

## Research Article

Young-Chul Kim and Doojin Ryu\*

# Segregation, Education Cost, and Group Inequality

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**Abstract:** This study analyzes the interplay between the segregation level, education cost, and the evolution of group inequality. In a market economy, individuals have incentives to invest in skill acquisition because of wage differentials. Because skill achievement is costly, a person with a higher inherent ability or a better community background is more likely to invest. Bowles, Loury, and Sethi (2014) show the possibility of group inequality evolution with a high level of segregation when network externalities over the skill acquisition period affect an individual's decision of skill achievement. This study emphasizes the effect of education costs on the evolution of group inequality. Even when the level of segregation is high, if the societal education cost of skill acquisition is not sufficiently large, group skill disparity may not evolve. Observing that education costs vary significantly across countries depending on the structure of their educational institutions, this theoretical analysis suggests that some countries may suffer more from between-group disparity than others because their education systems impose higher costs on individuals.

**Keywords:** group inequality, segregation, peer effects, education cost

**MSC 2020:** I24, I30, J15

## 1 Introduction

Socioeconomic disparities between social groups constitute a challenge in many countries worldwide. Although various

social groups may educate their children within identical educational systems and work in the same market economy, their skill achievement ratios and wage levels may differ significantly. It is thus difficult to determine a single root cause of the inequality between groups because the manner in which social groups are formed is unique to each society. For instance, groups form along racial lines in societies such as the United States, South Africa, and Australia but along religious lines in Turkey, Pakistan, and Northern Ireland. While ethnicity is important in countries such as Singapore, Indonesia, and Balkan countries, we often see caste-like social divisions in India and historical minorities such as gypsies in Europe. Furthermore, in many Western countries, the population is divided into immigrant and nonimmigrant groups.

Although these cases are distinct, a salient feature is consistent for all of them: divided social interactions between groups over their entire lifetime. The social network externalities around the skill acquisition period and the consequent development bias between groups have been discussed since the pioneering work of Loury (1977). According to this theory, the development of human beings is socially situated in the sense that communal resources influence a person's acquisition of human capital, which includes training resources, nutritional provision, after-school parenting, peer influences, mentoring, and role models. Loury (1977)'s theory is supported by numerous empirical works such as peer effects (Anderson, 2013), community effects (Cutler & Glaeser, 1997; Weinberg et al., 2004), racial network effects (Hoxby, 2002; Hanushek et al., 2009), and academic peer effects (Winston & Zimmerman, 2004).<sup>1</sup>

Several subsequent theoretical studies have discussed development bias, emphasizing network externalities over

\* **Corresponding author: Doojin Ryu**, Department of Economics, Sungkyunkwan University, Seoul 03063, Republic of Korea, e-mail: sharpjin@skku.edu

**Young-Chul Kim:** Department of Economics, Sogang University, Seoul 04107, Republic of Korea, e-mail: yckim@sogang.ac.kr

<sup>1</sup> Even after the skill acquisition period, opportunities travel along the synapses of social networks, and the benefits of skill development are influenced by social affiliations. For instance, an individual's social connections can influence one's career success via various routes, such as job referrals (Blau & Robins, 1990; Munshi, 2003), information channeling (Holzer, 1988; Rees, 1966), mentoring (Castilla, 2005; Rockoff, 2008), and business opportunities (Fafchamps & Minten, 1999; Khwaja et al., 2011). For more details, refer to Durlauf and Fafchamps (2005) and Ioannides and Loury (2004).

the skill acquisition period. For instance, Becker and Tomes (1979) and Loury (1981) focus on the effects of parental income on their offspring's education to explain the intergenerational dynamics of inequality. Lundberg and Startz (1998) consider a spillover effect between social groups where the average level of human capital in a community affects the skill investment decisions of the following generations. Benabou (1996) and Durlauf (1996) discuss the endogenous sorting of agents into homogeneous communities, given the local spillover in human capital investment.

More recently, Bowles et al. (2014), by focusing on interpersonal spillovers in human capital accumulation, have proved the instability of an equal society in a highly segregated economy under production complementarity between high- and low-skilled labor. They argue that the instability condition requires three factors – a high segregation level, strong interpersonal spillovers, and production complementarity – among which the extent of social segregation plays a critical role in determining whether group inequality can emerge.

We extend these arguments in Bowles et al. (2014) by exploring how the cost of education in a society that individuals pay for training their children is associated with the instability of an equal society. To this end, we first elaborate a concrete market structure by (1) incorporating a neoclassical production function that encompasses high- and low-skill complementarity and (2) implementing a wage redistribution scheme that reflects both the strength of spillovers and the level of segregation between social groups. Second, based on the elaborated market structure, we investigate the existence condition of a (nontrivial) symmetric steady state that represents an equal society among social groups and examine under what conditions this symmetric steady state is stable.<sup>2</sup>

Our results show that when the segregation level is sufficiently low and the spillover effect is strong enough, the symmetric steady state is stable regardless of the level of societal training costs, which is consistent with the findings of Bowles et al. (2014). However, if the conditions are not satisfied, the level of societal training costs can be the key to producing the stability of the symmetric steady state, in addition to the extent of social segregation. The higher the training costs, the more likely the symmetric steady state is unstable, indicating that group inequality

may emerge in a segregated society with high training costs. In other words, even in a highly segregated society, between-group skill disparities may not emerge at a sufficiently low societal training cost. This point is a meaningful extension of the arguments of Bowles et al. (2014).

Using the proposed dynamic model, we identify the snowball effect as a major force causing severe disparity: a small difference at the beginning results in a large difference at the end when an economy with skill complementarities has a strong externality of peer effects.<sup>3</sup> Suppose that two groups, A and B, have equal skill compositions at time  $t$ . A small advantage of the children belonging to group A provides the group with more skilled workers at time  $t + 1$ , whereas group B provides fewer skilled workers because of the global complementarities in neoclassical economies (i.e., a higher supply of skilled workers from group A leads to a wage differential decline, causing a decrease in the supply of skilled workers from group B). When the peer effect externality is strong, group A can provide even more skilled workers in the next generation, whereas group B provides even fewer workers (Loury, 1977). Subsequently, the skill difference between the two groups may increase and become larger due to the snowball effect even when the total fraction of the number of skilled workers in society does not vary significantly, implying the instability of an equal society. However, as emphasized above, the vulnerability of the initial equal state partly depends on the level of training costs in a given society. That is, with a lower training cost, the small advantage of the children belonging to group A will not induce a significant behavioral change to generate a permanent impact on group disparity.<sup>4</sup> However, an equal state in a segregated society may become unstable as societal training costs increase (e.g., advancement of skill-biased technology).

The above theoretical results have various implications for real-world situations. In the United States, residential and schooling segregation is widely known to be one of the major causes of unequal opportunities available to African Americans. As the degree of segregation between the two racial groups has declined since the civil rights movement in the 1960s, group inequality has also declined in the 1970s and 1980s. However, inequality in skill composition appears to persist throughout these decades (Loury,

<sup>2</sup> In expanding the findings in Bowles et al. (2014), Kim and Loury (2014) consider interpersonal spillovers in both the human capital investment stage and subsequent career stages and show that the coordinated expectations regarding future networks determine a social group's overall skill investment activities.

<sup>3</sup> Please refer to Benabou (1993)'s work, "Working of a City," for the mechanism of the snowball effect.

<sup>4</sup> In the extreme case, we can conjecture that there will be no skill disparity between groups with negligible training cost for skill achievement, as far as fundamental ability distributions are identical to each other.

2002).<sup>5</sup> This persistent gap is more puzzling because the degree of segregation continued to decline over the above-mentioned period. According to the dissimilarity index,<sup>6</sup> which is the most commonly used measure of segregation between two groups, the indices of segregation between African Americans and non-Hispanic whites in US metropolitan areas in 1980, 1990, 2000, and 2010 were 73.1, 67.7, 64.2, and 59.4, respectively (De la Roca et al., 2014).<sup>7</sup>

One possible reason for this persistent racial gap is the less affordable skill acquisition in the US labor market. In the 1970s, high school education was sufficient to be classified as a skilled worker, and it did not cost too much, thus making it available for poor families to train children as skilled workers at minor educational costs. However, since the 1990s, college education has replaced high school education. Without obtaining a B.A. degree, it is difficult to classify as a skilled worker. In contrast to high school education, a college education is not affordable to a considerable number of poor families. For instance, while the high school completion rate is currently on par for African American and white students, a large gap of more than 10% is maintained in terms of college graduation rates.<sup>8</sup> Therefore, the opportunities for children from poor African American households to develop their talents are more restricted these days by the increased “skill training cost” in the United States compared to the 1970s.

A notable contrast is also evident when comparing college education in the United States and Europe. In most parts of continental Europe, college education is extensively subsidized by governments and thus widely accessible to families with modest incomes. Provided that children from disadvantaged backgrounds are willing to

work diligently and possess talent, they are provided with opportunities to receive an affordable college education and become skilled workers. Consequently, in Europe, group inequality is less likely to grow compared to the United States.

In South Korea, educational costs have increased significantly since the early 1990s, when the Korean SAT was reformed. Although Korea adopts a strict public school system for high school education, parents have started to spend significant amounts on after-school private academies to improve their children’s college admission chances.<sup>9</sup> Currently, poor families cannot afford private tutoring, so they simply send their children to government-funded schools without providing extra education from private academies. Rich families send their children to intensive private education centers after school, where they further develop their talents. The sharp disparity between the “rich” south of Seoul (Gangnam) and the “modest” north (Gangbuk) in terms of college admission rates reflects how family background affects opportunities for children to develop their talent in a society with high training costs.<sup>10</sup> Furthermore, this implies that the skill disparity between south and north Seoul may increase over time because the richer southern communities can train more children, and skilled children bring more wealth to the community after joining the workplace, and they may, in turn, train more children in the next generation, and so on.

Finally, the proposed model has important implications for meritocracy. Even in a highly segregated society, the merit system can survive as long as the training cost is not too burdensome because society may converge to a symmetric steady state if talented children from both groups are given similar opportunities to develop their skills. However, in a society where training costs are high, children from advantaged and disadvantaged groups will not be given equal opportunities to develop their skills. Therefore, for the same wage differential expected in both groups, the supply of skilled workers from the rich group is greater than that from the poor group. Supply differences can widen over generations if the wealth of one’s social network becomes more deterministic of skill development

<sup>5</sup> According to a nationwide survey (i.e., the Current Population Survey), the median income for African American households in 2018 was \$51,600, while the median income for white households was \$84,600. Moreover, the median of the former households was 61% of the median income of the latter in 2018, down from 63% in 2007 (Schaeffer, 2020).

<sup>6</sup> The dissimilarity index measures the percentage of one group that would have to move across neighborhoods to be distributed in the same way as a comparison group. Please refer to White (1988) for details.

<sup>7</sup> The index number 59.4 implies that around 60% of African American (or White) households must move to achieve zero segregation.

<sup>8</sup> The percentage of African American 18- to 24-year-olds with a high school degree was nearly the same as that of their white peers, according to data from the National Center for Education Statistics. In 2017, the percentage was 94.8% for white students and 93.8% for African American students. According to the Current Population Survey, in 2015, more than a third (36%) of white people aged 25 years or older held a bachelor’s degree, compared with 23% of African American people (Pew Research Center, 2016).

<sup>9</sup> According to the 2019 Survey of Private Education Expenditures, the supplementary tutoring market of South Korea is worth about US\$20 billion, which is equivalent to approximately 50% of the government’s entire annual budget for primary and secondary education.

<sup>10</sup> Around 25% of the Seoul National University (SNU) admissions, which is known as the best college in Korea, go to rich residents in the south of Seoul (Gangnam people), which constitute only a small percent of the Korean population. Before education costs increased, only 10–15% of the Seoul National University admissions went to residents in Gangnam.

than one's inherent abilities. Therefore, the high training costs in society may result in the failure of the merit system and increase group inequality.

All examples mentioned above emphasize the significant relationship between the evolution of group inequality and the size of the training cost. The remainder of this article is organized as follows. Section 2 describes the theoretical framework of the proposed model. Section 3 examines the human development dynamics when the two social groups are indistinguishable or fully integrated. Section 4 evaluates the evolution of group disparity in an economy with distinguishable social groups. Finally, Section 5 presents the conclusions of this study.

## 2 Framework

Consider an economy composed of two social groups. We denote these as group  $b$  and group  $w$ . The proportion of each group is  $\beta^b$  and  $\beta^w$ , respectively, whose sum is one, where  $\beta^b \leq \beta^w$ . In addition, suppose there are two occupations: skilled ( $h$ ) and unskilled ( $l$ ) workers. Each agent lives for two periods: training and working periods. Generations overlap. Parents decide whether to train their children after observing a child's ability  $a$  at time  $t$ . The training cost for a skilled job is  $k$ ; however, there is no cost for an unskilled job. Children earn at time  $t + 1$ , and their wages depend on occupation, being  $w_h$  for a skilled job and  $w_l$  for unskilled jobs. For convenience, we assume there is a single parent and a single child in each family.

The proportion of skilled workers in group  $i \in \{b, w\}$  at time  $t$  is denoted as  $x_h^i(t)$ , and the proportion of unskilled workers as  $x_l^i(t)$ , implying  $x_h^i(t) + x_l^i(t) = 1$ . Therefore, the proportion of each occupation  $j \in \{h, l\}$  in the economy at time  $t$  is

$$x_j(t) = \beta^b x_j^b(t) + \beta^w x_j^w(t).$$

A neoclassical production function is given by  $f(x_h(t), x_l(t))$ , which is homogeneous of degree one. The wage for occupation  $j$  at time  $t$ ,  $w_j(t)$ , is the marginal productivity of the occupation,

$$w_j(t) = \frac{\partial f(x_h(t), x_l(t))}{\partial x_j(t)} (\equiv f_j(x_h(t), x_l(t))).$$

The marginal productivity of occupation  $j$  diminishes as  $x_j(t)$  increases,  $f_{jj} < 0$ . However, it increases with increased quantities of another input,  $f_{hl} > 0$ . The total product is zero when there are no skilled workers,  $f(0, 1) = 0$ . The

average wage in the economy equals the per capita output,  $f(x_h(t), x_l(t))$ , which is expressed as follows:

$$\bar{w}(t) = x_h(t)w_h(t) + x_l(t)w_l(t) (\equiv f(x_h(t), x_l(t))). \quad (1)$$

The average wage of group  $i$  is

$$\bar{w}^i(t) = x_h^i(t)w_h(t) + x_l^i(t)w_l(t). \quad (2)$$

Note that the average wage in the economy is the weighted sum of each group's average wage,  $\bar{w}(t) = \beta^b \bar{w}^b(t) + \beta^w \bar{w}^w(t)$ .

The wage difference is  $\Delta w(t) = w_h(t) - w_l(t)$ , which should be positive at market equilibrium because of the training cost required to achieve the occupation  $h$ . Suppose that the wage difference goes to infinity as  $x_h$  approaches zero, while it disappears as  $x_h$  approaches  $\bar{x}$ . Then, the average wage is an increasing function of  $x_h$  for any  $x_h \in (0, \bar{x})$  because the per capita total product,  $f(x_h, 1 - x_h)$ , is concave with respect to  $x_h$  and is maximized at  $\bar{x}$ .

### 2.1 Peer Effects

Peer effects in the economy relate to the redistribution of wages in each period between skilled ( $h$ ) and unskilled ( $l$ ) workers and between workers in groups  $b$  and  $w$ . We define the parent's effective wages in group  $i$  for occupation  $j$  as follows:

$$\tilde{w}_j^i(t) = (1 - \gamma)w_j(t) + \gamma\{\eta\bar{w}^i(t) + (1 - \eta)\bar{w}(t)\}, \quad (3)$$

where  $\gamma \in [0, 1]$  and  $\eta \in [0, 1]$  indicate the strength of the spillovers and the segregation level of social network for each. Note that the weighted average of the effective wages is the average wage in the economy.

$$\sum_{i \in b, w} \sum_{j \in h, l} \beta^i x_j^i(t) \tilde{w}_j^i(t) = \bar{w}(t).$$

Therefore, we have a production-neutral peer effect model. In other words, the total product of  $x_h(t)$  does not vary with  $\gamma$  or  $\eta$ . Instead, the total output reaped in the economy is redistributed among workers according to the size of spillover ( $\gamma$ ) and level of segregation ( $\eta$ ). Furthermore, the effective wealth of group  $i$  can be denoted as the average wage of group  $i$  and the average wage in the entire economy with weights  $\eta'$  and  $1 - \eta'$ :

$$\tilde{w}^i(t) = \eta' \bar{w}^i(t) + (1 - \eta') \bar{w}(t), \quad (4)$$

where  $\eta' = 1 - \gamma + \gamma\eta$ .

Let us call  $\eta'$  the effective segregation level because it reflects the degree of wealth transfer through peer effects between the two social groups. For simplicity, the symbol



$w(b)$  indicates the advantaged (disadvantaged) group at time  $t = 0$ :  $\bar{w}^w|_{t=0} \geq \bar{w}^b|_{t=0}$ .

## 2.2 A Parent's Decision

Suppose that a child's ability is distributed with a cumulative distribution function (CDF)  $G$  and its probability density function (PDF)  $g$ . There are no significant differences between the groups in terms of innate ability.<sup>11</sup> Training costs for a child with an ability level  $K(a)$  is assumed to be  $K(a) = k - a$ . Given the effective wage of a parent belonging to group  $i$  with occupation  $j$ ,  $\tilde{w}_j^i(t)$ , the parent trains his/her child only if utility decreases at time  $t$  after spending the training cost  $K(a)$  is less than or equal to the discounted wage differential at time  $t + 1$ :

$$u(\tilde{w}_j^i(t)) - u(\tilde{w}_j^i(t) - K(a)) \leq \delta \Delta w(t + 1).$$

Parents' marginal utility of consumption diminishes as consumption increases. Therefore, we assume that the parent's utility function is  $\ln(w)$ , which is concave. Then, a parent with  $(i, j)$  at time  $t$  trains his/her child if and only if  $K(a) \leq \tilde{w}_j^i(t)(1 - e^{-\delta \Delta w(t+1)})$ . Therefore, he/she trains the child when the child's ability exceeds some threshold  $(\tilde{a}_{j,t}^i)$ :

$$\text{Train if } a \geq k - \tilde{w}_j^i(t)(1 - e^{-\delta \Delta w(t+1)}) \quad (\equiv \tilde{a}_{j,t}^i). \quad (5)$$

Given the ability distribution  $\tilde{G}_{j,t}^i$  of children of parents with occupation  $j$  in group  $i$  in period  $t$ , the probability that the  $(i, j)$  parent trains his/her child is  $\Pr[(i, j)_t \rightarrow h_{t+1}] = 1 - \tilde{G}_{j,t}^i(\tilde{a}_{j,t}^i)$ . Hence, the fraction of skilled workers in group  $i$  in period  $t + 1$  is:

$$x_h^i(t + 1) = \sum_{j \in h, l} x_j^i(t) [1 - \tilde{G}_{j,t}^i(\tilde{a}_{j,t}^i)]. \quad (6)$$

The proportion of skilled workers in the economy at time  $t + 1$  is determined by  $x_h^b(t)$  and  $x_h^w(t)$ :

$$\begin{aligned} x_h(t + 1) &= \sum_{i \in b, w} \beta^i x_h^i(t + 1) \\ &= \sum_{i \in b, w} \sum_{j \in h, l} \beta^i x_j^i(t) [1 - \tilde{G}_{j,t}^i(\tilde{a}_{j,t}^i)] \\ &= \sum_{i \in b, w} \sum_{j \in h, l} \beta^i x_j^i(t) [1 - \tilde{G}_{j,t}^i(k - \tilde{w}_j^i(t) \\ &\quad \times (1 - e^{-\delta \Delta w(t+1)}))]. \end{aligned} \quad (7)$$

For any given  $(x_h^b(t), x_h^w(t))$ , the skilled labor supply  $x_h(t + 1)$  is determined for each level of  $\Delta w(t + 1)$  using the above equation. This labor supply curve slopes upward

with respect to  $\Delta w(t + 1)$ . In addition, the wage difference  $\Delta w(t + 1)$  is simply a decreasing function with respect to supplied skilled labor  $x_h(t + 1) \in (0, \bar{x})$ . Since  $\Delta w(t + 1) \rightarrow \infty$  as  $x_h(t + 1) \rightarrow 0$  and  $\Delta w(t + 1) \rightarrow 0$  as  $x_h(t + 1) \rightarrow \bar{x}$ , there always exists a unique market equilibrium  $x_h(t + 1)^*$  and  $\Delta w(t + 1)^*$  given  $(x_h^b(t), x_h^w(t))$ , which uniquely determines both  $x_h^b(t + 1)^*$  and  $x_h^w(t + 1)^*$  according to equations (5) and (6). Therefore, the state  $(x_h^b(t), x_h^w(t))$  for  $t = 1, 2, 3, \dots$  is uniquely determined by the initial state  $(x_h^b(0), x_h^w(0))$ :  $\{x_h^b(t), x_h^w(t); t = 1, 2, 3, \dots | (x_h^b(0), x_h^w(0))\}$ .

## 2.3 Ability Distribution

To simplify the dynamic structure without loss of generality (WLOG), we impose the following assumption.

**Assumption 1.** WLOG, child ability distribution is identical for any  $(i, j)$  parent group and for any period  $t$ :

$$G(a) = \tilde{G}_{j,t}^i(a), \quad \forall (i, j, t).$$

Suppose  $G(a)$  is a uniform distribution in  $[0, B]$ , and the training cost  $(k - a)$  is positive for every student:

$$G(a) = \frac{a}{B}, \quad \text{where } B < k. \quad (8)$$

Even the highest-ability students incur some cost  $(k - B)$  to achieve the necessary skills, whereas the skill acquisition cost for the least-talented ones is  $k$ . The average societal training cost (TC) of generation  $t$  agents is  $k - 0.5B$ . It is noteworthy that TC is an increasing function of  $k$ .

Using uniform distribution  $G(a)$ , we obtain the following result using equations (5) and (6):

$$x_h^i(t + 1) = 1 - G(k - \tilde{w}_j^i(t)(1 - e^{-\delta \Delta w(t+1)})). \quad (9)$$

Therefore, the transition matrix in equation (7) can be simplified in a more tractable manner as follows:

$$\begin{aligned} x_h(t + 1) &= \beta^w x_h^w(t + 1) + \beta^b x_h^b(t + 1) \\ &= \sum_{i \in w, b} \beta^i [1 - G(k - \tilde{w}_j^i(t)(1 - e^{-\delta \Delta w(t+1)}))]. \end{aligned} \quad (10)$$

In the following analysis, for notational simplicity, we use  $x_t^b$  instead of  $x_h^b(t)$  and  $x_t^w$  instead of  $x_h^w(t)$ , implying  $x_j^i(t) \equiv 1 - x_t^i$ . In addition, we denote  $x_h(t)$  as  $x_t$ , implying  $x_t(t) \equiv 1 - x_t$ .

## 3 Homogeneous Economy

First, let us consider the simplest structure of the economy, in which the two social groups are indistinguishable or

<sup>11</sup> Accordingly, we follow anti-essentialism adopted by Loury (2002).

fully integrated. Therefore, we impose a zero-segregation level  $\eta = 0$ .

In this homogeneous economy, the fraction of highly skilled workers evolves in the following manner:

$$x_{t+1} = x_t[1 - G(\tilde{a}_t)] + (1 - x_t)[1 - G(\tilde{a}_t)], \quad (11)$$

where  $\tilde{a}_j = k - \tilde{w}_j(t)(1 - e^{-\delta\Delta w(t+1)})$ .

As  $x_t \rightarrow 0$ , we have  $\tilde{w}(t) (\equiv f(x_t, 1 - x_t)) \rightarrow 0$ , which implies  $\tilde{w}_t(t) \rightarrow 0$  and  $G(\tilde{a}_t) \rightarrow 1$  from equation (1). Therefore, we can conclude that  $x^* = 0$  is a trivial steady state. Generally, given the uniform distribution of  $G(a)$ ,  $x_t$  evolves according to equation (9):

$$x_{t+1} = 1 - G(k - \tilde{w}(t)(1 - e^{-\delta\Delta w(t+1)})) \\ = -\frac{k - B}{B} + \tilde{w}(t) \frac{1 - e^{-\delta\Delta w(t+1)}}{B}, \quad (12)$$

where  $\tilde{w}(t) = \bar{w}(t)$  from equation (4). We can rewrite the results as follows:

$$\frac{Bx_{t+1} + k - B}{1 - e^{-\delta\Delta w(t+1)}} = \bar{w}(t) (\equiv f(x_t, 1 - x_t)). \quad (13)$$

The LHS is an increasing function with respect to  $x_{t+1}$  and the LHS at  $x_{t+1} = 0$  equals  $k - B$ , while the RHS is also an increasing function with respect to  $x_t \in [0, \bar{x}]$  and the RHS at  $x_t = 0$  is zero. As  $x_{t+1} \rightarrow \bar{x}$ , the LHS approaches infinity because  $\Delta w(t + 1) \rightarrow 0$ . As  $x_t \rightarrow \bar{x}$ , the RHS approaches  $\max\{f(x, 1 - x)\}$ .

It is noteworthy that when  $k$  is too large, the cost incurred for skill achievement,  $(k - a)$ , is too high for all workers, and no one invests in skills, implying that a trivial steady state  $x^* = 0$  is a unique steady state. However, when the cost is not too high, it is possible to have multiple steady states that satisfy the dynamic equation (13), as illustrated

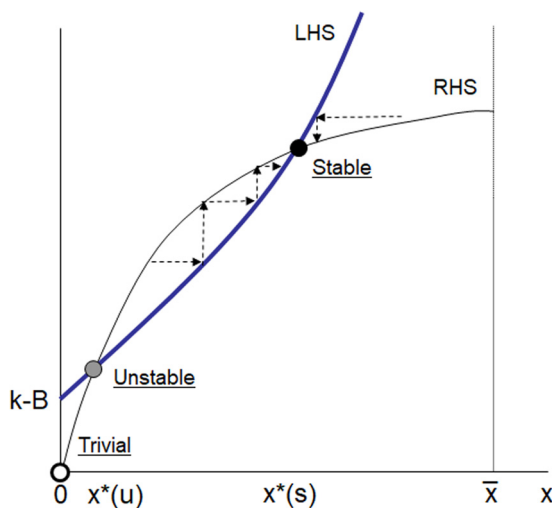


Figure 1: Stability of steady states in the homogeneous economy.

in Figure 1. In the following analysis, we assume a multiplicity of steady states with a sufficiently small  $k$ , which is guaranteed by a sufficiently high sensitivity of the per capita output  $\partial f(x_t, 1 - x_t)/\partial x_t (\equiv \partial \bar{w}(t)/\partial x_t)$  around  $x_t = 0$ .

Therefore, there must be a threshold level of  $k$  denoted by  $\hat{k}$  above which the multiplicity of steady states disappears, and the steady state is uniquely defined by a trivial steady state  $x^* = 0$ . When  $k$  is equal to threshold  $\hat{k}$ , there exists a trivial steady state ( $x^* = 0$ ) and a nontrivial steady state, denoted by  $\hat{x}$ . For any  $k$  below  $\hat{k}$ , there exists a trivial steady state ( $x^* = 0$ ) and more than one nontrivial steady state. In the case of  $k < \hat{k}$ , without loss of generality, we assume that there are three steady states, including a trivial steady state  $x^* = 0$ . We denote the two nontrivial steady states as  $x^*(u)$  and  $x^*(s)$ , where  $x^*(u) < \hat{x} < x^*(s)$ . As illustrated in Figure 1,  $x^*(u)$  is unstable, and  $x^*(s)$  is stable in the given dynamic system (13). Therefore, the final state is either  $x^*(s)$  or zero, depending on the initial state in the long run.

Note that there is an alternative method for searching steady states. Equation (12) implies that the steady states are the  $x$  values of  $(x, \tilde{a})$ s that satisfy the following equation:

$$x(\tilde{a}) (\equiv 1 - G(\tilde{a})) = 1 - \max\left\{\min\left\{\frac{\tilde{a}}{B}, 1\right\}, 0\right\}, \quad (14)$$

$$\tilde{a}(x) = k - \bar{w}[1 - e^{-\delta\Delta w}], \quad (15)$$

where both  $\bar{w}$  and  $\Delta w$  are functions of  $x$ . The blue kinked line in Figure 2 represents exactly the  $(x, \tilde{a})$  values that satisfy  $x = 1 - G(\tilde{a})$  in equation (14). As Figure 2 shows,  $\tilde{a}(x)$  approaches  $k$  in equation (15) as  $x \rightarrow 0$  or  $x \rightarrow \bar{x}$  because  $\bar{w} = 0$  with  $x = 0$  and  $\Delta w = 0$  with  $x = \bar{x}$ . We also know that  $\tilde{a}(x) < k$  for any  $x \in (0, \bar{x})$ . When  $k$  is too large, curve  $\tilde{a}(x)$  in Figure 2 moves significantly upward, and there cannot be a combination  $(x, \tilde{a})$  that satisfies equations (14) and (15), except for trivial point  $(0, k)$ . When  $k$  is not too large, we have multiple combination points  $(x, \tilde{a})$  that

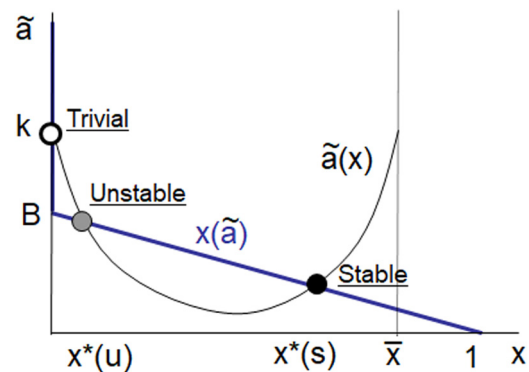


Figure 2: Search for steady states in the homogeneous economy.

satisfy the two equations. As mentioned above, without loss of generality, we assume three combination points, given  $k < \hat{k}$ . The first is a trivial point,  $(0, k)$ . The others are  $(x^*(u), \tilde{a}(x^*(u)))$  and  $(x^*(s), \tilde{a}(x^*(s)))$ , as illustrated in Figure 2.

Because trivial steady state  $x^* = 0$  is not of interest, we limit our analysis to multiple steady states  $(0, x^*(u), x^*(s))$  with  $k < \hat{k}$  in the following discussion. For  $k > B$ , we assume  $k \in (B, \hat{k})$ .

## 4 Heterogeneous Economy

Now, let us consider an economy with distinguishable social groups in which the segregation level is nonzero ( $\eta > 0$ ). Since equations (6) and (11) are identical, the three steady states in a homogeneous economy  $(0, x^*(u), x^*(s))$  must correspond to the symmetric steady states in the heterogeneous economy  $((0, 0), (x^*(u), x^*(u)), (x^*(s), x^*(s)))$ .

One of the nontrivial symmetric steady states,  $(x^*(u), x^*(u))$ , is unstable in the heterogeneous economy, as  $x^*(u)$  is unstable in the homogeneous economy. For example, a small symmetric perturbation from the state,  $(x^*(u) + \varepsilon, x^*(u) + \varepsilon)$  results in divergence because the dynamics along the symmetric path are determined through equation (13).

Therefore, the main focus of this analysis is whether the other nontrivial symmetric steady state,  $(x^*(s), x^*(s))$ , is still stable in a heterogeneous economy. By using equations (4), (8), and (9), the group skill difference at time  $t + 1$ ,  $\Delta x(t + 1)$ , is expressed as follows:

$$\begin{aligned} & |x_h^w(t + 1) - x_h^b(t + 1)| \\ &= |\tilde{w}^w(t) - \tilde{w}^b(t)| \cdot \frac{(1 - e^{-\delta \Delta w(x_{t+1})})}{B} \quad (\because \text{equations (8) and (9)}) \quad (16) \\ &= \eta' |\bar{w}^w(t) - \bar{w}^b(t)| \cdot \frac{(1 - e^{-\delta \Delta w(x_{t+1})})}{B} \quad (\because \text{equation (4)}) \end{aligned}$$

Applying equation (2), we obtain:  $\bar{w}^w(t) - \bar{w}^b(t) = (x_h^w(t) - x_h^b(t)) \cdot (w_h(t) - w_l(t))$ . The skill difference then evolves as follows:

$$\begin{aligned} & |x_h^w(t + 1) - x_h^b(t + 1)| \\ &= |x_h^w(t) - x_h^b(t)| \cdot \frac{\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})}) \cdot (1 - \gamma + \gamma \eta)}{B}. \quad (17) \end{aligned}$$

Therefore, when  $\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})}) \cdot (1 - \gamma + \gamma \eta)$  is greater than  $B$ , group skill difference  $\Delta x$  diverges over time. Let us consider the value of  $\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})}) \cdot (1 - \gamma + \gamma \eta)$  at a symmetric

steady state  $(x^*, x^*)$ :  $\Delta w(x^*)(1 - e^{-\delta \Delta w(x^*)}) \cdot (1 - \gamma + \gamma \eta)$ , which is infinite when  $x^* = 0$  and zero when  $x^* = \bar{x}$ . Because the value strictly decreases with respect to  $x^*$ , there exists a unique  $x^\eta \in (0, \bar{x})$  that satisfies  $\Delta w(x^\eta)(1 - e^{-\delta \Delta w(x^\eta)}) \cdot (1 - \gamma + \gamma \eta) = B$  for any given segregation level  $\eta$ . When  $x^*$  is greater (smaller) than the threshold  $x^\eta$ , given symmetric steady state  $(x^*, x^*)$  is stable (unstable) because  $\Delta w(x^*)(1 - e^{-\delta \Delta w(x^*)}) \cdot (1 - \gamma + \gamma \eta) < (>) B$ . As  $x^*(s)$  is a decreasing function of  $k$ , we conjecture that an increase in the societal skill acquisition cost captured by  $k$  tends to result in the instability of the nontrivial symmetric steady state,  $(x^*(s), x^*(s))$ .

If  $k$  is equal to threshold  $\hat{k}$ , there exists only one nontrivial symmetric steady state,  $(\hat{x}, \hat{x})$ . Given assumed condition  $k \in (B, \hat{k})$ , we always have  $x^*(u) < \hat{x} < x^*(s)$ . Therefore, when  $x^\eta$  is even smaller than  $\hat{x}$ , we must have  $x^\eta < x^*(s)$  regardless of the size of the societal cost level  $k \in (B, \hat{k})$ , which implies that a nontrivial symmetric steady state  $(x^*(s), x^*(s))$  is always stable.

Notably, threshold  $x^\eta$  becomes smaller than  $\hat{x}$  when  $(1 - \gamma + \gamma \eta)$  is sufficiently small because  $x^\eta$  satisfies  $\Delta w(x^\eta)(1 - e^{-\delta \Delta w(x^\eta)}) \cdot (1 - \gamma + \gamma \eta) = B$ . Noting that  $(1 - \gamma + \gamma \eta)$  is an increasing function of the segregation level  $\eta$  and a decreasing function of the spillover level  $\gamma$ , we obtain the following result:

**Proposition 1.** *If segregation level  $\eta$  is sufficiently low and the spillover effect measured by  $\gamma$  is strong enough that  $x^\eta < \hat{x}$  holds, nontrivial symmetric steady state  $(x^*(s), x^*(s))$  is stable regardless of cost parameter  $k \in (B, \hat{k})$ .*

However, when  $x^\eta$  exceeds  $\hat{x}$ , the stability of  $(x^*(s), x^*(s))$  depends on  $k \in (B, \hat{k})$ . Suppose that  $x^\eta > \hat{x}$  holds. Then, as  $k$  approaches  $\hat{k}$ ,  $x^*(s)$  approaches  $\hat{x}$ , implying that  $x^*(s)$  becomes smaller than  $x^\eta$  and the nontrivial symmetric steady state  $(x^*(s), x^*(s))$  becomes unstable. We also observe that  $x^*(s)$  becomes greater than  $x^\eta$  as  $k$  approaches  $B$  according to the following corollary, implying that the nontrivial symmetric steady state  $(x^*(s), x^*(s))$  becomes stable.

**Corollary 1.** *When  $k = B$ , a unique nontrivial steady state  $x^*$  is greater than  $x^\eta$  in a homogeneous economy.*

**Proof.** When  $k = B$ , a nontrivial steady state  $x^*$  must satisfy  $\frac{Bx^*}{1 - e^{-\delta \Delta w(x^*)}} = \bar{w}(x^*)$  according to equation (13). By definition,  $x^\eta$  satisfies  $\Delta w(x^\eta)(1 - e^{-\delta \Delta w(x^\eta)}) \cdot (1 - \gamma + \gamma \eta) = B$ . Suppose  $x^* \leq x^\eta$ , meaning that  $(1 - e^{-\delta \Delta w(x^*)}) \geq (1 - e^{-\delta \Delta w(x^\eta)})$ . Then, we must have  $\frac{Bx^*}{\bar{w}(x^*)} \geq \frac{B}{\Delta w(x^\eta)(1 - \gamma + \gamma \eta)}$ , equivalently  $x^* \Delta w(x^\eta)(1 - \gamma + \gamma \eta) \geq \bar{w}(x^*) = x^* \Delta w(x^*) + w_l(x^*)$ ,

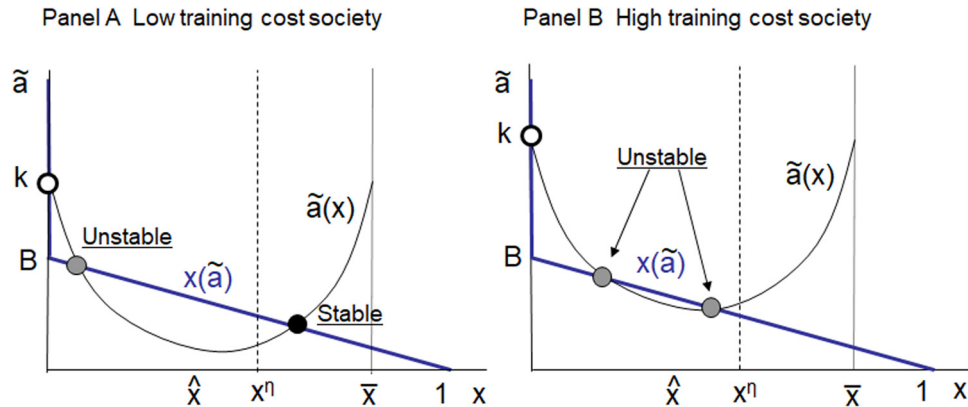


Figure 3: Stability of the symmetric steady states (given  $x^\eta > \hat{x}$ ).

implying  $\Delta w(x^\eta) > \Delta w(x^*)$ . However, this result contradicts the proposition that  $x^* \leq x^\eta$  because  $\Delta w(x)$  is a decreasing function of  $x$ . Therefore, we can conclude that  $x^* > x^\eta$  when  $k = B$ .  $\square$

Therefore, given  $x^\eta > \hat{x}$ , the nontrivial symmetric steady state  $(x^*(s), x^*(s))$  is unstable with a sufficiently large cost parameter  $k$ , whereas it is stable with a sufficiently small cost parameter  $k$ . As  $x^*(s)$  monotonically decreases in  $k$ , we obtain the following result:

**Theorem 1.** *Given  $x^\eta > \hat{x}$ , there exists a threshold level  $\tilde{k} \in (B, \hat{k})$  such that nontrivial symmetric steady state  $(x^*(s), x^*(s))$  is unstable if  $k > \tilde{k}$  and stable if  $k < \tilde{k}$ .*

In Figure 3, we compare a society with a low training cost of  $k < \tilde{k}$  (Panel A) with a society with a high training cost of  $k > \tilde{k}$  (Panel B). In particular, no stable (nontrivial) symmetric steady states exist in Panel B.

From equation (10), we obtain the following transition in the overall skill rate ( $x_t$ ) in the economy:

$$x_{t+1} = -\frac{k-B}{B} + \bar{w}(x_t) \frac{1 - e^{-\delta \Delta w(x_{t+1})}}{B}.$$

This rearrangement yields the following:

$$\frac{Bx_{t+1} + k - B}{1 - e^{-\delta \Delta w(x_{t+1})}} = \bar{w}(x_t), \quad (18)$$

which is exactly the same as the transition structure of the homogeneous economy noted in equation (13). Therefore,  $x_t$  uniquely determines the next skill rate,  $x_{t+1}$ , even in a heterogeneous economy. The iso- $x$  lines representing the  $(x^b, x^w)$  points satisfying  $\beta^b x^b + \beta^w x^w = x$  are shown in Figure 4 for each level of  $x \in (0, \bar{x})$ . State  $(x_t^b, x_t^w)$  moves along the iso- $x$  line that passes through a nontrivial symmetric steady state  $(x^*(s), x^*(s))$  when initial state  $(x_0^b, x_0^w)$

is on the line. The same holds for  $(x^*(u), x^*(u))$ . Between the two iso- $x$  lines, the overall skill rate  $x_t$  increases over time, whereas it decreases outside the two lines.

Figure 4 illustrates the case of the high training cost society depicted in Panel B of Figure 3, in which both nontrivial symmetric steady states are unstable. Because state  $(x_t^b, x_t^w)$  diverges, moving along the iso- $x$  line that passes the nontrivial symmetric steady state  $(x^*(s), x^*(s))$ , we can conjecture that stable steady states are obtained at both ends of the iso- $x$  line. For example, as shown in Figure 4, the following are stable steady states:  $(0, \frac{x^*(s)}{\beta^w})$  and  $(1, \frac{x^*(s) - \beta^b}{\beta^w})$  as far as  $\beta^b < x^*(s) < \beta^w$ . Thus, we obtain the following result:

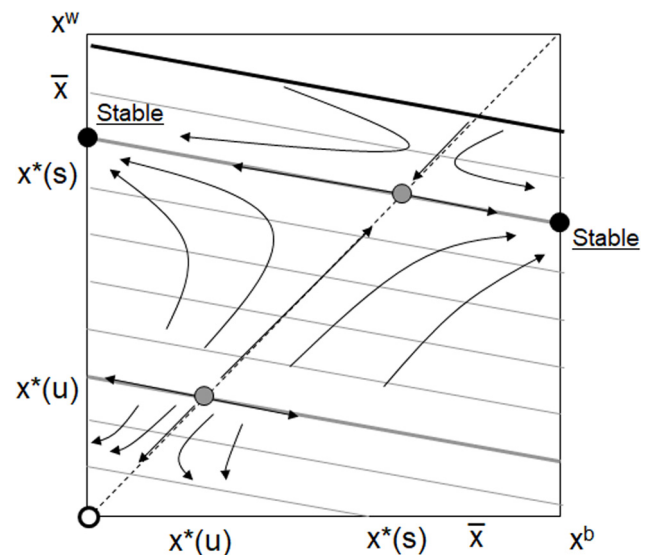


Figure 4: Unstable symmetric steady states.



**Proposition 2.** Given  $x^\eta > \hat{x}$ , in a high training cost society with  $k > \tilde{k}$ , the skill rate difference between the two groups,  $|x^w - x^b|$ , diverges along the iso- $x$  line satisfying  $\beta^b x^b + \beta^w x^w = x^*(s)$ .

However, in the case of the low training cost society depicted in Panel A of Figure 3, there is a stable symmetric steady state  $(x^*(s), x^*(s))$ , and any initial state  $(x_0^w, x_0^b)$  converges to the state when the initial societal skill share  $x_0$ , that is,  $\beta^w x_0^w + \beta^b x_0^b$ , is above a certain level,  $(x^*(u))$ . In particular, as long as the training cost parameter  $k$  is close to  $B$ , a stable symmetric steady state  $(x^*(s), x^*(s))$  always exists, regardless of segregation level  $\eta$ , according to Corollary 1. This point has been largely ignored in previous studies, including Bowles et al. (2014).

## 5 Conclusion

This study discusses the importance of education costs for the evolution of group skill disparity. Because the cost of skill achievement is affected by one's inherent ability and the quality of one's social network, the degree of integration between social groups must be considered in the analysis of group disparity (Chaudhuri & Sethi, 2008). Three factors – a level of integration, the size of education costs, and the neo-classical market system – are intricately interwoven in the analysis. The theoretical work presented in this study successfully manages to show how these factors affect each other to determine the emergence of group skill disparity.

In particular, from the analysis, we show that the size of education costs can play a key role in the evolution of group inequality. Even in a highly segregated society, low education costs may prevent group disparities in the neo-classical market economy. However, we do not insist that *integration* is a less important policy measure to solve the problem of group inequality (Bowles et al., 2014; Sethi & Somanathan, 2004). Rather, we suggest that, observing significant variances in educational systems across countries worldwide, countries with high private education costs may suffer more from growing group inequality, and the market system alone cannot stop the failure of the merit system. In addition, the emerging trend of a skill-biased economy will challenge group disparities in the future.

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## References

- Anderson, E. (2013). *Streetwise: Race, class, and change in an urban community*. University of Chicago Press.
- Becker, G. S., & Tomes, N. (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of Political Economy*, 87(6), 1153–1189.
- Benabou, R. (1993). Workings of a city: Location, education, and production. *The Quarterly Journal of Economics*, 108(3), 619–652.
- Benabou, R. (1996). Equity and efficiency in human capital investment: The local connection. *The Review of Economic Studies*, 63(2), 237–264.
- Blau, D. M., & Robins, P. K. (1990). Job search outcomes for the employed and unemployed. *Journal of Political Economy*, 98(3), 637–655.
- Bowles, S., Loury, G. C., & Sethi, R. (2014). Group inequality. *Journal of the European Economic Association*, 12(1), 129–152.
- Castilla, E. J. (2005). Social networks and employee performance in a call center. *American Journal of Sociology*, 110(5), 1243–1283.
- Chaudhuri, S., & Sethi, R. (2008). Statistical discrimination with peer effects: Can integration eliminate negative stereotypes? *The Review of Economic Studies*, 75(2), 579–596.
- Cutler, D. M., & Glaeser, E. L. (1997). Are ghettos good or bad? *The Quarterly Journal of Economics*, 112(3), 827–872.
- De la Roca, J., Ellen, I. G., & O'Regan, K. M. (2014). Race and neighborhoods in the 21st century: What does segregation mean today? *Regional Science and Urban Economics*, 47, 138–151.
- Durlauf, S. N. (1996). A theory of persistent income inequality. *Journal of Economic growth*, 1(1), 75–93.
- Durlauf, S. N., & Fafchamps, M. (2005). Social capital. In P. Aghion & S. Durlauf, (Eds.), *Handbook of Economic Growth* (pp. 1639–1699). Elsevier.
- Fafchamps, M., & Minten, B. (1999). Relationships and traders in Madagascar. *The Journal of Development Studies*, 35(6), 1–35.
- Hanushek, E. A., Kain, J. F., & Rivkin, S. G. (2009). New evidence about Brown v. Board of Education: The complex effects of school racial

- composition on achievement. *Journal of Labor Economics*, 27(3), 349–383.
- Holzer, H. J. (1988). Search method use by unemployed youth. *Journal of Labor Economics*, 6(1), 1–20.
- Hoxby, C. M. (2002). The power of peers: How does the makeup of a classroom influence achievement? *Education Next*, 2(2), 57–64.
- Ioannides, Y. M., & Loury, L. D. (2004). Job information networks, neighborhood effects, and inequality. *Journal of Economic Literature*, 42(4), 1056–1093.
- Khwaja, A. I., Mian, A., & Abid, Q. (2011). Bank credit and business networks. *HKS Faculty Research Working Paper Series*.
- Kim, Y.-C., & Loury, G. C. (2014). Social externalities, overlap and the poverty trap. *The Journal of Economic Inequality*, 12(4), 535–554.
- Loury, G. C. (1977). A dynamic theory of racial income differences. In P. A. Wallace & A. M. LaMond, (Eds.), *Women, minorities, and employment discrimination*. ERIC.
- Loury, G. C. (1981). Intergenerational transfers and the distribution of earnings. *Econometrica*, 49(4), 843–867.
- Loury, G. C. (2002). *The Anatomy of Racial Inequality*. Harvard University Press.
- Lundberg, S., & Startz, R. (1998). On the persistence of racial inequality. *Journal of Labor Economics*, 16(2), 292–323.
- Munshi, K. (2003). Networks in the modern economy: Mexican migrants in the US labor market. *The Quarterly Journal of Economics*, 118(2), 549–599.
- Pew Research Center. (2016). *On views of race and inequality, blacks and whites are worlds apart*. Pew Research Center.
- Rees, A. (1966). Information networks in labor markets. *American Economic Review*, 56(1/2), 559–566.
- Rockoff, J. E. (2008). *Does mentoring reduce turnover and improve skills of new employees? evidence from teachers in New York City*. Technical report, NBER Working Papers No. 13868.
- Schaeffer, K. (2020). *6 facts about economic inequality in the U.S.* Pew Research Center.
- Sethi, R., & Somanathan, R. (2004). Inequality and segregation. *Journal of Political Economy*, 112(6), 1296–1321.
- Weinberg, B. A., Reagan, P. B., & Yankow, J. J. (2004). Do neighborhoods affect hours worked? evidence from longitudinal data. *Journal of Labor Economics*, 22(4), 891–924.
- White, M. J. (1988). *American neighborhoods and residential differentiation*. Russell Sage Foundation.
- Winston, G., & Zimmerman, D. (2004). Peer effects in higher education. In: *College choices: The economics of where to go, when to go, and how to pay for it* (pp. 395–424). University of Chicago Press.