.

Lately, Dubois and Parade (1988) (Dubois & Prade, 1988) introduced the fuzzy inventory model (FIM). Unlike traditional IM that assume precise and deterministic parameters, a FIM allows for the representation of vague and ambiguous information, enabling decision-makers to make more robust and flexible inventory management decisions. Leopoldo Eduardo Cárdenas-Barrón (2011) (Cárdenas-Barrón, 2011) made a noteworthy contribution by developing EOQ/EPQ inventory models that incorporated dual backorder costs, utilizing the powerful tools of analytic geometry and algebra. Building on this foundation, Harun Sulak (2016) (Sulak, 2015) introduced an EOQ model that addressed the complexities of defective items and shortages within a fuzzy sets framework, offering a more realistic perspective on inventory management. In 2018, Gani and his colleagues (Gani & Rafi, A new method to discussing the manufacturing defects in EOQ/EPQ inventory models with shortages using fuzzy techniques, 2020) ventured into the application of the Arithmetic Geometric Mean (AGM) inequality method for calculating EOQ/EPQ within a fuzzy environment, opening up new avenues for precision in inventory calculations. Alfares and Ghaithan (2018) (Alfares & Ghaithan, 2019) provided a comprehensive review of state-of-the-art EOQ and EPQ production-IM, with a particular focus on incorporating variable holding costs, a critical factor in modern supply chain dynamics. Thinakaran et al. (2019) (Thinakaran, Jayaprakas, & Elanchezhian, 2019) undertook an extensive survey exploring inventory models for EOQ and EPQ, with a specific emphasis on addressing the challenges of partial backorders, shedding light on a common yet often complex scenario faced by businesses. In the same year, Scott Shu-Cheng Lin (2019) (Lin, 2019) made observations regarding the formulation of EOQ/EPQ IM with dual backorder costs, leveraging analytic geometry and algebra to refine existing approaches. Gani and Rafi (2019) (Gani & Rafi, A simplistic method to work out the EOQ/EPQ with shortages by applying algebraic method and arithmetic geometric mean inequality in fuzzy atmosphere, 2019) presented a simplified approach for calculating EOQ/EPQ while considering shortages in a fuzzy environment, introducing algebraic and AGM Inequality methods to streamline the decision-making process. In 2020, they further expanded their contributions by introducing a novel approach to address manufacturing defects in EOQ/EPQ IM with shortages through the application of fuzzy techniques, further enhancing the precision and robustness of inventory management strategies. Collectively, these scholars have made significant strides in advancing the theory and application of inventory management, addressing various complexities and uncertainties that modern businesses face. So far, we have discussed the FIM, but some challenges in FIM. Challenges in FIM involve handling fluctuating demand patterns, addressing imprecise or incomplete data, optimizing inventory allocation and distribution across complex supply chains, and adapting to unforeseen disruptions, whereas extended fuzzy inventory methodologies refine traditional approaches by integrating higher-order fuzzy reasoning, multiple granularities, and dynamic adjustments, enabling more adaptive, resilient, and agile inventory control strategies in volatile and uncertain environments.

However, in 1999, F. Smarandache (Smarandache, 1999) introduced the concept of a neutrosophic set and highlighted some of its specific characteristics that make it superior to the classical model and fuzzy model. In the context of this progression, we will now proceed to discuss the neutrosophic inventory model (NIM).

After reading many research books and articles, it has become clear that having a lot of knowledge about management is very important. Also, it is very important to understand that uncertainty is a big part of real-life problems. Because of these things, the present study aims to explore NIM as a cost-effective approach.

The study aims to explore NIM as a cost-effective approach. To achieve this, the following objectives have been set:

(1) Conducting a comprehensive literature survey on inventory management systems to enhance understanding of the overall management framework.
(2) Proposing a new method to effectively address and manage uncertainty within the IM context.
(3) Comparing the proposed method with existing approaches to establish its superiority.
(4) Conducting a thorough comparison, employing logical, graphical, and tabular analyses, with existing methods in the field.
(5) Discussing the practical applications of the proposed method within the inventory management system.
(6) Introducing a new method designed not only to address existing numerical challenges but also to tackle novel problem types.

This research endeavours to contribute to the existing body of knowledge in the field by addressing the identified gaps and offering innovative solutions to enhance the efficacy of IM systems in the face of uncertainty.

The paper consists of six sections. The first section is the introduction, which provides an overview of the study. The second section discusses neutrosophic numbers and their arithmetic and logical operators. The third section presents the proposed NIM Model. The fourth section is dedicated to numerical analysis. The fifth section presents the results and discussion. Finally, the sixth section concludes the paper.

2. Neutrosophic number and its Arithmetic and logical operators

Definition 2.1: Normalized Fuzzy set (Zadeh, 1965) **:** A fuzzy set $\tilde{\xi} = \left\{ \left(\omega, \sigma_{\tilde{\xi}}(\omega) \right) : \omega \in W, \sigma_{\tilde{\xi}}(\omega) \in [0,1] \right\}$ is called a normalized fuzzy set if and only if $\underset{\omega \in W}{Sup}\left\{ \sigma_{\tilde{\xi}}(\omega) \right\} = 1$.

Definition 2.2. Neutrosophic Set (Wang, Smarandache, Zhang, & Sunderraman, 2010) (Wang, Zhang, Sunderraman, & Smarandache, 2011)**:** A set $neuS$ in the universal discourse W, symbolically denoted by ω it is called Neutrosophic Set if $neuS = \left\{ \left(\omega, \left[t_{neuS}(\omega), i_{neuS}(\omega), f_{neuS}(\omega) \right] \right) : \omega \in W \right\}$ Where truth, Falsity, indeterminacy , membership function which has the degree of belongingness $t_{neuS}(\omega) : W \to [0,1]$, $f_{neuS}(\omega) : W \to [0,1]$,and $i_{neuS}(\omega) : W \to [0,1]$ of the decision maker. $T_{neuS}(\omega),, F_{neuS}(\omega), I_{neuS}(\omega)$ exhibits the following relation. $0 \le Sup\left\{ T_{neuS}(\omega) \right\} + Sup\left\{ F_{neuS}(\omega) \right\} + Sup\left\{ I_{neuS}(\omega) \right\} \le 3$.

Definition 2.3. (Liang, Wang, & Li, Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information, 2018) (Liang, Wang, & Zhang, A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information., 2018)**:** Let $T_{\hat{d}}, I_{\hat{d}}, F_{\hat{d}} \in [0,1]$, then a Single-valued trapezoidal neutrosophic (SVTpN) number $\hat{d} = \left\langle \left[\hat{d}^a, \hat{d}^s, \hat{d}^h, \hat{d}^o \right], (T_{\hat{d}}, I_{\hat{d}}, F_{\hat{d}}) \right\rangle$ is a special Ns on the real number set R, whose indeterminacy-MF $\xi_{\hat{d}}(x)$, truth-MF $\psi_{\hat{d}}(x)$, and falsity-MF $\zeta_{\hat{d}}(x)$ are given as follows:

$$\xi_{\hat{d}}(x) = \begin{cases} \dfrac{(\hat{d}^s - x + I_{\hat{d}}(x - \hat{d}^a))}{(\hat{d}^s - \hat{d}^a)}, \hat{d}^a \le x \le \hat{d}^s \\ I_{\hat{d}}, \hat{d}^s \le x \le \hat{d}^h \\ \dfrac{(x - \hat{d}^h + I_{\hat{d}}(\hat{d}^o - x))}{(\hat{d}^o - \hat{d}^h)}, \hat{d}^h \le x \le \hat{d}^o \\ 1, otherwise \end{cases}, \psi_{\hat{d}}(x) = \begin{cases} \dfrac{T_{\hat{d}}(x - \hat{d}^a)}{(\hat{d}^s - \hat{d}^a)}, \hat{d}^a \le x \le \hat{d}^s \\ T_{\hat{d}}, \hat{d}^s \le x \le \hat{d}^h \\ \dfrac{T_{\hat{d}}(\hat{d}^o - x)}{(\hat{d}^o - \hat{d}^h)}, \hat{d}^h \le x \le \hat{d}^o \\ 0, otherwise \end{cases},$$

$$\zeta_{\hat{d}}(x) = \begin{cases} \dfrac{(\hat{d}^s - x + F_{\hat{d}}(x - \hat{d}^a))}{(\tilde{d}^s - \tilde{d}^a)}, \hat{d}^a \le x \le \hat{d}^s \\ F_{\hat{d}}, \hat{d}^s \le x \le \hat{d}^h \\ \dfrac{(x - \hat{d}^h + F_{\hat{d}}(\hat{d}^o - x))}{(\hat{d}^o - \hat{d}^h)}, \hat{d}^h \le x \le \hat{d}^o \\ 1, otherwise \end{cases}$$

Definition 2.4 (Ye, 2017)**:** Let $\underline{\tilde{A}} = \left\langle [\tilde{a}_1^a, \tilde{a}_2^n, \tilde{a}_3^k, \tilde{a}_4^i]; (T_{\underline{\tilde{A}}}, I_{\underline{\tilde{A}}}, F_{\underline{\tilde{A}}}) \right\rangle$ and $\underline{\tilde{B}} = \left\langle [\tilde{b}_1^a, \tilde{b}_2^n, \tilde{b}_3^k, \tilde{b}_4^i]; (T_{\underline{\tilde{B}}}, I_{\underline{\tilde{B}}}, F_{\underline{\tilde{B}}}) \right\rangle$ are two trapezoidal neutrosophic numbers, then

$$\frac{\tilde{\underline{A}}}{\tilde{\underline{B}}}=\begin{cases}\left\{\left\langle\left(\frac{a_1^a}{b_4^i},\frac{a_2^n}{b_3^k},\frac{a_3^k}{b_2^n},\frac{a_4^i}{b_1^a}\right);T_{\tilde{\underline{A}}}\wedge T_{\tilde{\underline{B}}},I_{\tilde{\underline{A}}}\vee I_{\tilde{\underline{B}}},F_{\tilde{\underline{A}}}\vee F_{\tilde{\underline{B}}}\right\rangle,if a_4^i>0,b_4^i>0\right.\\ \left\{\left\langle\left(\frac{a_4^i}{b_4^i},\frac{a_3^k}{b_3^k},\frac{a_2^n}{b_2^n},\frac{a_1^a}{b_1^a}\right);T_{\tilde{\underline{A}}}\wedge T_{\tilde{\underline{B}}},I_{\tilde{\underline{A}}}\vee I_{\tilde{\underline{B}}},F_{\tilde{\underline{A}}}\vee F_{\tilde{\underline{B}}}\right\rangle,if a_4^i<0,b_4^i>0\right.\\ \left\{\left\langle\left(\frac{a_4^i}{b_1^a},\frac{a_3^k}{b_2^n},\frac{a_2^n}{b_3^k},\frac{a_1^a}{b_4^i}\right);T_{\tilde{\underline{A}}}\wedge T_{\tilde{\underline{B}}},I_{\tilde{\underline{A}}}\vee I_{\tilde{\underline{B}}},F_{\tilde{\underline{A}}}\vee F_{\tilde{\underline{B}}}\right\rangle,if a_4^i<0,b_4^i<0\right.\end{cases}$$

$$\tilde{\underline{A}}\otimes\tilde{\underline{B}}=\left\langle[\tilde{a}_1^a\tilde{b}_1^a,\tilde{a}_2^n\tilde{b}_2^n,\tilde{a}_3^k\tilde{b}_3^k,\tilde{a}_4^i\tilde{b}_4^i];(T_{\tilde{\underline{A}}}\wedge T_{\tilde{\underline{B}}},I_{\tilde{\underline{A}}}\vee I_{\tilde{\underline{B}}},F_{\tilde{\underline{A}}}\vee F_{\tilde{\underline{B}}})\right\rangle$$

$$\alpha\otimes\tilde{\underline{A}}=\left\langle[\alpha\tilde{a}_1^a,\alpha\tilde{a}_2^n,\alpha\tilde{a}_3^k,\alpha\tilde{a}_4^i];(T_{\tilde{\underline{A}}},I_{\tilde{\underline{A}}},F_{\tilde{\underline{A}}})\right\rangle,\alpha\geq 0$$

$$\alpha\otimes\tilde{\underline{A}}=\left\langle[\alpha\tilde{a}_4^i,\alpha\tilde{a}_3^k,\alpha\tilde{a}_2^n,\alpha\tilde{a}_1^a];(T_{\tilde{\underline{A}}},I_{\tilde{\underline{A}}},F_{\tilde{\underline{A}}})\right\rangle,\alpha<0$$

$$\tilde{\underline{A}}\oplus\tilde{\underline{B}}=\left\langle[\tilde{a}_1^a+\tilde{b}_1^a,\tilde{a}_2^n+\tilde{b}_2^n,\tilde{a}_3^k+\tilde{b}_3^k,\tilde{a}_4^i+\tilde{b}_4^i];(T_{\tilde{\underline{A}}}\wedge T_{\tilde{\underline{B}}},I_{\tilde{\underline{A}}}\vee I_{\tilde{\underline{B}}},F_{\tilde{\underline{A}}}\vee F_{\tilde{\underline{B}}})\right\rangle$$

Definition 2.5 (Ye, 2017)**:** Let $\tilde{A}_d=\left\langle(a^a,b^n,c^k,d^i);(T_{\tilde{A}_d},I_{\tilde{A}_d},F_{\tilde{A}_d})\right\rangle$ be TpFN then the score function of $\tilde{A}_d$ is defined as $Sc(\tilde{A}_d)=\frac{1}{12}(a^a+b^n+c^k+d^i)(2+T_{\tilde{A}_d}-I_{\tilde{A}_d}-F_{\tilde{A}_d})$

Let $\tilde{A}_d=\left\langle(a_1^a,b_1^n,c_1^k,d_1^i);(T_{\tilde{A}_d},I_{\tilde{A}_d},F_{\tilde{A}_d})\right\rangle$ and $\tilde{B}_d=\left\langle(a_2^a,b_2^n,c_2^k,d_2^i);(T_{\tilde{B}_d},I_{\tilde{B}_d},F_{\tilde{B}_d})\right\rangle$ be two SVTN-number. Then,

1. If $Sc(\tilde{A}_d)<Sc(\tilde{B}_d)$, then $\tilde{A}_d$ is smaller than $\tilde{B}_d$, denoted by $\tilde{A}_d<\tilde{B}_d$.
2. If $Sc(\tilde{A}_d)=Sc(\tilde{B}_d)$;

   (a) If $Ac(\tilde{A}_d)<Ac(\tilde{B}_d)$, then $\tilde{A}_d$ is smaller than $\tilde{B}_d$, denoted by $\tilde{A}_d<\tilde{B}_d$.

   (b) If $Ac(\tilde{A}_d)=Ac(\tilde{B}_d)$, then $\tilde{A}_d$ and $\tilde{B}_d$ are the same, denoted by $\tilde{A}_d=\tilde{B}_d$.

3. Proposed Neutrosophic Inventory Management Model

3.1 NOTATIONS:

Define the following parameters used in inventory model:

τ **stands for** "Total cost";

ℓ **stands for** "Length of the cycle";

ν^* **stands for** "Optimal order quantity";

ν **stands for** "Order quantity per cycle";

n **stands for** "Ordering cost per cycle";

$\tilde{\tau}^{neu}$ **stands for** "Neutrosophic total cost";

ν_s **stands for** "Shortage quantity per cycle";

a **stands for** "Annual demand in period $[0,\ell]$";

i **stands for** "Holding cost for one unit per day".

$\tilde{n}^{neu}$ **stands for** "Neutrosophic ordering cost per cycle".

$(\nu^*)^{neu}$ **stands for** "Neutrosophic optimal order quantity".

k **stands for** "Shortage (backorder) cost per unit per day".

$(\nu_s^*)^{neu}$ **stands for** "Neutrosophic optimal shortage quantity".

$\tilde{i}^{neu}$ **stands for** "Neutrosophic holding cost for one unit per day".

$\tilde{k}^{neu}$ **stands for** "Neutrosophic shortage (backorder) cost per unit per day".

3.2 ASSUMPTIONS-

In this IM, the following norms are considered i.e., Total demand and time of plan are constant, and shortage in inventory is allowed.

3.3 MATHEMATICAL FORMATION AND SOLUTION OF MODEL-

3.3.1 EOQ Model in Classical Environment-

For a crisp IM with shortage quantity, then IM in classical sense is,

Total cost = Ordering Cost + Holding Cost + Shortage Cost

$$\tau = i\frac{(\nu-\nu_s)^2}{2\nu}\ell + k\frac{\nu_s{}^2}{2\nu}\ell + n\frac{a}{\nu} \quad \textbf{(1)}$$

$\tau(\nu)$ is minimum when to optimize the total cost we can use $\frac{d\tau(\nu)}{d\nu}=0, \frac{d^2\tau(\nu)}{d\nu^2}>0$

Now, differentiating equation (1) w.r.t. ν, we have, the optimal order quantity is

$$\nu^* = \sqrt{\frac{2(i+k)na}{ki\ell}} \quad \textbf{(2)}$$

And optimal shortage quantity is

$$\nu_s{}^* = \sqrt{\frac{2.ina}{k(i+k)\ell}} \quad \textbf{(3)}$$

From Eq. (1), the minimal total cost is

$$\tau^* = \sqrt{\frac{2.ikna\ell}{i+k}} \quad \textbf{(4)}$$

3.3.2 EOQ Model in Neutrosophic Environment-

In an environment characterized by its clarity and precision, we can determine the total cost using Equation (1). However, in real-world scenarios, this cost may exhibit slight fluctuations, thereby impacting the ordering quantity (n), shortage quantity (ν_s), and holding quantity (i). To address this variability, we adopt a model that accounts for permissible shortages within a Neutrosophic framework. This decision is informed by the fact that the Neutrosophic nature of the ordering cost per order, carrying cost (or holding cost) per unit quantity per unit time, and shortage cost per unit quantity necessitates a Neutrosophic approach. Consequently, we transform the values of o, s, and h into Neutrosophic numbers using a trapezoidal Neutrosophic representation as follows:

$\tilde{n}^{neu} = \left\langle (n_1{}^{neu}, n_2{}^{neu}, n_3{}^{neu}, n_4{}^{neu}); (T_{\tilde{n}^{neu}}, I_{\tilde{n}^{neu}}, F_{\tilde{n}^{neu}}) \right\rangle$, $\tilde{k}^{neu} = \left\langle (\tilde{k}_1{}^{neu}, \tilde{k}_2{}^{neu}, \tilde{k}_3{}^{neu}, \tilde{k}_4{}^{neu}); (T_{\tilde{k}^{neu}}, I_{\tilde{k}^{neu}}, F_{\tilde{k}^{neu}}) \right\rangle$ and

$\tilde{i}^{neu} = \left\langle (\tilde{i}_1{}^{neu}, \tilde{i}_2{}^{neu}, \tilde{i}_3{}^{neu}, \tilde{i}_4{}^{neu}); (T_{\tilde{i}^{neu}}, I_{\tilde{i}^{neu}}, F_{\tilde{i}^{neu}}) \right\rangle$

Proposed model for find total cost while considering the uncertainty

The neutrosophic total cost is denoted as $\tilde{\tau}^{neu}$ **, where** $\tilde{i}^{neu}, \tilde{k}^{neu}$ & $\tilde{n}^{neu}$ **are neutrosophic variables**

$$\tilde{\tau}^{neu} = \tilde{i}^{neu}\frac{(\nu-\nu_s)^2}{2\nu}\ell + \tilde{k}^{neu}\frac{(\nu_s)^2}{2\nu}\ell + \tilde{n}^{neu}\frac{a}{\nu} \quad \textbf{(5)}$$

$$\tilde{\tau}^{neu} = \begin{bmatrix} \left\langle \left[\tilde{i}_1{}^{neu}, \tilde{i}_2{}^{neu}, \tilde{i}_3{}^{neu}, \tilde{i}_4{}^{neu}\right]; \left(T_{\tilde{i}^{neu}}, I_{\tilde{i}^{neu}}, F_{\tilde{i}^{neu}}\right) \right\rangle \frac{(\nu-\nu_s)^2}{2\nu}\ell \oplus \left\langle \left[\tilde{k}_1{}^{neu}, \tilde{k}_2{}^{neu}, \tilde{k}_3{}^{neu}, \tilde{k}_4{}^{neu}\right]; \left(T_{\tilde{k}^{neu}}, I_{\tilde{k}^{neu}}, F_{\tilde{k}^{neu}}\right) \right\rangle \frac{(\nu_s)^2}{2\nu}\ell \\ \oplus \left\langle \left[n_1{}^{neu}, n_2{}^{neu}, n_3{}^{neu}, n_4{}^{neu}\right]; \left(T_{\tilde{n}^{neu}}, I_{\tilde{n}^{neu}}, F_{\tilde{n}^{neu}}\right) \right\rangle \frac{a}{\nu} \end{bmatrix}$$

(6)

Now applying the **def. 2.4 and def. 2.5 respectively on equation (6), we get**

$$\tilde{\tau}^{neu} = \left\langle \begin{matrix} \left[\tilde{i}_1^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell, \tilde{i}_2^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell, \tilde{i}_3^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell, \tilde{i}_4^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell\right];\left(T_{\tilde{i}^{neu}}, I_{\tilde{i}^{neu}}, F_{\tilde{i}^{neu}}\right) \\ \oplus\left[\tilde{k}_1^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell, \tilde{k}_2^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell, \tilde{k}_3^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell, \tilde{k}_4^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell\right];\left(T_{\tilde{k}^{neu}}, I_{\tilde{k}^{neu}}, F_{\tilde{k}^{neu}}\right) \\ \oplus\left[n_1^{neu}\cdot\frac{a}{\nu}, n_2^{neu}\frac{a}{\nu}, n_3^{neu}\frac{a}{\nu}, n_4^{neu}\frac{a}{\nu}\right];\left(T_{\tilde{n}^{neu}}, I_{\tilde{n}^{neu}}, F_{\tilde{n}^{neu}}\right) \end{matrix} \right\rangle$$

$$\tilde{\tau}^{neu} = \left[\left\langle \begin{matrix} \left[\tilde{i}_1^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell\oplus\tilde{k}_1^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell\oplus\tilde{n}_1^{neu}\cdot\frac{a}{\nu}\right] \\ \left[\tilde{i}_2^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell\oplus\tilde{k}_2^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell\oplus\tilde{n}_2^{neu}\frac{a}{\nu}\right] \\ \left[\tilde{i}_3^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell\oplus\tilde{k}_3^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell\oplus\tilde{n}_3^{neu}\frac{a}{\nu}\right] \\ \left[\tilde{i}_4^{neu}\cdot\frac{(\nu-\nu_s)^2}{2\nu}\ell\oplus\tilde{k}_4^{neu}\cdot\frac{(\nu_s)^2}{2\nu}\ell\oplus\tilde{n}_4^{neu}\frac{a}{\nu}\right] \end{matrix} \right\rangle;\begin{pmatrix} T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}}, \\ I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}}, \\ F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}} \end{pmatrix}\right]$$

$$\left(\tilde{\tau}^{neu}\right)^R = \frac{1}{12}\left[\left\langle \left[\begin{matrix} \left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\cdot\begin{pmatrix} 2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}}) \\ -(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}}) \\ -(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}}) \end{pmatrix}\frac{(\nu-\nu_s)^2}{2\nu}\ell \\ \oplus\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\cdot\begin{pmatrix} 2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}}) \\ -(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}}) \\ -(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}}) \end{pmatrix}\frac{(\nu_s)^2}{2\nu}\ell \\ \oplus\left(\tilde{n}_1^{neu}+\tilde{n}_2^{neu}+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\right)\cdot\begin{pmatrix} 2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}}) \\ -(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}}) \\ -(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}}) \end{pmatrix}\frac{a}{\nu} \end{matrix} \right]\right\rangle\right]$$

$$= \left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s) \text{ (Say)}$$

(7)

$\left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s)$ is minimum when $\dfrac{d\left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s)}{d\nu}=0, \dfrac{d^2\left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s)}{d\nu^2}>0$

i.e. $\dfrac{\partial\left(\tilde{\tau}^{neu}\right)^R}{\partial\nu}=0$

$$\frac{\partial\left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s)}{\partial\nu}=\frac{1}{12}\left[\left\langle\left[\begin{array}{l}\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left\{\frac{2\nu(\nu-\nu_s)-(\nu-\nu_s)^2}{2\nu^2}\right\}\ell\\ \oplus\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left\{\frac{-(\nu_s)^2}{2\nu^2}\right\}\ell\\ \oplus\left(\tilde{n}_1^{neu}+\tilde{n}_2^{neu}+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left(-\frac{a}{\nu^2}\right)\end{array}\right]\right\rangle\right]=0$$

Now differentiating Equation (7) w.r.t $\nu\,\&\,\nu_s$ respectively to get eq. (8) and (9).

$$\left[\begin{array}{l}\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet\nu^2\\ -\left[\begin{array}{l}\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\\ \oplus\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\end{array}\right]\bullet\nu_s^2\end{array}\right]=\frac{2.\begin{pmatrix}\tilde{n}_1^{neu}+\\\tilde{n}_2^{neu}\\+\tilde{n}_3^{neu}\\+\tilde{n}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet a}{\ell}$$

$$\nu^2=\frac{1}{\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}}\otimes\left[\begin{array}{l}\frac{2\begin{pmatrix}\tilde{n}_1^{neu}+\tilde{n}_2^{neu}\\+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet a}{\ell}\\ \otimes\left[\begin{array}{l}\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\\ \oplus\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\end{array}\right]\bullet\nu_s^2\end{array}\right] \quad \textbf{(8)}$$

Similarly differentiate wrt to vs i.e., , $\frac{\partial\left(\tilde{\tau}^{neu}\right)^R}{\partial\nu_s}=0$

$$\frac{\partial\left(\tilde{\tau}^{neu}\right)^R(\nu,\nu_s)}{\partial\nu_s}=-\frac{\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}\\+\tilde{i}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}}{\nu}\bullet\left(\nu-\nu_{s)}\right)\ell+\frac{\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}\\+\tilde{k}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}}{\nu}\bullet\nu_s\ell=0$$

$$\nu_s = \frac{\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\cdot\left(2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\right).\nu}{\left[\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\oplus\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\right]}$$

Putting the value of ν_s in equation (8) we have

$$\nu = \sqrt{\frac{\begin{matrix}2\left[\left\{\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\oplus\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\right\}\right]\bullet\left(\tilde{n}_1^{neu}+\tilde{n}_2^{neu}+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\right)\\ \cdot\left(2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\right)\bullet a\end{matrix}}{\left[\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\bullet\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\right]\bullet\ell}}$$

Similarly,

$$\nu_s = \sqrt{\frac{2\left[\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\bullet\left(\tilde{n}_1^{neu}+\tilde{n}_2^{neu}+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\right]\bullet a}{\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\bullet\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet\left[\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}+\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\right]\bullet\ell}}$$

Therefore, optimal quantity while considering uncertainty

$$\tilde{\nu}^{neu} = \sqrt{\frac{2\left[\left\{\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\oplus\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\right\}\right]\bullet\begin{pmatrix}\tilde{n}_1^{neu}+\tilde{n}_2^{neu}\\+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet a}{\left[\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\bullet\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\right]\bullet\ell}} \quad \textbf{(9)}$$

And, similarly optimal shortage quantity while considering uncertainty:

$$\tilde{\nu}_s^{neu} = \sqrt{\frac{2\left[\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}\bullet\begin{pmatrix}\tilde{n}_1^{neu}+\tilde{n}_2^{neu}\\+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\end{pmatrix}\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\right]\bullet a}{\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\bullet\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\bullet\left[\begin{pmatrix}\tilde{i}_1^{neu}+\tilde{i}_2^{neu}\\+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\end{pmatrix}+\begin{pmatrix}\tilde{k}_1^{neu}+\tilde{k}_2^{neu}\\+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\end{pmatrix}\right]\bullet\ell}} \quad \textbf{(10)}$$

This shows that $\psi(\nu,\nu_s)$ is minimum at $\tilde{\nu}^{neu}\,\&\,\tilde{\nu}_s^{neu}$.

Optimal (minimum) total cost while considering uncertainty,

$$\frac{\partial\left(\tilde{\tau}^{neu}\right)^{R}(\nu,\nu_s)}{\partial \nu}=\frac{1}{12}\left[\left\langle\left[\begin{array}{l}\left(\tilde{i}_1^{neu}+\tilde{i}_2^{neu}+\tilde{i}_3^{neu}+\tilde{i}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left\{\frac{2\nu(\nu-\nu_s)-(\nu-\nu_s)^2}{2\nu^2}\right\}\ell\\ \oplus\left(\tilde{k}_1^{neu}+\tilde{k}_2^{neu}+\tilde{k}_3^{neu}+\tilde{k}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left\{\frac{-(\nu_s)^2}{2\nu^2}\right\}\ell\\ \oplus\left(\tilde{n}_1^{neu}+\tilde{n}_2^{neu}+\tilde{n}_3^{neu}+\tilde{n}_4^{neu}\right)\cdot\begin{pmatrix}2+(T_{\tilde{i}^{neu}}\wedge T_{\tilde{k}^{neu}}\wedge T_{\tilde{n}^{neu}})\\-(I_{\tilde{i}^{neu}}\vee I_{\tilde{k}^{neu}}\vee I_{\tilde{n}^{neu}})\\-(F_{\tilde{i}^{neu}}\vee F_{\tilde{k}^{neu}}\vee F_{\tilde{n}^{neu}})\end{pmatrix}\cdot\left(-\frac{a}{\nu^2}\right)\end{array}\right]\right\rangle\right]$$

(11)

4. Numerical Analysis

Example 4.1: Comparison with the existing method: As per the Rajput et.al (2019) (Rajput, Singh, & Pandey, 2019) consideration, A manufacturing factory needs to develop an EOQ model for optimize the total cost of their product. The length of cycle is 6 months, ordering cost is Rs.20 per unit, holding cost is Rs.04 per unit and shortage cost is Rs.10 per unit.

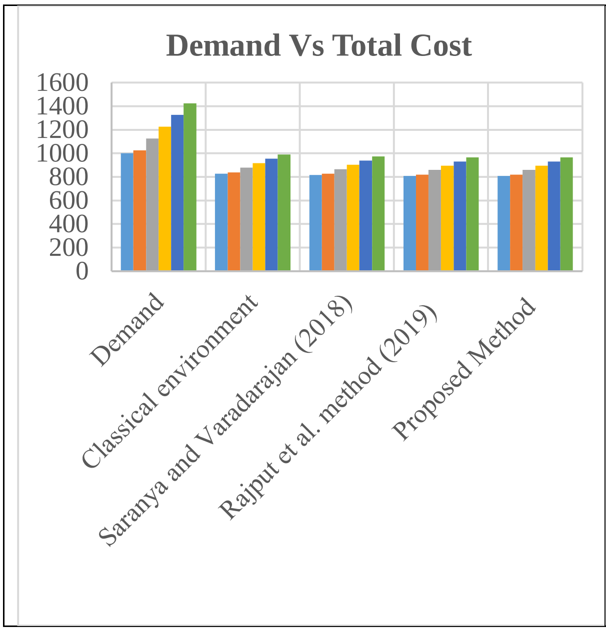
We use a SVTpN membership function to capture the data's inherent uncertainty when addressing this challenge using Neutrosophic parameters. For every feasible cost, the corresponding membership functions are specified as follows:

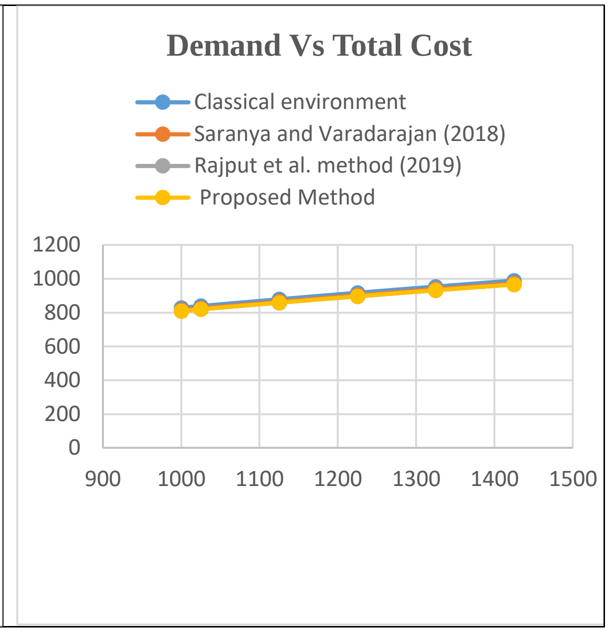
$\tilde{i}^{neu}=\langle[1,3,5,6];(1,0,0)\rangle$, $\tilde{k}^{neu}=\langle[8,9,11,12];(1,0,0)\rangle$ and $\tilde{n}^{neu}=\langle[15,18,22,25];(1,0,0)\rangle$

Table 1: Tabular comparison study with some of the existing methods such as Sen and Malakar (2015) (Sen & Malakar, 2015), Saranya and Varadarajan (2018) (Saranya & Varadarajan, 2018), Rajput et al. method (2019) (Rajput, Singh, & Pandey, 2019).

	Total Cost				
Demand	Classical environment	Sen and Malakar (2015) (Sen & Malakar, 2015)	Saranya and Varadarajan (2018) (Saranya & Varadarajan, 2018)	Rajput et al. method(2019) (Rajput, Singh, & Pandey, 2019)	Proposed Method
1000	828.078	NA	815.5122568	809.039	809.039
1025	838.3657572	NA	825.6432323	819.0904	819.0904
1125	878.3100657	NA	864.9813704	858.1163	858.1163
1225	916.515139	NA	902.6066669	895.443	895.443
1325	953.1901324	NA	938.7251031	931.2748	931.2748
1425	988.5053653	NA	973.5044136	965.7781	965.7781

In addition to tabular comparison, we also conducted a pictorial comparison study with some of the existing methods such as Sen and Malakar (2015) (Sen & Malakar, 2015), Saranya and Varadarajan (2018) (Saranya & Varadarajan, 2018), and Rajput et al. method (2019) (Rajput, Singh, & Pandey, 2019).





Finally, after doing the logical comparison i.e., classical total cost is greater than Rajput et al. (2019) (Rajput, Singh, & Pandey, 2019) proposed method but it is equal to our proposed method.

In Example 4.1, many authors have proposed different methods to solve Rajput's numerical problem. In the comparison study of tabular, pictorial, and logical methods, it is observed that our proposed method provides an optimal solution similar to Rajput et al (2019) (Rajput, Singh, & Pandey, 2019). Our proposed method not only solves existing problems but also solves a new type of environment, which is discussed below in Example 4.2.

Example 4.2: Transforming the environment into Type I Neutrosophic environment.

Table 2: Finding the total optimal cost under different cases

Different Cases	$\tilde{i}^{neu} = \langle (\tilde{i}_1^{neu}, \tilde{i}_2^{neu}, \tilde{i}_3^{neu}, \tilde{i}_4^{neu}); (T_{\tilde{i}^{neu}}, I_{\tilde{i}^{neu}}, F_{\tilde{i}^{neu}}) \rangle$, $\tilde{k}^{neu} = \langle (\tilde{k}_1^{neu}, \tilde{k}_2^{neu}, \tilde{k}_3^{neu}, \tilde{k}_4^{neu}); (T_{\tilde{k}^{neu}}, I_{\tilde{k}^{neu}}, F_{\tilde{k}^{neu}}) \rangle$, and $\tilde{n}^{neu} = \langle (n_1^{neu}, n_2^{neu}, n_3^{neu}, n_4^{neu}); (T_{\tilde{n}^{neu}}, I_{\tilde{n}^{neu}}, F_{\tilde{n}^{neu}}) \rangle$
Case 1:	$\langle (1,3,5,6);(0.99,0.98,0.73) \rangle$, $\langle (8,9,11,12);(0.98,0.91,0.71) \rangle$, $\langle (15,18,22,25);(0.99,0.97,0.7) \rangle$
Case 2:	$\langle (7,9,11,12);(0.83,0.82,0.61) \rangle$, $\langle (14,15,17,18);(0.83,0.85,0.56) \rangle$, $\langle (21,24,28,3);(0.85,0.82,0.53) \rangle$
Case 3:	$\langle (8,10,12,13);(0.79,0.81,0.58) \rangle$, $\langle (15,16,18,19);(0.81,0.81,0.54) \rangle$, $\langle (22,25,29,32);(0.82,0.79,0.51) \rangle$
Case 4:	$\langle (9,11,13,14);(0.77,0.79,0.57) \rangle$, $\langle (16,17,19,20);(0.79,0.78,0.53) \rangle$, $\langle (23,26,30,33);(0.8,0.75,0.49) \rangle$
Case 5:	$\langle (10,12,14,15);(0.73,0.77,0.55) \rangle$, $\langle (17,18,20,21);(0.77,0.75,0.51) \rangle$, $\langle (24,27,31,34);(0.79,0.73,0.46) \rangle$
Case 6:	$\langle (11,13,15,16);(0.71,0.75,0.51) \rangle$, $\langle (18,19,21,22);(0.74,0.71,0.49) \rangle$, $\langle (25,28,32,35);(0.7,0.69,0.44) \rangle$

Table 3: Tabular comparison study with some of the existing methods such as Sen and Malakar (2015) (Sen & Malakar, 2015), Saranya and Varadarajan (2018) (Saranya & Varadarajan, 2018), Rajput et al. method (2019) (Rajput, Singh, & Pandey, 2019).

Demand	Sen and Malakar (2015) (Sen & Malakar, 2015), Saranya and Varadarajan (2018) (Saranya & Varadarajan, 2018), Rajput et al. method (2019) (Rajput, Singh, & Pandey, 2019)	Proposed Method
1000	NA	342.4935301
1025	NA	632.5781217

1125	NA	719.6689366
1225	NA	800.3864355
1325	NA	881.5672484
1425	NA	981.3623184

We compared our proposed method with some of the existing methods such as Sen and Malakar (2015) (Sen & Malakar, 2015), Saranya and Varadarajan (2018), and Rajput et al. method (2019) (Rajput, Singh, & Pandey, 2019) using both tabular and pictorial comparison studies.

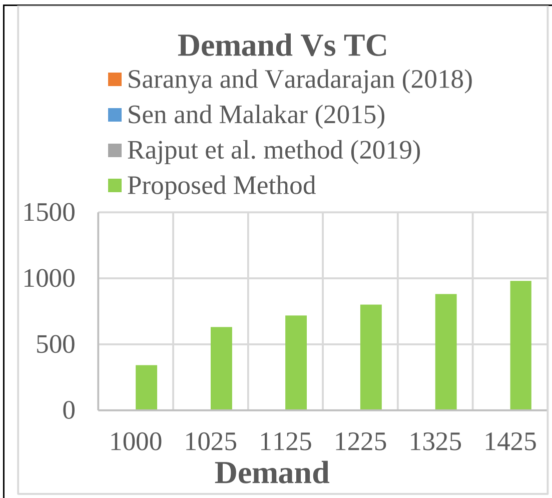


Figure 1

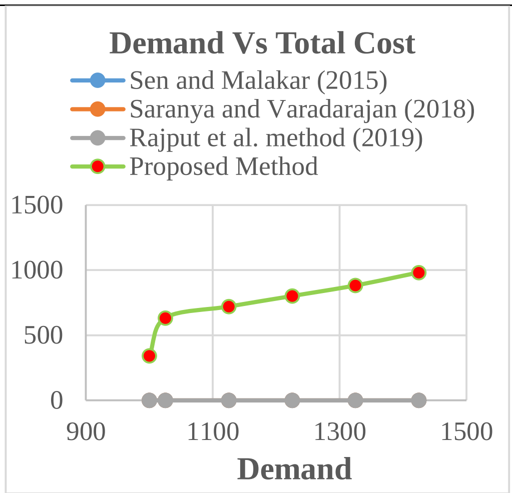


Figure 2:

In example 4.2 it is clear evident that our proposed method not only to address existing numerical challenges but also to tackle a new types of uncertain problem types. Moreover, from the above comparison it is clear that our proposed method is superior than some of the existing method. To explain more about our method, we have also conducted a sensitive analysis in the below section 5.

5. Sensitive Analysis.

A manufacturing factory needs to develop an EOQ model for optimize the total cost of their product. The length of cycle is 6 months, ordering cost is Rs. $\langle [15,18,22,25];\ (0.99,0.97,0.70)\rangle$ per unit, holding cost is Rs. $\langle [1,3,5,6];\ (0.99,0.98,0.73)\rangle$ per unit and shortage cost is Rs. $\langle [8,9,11,12];\ (0.98,0.91,0.71)\rangle$ per unit.

Table 4: Sensitive Analysis. On proposed method

Parameter	**Change in Parameters**	v	v_s	**Proposed TC**
		Proposed Method	Proposed method	Proposed Method
$\tilde{i}^{neu}$ $\langle [1,3,5,6];\ (0.99,0.98,0.73)\rangle$	13%	47.32780723	14.08615158	357.7882502
	29%	44.83152683	14.87048766	377.7103867
	37%	43.0753968	15.47673884	393.1091665
	-13%	48.68631097	13.69310292	347.804814
	-29%	53.37369196	12.4905481	317.2599217
	-37%	57.98788415	11.49665446	292.0150232
$\tilde{k}^{neu}$ $\langle [8,9,11,12];\ (0.98,0.91,0.71)\rangle$	13%	48.65951376	12.12446356	347.9963531
	29%	47.90170815	10.78867301	353.5016597
	37%	47.58566783	10.22614638	355.8494418
	-13%	50.43868855	15.19237607	335.7211264

	-29%	52.12240319	18.01465548	324.8762969
		53.25390911	19.87086161	317.9735275
$\tilde{n}^{neu}$ $\langle [15,18,22,25];\ (0.99,0.97,0.70) \rangle$	13%	52.55684753	14.33368569	364.0756165
	29%	56.15454864	15.3148769	388.9978733
	37%	57.86958518	15.78261414	400.8783992
	-13%	46.11579628	12.57703535	319.4566979
	-29%	41.65999947	11.36181804	288.5901781
		39.24283374	10.70259102	271.8458119

Logical comparison

Table 5: logical comparison with TC

Examples	**Comparison**
Example 4.1	$ClassicalTC(828.078) \succ FuzzyTC(809.039) \approx OurproposedTC(809.039)$
Example 4.2	$ClassicalTC(828.078) \succ FuzzyTC(809.039) \succ OurproposedTC(342.4935)$

In Table 5, we have conducted a comparative analysis of total cost across various environmental scenarios. Our observations reveal that in a fuzzy environment, the value of total cost is lower than that in a classical environment. Furthermore, in our proposed model, the total cost is observed to be lower than both classical and fuzzy environments.

Conclusion:

Neutrosophic set theory is a powerful tool in the domain of inventory management. It is especially useful when dealing with uncertainty, inaccuracies, and imprecise data in factors such as holding costs, shortage costs, and ordering costs. Our proposed model outperforms existing methods, offering a more effective approach to inventory optimization. Our research contributes valuable insights to enhance inventory management strategies and moderate uncertainties in EPQ operations through graphical, logical, and tabular comparisons.

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