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Segregation, Education Cost and the Group Inequality

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Abstract

This paper deals with the interplay between the segregation level, the size of education cost, and the evolution of group inequality. In the market economy, everybody has an incentive to invest in skill acquisition due to the wage differential between a skilled worker and an unskilled worker. Since the skill achievement is costly, the person with the higher inherent ability or better community background is more likely to invest. Bowles et al. (2014) shows the possibility of group inequality evolution with a high level of segregation when the network externality over the skill acquisition period affects an individual's decision for skill achievement. In this paper, I emphasize the importance of the level of education cost on the evolution of group inequality. Even when the level of segregation is high, if the societal education cost for skill acquisition is not big enough, the group skill disparity may not evolve in a competitive market economy. Observing that the education cost varies significantly across countries depending on the structure of educational institutions, the theoretic work suggests some countries may suffer more than others from the between-group disparity due to their education systems that impose higher costs on individuals.

KEYWORDS: Group Inequality, Segregation, Education Cost, Merit System.

1 Introduction

This work shows how the education cost in a society that individuals pay to train their kids is associated with the problem of group inequality. I suggest that the higher the education cost, the more likely group disparity will grow. First, I check the stability of symmetric steady states and show how those are vulnerable in a high training cost society with a high segregation level. Bowles et al. (2014) show that a high segregation level in a society tends to bring group skill disparity. Unlike the abstract model in their paper, I elaborate a concrete story about the instability of symmetric steady states and search for the sufficient conditions that cause the instability. Secondly, I suggest that the societal training cost is a key to produce the instability of symmetric steady states. The higher the training cost, the more likely the group inequality will grow in a society.

The theoretical suggestions have various implications about the real world. In the US, group inequality between black and white got shrunk over the 70s and 80s, but the inequality in skill composition seems to have been persistent throughout these decades (Loury, 2002). One possible reason could be the less affordable "higher" education in the US society. In the 70s, high school education was enough to be classified as a skilled worker and it did not cost too much, so that poor families were able to train their kids to be skilled workers with minor education cost. However, since the 90s, college education replaced the position of high school education. Without getting a BA degree, it became hard to be classified as skilled workers. In contrast to a high school education, a college education is not affordable to a considerable number of poor families. Thus, the opportunities for kids from poor families to develop their talents are more restricted these days by the increased "skill training cost" in the US, compared to days in the 70s.

If we compare a college education in the US and Europe, we may observe a clear difference between them. In Europe, it is almost a public good. Poor people are given almost equal chance to rich people to develop their talents because a college education is free in many parts of Europe. As long as kids are willing to study hard and are talented, they can have training opportunities to be skilled workers through affordable college education. Thus, it seems that in Europe group inequality is less likely to grow compared to the US.

In South Korea, education cost started to increase significantly since the early 90s when the Korean SAT test was reformed. Even though Korea adopted a strict public school system up to high school education, parents started to spend incredible amounts for after-school private education toward their children's college admission. Currently, poor families can not afford the cost of private education, so they just send their kids to public schools without providing extra education from private academies. Rich families send their kids to various private education centers after school, where they develop their talents further. The sharp disparity between the "rich" south of Seoul (Kangnam) and the "poor" north (Kangbuk) in terms of best college admission rate reflects how family background can affect the opportunities for kids to develop their talents in a high training cost society.[1] Furthermore, this implies the skill disparity between the south of Seoul and the north may grow more and more over generations under the high education cost structure, because the richer south community is able to train more kids, the skilled kids bring more wealth to the community after joining the workplace, and they may train more kids than before in the next generation, and so on. At the same time, more poor families in the north of Seoul may give up training their kids over generations.

All those examples mentioned above emphasize the significant relationship between the evolution of group inequality and the size of training cost. We have further comments:

Snowball Effect. As in Benabou(1993)'s work, "Working of a City", the model in this paper suggests a snowball effect as a major force to bring the severe disparity- a small difference in the beginning can cause a huge difference in the end when the economy carries a strong externality of

[1] About 25 percent of Seoul National University admission, which is known as the best college in Korea, goes to rich residents in the south of Seoul (Kangnam people), which constitutes just a tiny fraction of the total Korean population. Before the education cost increased, the coverage of admission was just around 10 to 15 percent.

peer effects. Suppose two groups with equal skill composition at time t. A small advantage to kids belonging to group A will make the group provide more skilled workers at time t+1, while group B will provide less skilled workers due to the global complimentarities in the neoclassical economy. (Note that higher supply of skilled workers from group A leads to the wage differential decline, which causes the decrease in the supply of skilled workers from group B.) When peer effects externality is strong, group A will be able to provide even more skilled workers in the next generation, while group B can provide even less workers (Loury, 1977). Then, the skill difference between two groups may get bigger and bigger like a snowball effect, even when the total fraction of skilled workers in the society does not vary too much.

Failure of a Merit System. Even in a highly segregated society, the society may converge to a symmetric steady state if talented kids from both groups are given equal opportunities to develop their skills. If this merit system works, the unequal skill composition between two groups may disappear over generations. However, in a society where training cost is so high, the kids from an advantaged group and the disadvantaged group will not be given equal opportunities to develop skills. Kids from the rich group are given more chances to develop skills while kids from the poor group are given less chances to develop skills. Thus, for the same wage differential at time t+1 that both groups expect, the supply of skilled workers from the rich group is bigger than that from the poor group. Since an economy which is defined by a neoclassical production function determines the faction of skilled workers in a supply and demand equilibrium, the richer group provides more skilled workers, and the poorer group less skilled workers in the next generation. The difference can widen over generations if a kid's inherent ability is not critical in the determination of chances to develop his skill, while the wealth of his social network is rather deterministic for skill development. Therefore, the high training cost in a society may result in the failure of the merit system and growing group inequality.

This paper is organized in the following way. Section 2 presents a theoretical framework of the model. Section 3 shows main arguments using a simple uniform distribution of inherent ability. Section 4 shows the same arguments without relying one any specific functional form of ability function. Section 5 concludes.

2 Framework

Suppose that the economy is composed of two groups, $i \in b, w$, and the proportion of each group is β^b and β^w, whose sum is one. Also, suppose that there are two occupations: skilled and unskilled, $j \in h, l$. Each agent lives two periods, training and working periods. generations are overlapped. Parents decide whether to train their children or not after observing the child's ability a at time t. The training cost for the skilled job is k, but no cost for the unskilled job. Children earn at time $t+1$, whose wages depend on the occupation, w_h for the skilled job and w_l for the unskilled job. For convenience, we assume a single parent and a single child in each family.

The fraction of skilled workers in group i at time t is denoted as $x_h^i(t)$ and the fraction of unskilled workers as $x_l^i(t)$. Thus, the fraction of each occupation j in the whole economy at time t is

$$x_j(t) = \beta^b x_j^b(t) + \beta^w x_j^w(t).$$

A neoclassical production function $f(x_h(t), x_l(t))$ is given. The wage for occupation j at time t, $w_j(t)$, is marginal productivity of the occupation,

$$w_j(t) = \frac{\partial f(x_h(t), x_l(t))}{\partial x_j(t)} (= f_j(x_h(t), x_l(t))).$$

The marginal productivity of occupation j diminishes as $x_j(t)$ increases: $f_{jj} < 0$ and $f_{hl} > 0$. The wage difference is $\Delta w(t) = w_h(t) - w_l(t)$, which should be positive because of the training cost to achieve the occupation h. The average wage in the economy equals to per capita output and is expressed as

$$\bar{w}(t) = x_h(t)w_h(t) + x_l(t)w_l(t).$$

The average wage of group i is

$$\bar{w}^i(t) = x_h^i(t)w_h(t) + x_l^i(t)w_l(t).$$

Note that the average wage in the economy is the weighted sum of each group's average wages, $\bar{w}(t) = \beta^b\bar{w}^b(t) + \beta^w\bar{w}^w(t)$.

2.1 Peer Effects

Peer effects in the economy is about a redistribution of the wages at each period between skilled and unskilled workers, and workers in group b and w. Let us define the effective wage of a parent in group i at occupation j as follows.

$$\tilde{w}_j^i(t) = (1-\gamma)w_j(t) + \gamma\{\eta\bar{w}^i(t) + (1-\eta)\bar{w}(t)\} \tag{1}$$

, where $\gamma \in [0,1]$ and $\eta \in [0,1]$ indicate the strength of spillovers and the segregation level of social network for each. Note that the weighted average of effective wages equals to that of real wages:

$$\sum_{i\in b,w}\sum_{j\in h,l}\beta^i x_j^i(t)\tilde{w}_j^i(t) = \bar{w}(t).$$

Thus, we have a peer effects model in a production-neutral way. Productivity at $x(t)$ does not vary with γ or η. The total production is simply redistributed among workers according to the size of spillover (γ) and the level of segregation (η). Also, effective wealth of group i can be denoted by the average wage of group i and the average wage in the whole economy:

$$\tilde{w}^i(t) = \eta'\bar{w}^i(t) + (1-\eta')\bar{w}(t), \quad \text{where } \eta' = 1-\gamma+\gamma\eta. \tag{2}$$

Let us call η' as an effective segregation level because it reflects the degree of wealth transfer through peer effects between two social groups. For simplicity, let us denote that w indicates the advantaged group at time $t=0$: $\bar{w}^w|_{t=0} \geq \bar{w}^b|_{t=0}$.

2.2 A Parent's Decision

Suppose that child ability is distributed with CDF function G and its PDF g. There is no difference between two groups in terms of the innate ability. The training cost for a child with ability level $K(a)$ is assumed as $K(a) = k - a$. If the effective wage of a parent belonged to group i with occupation j is $\tilde{w}_j^i(t)$, the parent trains her child only if the decrease in her utility at time t after spending the training cost $K(a)$ is less than or equal to the discounted wage differential at time $t+1$:

$$u(\tilde{w}_j^i(t)) - u(\tilde{w}_j^i(t) - K(a)) \leq \delta\Delta w(t+1).$$

Parents' marginal utility of consumption diminishes as consumption increases. Let us assume a parent's utility function as $ln(w)$. Then a parent with (i,j) at time t trains her child if and only if

$$K(a) \leq \tilde{w}_j^i(t)(1 - e^{-\delta\Delta w(t+1)}).$$

Therefore, she trains her child when her child ability is above some threshold ($\tilde{a}_{j,t}^i$):

$$\textit{Train if} \quad a \geq k - \tilde{w}_j^i(t)(1 - e^{-\delta\Delta w(t+1)}) \; (= \tilde{a}_{j,t}^i). \tag{3}$$

Definition 1. ***Full Training Condition.*** *If the threshold ability level* $(\tilde{a}^i_{j,t})$ *is as low as* $min\{G^{-1}\}$ *for group i and occupation j parents, I define the parents group subject to the full training condition, which means that they train all their children, regardless of their inherent ability levels.*

I define the following property that is useful for the following analysis.

Lemma 1. *A state* (x^w, x^b) *can not be a stable state if at least one parents group of group i and occupation j in the society is subject to the fulling training condition.*

Thus, given the ability distribution $\tilde{G}^i_{j,t}$ of children of occupation j parents at group i at period t, the probability that the (i, j) parent trains her child is

$$Pr[(i,j)_t \rightarrow h_{t+1}] = 1 - \tilde{G}^i_{j,t}(\tilde{a}^i_{j,t}).$$

Hence, the fraction of skilled workers in group i at period $t + 1$ is

$$x^i_h(t+1) = \sum_{j \in h,l} x^i_j(t)[1 - \tilde{G}^i_{j,t}(\tilde{a}^i_{j,t})]. \tag{4}$$

As far as there exists a meaningful correlation between a parent's ability and a child's ability, $\tilde{G}^i_{h,t}$ can not be equal to $\tilde{G}^i_{l,t}$, and $\tilde{G}^i_{l,t}(a) \geq \tilde{G}^i_{h,t}(a)$. Also, for every period t, the followings should hold

$$\begin{aligned} G^i(a) &= x^i_t \tilde{G}^i_{h,t}(a) + (1 - x^i_t)\tilde{G}^i_{l,t}(a), \ \forall t \\ G^w(a) &= G^b(a). \end{aligned} \tag{5}$$

The first equation supports the identical ability distribution across generations. The second equation supports anti-essentialism. The biological transfer of innate abilities between generations make the analysis get complicated. In order to simplify the dynamic structure without loss of generality, I impose the following assumption.

Assumption 1. *WLOG, child ability distribution is identical for any* (i, j) *parent group and for any period* t*:*

$$G(a) = \tilde{G}^i_{j,t}(a), \ \forall (i, j, t).$$

Assuming the identical ability distribution, we are able to constitute the transition matrix from $(x^b_h(t), x^w_h(t))$ to $(x^b_h(t+1), x^w_h(t+1))$ using $G(a)$ function. Thus, the state $(x^b_h(t), x^w_h(t))$ is uniquely determined by the initial state $(x^b_h(0), x^w_h(0))$:

$$\{x^b_h(t), x^w_h(t); t = 1, 2, 3, ...|(x^b_h(0), x^w_h(0))\}.$$

The fraction of skilled workers in the whole economy at time $t + 1$ is determined by $x^b_h(t)$ and $x^w_h(t)$:

$$\begin{aligned} x_h(t+1) &= \sum_{i \in b,w} \beta^i x^i_h(t+1) \\ &= \sum_{i \in b,w} \sum_{j \in h,l} \beta^i x^i_j(t)[1 - G(\tilde{a}^i_{j,t})]. \end{aligned} \tag{6}$$

In order to understand the history determinacy, refer to the Appendix Figure 1, where I depict the supply and demand curves and the market equilibrium in each period $t+1$. For any given x_t, the supply x_{t+1} is determined for each level of Δw_{t+1} by

$$\begin{aligned} x_{t+1} &= \beta^w x^w_{t+1} + \beta^b x^b_{t+1} \\ &= \beta^w[1 - G(k - \tilde{w}^w(x_t)(1 - e^{-\delta \Delta w_{t+1}}))] + \beta^b[1 - G(k - \tilde{w}^b(x_t)(1 - e^{-\delta \Delta w_{t+1}}))]. \end{aligned}$$

This is a supply curve, which is a increasing function as depicted in the figure. The demand curve is simply $\Delta w_{t+1} = \Delta w(x_{t+1})$, which is a decreasing function. Therefore, there exists a unique market equilibrium for each period.

3 Dynamics in Uniform G Function

3.1 Steady States

Suppose G is a uniform distribution in $[A, A+B]$, where $A \geq 0$ and $A + B < k$. This implies that even the highest ability student should incur some cost $(K - A - B)$ to achieve skills:

$$G(a) = \frac{a - A}{B}, \text{where } A + B < k. \tag{7}$$

In this simple model with uniform ability distribution, the overall training cost in the society is measured by the size of B given A: An average societal training cost (TC) among generation t agents is $k - A - .5B$. Note that TC is a decreasing function of B.

The fraction of high skill workers in group i evolves in the following way,

$$x_h^i(t+1) = x_h^i(t)[1 - G(\tilde{a}_h^i)] + x_l^i(t)[1 - G(\tilde{a}_l^i)],$$

with

$$\begin{aligned} \tilde{a}_h^i &= k - \tilde{w}_h^i(1 - e^{-\delta \Delta w(t+1)}) \\ \tilde{a}_l^i &= k - \tilde{w}_l^i(1 - e^{-\delta \Delta w(t+1)}). \end{aligned} \tag{8}$$

For simplicity, let us denote $x_h^b(t)$ as x_t^b and $x_h^w(t)$ as x_t^w, consequently $x_l^b(t)$ as $1 - x_t^b$ and $x_l^w(t)$ as $1 - x_t^w$. Also, denote $x_h(t)$ as x_t.

Therefore, x_t evolves:

$$\begin{aligned} x_{t+1} &= \beta^b x_{t+1}^b + \beta^w x_{t+1}^w \\ &= -\frac{k - A - B}{B} + w(x_t)\frac{1 - e^{-\delta \Delta w(t+1)}}{B}. \end{aligned} \tag{9}$$

We can rearrange the result as

$$\frac{Bx_{t+1} + k - A - B}{1 - e^{-\delta \Delta w(t+1)}} = w(x_t). \tag{10}$$

LHS and RHS are described in Figure 1. Assuming LHS is convex, we can find three steady states, among which one is stable and the others are unstable if it were a homogeneous economy. (Note that the steady state with $x^* = 0$ is unstable because it is subject to the "full training condition".) Let us denote two steady states which are not subject to the full training condition as $x^*(s)$ $(x^*(u))$, which is stable (unstable) in the homogeneous economy.

The group skill difference at time $t+1$, $\Delta x(t+1)$, is expressed as

$$\begin{aligned} |x_h^w(t+1) - x_h^b(t+1)| &= |(x_h^w \tilde{w}_h^w + x_l^w \tilde{w}_l^w) - (x_h^b \tilde{w}_h^b + x_l^b \tilde{w}_l^b)| \cdot \frac{(1 - e^{-\delta \Delta w(x_{t+1})})}{B} \\ &= |x_h^w(t) - x_h^b(t)| \cdot \frac{\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})})}{B} \cdot (1 - \gamma + \gamma\eta). \end{aligned} \tag{11}$$

Therefore when $\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})})$ is greater than B, $\Delta x(t)$ diverges with high enough $\eta'(= 1 - \gamma + \gamma\eta)$. Because x_{t+1} is monotonic and an increasing function of x_t, $\Delta w(x_t)(1 - e^{-\delta \Delta w(x_{t+1})})$ is a decreasing function of x_t, and approaches to positive infinite as x_t converges to zero and to zero as

x_t converges to $\bar{x}$. Therefore, there exists a unique $\tilde{x}$ such that $\Delta x(t)$ diverges over time with $\eta' = 1$ when $x_t < \tilde{x}$, and $\tilde{x}$ is a decreasing function of B: $\tilde{x}(B)$. (Refer to Figure 5 for an example of $\tilde{x}$.)

In Figure 2, I describe an alternative way to search steady states. The steady states are the set of $(x, \tilde{a})$ that satisfies the following two equations for any given A and B:

$$\begin{aligned} x(= 1 - G(\tilde{a})) &= 1 - \frac{\tilde{a} - A}{B} \\ \tilde{a}(x) &= k - w(x)[1 - e^{-\delta \Delta w(x)}]. \end{aligned} \quad (12)$$

As Figure 2 illustrates, $\tilde{a}(x)$ is close to the convex shape as far as the curvature of $\Delta w(x)$ is not so strong. Thus, I impose the following assumption on the production function for simplicity of the analysis.

Assumption 2. $\tilde{a}(\Delta w(x), w(x))$ *is convex w.r.t.* x.

Note that the convexity of $\tilde{a}(x)$ is achieved when the following condition is satisfied

$$\Delta w'' < \delta(\Delta w')^2 - 2\Delta w' \frac{w'}{w}.$$

RHS is positive always because $\Delta w'$ is negative. The condition holds when Δw curve is not bent too much (eg. $\Delta w''$ is zero when Δw is linear). Also let us denote $\hat{x}$ as

$$\hat{x} = argmin\{\tilde{a}(x)\}. \quad (13)$$

Note that the blue kinked line exactly represents ability distribution function $G(a)$ if you interchange x and y coordinates. This is a useful graph because the black curve $\tilde{a}(x)$ is always fixed as $G(a)$ changes. We can easily observe how locations of steady states change as $G(a)$ has different shapes.

3.2 Stability of Steady States

For further analysis, let us fix A as a constant and varies B, which reflects the different levels of training cost because the societal training cost is defined as $k - A - .5B$ in this simple linear model. (Note that $k - A$ is the training cost for the lowest ability kid, $k - A - B$ is that for the highest ability kid.) In Figure 3, I summarize the steady states for any given B in blue curve. Also, I depict the $\Delta w(x^*)(1 - e^{-\delta \Delta w(x^*)})$ locus for given steady state x^*. We can find the threshold level of x (x^c) and its corresponding B (B^c). When B is smaller than B^c for given A, all symmetric steady states are unstable. (Identically, when all steady states are located in the lefthand side of the threshold level x^c, they all are unstable.) Since we know that the smaller B is, the higher the societal training cost (TC) is, we have the following result.

Proposition 1. *When the level of societal training cost (TC) is high enough, there does not exist any stable symmetric steady state in a highly segregated society:*

$$\textit{No Stable Symmetric Steady State with } \eta' = 1 \textit{ if } \quad TC > k - (A + .5B^c). \quad (14)$$

Corollary 1. *The society converges to a stable symmetric steady state if the segregation level* (η') *is small enough.*

Also, as Figure 4 depicts, the following two are stable steady states: $(0, \frac{x^*}{\beta^w})$ and $(1, \frac{x^* - \beta^b}{\beta^w})$ with high enough η'. Using equation (11), we have the following proposition.

Proposition 2. *In a highly segregated society, when the societal training cost (TC) is greater than* $k - (A + .5B^c)$*, the hierarchal stable steady state* $(0, \frac{x^*}{\beta^w})$ *or* $(1, \frac{x^* - \beta^b}{\beta^w})$ *can emerge with any tiny skill difference at time zero.*

The whole dynamics is described in Figure 5. In Figure 6, I compare a society with low training cost (Panel A) and that with high training cost (Panel B). As you can observe, all three steady states are located in the lefthand side of the threshold level x^c in Panel B. Therefore, there exists no stable symmetric steady state in a high training cost society. In a low training cost society, there exists a stable symmetric steady state, and any initial state (x_0^w, x_0^b) converges to the state if the initial societal skill share x_0, which is $\beta^w x_0^w + \beta^b x_0^b$, is above some level $(x^*(u))$ as described in Figure 5.

The following argument briefly stresses about how the stability of symmetric steady states can be so vulnerable in high training cost society. Equation (12) shows that, at a steady state x^*, the following is true:

$$x^* = 1 - \frac{k - w(x^*)[1 - e^{-\delta \Delta w(x^*)}] - A}{B}.$$

If you plug this into equation (11), we have the following result:

$$|x_h^w(t+1) - x_h^b(t+1)| = |x_h^w(t) - x_h^b(t)| \cdot \frac{\Delta w(x^*)(Bx^* + k - A - B)}{Bw(x^*)} \cdot (1 - \gamma + \gamma\eta). \qquad (15)$$

Therefore, x^* is unstable with $\eta' = 1$ if $\Delta w(x^*)(Bx^* + k - A - B) > Bw(x^*)$. Since $w(x) = x\Delta w(x) + w_l(x)$, it is unstable if $\Delta w(x^*)(Bx^* + k - A - B) > B(x^*\Delta w(x^*) + w_l(x^*))$. Rearranging the inequality formula, we have the following result.

Corollary 2. *In a highly segregated society, a symmetric steady state (x^*, x^*) is unstable if*

$$B < \frac{\Delta w(x^*)}{w_h(x^*)}(k - A).$$

We can easily find B that makes the inequality hold.

4 Dynamics in General G Function

In this section, I expand the previous analysis to the general G functions. G function does not take any specific functional form anymore. It could be normal, exponential, linear or something else. I show that, even when no specific functional form is taken, we have the similar results to the uniform G case.

4.1 A Newborn Child's Environment

I suppose G(a) is defined in $[A, A+B]$, where $A \geq 0$ and $A + B < k$, as I did in the previous analysis. However, I take a slight different framework from the one I imposed in the previous analysis with linear G function, in order to make the analysis with non-functional form of G tractable. Suppose that a newborn child ability is distributed with CDF G and its PDF g for any period t. There is no difference between two groups in terms of the innate ability distribution consistent with the Anti-essentialism (Loury, 2002). The training cost for a child with ability level $K(a)$ is assumed as $K(a) = k - a$. Suppose that the parent with her wealth level w trains her child only if the decrease in her utility at time t after spending the training cost $K(a)$ is less than or equal to the discounted wage differential that her child gets benefited with at time t+1:

$$u(w(t)) - u(w(t) - K(a)) \leq \delta \Delta w(t+1).$$

Suppose any two children born to different social groups b and w at period t. They will face different chances to develop their skills unless the wealth levels of two groups are identical to each other. A child born to group i can expect to have her parent with wealth level $\tilde{w}^i(t)$, because $E(w)_t^i =$

$x_h^i \tilde{w}_h^i(t) + x_l^i \tilde{w}_l^i(t) = \tilde{w}^i(t)$. Therefore, we can expect that a child born to group i at period t has an opportunity to train her skill only when her innate ability level satisfies the following condition:

$$u(E(w)_t^i) - u(E(w)_t^i - K(a)) \leq \delta \Delta w(t+1),$$

where $E(w)_t^i = \tilde{w}^i(t)$.

Parents' marginal utility of consumption diminishes as consumption increases. Let us suppose that parents' utility function as $ln(w)$. Then a newborn child in group i can expect to have an opportunity to train her skill only when her ability is above some threshold level ($\tilde{a}^i(t)$):

$$\textit{Trained if} \quad a \geq k - \tilde{w}^i(t)(1 - e^{-\delta \Delta w(t+1)}) \ (= \tilde{a}^i(t)).$$

Therefore, the fraction of skilled workers in group i at period $t+1$ is

$$\begin{aligned} x_h^i(t+1) &= 1 - G(\tilde{a}^i(t)) \\ &= 1 - G(k - \tilde{w}^i(t)(1 - e^{-\delta \Delta w(t+1)})). \end{aligned} \tag{16}$$

Note that the ability thresholds for group b and w are different if the group effective wealth levels, $\tilde{w}^b(t)$ and $\tilde{w}^w(t)$, are different, even though the newborn child's abilities are equally distributed. In this setup, we are allowing the intergenerational innate ability transfer (positive correlation between a parent's ability and a child's ability), which was ignored taking Assumption 1 in the previous section. Instead, we focus here on the expected parent wealth level given the group-neutral ability distribution G. (Under this framework, we have jumped out of the complex ability structure coming with the intergenerational ability transfer ($\tilde{G}_{l,t}^i \geq \tilde{G}_{h,t}^i$) noted in equations (5).)

Thus, now we are ready to constitute the transition matrix from $(x_h^b(t), x_h^w(t))$ to $(x_h^b(t+1), x_h^w(t+1))$ The state $(x_h^b(t), x_h^w(t))$ is uniquely determined by the initial state $(x_h^b(0), x_h^w(0))$:

$$\{x_h^b(t), x_h^w(t); t = 1, 2, 3, ... | (x_h^b(0), x_h^w(0))\}.$$

The fraction of skilled workers in the whole economy at time $t+1$ is determined by $x_h^b(t)$ and $x_h^w(t)$:

$$x_h(t+1) = \sum_{i \in b,w} \beta^i x_h^i(t+1). \tag{17}$$

4.2 Symmetric Steady State

I do not impose any specific G function. I just suppose that G is continuous and differentiable. (Suppose that $G(k) > \bar{x}$, where $\Delta w(\bar{x}) = 0$.) Note that the threshold ability $\tilde{a}^i$ is a decreasing function of both arguments, Δw_{t+1} and $\tilde{w}^i(t)$:

$$\tilde{a}^i = \tilde{a}(\Delta w_{t+1}, \tilde{w}^i(t)).$$

For simplicity, let us denote $x_h^b(t)$ as x_t^b and $x_h^w(t)$ as x_t^w, consequently $x_l^b(t)$ as $1 - x_t^b$ and $x_l^w(t)$ as $1 - x_t^w$. A competitive equilibrium path is a steady state if (x_t^b, x_t^w)=(x_{t+1}^b, x_{t+1}^w) for all periods t. Our main interest is the stability of a symmetric steady state. At a symmetric steady state, the following should hold according to equation (16),

$$\frac{k - G^{-1}(1-x)}{1 - e^{-\delta \Delta w(x)}} = w(x). \tag{18}$$

In order to get LHS, figure 7 shows two separate functions whose product is LHS. Since both are increasing functions, LHS is an increasing function as shown in Figure 8. Three symmetric steady states are described in the figure. Another way to get steady states is to find x that satisfies the following two equations, as shown in Panel A of Figure 9:

$$\begin{aligned} x &= 1 - G(\tilde{a}) \\ \tilde{a}(x) &= k - w(x)[1 - e^{-\delta \Delta w(x)}]. \end{aligned} \tag{19}$$

Note that the following first derivatives of $\tilde{a}$ with respect to its two arguments hold at the symmetric steady state:

$$\tilde{a}_1^* = -\delta w(x^*) e^{-\delta \Delta w(x^*)} \tag{20}$$

$$\tilde{a}_2^* = -1 + e^{-\delta \Delta w(x^*)}. \tag{21}$$

4.3 Stability Condition

In this section, I specify the condition to make a symmetric state state unstable. The dynamic system under the current setup can be described as, using functions ϕ^b and ϕ^w,

$$\begin{aligned} x_{t+1}^b &= \phi^b(x_t^b, x_t^w) \\ x_{t+1}^w &= \phi^w(x_t^b, x_t^w). \end{aligned} \tag{22}$$

Note that the following holds by equation (2),

$$\tilde{w}_t^i = (\eta' + (1-\eta')\beta^i)\Delta w x_t^i + ((1-\eta')\beta^j)\Delta w x_t^j + w_l, \quad \text{where } \eta' = 1 - \gamma + \gamma\eta. \tag{23}$$

From equations (16) and (23),we have

$$\begin{aligned} \phi^b &= 1 - G(\tilde{a}(\Delta w(x_{t+1}), \tilde{w}_t^b) \\ &= 1 - G(\tilde{a}(\Delta(\beta^b\phi^b + \beta^w\phi^w), (\eta' + (1-\eta')\beta^b)\Delta w x_t^b + ((1-\eta')\beta^w)\Delta w x_t^w + w_l) \qquad (24) \\ \phi^w &= 1 - G(\tilde{a}(\Delta w(x_{t+1}), \tilde{w}_t^w) \\ &= 1 - G(\tilde{a}(\Delta(\beta^b\phi^b + \beta^w\phi^w), (\eta' + (1-\eta')\beta^w)\Delta w x_t^w + ((1-\eta')\beta^b)\Delta w x_t^b + w_l). \qquad (25) \end{aligned}$$

In order to have Jacobian Matrix for the dynamic system, we need to find the first derivatives of ϕ^b and ϕ^w,

$$\mathbf{J} = \begin{bmatrix} \phi_1^b & \phi_2^b \\ \phi_1^w & \phi_2^w \end{bmatrix}.$$

The followings are the first derivatives of functions ϕ^b and ϕ^w with respect to x_t^b and x_t^w,

$$\begin{aligned} \phi_1^b &= -G'[\tilde{a}_1\Delta w'(\beta^b\phi_1^b + \beta^w\phi_1^w) + \tilde{a}_2(\eta' + (1-\eta')\beta^b)\Delta w + \tilde{a}_2(\eta' + (1-\eta')\beta^b)\Delta w'\beta^b x_t^b \\ &\quad + \tilde{a}_2(1-\eta')\beta^w\Delta w'\beta^b x_t^w] \qquad (26) \\ \phi_2^b &= -G'[\tilde{a}_1\Delta w'(\beta^b\phi_2^b + \beta^w\phi_2^w) + \tilde{a}_2(1-\eta')\beta^w\Delta w + \tilde{a}_2(1-\eta')\beta^w\Delta w'\beta^w x_t^w \\ &\quad + \tilde{a}_2(\eta' + (1-\eta')\beta^b)\Delta w'\beta^w x_t^b] \qquad (27) \\ \phi_1^w &= -G'[\tilde{a}_1\Delta w'(\beta^b\phi_1^b + \beta^w\phi_1^w) + \tilde{a}_2(1-\eta')\beta^b\Delta w + \tilde{a}_2(1-\eta')\beta^b\Delta w'\beta^b x_t^b \\ &\quad + \tilde{a}_2(\eta' + (1-\eta')\beta^w)\Delta w'\beta^b x_t^w] \qquad (28) \\ \phi_2^w &= -G'[\tilde{a}_1\Delta w'(\beta^b\phi_2^b + \beta^w\phi_2^w) + \tilde{a}_2(\eta' + (1-\eta')\beta^w)\Delta w + \tilde{a}_2(\eta' + (1-\eta')\beta^w)\Delta w'\beta^w x_t^w \\ &\quad + \tilde{a}_2(1-\eta')\beta^b\Delta w'\beta^w x_t^1]. \qquad (29) \end{aligned}$$

Using equations (26) and (28), we can find that

$$\beta^b\phi_1^b + \beta^w\phi_1^w = \frac{-G'\tilde{a}_2(\Delta w + \Delta w' w)}{1 + G'\tilde{a}_1\Delta w'}\beta^b.$$

Using equations (27) and (29), we can find that

$$\beta^b\phi_2^b + \beta^w\phi_2^w = \frac{-G'\tilde{a}_2(\Delta w + \Delta w' w)}{1 + G'\tilde{a}_1\Delta w'}\beta^w.$$

Replacing these results in equations (26), (27), (28) and (29), we finally have the Jacobian Matrix for the given dynamic system,

$$\begin{aligned}
\phi_1^b &= -G'\tilde{a}_2[\eta'\Delta w + \beta^b((1-\eta')\Delta w - \mu) + \beta^b\Delta w'(\eta' x_t^b + (1-\eta')w_t)] \\
\phi_2^b &= -G'\tilde{a}_2[\beta^w((1-\eta')\Delta w - \mu) + \beta^w\Delta w'(\eta' x_t^b + (1-\eta')w_t)] \\
\phi_1^w &= -G'\tilde{a}_2[\beta^b((1-\eta')\Delta w - \mu) + \beta^b\Delta w'(\eta' x_t^w + (1-\eta')w_t)] \\
\phi_2^w &= -G'\tilde{a}_2[\eta'\Delta w + \beta^w((1-\eta')\Delta w - \mu) + \beta^w\Delta w'(\eta' x_t^w + (1-\eta')w_t)],
\end{aligned}$$

where

$$\begin{aligned}
\mu &= \frac{G'\tilde{a}_1\Delta w'(\Delta w + \Delta w' w)}{1 + G'\tilde{a}_1\Delta w'} \\
\eta' &= 1 - \gamma + \gamma\eta.
\end{aligned}$$

Out of the Jacobian Matrix, we can find the following two eigenvalues of the given dynamic system:

$$\begin{aligned}
\lambda^1 &= -G'\tilde{a}_2\Delta w\eta' \\
&= -G'\tilde{a}_2\Delta w(1-\gamma+\gamma\eta) && (30) \\
\lambda^2 &= -G'\tilde{a}_2(\Delta w + \Delta w' w - \mu) \\
&= \frac{-G'\tilde{a}_2 w'}{1 + G'\tilde{a}_1\Delta w'} \cdot \frac{\Delta w + \Delta w' w}{w'}. && (31)
\end{aligned}$$

Using equations (20) and (21), λ^1 and λ^2 at a symmetric steady state x^* is expressed as

$$\begin{aligned}
\lambda^{1*} &= G'(a^*)[1 - e^{-\delta\Delta w(x^*)}]\Delta w(w^*) \cdot (1-\gamma+\gamma\eta) \\
\lambda^{2*} &= \frac{G'(a^*)[1 - e^{-\delta\Delta w(x^*)}]w'(x^*)}{1 - \delta G'(a^*)w(x^*)e^{-\delta\Delta w(x^*)}\Delta w'(x^*)} \cdot \frac{\Delta w(x^*) + \Delta w'(x^*)w(x^*)}{w'(x^*)}, && (32)
\end{aligned}$$

where $G'(a^*) = g(a^*)$ given $x^* = 1 - G(a^*)$.

Lemma 2. *The dynamic system* (22) *is locally stable at a symmetric steady state* x^* *if both* $|\lambda^{1*}|$ *and* $|\lambda^{2*}|$ *are less than one, and locally unstable if either one of them is greater than one.*

4.4 Stability of Steady States

According to Lemma 2, the sufficient condition for the instability of x^* is that λ^1 is greater than 1. Note that $\lambda^1 > 1$ when $G'(a^*) > [\Delta w(x^*)(1 - e^{-\delta\Delta w(x^*)})]^{-1}$ with $x^* = 1 - G(a^*)$. All three steady states are unstable in the given case in Figures 8 and 9 because those are located in the left hand side of the threshold level x^u, as shown in Panel B of Figure 9. (Note that the steady state with $x^* = 0$ is unstable because it is subject to the "full training condition".)

Figure 10 shows the possible steady states in the heterogeneous economy. One of curves indicates a possible $\Delta x^b|_{x^w} = 0$ locus and the other is a possible $\Delta x^w|_{x^b} = 0$ locus. Note that we can prove that two loci meet at (x^*, x^*). Since the symmetric steady states are not locally stable, if there is at least one stable steady state, it should be an asymmetric steady state, implying the persistent group inequality. The exact number of asymmetric steady states and their stabilities are hard to be identified due to the complexity of the current dynamic system. (We would be able to identify the exact steady states if the functional forms of $G(a)$ and $w(x)$ are defined.)

In Figure 11, we compare the high and low training cost societies for a specific G function case, where $G(a)$ is defined in $[A, A+B]$. Note that $x = 1 - G(a)$ curves in Panels A and B of the figure are simply those scaled up and down of the $x = 1 - G(a)$ curve in Panel A of Figure 9, where the scaling reflects the different training cost levels in two societies. As the $x = 1 - G(a)$ curve is scaled up (or down), $g(a)$ curve in Panel B of Figure 9 is scaled down (or up) as well. The small diagram in each panel of Figure 11 indicates how $g(a)$ is scaled up or down and how the new threshold level x^u is determined accordingly. (For example, when the $x = 1 - G(a)$ curve is scaled up, the $g(a)$ curve is scaled down as described in Panel A of Figure 11, and the threshold level x^u moves to the left.) As you can see in each panel, there is no stable symmetric steady state in a high training cost society because all symmetric steady states are located in the left hand side of x^u, which is consistent to Proposition 1.

Proposition 3. *When the level of societal training cost high enough, there does not exist any stable symmetric steady state in a highly segregated society, regardless of a functional form of G taken.*

Although this theoretic work has a similar structure to Bowles et al. (2014), it is a significant development from their work with the following reasons: 1) the result in Bowles et al. comes out of the rather strong assumptions that 1) there exists a unique symmetric steady state and 2) some market condition (implied in equation (9) in their paper) derives the instability of the symmetric steady state. The theoretic work in this paper deals with the same issue without imposing the first assumption. Also, I provide the concrete economic meaning of the instability condition, which is the degree of sensitivity determined by the structure of education cost.

5 Conclusion

This paper discusses the importance of eduction cost on the evolution of group skill disparity. Since the cost of skill achievement is affected not only by one's inherent ability, but also by the quality of his social network, the degree of integration between social groups must be counted in the analysis of group disparity (Chaudhuri and Sethi, 2008). Three factors, the level of integration, the size of education cost, and neoclassical market system, are interwoven together with complexity in the analysis. The theoretic work presented in this paper successfully manage how those factors affect each other to determine the emergence of group skill disparity. Out of the analysis, I show that the level of education cost as well as the level of integration plays a key role in the evolution of group inequality. Even in a highly segregated society, low enough education cost may prevent the emergence of group disparity in the neoclassical market economy. However, I do not insist that *integration* is a less important policy measure to cure the problem of group inequality (Sethi and Somanathan, 2004). I suggest that, observing significant variances of educational systems across countries around the world, countries with high private education costs may suffer more from the growing group inequality and the market system can not alone stop the failure of merit system.

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Figure 1. Steady States in Homogeneous Economy

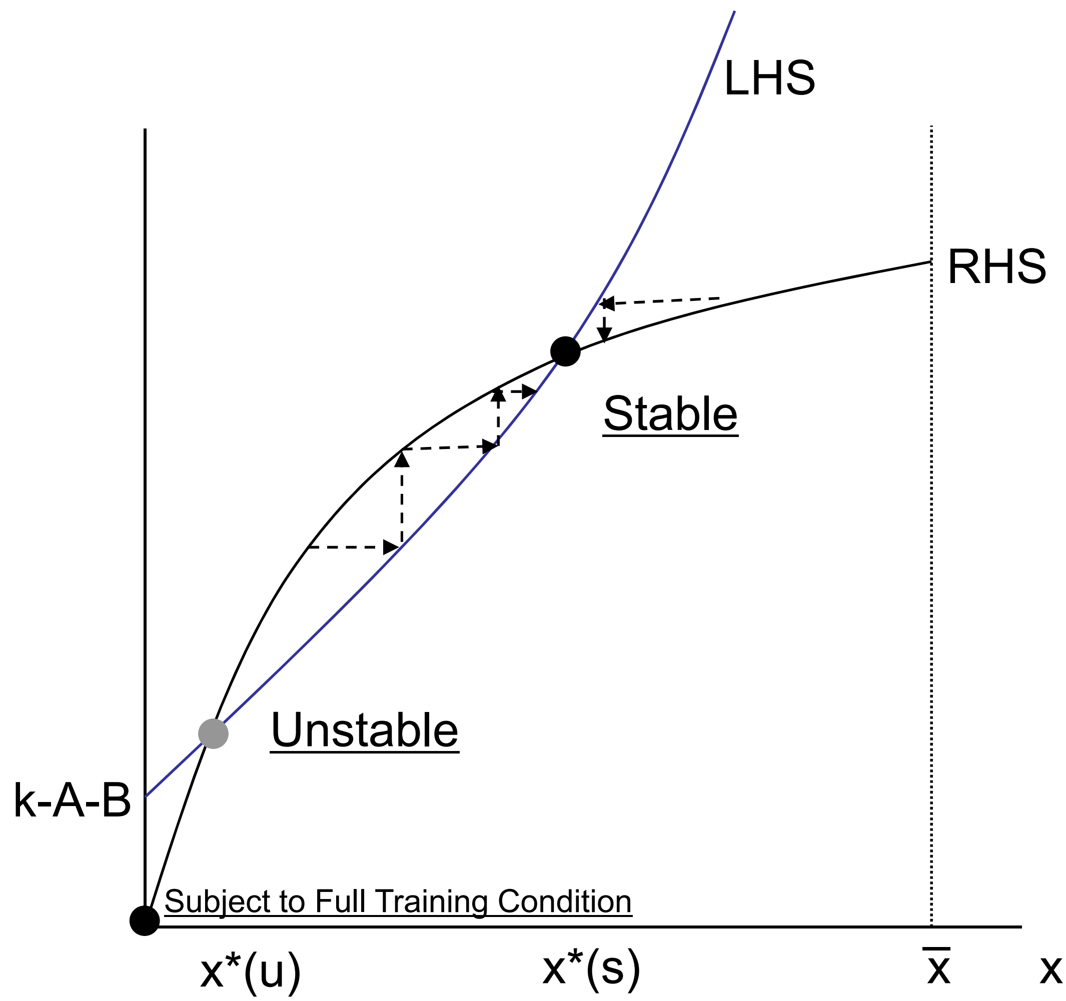


Figure 2. Alternative Way to Find Steady States

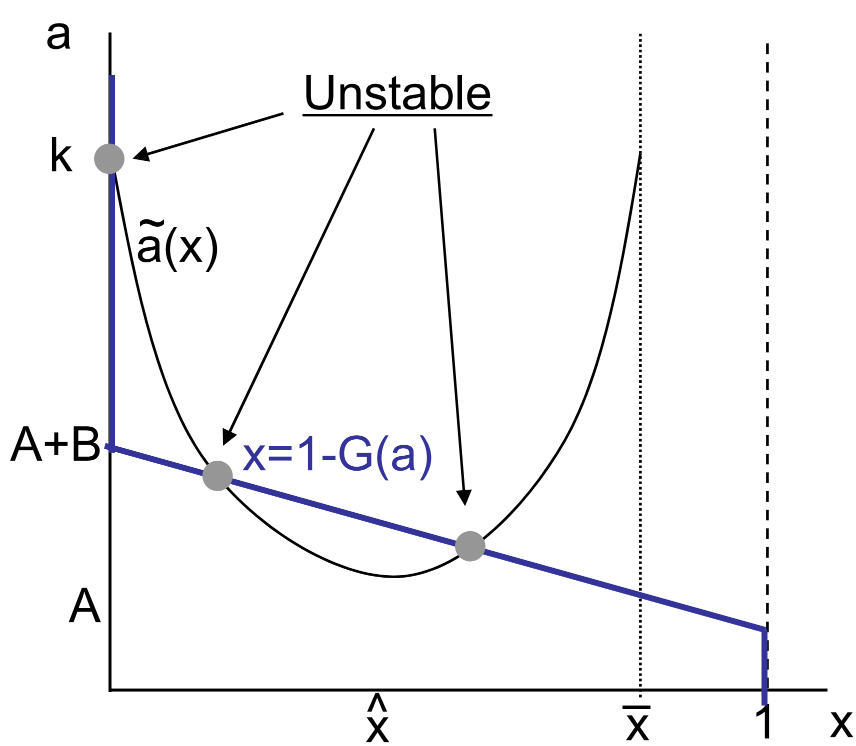


Figure 3. Threshold level of B to Make Unstable x*

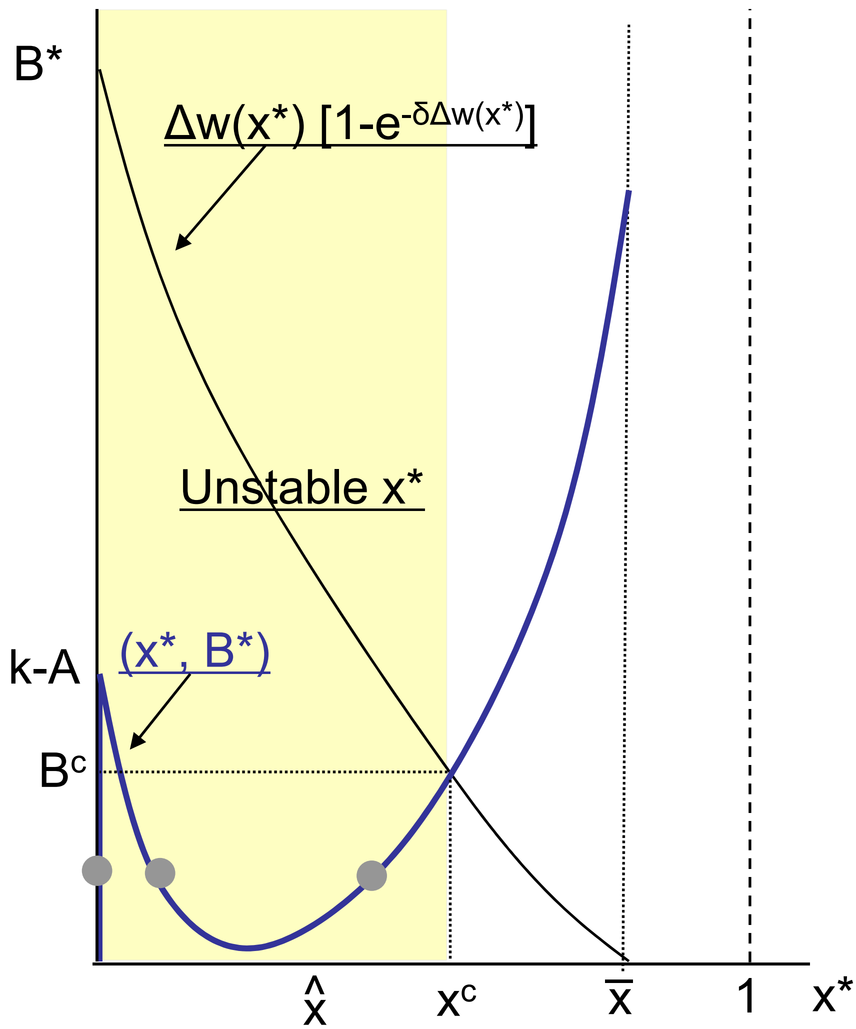


Figure 4. Steady States in Heterogeneous Economy

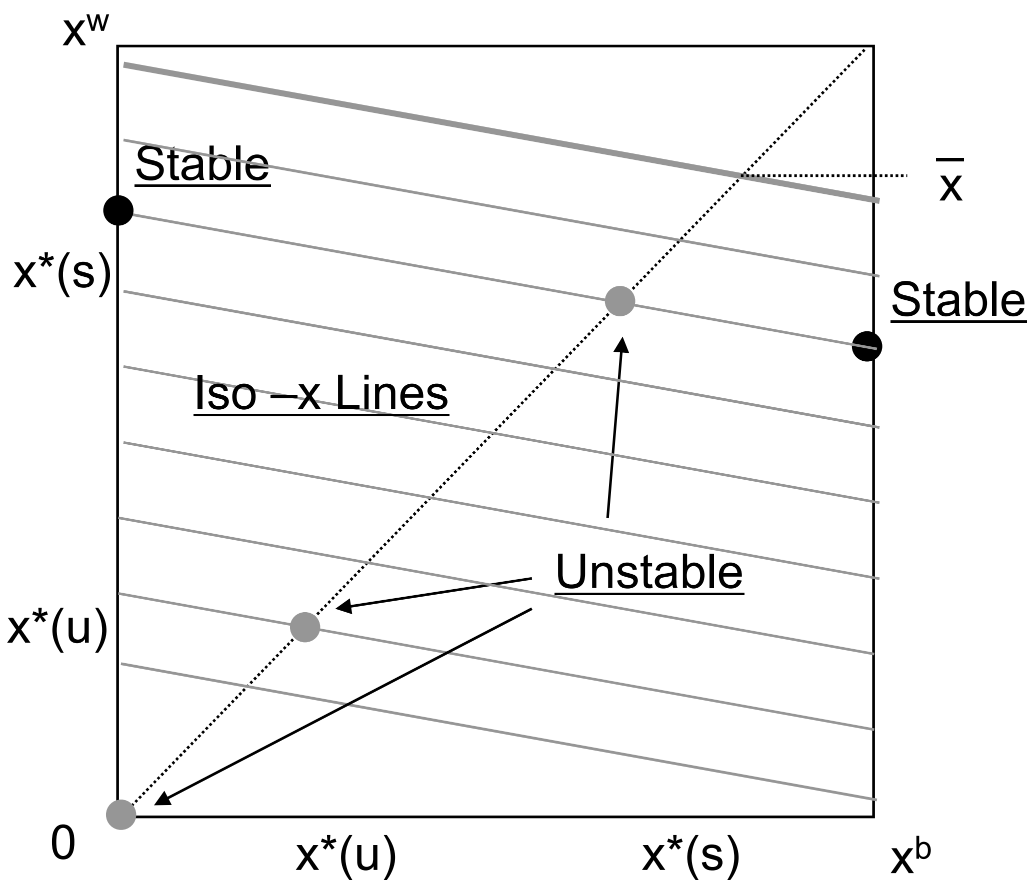


Figure 5. Moving Directions in Heterogeneous Economy

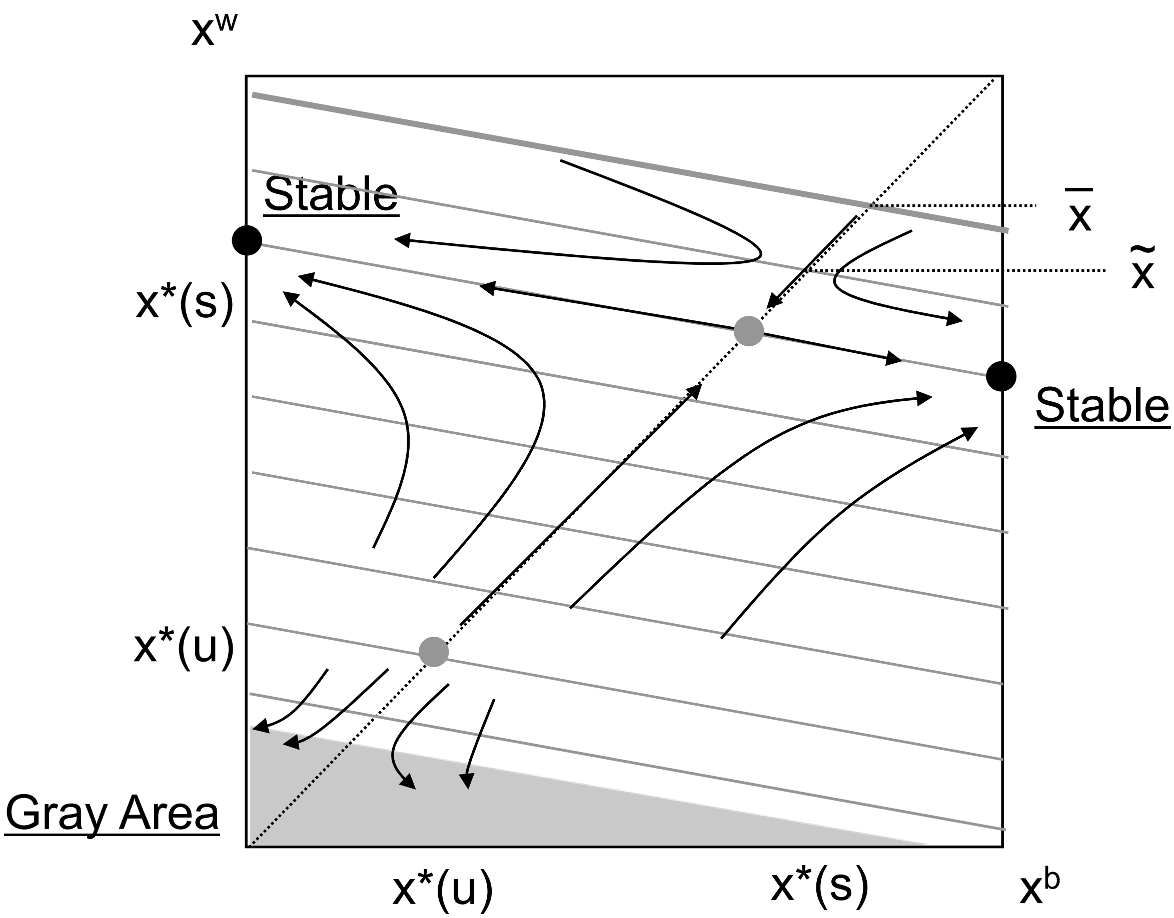
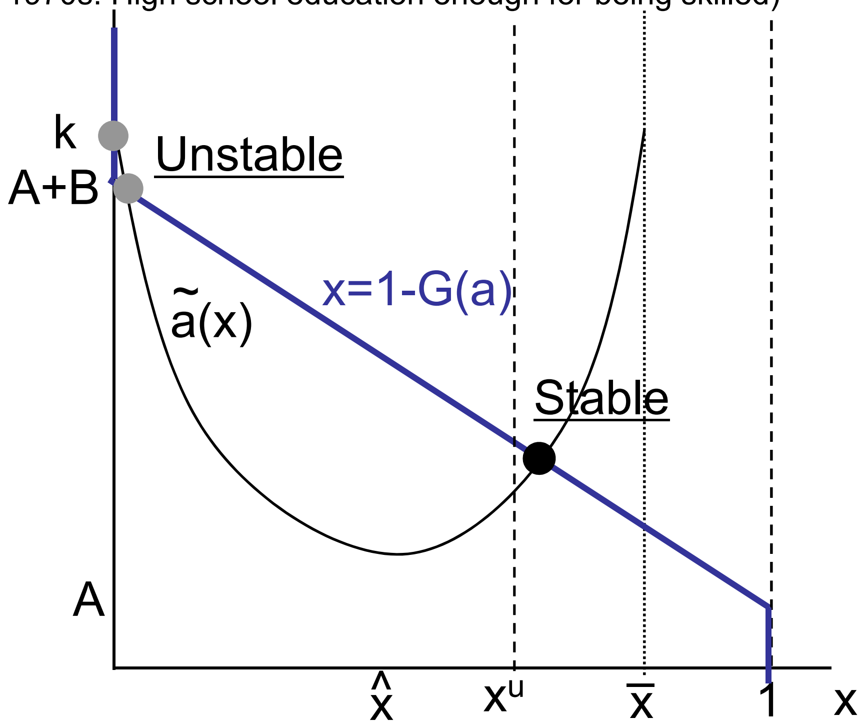


Figure 6. Compare High and Low Training Cost Societies

Panel A Low training cost society

(Ex, US in 1970s: High school education enough for being skilled)



Panel B High training cost society

(Ex, US in 1990s: College education required to be a skilled worker)

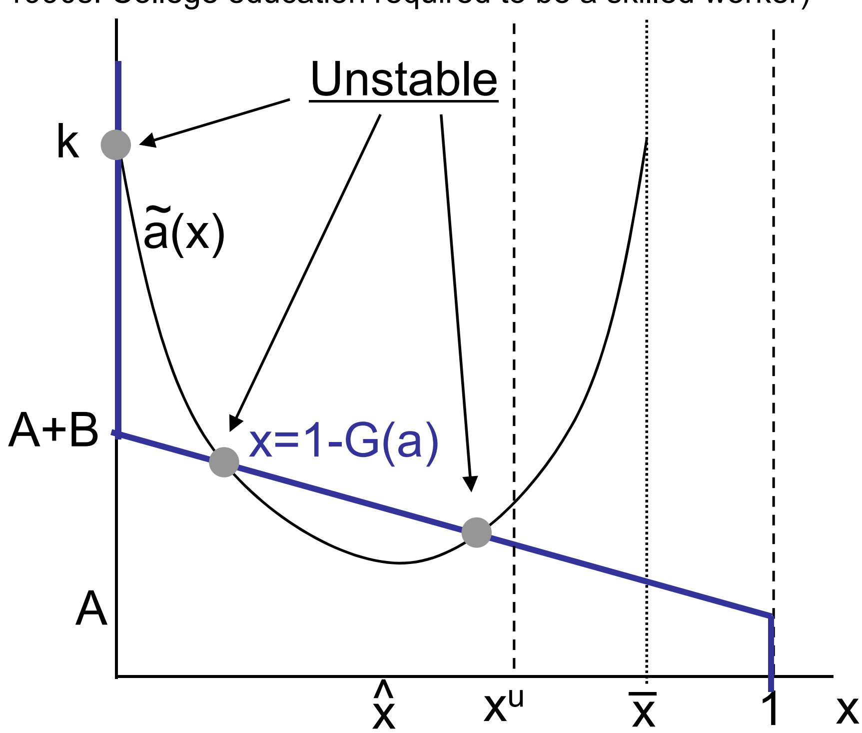
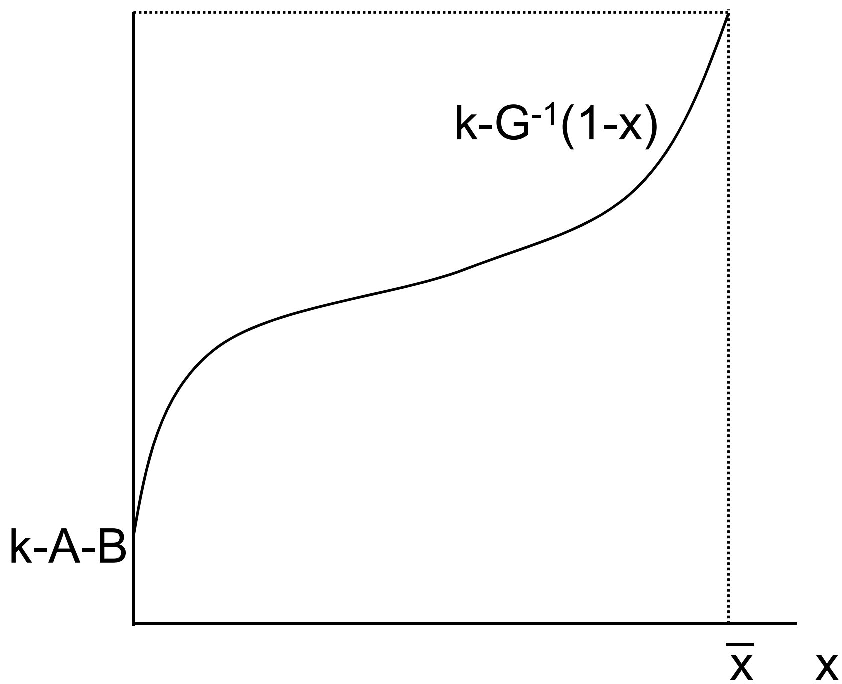


Figure 7. Find LHS with General G functions

* LHS is a product of following two functions, given G is CDF in [A, A+B].



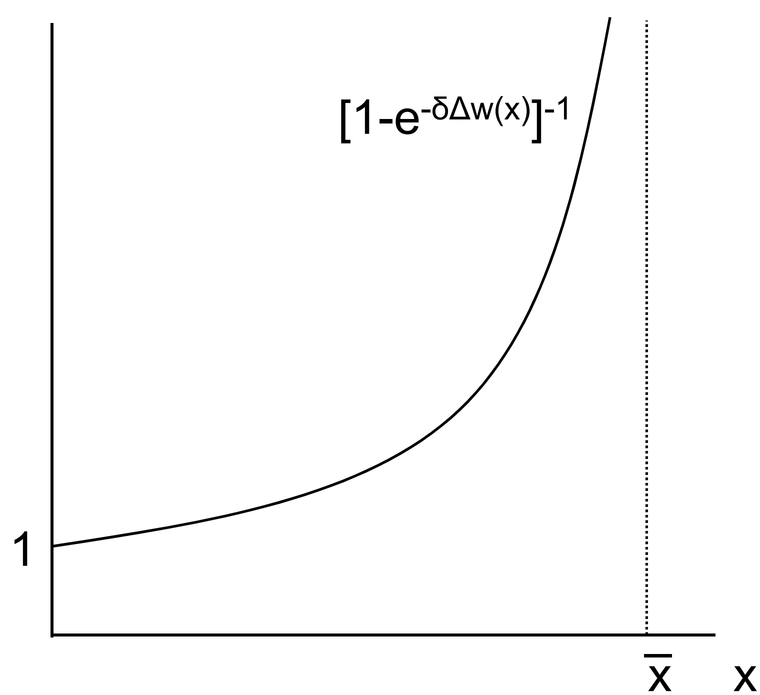


Figure 8. Symmetric Steady States with General G functions

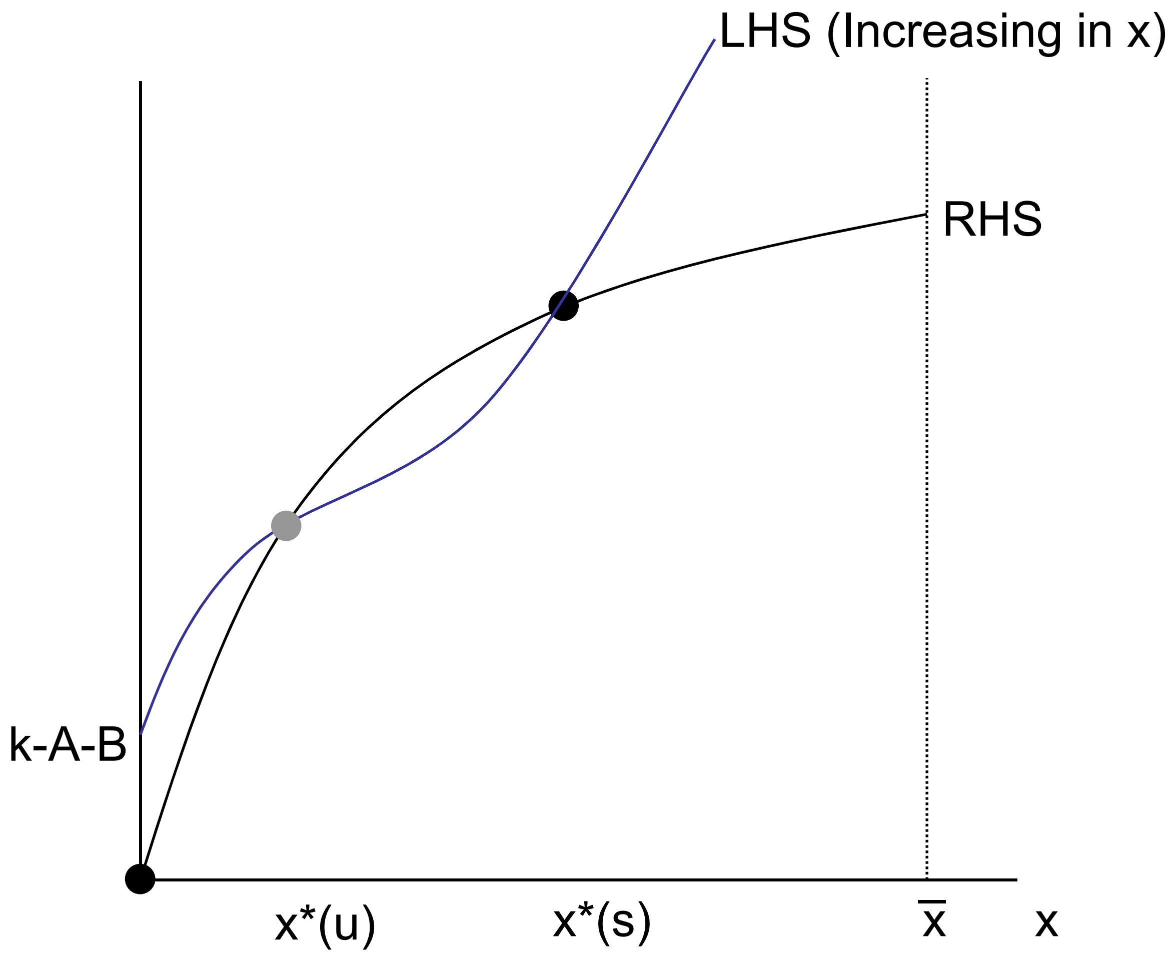
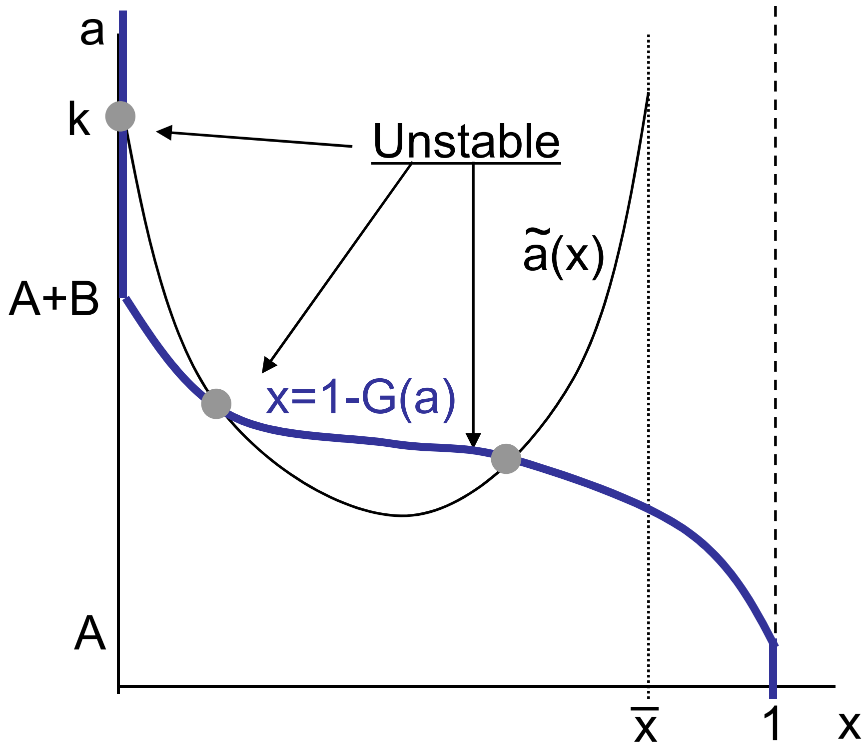


Figure 9. Stability Check for General G functions

Panel A Symmetric Steady States



Panel B Range of x* that makes $\lambda^1 > 1$ with η'=1

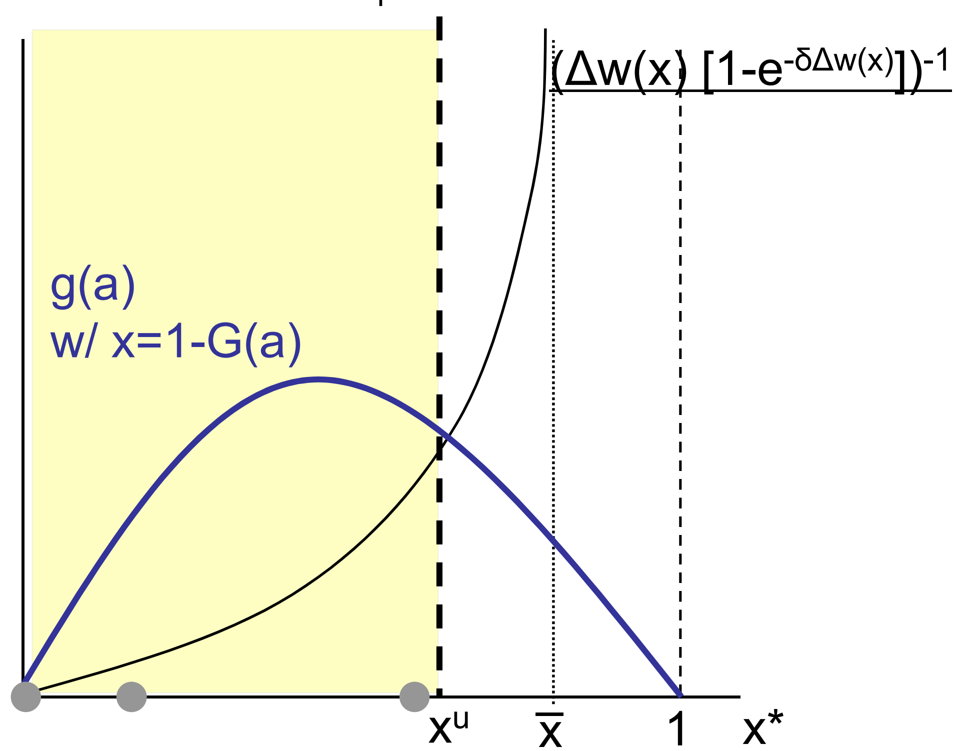


Figure 10. Steady States for General G functions

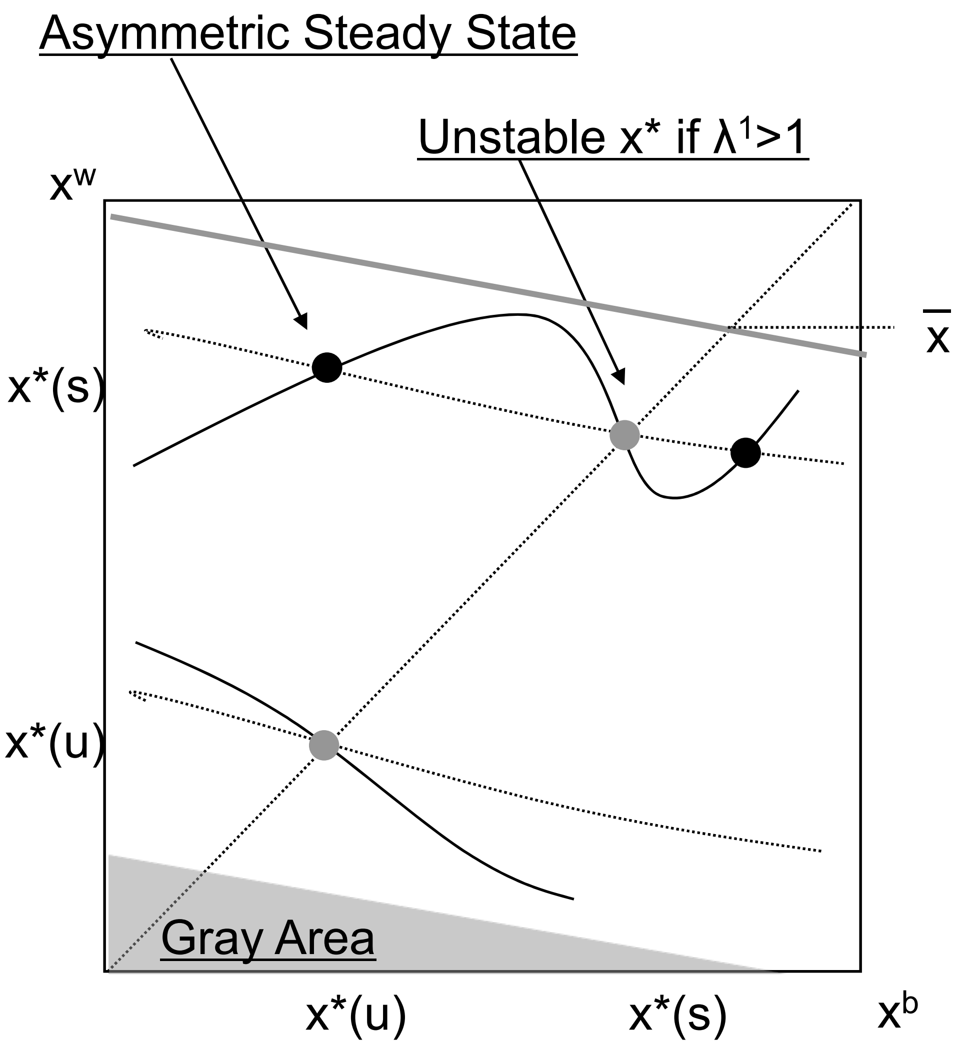
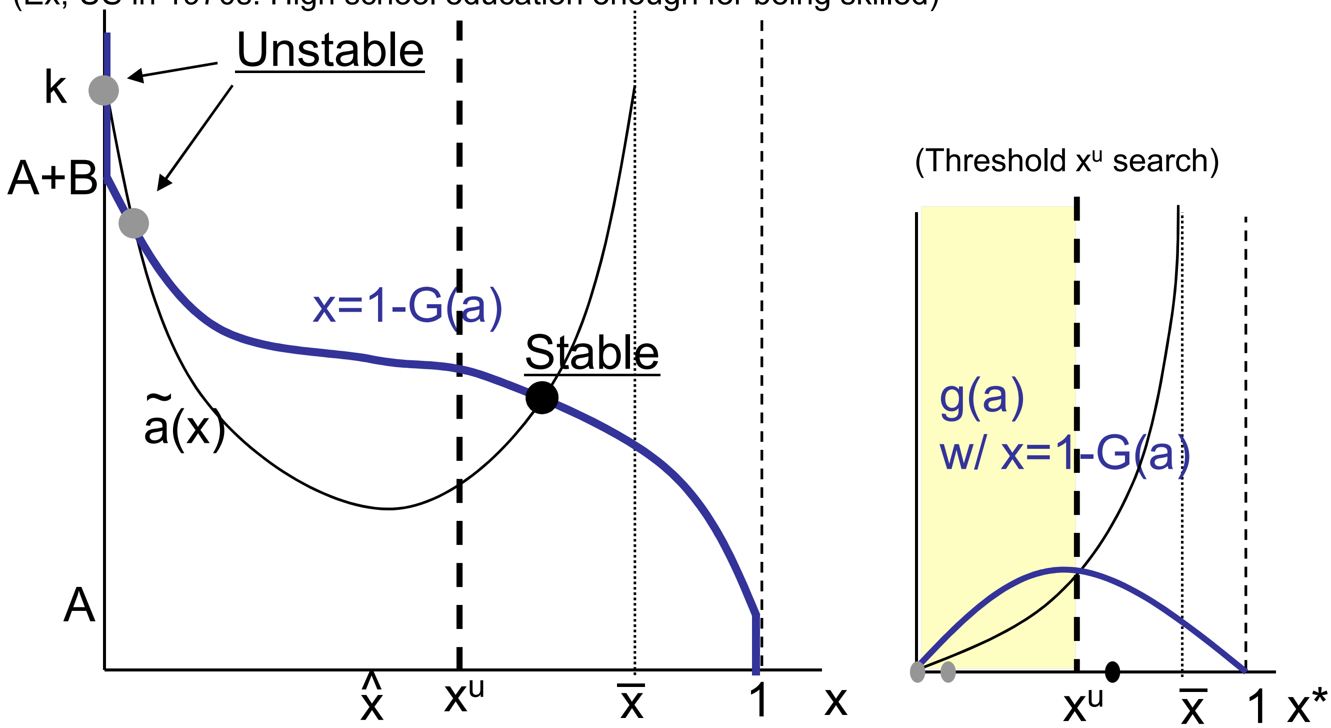


Figure 11. Compare High and Low Training Cost Societies

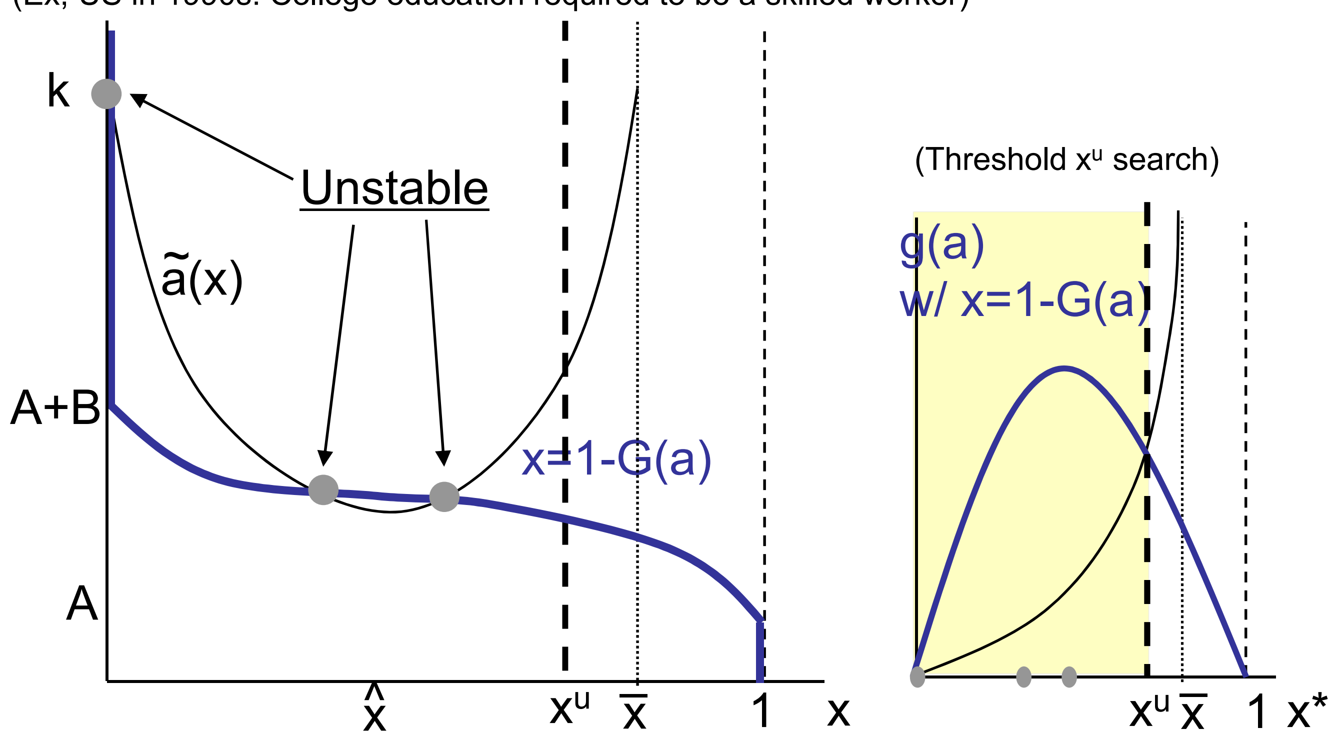
Panel A Low training cost society

(Ex, US in 1970s: High school education enough for being skilled)



Panel B High training cost society

(Ex, US in 1990s: College education required to be a skilled worker)



[Appendix] Figure 1 Labor Market Equilibrium