



Bob Aumann, circa 2000

## An interview with Robert Aumann<sup>1</sup>

by Sergiu Hart

*Who is Robert Aumann? Is he an economist or a mathematician? A rational scientist or a deeply religious man? A deep thinker or an easygoing person?*

*These seemingly disparate qualities can all be found in Aumann; all are essential facets of his personality. A pure mathematician who is a renowned economist, he has been a central figure in developing game theory and establishing its key role in modern economics. He has shaped the field through his fundamental and pioneering work, work that is conceptually profound, and much of it mathematically deep. He has greatly influenced and inspired many people: his students, collaborators, colleagues, and anyone who has been excited by reading his papers or listening to his talks.*

*Aumann promotes a unified view of rational behavior, in many different disciplines: chiefly economics, but also political science, biology, computer science, and more. He has broken new ground in many areas, the most notable being perfect competition, repeated games, correlated equilibrium, interactive knowledge and rationality, and coalitions and cooperation.*

*But Aumann is not just a theoretical scholar, closed in his ivory tower. He is interested in real-life phenomena and issues, to which he applies insights from his research. He is a devoutly religious man; and he is one of the founding fathers – and a central and most active member – of the multidisciplinary Center for the Study of Rationality at the Hebrew University in Jerusalem.*

*Aumann enjoys skiing, mountain climbing, and cooking – no less than working out a complex economic question or proving a deep theorem. He is a family man, a very warm and gracious person – of an extremely subtle and sharp mind.*

*This interview catches a few glimpses of Robert Aumann's fascinating world. It was held in Jerusalem on three consecutive days in September 2004. I hope the reader will learn from it and enjoy it as much as we two did.*

*SH, Jerusalem, January 2005*

**Sergiu HART:** Good morning, Professor Aumann. Let's start with your scientific biography, namely, what were the milestones on your scientific route?

**Robert AUMANN:** I did an undergraduate degree at City College in New York in mathematics, then on to MIT, where I did a doctorate with George Whitehead in algebraic topology, then on to a post-doc at Princeton with an operations research group affiliated with the math department. There I got interested in game theory. From there I went to the Hebrew University in Jerusalem, where I've been ever since. That's the broad outline.

Now to fill that in a little bit. My interest in mathematics actually started in high school – the Rabbi Jacob Joseph Yeshiva (Hebrew Day School) on the lower east side of New York City. There was a marvelous teacher of mathematics there, by the name of Joseph Gansler. The classes were very small; the high school had just started operating. He used to gather the students around his desk. What really turned me on was geometry, theorems and proofs. So all the credit belongs to Joey Gansler.

Then I went on to City College. Actually I did a bit of soul-searching when finishing high school, on

<sup>1</sup> This is a collection of excerpts (compiled by *DMV-Mitteilungen*) from the original interview. The full version has been published in *Macroeconomic Dynamics* 9 (2005), 5, 683–740, and is available at <http://www.ma.huji.ac.il/hart/abs/aumann.html>.

whether to become a Talmudic scholar, or study secular subjects at a university. For a while I did both. But after one semester it became too much for me and I made the hard decision to quit the yeshiva and study mathematics.

At City College, there was a very active group of mathematics students. A lot of socializing went on. There was a table in the cafeteria called the mathematics table. Between classes we would sit there and have ice cream and –

**H:** Discuss the topology of bagels?

**A:** Right, that kind of thing. A lot of chess playing, a lot of math talk. We ran our own seminars, had a math club. Some very prominent mathematicians came out of there – Jack Schwartz of Dunford–Schwartz fame, Leon Ehrenpreis, Alan Shields, Leo Flatto, Martin Davis, D. J. Newman. That was a very intense experience. From there I went on to graduate work at MIT, where I did a doctorate in algebraic topology with George Whitehead.

Let me tell you something very moving relating to my thesis. As an undergraduate, I read a lot of analytic and algebraic number theory. What is fascinating about number theory is that it uses very deep methods to attack problems that are in some sense very “natural” and also simple to formulate. A schoolchild can understand Fermat’s last theorem, but it took extremely deep methods to prove it. Another interesting aspect of number theory was that it was absolutely useless – pure mathematics at its purest.

In graduate school, I heard George Whitehead’s excellent lectures on algebraic topology. Whitehead did not talk much about knots, but I had heard about them, and they fascinated me. Knots are like number theory: the problems are very simple to formulate, a schoolchild can understand them; and they are very natural, they have a simplicity and immediacy that is even greater than that of Fermat’s last theorem. But it is very difficult to prove anything at all about them; it requires really deep methods of algebraic topology. And, like number theory, knot theory was totally, totally useless.

So, I was attracted to knots. I went to Whitehead and said, I want to do a PhD with you, please give me a problem. But not just any problem; please, give me an open problem in knot theory. And he did; he gave me a famous, very difficult problem – the “asphericity” of knots – that had been open for twenty-five years and had defied the most concerted attempts to solve.

Though I did not solve that problem, I did solve a special case. The complete statement of my result is not easy to formulate for a layman, but it does have an interesting implication that even a schoolchild can

understand and that had not been known before my work: alternating knots do not “come apart,” cannot be separated.

So, I had accomplished my objective – done something that i) is the answer to a “natural” question, ii) is easy to formulate, iii) has a deep, difficult proof, and iv) is absolutely useless, the purest of pure mathematics.

It was in the fall of 1954 that I got the crucial idea that was the key to proving my result. The thesis was published in the *Annals of Mathematics* in 1956; but the proof was essentially in place in the fall of 1954.

That’s Act I of the story. And now, the curtain rises on Act II – fifty years later, almost to the day. It’s 10 p.m., and the phone rings in my home. My grandson Yakov Rosen is on the line. Yakov is in his second year of medical school. “Grandpa,” he says, “can I pick your brain? We are studying knots. I don’t understand the material, and think that our lecturer doesn’t understand it either. For example, could you explain to me what, exactly, are ‘linking numbers’?” “Why are you studying knots?” I ask; “what do knots have to do with medicine?” “Well,” says Yakov, “sometimes the DNA in a cell gets knotted up. Depending on the characteristics of the knot, this may lead to cancer. So, we have to understand knots.”

I was completely bowled over. Fifty years later, the “absolutely useless” – the “purest of the pure” – is taught in the second year of medical school, and my grandson is studying it. I invited Yakov to come over, and told him about knots, and linking numbers, and my thesis.

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**H:** Okay, now that we are all tied up in knots, let’s untangle them and go on. You did your PhD at MIT in algebraic topology, and then what?

**A:** Then for my post-doc, I joined an operations research group at Princeton. This was a rather sharp turn because algebraic topology is just about the purest of pure mathematics and operations research is very applied. It was a small group of about ten people at the Forrestal Research Center, which is attached to Princeton University.

**H:** In those days operations research and game theory were quite connected. I guess that’s how you –

**A:** – became interested in game theory, exactly. There was a problem about defending a city from a squadron of aircraft most of which are decoys – do not carry any weapons – but a small percentage do carry nuclear weapons. The project was sponsored by Bell Labs, who were developing a defense missile.



Sergiu Hart, Mike Maschler, Bob Aumann, Bob Wilson, and Oskar Morgenstern, at the 1994 Morgenstern Lecture, Jerusalem

At MIT I had met John Nash, who came there in '53 after doing his doctorate at Princeton. I was a senior graduate student and he was a Moore instructor, which was a prestigious instructorship for young mathematicians. So he was a little older than me, scientifically and also chronologically. We got to know each other fairly well and I heard from him about game theory. One of the problems that we kicked around was that of dueling – silent duels, noisy duels, and so on. So when I came to Princeton, although I didn't know much about game theory at all, I had heard about it; and when we were given this problem by Bell Labs, I was able to say, this sounds a little bit like what Nash was telling us; let's examine it from that point of view. So I started studying game theory; the rest is history, as they say.

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**H:** Since you started talking about these topics, let's perhaps move to Mathematica, the United States Arms Control and Disarmament Agency (ACDA), and repeated games. Tell us about your famous work on repeated games. But first, what are repeated games?

**A:** It's when a single game is repeated many times. How exactly you model "many" may be important, but qualitatively speaking, it usually doesn't matter too much.

**H:** Why are these models important?

**A:** They model ongoing interactions. In the real world we often respond to a given game situation, not so much because of the outcome of that particular game, as because our behavior in a particular situation may affect the outcome of future situations in which a similar game is played. For example, let's say somebody promises something and we respond to

that promise and then he doesn't keep it – he double-crosses us. He may turn out a winner in the short term, but a loser in the long term: if I meet up with him again, I won't trust him. Whether he is rational, whether we are both rational, is reflected not only in the outcome of the particular situation in which we are involved today, but also in how it affects future situations.

Another example is revenge, which in the short term may seem irrational; but in the long term, it may be rational, because if you take revenge, then the next time you meet that person, he will not kick you in the stomach. Altruistic behavior, revengeful behavior, any of those things, make sense when viewed from the perspective of a repeated game, but not from the perspective of a one-shot game. So, a repeated game is often more realistic than a one-shot game: it models ongoing relationships.

In 1959 I published a paper on repeated games (*ContribGameTh* IV). The brunt of that paper is that cooperative behavior in the one-shot game corresponds to equilibrium or egotistic behavior in the repeated game. This is to put it very simplistically.

**H:** There is the famous "Folk Theorem." In the seventies you named it, in your survey of repeated games. The name has stuck.

**A:** The Folk Theorem is quite similar to my '59 paper, but a good deal simpler, less deep. I called it the Folk Theorem because its authorship is not clear, like folk music, folk songs. It was in the air in the late fifties and early sixties.

**H:** Yours was the first full formal statement and proof of something like this. Even Luce and Raiffa, in their very influential '57 book, *Games and Decisions*, don't have the Folk Theorem.

**A:** The first people explicitly to consider repeated non-zero-sum games of the kind treated in my '59 paper were Luce and Raiffa. But as you say, they didn't have the Folk Theorem. Shubik's book *Strategy and Market Structure*, published in '59, has a special case of the Folk Theorem, with a proof that has the germ of the general proof.

At that time people did not necessarily publish everything they knew – in fact, they published only a small proportion of what they knew, only really deep results or something really interesting and nontrivial in the mathematical sense of the word – which is not a good sense. Some very important things would be considered trivial by a mathematician.

For example, take the Cantor diagonal method. Perhaps it really is "trivial." But it is extremely important; inter alia, Gödel's famous incompleteness theorem is based on it.

So, even within pure mathematics the trivial may be important. But certainly outside of it, there are interesting observations that are mathematically trivial – like the Folk Theorem. I knew about the Folk Theorem in the late fifties, but was too young to recognize its importance. I wanted something deeper, and that is what I did in fact publish. That's my '59 paper. It's a nice paper – my first published paper in game theory proper. But the Folk Theorem, although much easier, is more important. So it's important for a person to realize what's important. At that time I didn't have the maturity for this.

Quite possibly, other people knew about it. People were thinking about long-term interaction. There are Shapley's stochastic games, Everett's recursive games, the work of Gillette, and so on. I wasn't the only person thinking about repeated games. Anybody who thinks a little about repeated games, especially if he is a mathematician, will very soon hit on the Folk Theorem. It is not deep.

**H:** That's '59; let's move forward.

**A:** In the early sixties Morgenstern and Kuhn founded a consulting firm called Mathematica, based in Princeton, not to be confused with the software that goes by that name today. In '65 they started working with the United States Arms Control and Disarmament Agency on a project that had to do with the Geneva disarmament negotiations: a series of negotiations with the Soviet Union, on arms control and disarmament. The people on this project included Kuhn, Gerard Debreu, Herb Scarf, Reinhard Selten, John Harsanyi, Jim Mayberry, Mike Maschler, Dick Stearns (who came in a little later), and me. What struck Maschler and me was that these negotiations were taking place again and again; a good way of modeling this is a repeated game. The only thing that distinguished it from the theory of the late fifties that we discussed before is that these were repeated games of incomplete information. We did not know how many weapons the Russians held, and the Russians did not know how many weapons we held. What we – the United States – proposed to put into the agreements might influence what the Russians thought or knew that we had, and this would affect what they would do in later rounds.

**H:** What you do reveals something about your private information. For example, taking an action that is optimal in the short run may reveal to the other side exactly what your situation is, and then in the long run you may be worse off.

**A:** Right. This informational aspect is absent from the previous work, where everything was open and above board, and the issues are how one's behavior affects future interaction. Here the question is how one's *behavior* affects the other player's *knowledge*.



At the 1994 Morgenstern Lecture, Jerusalem. Bob Aumann (front row), Don Patinkin, Mike Maschler, Ken Arrow (second row, left to right), Tom Schelling (third row, second from left); also Marshall Sarnat, Jonathan Shalev, Michael Beensstock, Dieter Balkenborg, Eytan Sheshinski, Edna Ullmann-Margalit, Maya Bar-Hillel, Gershon Ben-Shakhar, Benjamin Weiss, Reuben Gronau, Motty Perry, Menahem Yaari, Zur Shapira, David Budescu, Gary Bornstein

So Maschler and I, and later Stearns, developed a theory of repeated games of incomplete information. This theory was set forth in a series of research reports between '66 and '68, which for many years were unavailable.

**H:** Except to the aficionados, who were passing bootlegged copies from mimeograph machines. They were extremely hard to find.

**A:** Eventually they were published by MIT Press in '95, together with extensive postscripts describing what has happened since the late sixties – a tremendous amount of work. The mathematically deepest started in the early seventies in Belgium, at CORE, and in Israel, mostly by my students and then by their students. Later it spread to France, Russia, and elsewhere. The area is still active.

**H:** What is the big insight?

**A:** It is always misleading to sum it up in a few words, but here goes: in the long run, you cannot use information without revealing it; you can use information only to the extent that you are willing to reveal it. A

player with private information must choose between not making use of that information – and then he doesn't have to reveal it – or making use of it, and then taking the consequences of the other side finding it out. That's the big picture.

**H:** In addition, in a non-zero-sum situation, you may *want* to pass information to the other side; it may be mutually advantageous to reveal your information. The question is how to do it so that you can be trusted, or in technical terms, in a way that is incentive-compatible.

**A:** The bottom line remains similar. In that case you can use the information, not only if you are willing to reveal it, but also if you actually *want* to reveal it. It may actually have positive value to reveal the information. Then you use it *and* reveal it.

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**H:** Let's move to another major work of yours, "Markets with a Continuum of Traders" (*Econometrica* 1964): modeling perfect competition by a continuum.

**A:** At Princeton in '60-'61, the Milnor-Shapley paper "Oceanic Games" caught my fancy. It treats games with an ocean – nowadays we call it a continuum – of small players, and a small number of large players, whom they called atoms. Then in the fall of '61, at the conference at which Kissinger and Lloyd Shapley were present, Herb Scarf gave a talk about large markets. He had a countable infinity of players. Before that, in '59, Martin Shubik had published a paper called "Edgeworth Market Games," in which he made a connection between the core of a large market game and the competitive equilibrium. Scarf's model somehow wasn't very satisfactory, and Herb realized that himself; afterwards, he and Debreu proved a much more satisfactory version, in their *IER* 1963 paper. The bottom line was that, under certain assumptions, the core of a large economy is close to the competitive solution, the solution to which one is led from the law of supply and demand. I heard Scarf's talk, and, as I said, the formulation was not very satisfactory. I put it together with the result of Milnor and Shapley about oceanic games, and realized that *that* has to be the right way of treating this situation: a continuum, not the countable infinity that Scarf was using. It took a while longer to put all this together, but eventually I did get a very general theorem with a continuum of traders. It has very few assumptions, and it is not a limit result. It simply says that the core of a large market is the *same* as the set of competitive outcomes.

**H:** Indeed, the introduction of the continuum idea to economic theory has proved indispensable to the advancement of the discipline. In the same way as in

most of the natural sciences, it enables a precise and rigorous analysis, which otherwise would have been very hard or even impossible.

**A:** The continuum is an approximation to the "true" situation, in which the number of traders is large but finite. The purpose of the continuous approximation is to make available the powerful and elegant methods of the branch of mathematics called "analysis," in a situation where treatment by finite methods would be much more difficult or even hopeless – think of trying to do fluid mechanics by solving  $n$ -body problems for large  $n$ .

**H:** The continuum is the best way to start understanding what's going on. Once you have that, you can do approximations and get limit results.

**A:** Yes, these approximations by finite markets became a hot topic in the late sixties and early seventies. The '64 paper was followed by the *Econometrica* '66 paper on existence of competitive equilibria in continuum markets; in '75 came the paper on values of such markets, also in *Econometrica*. Then there came later papers using a continuum, by me with or without coauthors, by Werner Hildenbrand and his school, and by many, many others.

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**H:** Let's make a big jump. In 1991, the Center for Rationality was established at the Hebrew University.

**A:** Yoram Ben-Porath, who was rector of the university, asked Menahem Yaari, Itamar Pitowsky, Motty Perry, and me to make a proposal for establishing an interdisciplinary center. What came out was the Center for Rationality, which you, Sergiu, directed for its first eight critical years; it was you who really got it going and gave it its oomph. The Center is really unique in the whole world in that it brings together very many disciplines. Throughout the world there are several research centers in areas connected with game theory. Usually they are associated with departments of economics: the Cowles Foundation at Yale, CORE in Louvain, the late Institute for Mathematical Studies in the Social Sciences at Stanford. The Center for Rationality at the Hebrew University is quite different, in that it is much broader. The basic idea is "rationality": behavior that advances one's own interests. This appears in many different contexts, represented by many academic disciplines. The Center has members from mathematics, economics, computer science, evolutionary biology, general philosophy, philosophy of science, psychology, law, statistics, the business school, and education. There is nothing in the world even approaching the breadth of coverage of the Center for Rationality.



Sergiu Hart and Bob Aumann, 2005

It is broad but nevertheless focused. There would seem to be a contradiction between breadth and focus, but our Center has both – breadth and focus. The breadth is in the number and range of different disciplines that are represented at the Center. The focus is, in all these disciplines, on rational, self-interested behavior – or the lack of it. We take all these different disciplines, and we look at a certain segment of each one, and at how these various segments from this great number of disciplines fit together.

**H:** Can you give a few examples? Readers may be surprised to hear about some of these connections.

**A:** I'll try; let's go through some applications. In computer science we have distributed computing, in which there are many different processors. The problem is to coordinate the work of these processors, which may number in the hundreds of thousands, each doing its own work.

**H:** That is, how processors that work in a decentralized way reach a coordinated goal.

**A:** Exactly. Another application is protecting computers against hackers who are trying to break down the computer. This is a very grim game, but it is a game. Still another kind comes from computers that solve games, play games, and design games – like auctions – particularly on the Web.

Biology is another example where one might think that games don't seem particularly relevant. But they are! There is a book by Richard Dawkins called *The Selfish Gene*. This book discusses how evolution makes organisms operate as if they were promoting their self-interest, acting rationally. What drives this is the survival of the fittest. If the genes that organisms have developed in the course of evolution are not optimal, are not doing as well as other genes, then they will not survive. There is a tremendous

range of applications of game-theoretic and rationalistic reasoning in evolutionary biology.

Economics is of course the main area of application of game theory. The book by von Neumann and Morgenstern that started game theory rolling is called *The Theory of Games and Economic Behavior*. Psychology – that of decision-making – has close ties to game theory; whether behavior is rational or irrational – the *subject* is still rationality.

There is much political application of game theory in international relations, which we already discussed in connection with Kissinger. There also are national politics, like various electoral systems. Another aspect is forming a government coalition: if it is too small – a minimal winning coalition – it will be unstable; if too large, the prime minister will have too little influence. What is the right balance?

Law: more and more, we have law and economics, law and game theory. There are studies of how laws affect the behavior of people, the behavior of criminals, the behavior of the police. All these things are about self-interested, rational behavior.

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**H:** Let's move now to your personal biography.

**A:** I was born in 1930 in Frankfurt, Germany, to an orthodox Jewish family, the second of two boys. My father was a wholesale textile merchant – a fine, upright man, a loving, warm father. My mother was extraordinary. She got a bachelor's degree in England in 1914, at a time when that was very unusual for women. She was a medal-winning long-distance swimmer, sang Schubert lieder while accompanying herself on the piano, introduced us children to nature, music, reading. We would walk the streets and she would teach us the names of the trees. At night we looked at the sky and she taught us the names of the constellations. When I was about twelve, we started reading Dickens's *A Tale of Two Cities* together – until the book gripped me and I raced ahead alone. From then on, I read voraciously. She even introduced me to interactive epistemology; look at the "folk ditty" in *GEB* '96. She always encouraged, always pushed us along, gently, unobtrusively, always allowed us to make our own decisions.

We got away in 1938. Actually we had planned to leave already when Hitler came to power in 1933, but for one reason or another we didn't. People convinced my parents that it wasn't so bad; it will be okay, this thing will blow over. The German people will not allow such a madman to take over, etc., etc. A well-known story. But it illustrates that when one is in the middle of things it is very, very difficult to

see the future. Things seem clear in hindsight, but in the middle of the crisis they are very murky.

**H:** Especially when it is a slow-moving process, rather than a dramatic change: every time it is just a little more and you say, that's not much, but when you look at the integral of all this, suddenly it is a big change.

**A:** That is one thing. But even more basically, it is just difficult to see. Let me jump forward from 1933 to 1967. I was in Israel and there was the crisis preceding the Six-Day War. In hindsight it was "clear" that Israel would come out on top of that conflict. But at the time it wasn't at all clear, not at all. I vividly remember the weeks leading up to the Six-Day War, the crisis in which Nasser closed the Tiran Straits and massed troops on Israel's border; it wasn't at all clear that Israel would survive. Not only to me, but to anybody in the general population. Maybe our generals were confident, but I don't think so, because our government certainly was not confident. Prime Minister Eshkol was very worried. He made a broadcast in which he stuttered and his concern was very evident, very real. Nobody knew what was going to happen; people were very worried, and I, too, was very worried. I had a wife and three children and we all had American papers. So I said to myself, Johnny, don't make the mistake your father made by staying in Germany. Pick yourself up, get on a plane and leave, and save your skin and that of your family; because there is a very good chance that Israel will be destroyed and the inhabitants of Israel will be wiped out totally, killed, in the next two or three weeks. Pick yourself up and *GO*.

I made a conscious decision not to do that. I said, I am staying. Herb Scarf was here during the crisis. When he left, about two weeks before the war, we said good-bye, and it was clear to both of us that we might never see each other again.

This illustrates that it is very difficult to judge a situation from the middle of it. When you're swimming in a big lake, it's difficult to see the shore, because you are low, you are inside it. One should not blame the German Jews or the European Jews for not leaving Europe in the thirties, because it was difficult to assess the situation.

We did get away in time, in 1938. We left Germany, and made our way to the United States. In this passage, my parents lost all their money. They had to work extremely hard in the United States to make

ends meet, but nevertheless they gave their two children, my brother and myself, a good Jewish and a good secular education.

When the State of Israel was created in 1948, I made a determination eventually to come to Israel, but that didn't actually happen until 1956. In 1954 I met an Israeli girl, Esther Schlesinger, who was visiting the United States. We fell in love, got engaged, and got married. We had five children; the oldest, Shlomo, was killed in action in Lebanon in 1982. My other children are all happily married. Shlomo's widow also remarried and she is like a daughter to us. Shlomo had two children, the second one born after he was killed. Altogether I now have seventeen grandchildren and one great-grandchild. We have a very good family relationship, do a lot of things together. One of the things we like best is skiing. Every year I go with a different part of the family. Once in four or five years, all thirty of us go together.

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**H:** Any closing "words of wisdom"?

**A:** Just one: Game theory is ethically neutral. That is, game theorists don't necessarily advocate carrying out the normative prescriptions of game theory. Bacteriologists do not advocate disease, they study it; similarly, studying self-interested behavior is different from advocating it. Game theory says nothing about whether the "rational" way is morally or ethically right. It just says what rational – self-interested – entities will do; not what they "should" do, ethically speaking. If we want a better world, we had better pay attention to where rational incentives lead.

**H:** That's a very good conclusion to this fascinating interview. Thank you.

**A:** And thank *you*, Sergiu, for your part in this wonderful interview.

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