Research Article

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Dynamical property of hyperspace on uniform space

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Abstract: First, we introduce the concepts of equicontinuity, expansivity , ergodic shadowing property, and chain transitivity in uniform space. Second, we study the dynamical properties of equicontinuity, expansivity , ergodic shadowing property, and chain transitivity in the hyperspace of uniform space. Let (X, μ) be a uniform space, $(C(X), C^{\mu})$ be a hyperspace of (X, μ), and $f : X \to X$ be uniformly continuous. By using the relationship between original space and hyperspace, we obtain the following results: (a) the map f is equicontinous if and only if the induced map C^f is equicontinous; (b) if the induced map C^f is expansive, then the map f is expansive; (c) if the induced map C^f has ergodic shadowing property, then the map f has ergodic shadowing property; (d) if the induced map C^f is chain transitive, then the map f is chain transitive. In addition, we also study the topological conjugate invariance of (G, h)-shadowing property in metric G- space and prove that the map S has (G, h)-shadowing property if and only if the map T has (G, h)-shadowing property. These results generalize the conclusions of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in hyperspace.

Keywords: equicontinuity, expansivity, ergodic shadowing property, chain transitivity, (G, h)-shadowing property, uniform space

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1 Introduction

Uniform space is an important research branch of dynamical systems, and it has also been the focus and hotspot in recent years. After development, we achieved good research results (see [1–15]). Wu et al. [1] proved that the map f is uniformly rigid if and only if the induced map C^f is uniformly rigid in the hyperspace of uniform space; Yan and Zeng [2] proved that the map f is expansive if and only if, for any $k > 1$, f^k is expansive; Wu et al. [3] obtained that if there exists a transitive non-equicontinuity point in Hausdorff uniform space, then the map f is sensitive; Pirfalak et al. [4] proved that if the minimal point set is dense and f has topological average shadowing property, then it is topologically complete and strongly ergodic in uniform space; Ahmadi et al. [5] studied the relationship between topological entropy point and traceability in uniform space; In another study [6], Ahmadi et al. proved that f has shadowing property in a uniform space if and only if X is completely disconnected. In this article, we study the dynamical properties of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in the hyperspace of uniform space. Through reasoning, we obtain the following theorems:

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Theorem 1.1. *Let* (X, μ) *be a uniform space,* $(C(X), C^{\mu})$ *be a hyperspace of* (X, μ)*, and* $f : X \to X$ *be uniformly continuous. Then, the map* f *is equicontinous if and only if the induced map* C^f *is equicontinous.*

Theorem 1.2. *Let* (X, μ) *be a uniform space,* $(C(X), C^{\mu})$ *be a hyperspace of* (X, μ)*, and* $f : X \to X$ *be uniformly continuous. If the induced map* C^f *is expansive, then the map* f *is expansive.*

Theorem 1.3. *Let* (X, μ) *be a uniform space,* $(C(X), C^{\mu})$ *be a hyperspace of* (X, μ)*, and* $f : X \to X$ *be uniformly continuous. If the induced map* C^f *has ergodic shadowing property, then the map* f *has ergodic shadowing property.*

Theorem 1.4. *Let* (X, μ) *be a uniform space,* $(C(X), C^{\mu})$ *be a hyperspace of* (X, μ)*, and the mapping* $f : X$ *to* X *be uniformly continuous. If the induced map* C^f *is chain transitive, then the map* f *is chain transitive.*

In addition, metric G-space is also the focus of many scholars' research and the related research results are seen in [16–20]. In the study by Ekta and Tarun [16], it is shown that if f is a pseudo-equivariant minimal homeomorphism map, it lacks pseudo-orbit tracking property in metric G-space. Ji [17] proved that the self-map f has the G-asymptotic average tracking property if and only if the shift map σ has G-asymptotic average tracking property. Ahmadi [18] proved that chain transitivity is topological conjugate invariant in metric G-space. Ji [19] proved that if the self-map f is G-chaotic, the shift map σ is G-chaotic. In this article, we introduce the concept of (G, h)-shadowing property in metric G-space and obtain the following theorem:

Theorem 1.5. *Let* (X, d) *be a compact metric G-space,* (Y, ρ) *be a compact metric G-space,* $S : X \longrightarrow X$ *be continuous, and* $T : Y \longrightarrow Y$ *be continuous. If* S *is topologically* G *-conjugate to* T*, then the map* S *has* (G, h)*-shadowing property if and only if the map* T *has* (G, h)*-shadowing property.*

Now, we prove above theorem in Section 3.

2 Basic definitions

In this section, we gave some concepts that may be used in the following.

Definition 2.1. [1] Let X be nonempty set. The diagonal $\triangle$ of $X \times X$ is defined as follows:

$$\triangle = \{(x, x) : x \in X\}.$$

Given $U, V \subset X \times X$, $A \subset X$ and $x \in X$, we define

$$\begin{aligned} U^{-1} &= \{(x, y) : (y, x) \in U\}; \\ B(x, U) &= \{y \in X : (x, y) \in U\}; \\ B(A, U) &= \bigcup_{x \in X} B(x, U); \\ U \circ V &= \{(x, y) \in X \times X : \exists z \in X, (x, z) \in U, (z, y) \in V\}. \end{aligned}$$

Definition 2.2. [2] Let X be nonempty set and μ be a collection of subsets of $X \times X$. The (X, μ) is called a uniform space if the following conditions are satisfied:
(1) $(x, x) \in U$ for any $x \in U$ and $U \in \mu$;
(2) If $U \in \mu$ and $U \subset V \subset X \times X$, then $V \in \mu$;
(3) If $U \in \mu$ and $V \in \mu$, then $U \bigcap V \in \mu$;
(4) If $U \in \mu$, then $U^{-1} \in \mu$;
(5) For any $U \in \mu$, then there exists $V \in \mu$ such that $V \circ V \subset U$.

The μ is called uniform structure on a set X. A member of μ is called an entourage.

Definition 2.3. [1] Let (X, μ) be a uniform space. Write

$$\zeta = \{G \subset X : \forall x \in G, \exists U \in \mu, B(x, U) \subset G\}.$$

Then, the ζ is called uniform topology on a set X. The uniform topology ζ generated by μ is denoted by $|\mu|$.

Definition 2.4. [1] Let (X, μ) be a uniform space. Write

$$C^\mu = \{C^V : V \in \mu\},$$

where $C^V = \{(A, D) \in C(X) \times C(X) : A \subset B(D, U), D \subset B(A, U)\}$. Clearly, C^μ is a uniform structure on a set $C(X)$, and $(C(X), C^\mu)$ is a uniform space. Write

$$\xi = \{\upsilon(U_1, U_2, \dots, U_n)\},$$

where $\upsilon(U_1, U_2, \dots, U_n) = \{A \in C(X) : A \subset \bigcup U_i, A \bigcap U_i \neq \emptyset, 1 \leq i \leq n\}$ and U_i be finite open set of (X, μ). Then, ξ is a Vietoris topology on a set $C(X)$. According to Wu et al. [1], Vietoris topology ξ coincides with uniform topology $|C^\mu|$ generated by C^μ. Hence, $(C(X), C^\mu)$ is the hyperspace of (X, μ).

Let $f : X \longrightarrow X$ be uniformly continuous. Define the induced map C^f on $(C(X), C^\mu)$ by

$$C^f(A) = f(A), \forall A \in X.$$

Then, C^f is uniformly continuous from $C(X)$ to $C(X)$.

Definition 2.5. [2] Let (X, μ) be a uniform space and $f : X \to X$ be uniformly continuous. The map f is expansive if there exists $D \in \mu$ such that for $x \neq y$, there exists $n > 0$ satisfying

$$(f^n(x), f^n(y)) \in D.$$

Definition 2.6. [3] Let (X, μ) be a uniform space and $f : X \to X$ be uniformly continuous. The map f is equicontinous if, for any $U \in \mu$, there exists $D \in \mu$ such that $(x, y) \in D$ implies

$$(f^n(x), f^n(y)) \in U \quad \text{for any } n > 0.$$

Definition 2.7. [3] Let (X, μ) be a uniform space, $f : X \to X$ be uniformly continuous, and $V \in \mu$. The sequence $\{x_i\}_{i=0}^{\infty}$ is V-ergodic pseudo-orbit of f if

$$\lim_{n\to\infty} \frac{\text{Card}\{i : 0 \leq i < n, (f(x_i), x_{i+1}) \in U\}}{n} = 1.$$

Definition 2.8. [3] Let (X, μ) be a uniform space and $f : X \to X$ be uniformly continuous. The map f is said to have the ergodic shadowing property if, for any $U \in \mu$, there exists $V \in \mu$ such that for any V-ergodic pseudo-orbit $\{x_i\}_{i=0}^{\infty}$ there exists $x \in X$ satisfying

$$\lim_{n\to\infty} \frac{\text{Card}\{i : 0 \leq i < n, (f^i(x), x_i) \in U\}}{n} = 1.$$

Definition 2.9. [4] Let (X, μ) be a uniform space, $f : X \to X$ be uniformly continuous, and $U \in \mu$. The sequence $\{x_i\}_{i=0}^{i=m}$ is U-chain of f if

$$(f(x_i), x_{i+1}) \in U \quad \text{for any } 0 \leq i < m.$$

Definition 2.10. [4] Let (X, μ) be uniform space and $f : X \to X$ be uniformly continuous. The map f is chain transitive if for any $U \in \mu$ and $x, y \in X$, there exists U-chain $\{x_i\}_{i=0}^{i=m}$ from x to y.

Definition 2.11. [16] Let (X, d) be a metric G-space and f be a continuous map from X to X. f is said to be a pseudo-equivariant map if, for all $x \in X$ and $p \in G$, there exists $g \in G$ such that $f(px) = gf(x)$.

Definition 2.12. [16] Let (X, d) be a compact metric G-space, (Y, ρ) be a compact metric G-space, $S : X \longrightarrow X$ be be continuous, and $T : Y \longrightarrow Y$ be continuous. The map S is topologically G-conjugate to T if there exists a homeomorphic pseudo-equivariant map $h : X \longrightarrow Y$ such that $h \circ S = T \circ h$.

Definition 2.13. [18] Let (X, d) be a metric space, G be a topological group, and $\theta : G \times X \to X$ be a continuous map. The triple (X, G, θ) is called a metric G-space if the following conditions are satisfied:

(1) $\theta(e, x) = x$, where all $x \in X$ and e is the identity of G.

(2) $\theta(g_1, \theta(g_2, x)) = \theta(g_1 g_2, x)$ for all $x \in X$ and $g_1, g_2 \in G$.

If (X, d) is compact, then (X, G, θ) is also said to be compact metric G-space. For the convenience of writing, $\theta(g, x)$ is usually abbreviated as gx

Definition 2.14. Let (X, d) be a metric G-space and $f : X \to X$ be a continuous map. The map f is said to be have (G, h)-shadowing property if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that for any (G, δ)-chain $\{x_i\}_{i=0}^{i=m}$, there exists $y \in X$ and $g_i \in G$ satisfying

$$d(f^i(y), g_i x_i) < \varepsilon \quad \text{for any } 0 \le i < m;$$

$$f^m(y) = g_m x_m.$$

3 Equicontinous, expansive, ergodic shadowing property, and (G, h)-shadowing property

Now, we prove the main results in this section.

Theorem 3.1. *Let (X, μ) be a uniform space, $(C(X), C^\mu)$ be a hyperspace of (X, μ), and $f : X \to X$ be uniformly continuous. Then, the map f is equicontinous if and only if the induced map C^f is equicontinous.*

Proof. (Necessity) Suppose the map f is equicontinous. For any $U \in C^\mu$, there exists $V \in \mu$ such that $C^V \subset U$. Since f is equicontinous, for above $V \in \mu$, there exists $W \in \mu$ such that $(\acute{x}, \acute{y}) \in W$ implies

$$(f^n(\acute{x}), f^n(\acute{y})) \in V \quad \text{for any } n \ge 1. \tag{1}$$

Let $(A, D) \in C^W$. Then,

$$A \subset B(D, W).$$

$$B \subset B(A, W).$$

Hence, for any $x \in A$ and $y \in D$, we can obtain $(x, y) \in W$. Applying equation (1) implies

$$(f^n(x), f^n(y)) \in V \quad \text{for any } n \ge 1.$$

Thus, we have that

$$f^n(A) \subset B(f^n(D), V);$$

$$f^n(D) \subset B(f^n(A), V).$$

So,

$$(f^n(A), f^n(D)) \in C^V \subset U.$$

Hence, the induced map C^f is equicontinous.

(*Sufficiency*) Suppose the induced map C^f is equicontinous. For any $U \in \mu$, we can obtain $C^U \in C^\mu$. According to that C^f is equicontinous, there exists $C^W \in C^\mu (W \in \mu)$ such that $(A, D) \in C^W$ implies

$$((C^f)^n(A), (C^f)^n(D)) \in C^U \quad \text{for any } n \geq 1. \tag{2}$$

Let $(x, y) \in W$. Write $A = \{x\}$ and $D = \{y\}$. Then,

$$(A, D) \in C^W.$$

By equation (2), we can obtain

$$((C^f)^n(A), (C^f)^n(D)) \in C^U \quad \text{for any } n \geq 1.$$

That is,

$$(f^n(x), f^n(y)) \in U.$$

Thus, f is equicontinous. We end the proof. □

Theorem 3.2. *Let (X, μ) be a uniform space, $(C(X), C^\mu)$ be a hyperspace of (X, μ), and $f: X \to X$ be uniformly continuous. If the induced map C^f is expansive, then the map f is an expansive.*

Proof. Suppose the induced map C^f is expansive. Then, there exists $C^U \in C^\mu (U \in \mu)$ such that if, for any $n > 1$, $((C^f)^n(A), (C^f)^n(B)) \in C^U$, then we have

$$A = B. \tag{3}$$

Suppose $(f^n(x), f^n(y)) \in U$ for any $n \geq 1$. Write

$$A = \{x\}, B = \{y\}.$$

Then,

$$((C^f)^n(A), (C^f)^n(B)) \in C^U.$$

By equation (3), we can obtain $A = B$. Thus, $x = y$. Hence, f is expansive. Thus, we complete the proof. □

Theorem 3.3. *Let (X, μ) be a uniform space, $(C(X), C^\mu)$ be a hyperspace of (X, μ), and $f: X \to X$ be uniformly continuous. Then, C^f has ergodic shadowing property implying that f has ergodic shadowing property.*

Proof. Suppose the induced map C^f has ergodic shadowing property. For any $U \in \mu$, we can obtain $C^U \in C^\mu$. According to that, C^f has ergodic shadowing property, and for above $C^U \in C^\mu$, there exists $C^V \in C^\mu (V \in \mu)$ such that for any C^V-ergodic pseudo-orbit $\{A_i\}_{i=0}^\infty$, there exists $B \in C(X)$ satisfying

$$\lim_{n\to\infty} \frac{\text{Card}\{i : 0 \leq i < n, ((C^f)^i(B), A_i) \in C^U\}}{n} = 1. \tag{4}$$

Let $\{x_i\}_{i=0}^\infty$ be V-ergodic pseudo-orbit of f. Then,

$$\lim_{n\to\infty} \frac{\text{Card}\{i : 0 \leq i < n, (f(x_i), x_{i+1}) \in V\}}{n} = 1.$$

Write

$$D_i = \{x_i\};$$
$$K = \{i : 0 \leq i < n, (f(x_i), x_{i+1}) \in V\};$$
$$M = \{i : 0 \leq i < n, (C^f(D_i), D_{i+1}) \in C^V\}.$$

Suppose $i \in K$. Then,

$$(f(x_i), x_{i+1}) \in V.$$

Therefore, we can obtain that

$$(C^f(D_i), D_{i+1}) \in C^V.$$

Hence, $i \in M$ and $\mathrm{Card}(M) \geq \mathrm{Card}(K)$. Thus, we have,

$$\lim_{n\to\infty} \frac{\mathrm{Card}\{i : 0 \leq i < n, (C^f(D_i), D_{i+1}) \in C^V\}}{n} = 1.$$

So $\{D_i\}_{i=0}^{\infty}$ is C^V-ergodic pseudo-orbit of C^f. By equation (4), there exists $W \in C(X)$ satisfying

$$\lim_{n\to\infty} \frac{\mathrm{Card}\{i : 0 \leq i < n, ((C^f)^i(W), D_i) \in C^U\}}{n} = 1.$$

Let $y \in W$. Write

$$E = \{i : 0 \leq i < n, ((C^f)^i(W), D_i) \in C^U\};$$
$$F = \{i : 0 \leq i < n, (f^i(y), x_i) \in U\}.$$

Suppose $i \in E$. Then,

$$((C^f)^i(W), D_i) \in C^U.$$

That is,

$$(f^i(W), D_i) \in C^U.$$

Thus, we can obtain that

$$(f^i(y), x_i) \in U.$$

Hence, $i \in F$ and $\mathrm{Card}(F) \geq \mathrm{Card}(E)$. Thus, we have,

$$\lim_{n\to\infty} \frac{\mathrm{Card}\{i : 0 \leq i < n, (f^i(y), x_i) \in U\}}{n} = 1.$$

So the map f has ergodic shadowing property. Thus, we end the proof. □

Theorem 3.4. *Let (X, μ) be a uniform space, $(C(X), C^\mu)$ be a hyperspace of (X, μ), and $f : X \to X$ be uniformly continuous. If the induced map C^f is chain transitive, then the map f is chain transitive.*

Proof. Suppose the induced map C^f is chain transitive. For any $U \in \mu$, we can obtain $C^U \in C^\mu$. Let $x \in X$ and $y \in Y$. Then, $\{x\}$ and $\{y\}$ are singleton sets. According to that, C^f is chain transitive, and there exists C^U-chain $\{D_i\}_{i=0}^{m}$ of C^f from $\{x\}$ to $\{y\}$. Hence,

$$((C^f)(D_i), D_{i+1}) \in C^U \quad \text{for any } 0 \leq i < m.$$

That is,

$$(f(D_i), D_{i+1}) \in C^U \quad \text{for any } 0 \leq i < m.$$

Hence, for any $0 \leq i < m$, there exist $z_i \in D_i$ such that $\{z_i\}_{i=0}^{m}$ is a U-chain of f from x to y. So the map f is chain transitive. Thus, we complete the proof. □

Theorem 3.5. *Let (X, d) be a compact metric G-space, (Y, ρ) be a compact metric G-space, $S : X \longrightarrow X$ be continuous, and $T : Y \longrightarrow Y$ be continuous. If S is topologically G-conjugate to T, then the map S has (G, h)-shadowing property if and only if the map T has (G, h)-shadowing property.*

Proof. (*Necessity*) Since S is topologically G-conjugate to T, there exists a homeomorphic pseudo-equivariant map $h : X \longrightarrow Y$ such that

$$h \circ S = T \circ h. \tag{5}$$

According to the uniform continuity of h, for any $\varepsilon > 0$, there exists $\delta_1 > 0$ such that $d(z_1, z_2) < \delta_1$ implies

$$\rho(h(z_1), h(z_2)) < \varepsilon. \tag{6}$$

Suppose the map S has (G, h)-shadowing property. Then, for above $\delta_1 > 0$, there exists $\delta_2 > 0$ such that for any (G, δ_2)-chain $\{x_i\}_{i=0}^{i=m}$, there exists $y \in X$ and $g_i \in G$ satisfying

$$d(S^i(y), g_i x_i) < \delta_1 \quad \text{for any } 0 \le i < m;$$

$$S^m(y) = g_m x_m. \tag{7}$$

According to the uniform continuity of h^{-1}, for above $\delta_2 > 0$, there exists $\delta_3 > 0$ such that $\rho(z_1, z_2) < \delta_3$ implies

$$d(h^{-1}(z_1), h^{-1}(z_2)) < \delta_2. \tag{8}$$

Let $\{y_i\}_{i=0}^{i=m}$ be a (G, δ_3)-chain of T. Then, for any $0 \le i < m$, there exists $t_i \in G$ such that

$$\rho(t_i T(y_i), y_{i+1}) < \delta_3.$$

By equations (5) and (8), for any $0 \le i < m$, there exists $l_i \in G$ such that

$$d(l_i S(h^{-1}(y_i)), h^{-1}(y_{i+1}) < \delta_2.$$

Thus, $\{h^{-1}(y_i)\}_{i=0}^{i=m}$ is an (G, δ_2)-chain of S. According to equation (7), there exists $x \in X$ and $p_i \in G$ satisfying

$$d(S^i(x), p_i h^{-1}(y_i)) < \delta_1 \quad \text{for any } 0 \le i < m;$$

$$S^m(x) = p_m h^{-1}(y_m).$$

By equations (5) and (6), for any $0 \le i < m$, there exists $s_i \in G$ such that

$$\rho(T^i(h(x)), s_i y_i) < \varepsilon \quad \text{for any } 0 \le i < m;$$

$$T^m(h(x)) = s_m y_m.$$

Hence, the map T has (G, h)-shadowing property.

(*Sufficiency*) According to the uniform continuity of h^{-1}, for any $\varepsilon > 0$, there exists $\delta_1 > 0$ such that $\rho(z_1, z_2) < \delta_1$ implies

$$d(h^{-1}(z_1), h^{-1}(z_2)) < \varepsilon. \tag{9}$$

Suppose the map T has (G, h)-shadowing property. Then, for above $\delta_1 > 0$, there exists $\delta_2 > 0$ such that for any (G, δ_2)-chain $\{y_i\}_{i=0}^{i=m}$ of T, there exists $y \in X$ and $g_i \in G$ satisfying

$$d(T^i(y), g_i y_i) < \delta_1 \quad \text{for any } 0 \le i < m;$$

$$T^m(y) = g_m y_m. \tag{10}$$

According to the uniform continuity of h, for above $\delta_2 > 0$, there exists $\delta_3 > 0$ such that $d(z_1, z_2) < \delta_3$ implies

$$\rho(h(z_1), h(z_2)) < \delta_2. \tag{11}$$

Let $\{x_i\}_{i=0}^{i=n}$ be an (G, δ_3)-chain of S. Then, for any $0 \le i < m$, there exists $t_i \in G$ such that

$$d(t_i S(x_i), x_{i+1}) < \delta_3.$$

By equations (5) and (11), for any $0 \le i < m$, there exists $l_i \in G$ such that

$$\rho(l_i T(h(x_i)), h(x_{i+1})) < \delta_2.$$

Thus, $\{h(x_i)\}_{i=0}^{i=m}$ is an (G, δ_2)-chain of T. According to equation (10), there exists $y \in Y$ and $p_i \in G$ satisfying

$$\rho(T^i(y), p_i h(x_i)) < \delta_1 \quad \text{for any } 0 \le i < m;$$

$$T^m(y) = p_m h(x_m).$$

By equations (5) and (9), for any $0 \le i < m$, there exists $s_i \in G$ such that

$$d(S^i(h^{-1}(y)), s_i x_i) < \varepsilon \quad \text{for any } 0 \le i < m;$$

$$S^m(h^{-1}(y)) = s_m x_m.$$

Hence, the map S has (G, h)-shadowing. Thus, we complete the proof. □

4 Conclusion

First, we introduce the concepts of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in uniform space. Second, we study the dynamical properties of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in the hyperspace of uniform space. Let (X, μ) be a uniform space, $(C(X), C^{\mu})$ be a hyperspace of (X, μ), and $f : X \to X$ be uniformly continuous. By using the relationship between original space and hyperspace, we obtain the following results: (1) the map f is equicontinous if and only if the induced map C^f is equicontinous; (2) if the induced map C^f is expansive, then the map f is expansive; (3) if the induced map C^f has ergodic shadowing property, then the map f has ergodic shadowing property; and (4) if the induced map C^f is chain transitive, then the map f is chain transitive. In addition, we also study the topological conjugate invariance of (G, h)-shadowing property in metric G-space and prove that the map S has (G, h)-shadowing property if and only if the map T has (G, h)-shadowing property. These new results enrich the conclusions of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in the hyperspace. They are sharp and more accurate compared to the conclusions of equicontinuity, expansivity, ergodic shadowing property, and chain transitivity in the hyperspace. Most importantly, it provides the theoretical basis and scientific foundation for the application of equicontinuity, expansivity, ergodic shadowing property, chain transitivity, and shadowing property in computational mathematics, biological mathematics, nature, and society.

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References

[1] X. X. Wu, Y. Luo, X. Ma, and T. Lu, *Rigidity and sensitivity on uniform spaces*, Topol. Appl. **252** (2019), 145–157, DOI: https://doi.org/10.1016/j.topol.2018.11.014.

[2] K. S. Yan and F. P. Zeng, *Topological stability and pseudo-orbit tracing property for homeomorphisms on uniform spaces*, Acta Math. Sinica **38** (2022), no. 2, 431–432, DOI: https://doi.org/10.1007/s10114-021-0232-x.

[3] X. X. Wu, X. Ma, Z. Zhu, and T. Lu, *Topological ergodic shadowing and chaos on uniform spaces*, Int. J. Bifurcat. Chaos **28** (2018), no. 3, 1–9, DOI: https://doi.org/10.1142/S0218127418500438.

[4] F. Pirfalak, S. A. Ahmadi, X. X. Wu, and N. Kouhestani, *Topological average shadowing property on uniform spaces*, Qualitative Theory Dyn. Sys. **20** (2021), no. 2, 31–45, DOI: https://doi.org/10.1007/S12346-021-00466-W.

[5] S. A. Ahmadi, X. X. Wu, Z. H. Feng, X. Ma, and T. Lu, *On the entropy points and shadowing in uniform spaces*, Int. J. Bifurcat. Chaos **28** (2018), no. 12, 1–10, DOI: https://doi.org/10.1142/S0218127418501559.

[6] S. A. Ahmadi, X. X. Wu, and G. R. Chen, *Topological chain and shadowing properties of dynamical systems on uniform spaces*, Topol. Appl. **275** (2020), 1–11, DOI: https://doi.org/10.1016/j.topol.2020.107153.

[7] T. Rasham, A Shoaib, and S Alshoraify, *Study of multivalued fixed point problems for generalized contractions in double controlled dislocated quasi metric type space*, Filomat **31** (2017), no. 11, 11–3263, DOI: https://doi.org/10.1142/S0218127418501559.

[8] W. AlOmeri, M Salmi, M Noorani, et al., *α-local function and its properties in ideal topological spaces*, Fasciculi Mathematici **53** (2014), 5–15, DOI: https://doi.org/10.3934/DCDS.2018043.

[9] W. AlOmeri and T Noiri, *On almost e I continuous functions*, Demonstr. Math. **54** (2021), no. 1, 168–177, DOI: https://doi.org/10.1515/dema-2021-0014.

[10] V. Renukadevi and S. Tamilselvi, *Transitivity and sensitivity of semigroup actions on uniform spaces*, Asian-European J. Math. **15** (2022), no. 12, 22–26, DOI: https://doi.org/10.1142/S1793557122502096.

[11] F. A. Shirazi, Z. N. Ahmadabadi, B. Tajbakhsh, and K. Tajbakhsh, *Specification properties on uniform spaces*, J. Dyn. Control Sys. **27** (2020), no. 2, 1–13, DOI: https://doi.org/10.1007/s10883-020-09499-x.

[12] P. Das and T. Das, *A note on measure and expansiveness on uniform spaces*, Appl. General Topol. **20** (2019), no. 1, 19–31, DOI: https://doi.org/10.4995/agt.2019.8843.

[13] R. Fierro, *Noncompactness measure and fixed points for multi-valued functions on uniform spaces*, Mediterranean J. Math. **15** (2018), no. 3, 1–13, DOI: https://doi.org/10.1007/s00009-018-1140-x.

[14] C. A. Morales and V. Sirvent, *Expansivity for measures on uniform spaces*, Trans. Amer. Math. Soc. **368** (2015), no. 8, 5399–5414, DOI: https://doi.org/10.1090/tran/6555.
[15] F. Raul, *A condition on uniform spaces for the existence of maximal elements and fixed points*, J Fixed Point Theory Appl. **24** (2022), no. 3, 1–12, DOI: https://doi.org/10.1007/S11784-022-00976-3.
[16] S. Ekta and D. Tarun, *Consequences of shadowing property of G-spaces*, Math. Analysis **7** (2013), no. 9, 579–588, DOI: https://doi.org/10.12988/ijma.2013.13056.
[17] Z. J. Ji, *The G-asymptotic tracking property and G-asymptotic average tracking property in the inverse limit spaces under group action*, Discrete Dyn. Nature Soc. **2021** (2021), 1–6, DOI: https://doi.org/10.1155/2021/5576389.
[18] S. A. Ahmadi, *Invariants of topological G-conjugacy on G-spaces*, Math. Moravica **18** (2014), no. 1, 67–75, DOI: https://doi.org/10.5937/MatMor1401067A.
[19] Z. J. Ji, *The research of (G, w)-chaos and G-Lipschitz shadowing property*, AIMS Math. **7** (2022), no. 6, 10180–10194, DOI: https://doi.org/10.3934/math.2022566.
[20] A. Shoaib, M. Arshad, T. Rasham, and M. Abbas, *Unique fixed point results on closed ball for dislocated quasi G-metric spaces*, Trans. A. Razmadze Math. Institute **171** (2017), no. 2, 221–230, DOI: https://doi.org/10.1016/j.trmi.2017.01.002.