Research Article

Fawaz E. Alsaadi, Fernando E. Serrano*, and Larissa M. Batrancea

Neural network quaternion-based controller for port-Hamiltonian system

https://doi.org/10.1515/dema-2023-0131
received May 19, 2023; accepted October 26, 2023

Abstract: In this research article, a control approach for port-Hamiltonian PH systems based in a neural network (NN) quaternion-based control strategy is presented. First, the dynamics is converted by the implementation of a Poisson bracket in order to facilitate the mathematical model in order to obtain a feasible formulation for the controller design based on quaternion NNs. In this study, two controllers for this kind of of system are presented: the first one consists in the controller design for a PH system about its equilibrium points taking into consideration the position and momentum. This mean is achieved by dividing the quaternion neural controller into scalar and vectorial parts to facilitate the controller derivation by selecting a Lyapunov functional. The second control strategy consists in designing the trajectory tracking controller, in which a reference moment is considered in order to drive this variable to the final desired position according to a reference variable; again, a Lyapunov functional is implemented to obtain the desired control law. It is important to mention that both controllers take into advantage that the energy consideration and that the representation of many physical systems could be implemented in quaternions. Besides the angular velocity, trajectory tracking of a three-phase induction motor is presented as a third numerical experiment. Two numerical experiments are presented to validate the theoretical results evinced in this study. Finally, a discussion and conclusion section is provided.

Keywords: neural networks, quaternions, port-Hamiltonian systems, neural control

MSC 2020: 93D15, 93D20, 93D30, 93D05

1 Introduction

Due to its energy consideration, port-Hamiltonian systems has become an important mathematical representation for many types of physical systems such as fluid and particle dynamic systems, in which the port-Hamiltonian mathematical representation consists of two phase variables related to the position and the momentum of a particle ensemble. It is important to consider that port-Hamiltonian systems for the representation of mechanical systems have been increased nowadays due to the vast amount of novel robotic mechanisms. For this reason, it is important to obtain a feasible mathematical model for controller design purpose [1,2]. Among the kinds of controllers that can be designed for a port-Hamiltonian system are energy-based and passivity-based control.

*** Corresponding author: Fernando E. Serrano**, Instituto de Investigación en Energía IIE, Universidad Nacional Autónoma de Honduras (UNAH), Tegucigalpa, Honduras, e-mail: fernandoserr@gmail.com
Fawaz E. Alsaadi: Department of Information Technology, Communication Systems and Networks Research Group, Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah, Saudi Arabia, e-mail: fesalsaadi@kau.edu.sa
Larissa M. Batrancea: Department of Business, Babeş-Bolyai University, Cluj-Napoca, Romania, e-mail: larissa.batrancea@ubbcluj.ro

Neural network (NN) control has been implemented nowadays considering the flexibility and easiness to tune for the stabilization of many kinds of nonlinear dynamic systems. For these reasons, in this section, an establishment of the theoretical fundamentals for this study is presented.

NNs have become important nowadays for different kinds of applications that are not necessarily related to controller design. Among these applications, optimization, pattern recognition, classification, and prediction are found; so, for example, in articles like [3], the topological properties of NN from the graph-theory viewpoint are evinced. Then, in [4], a comparative analysis between feedforward neural-networks and radial-basis NN is performed. Meanwhile, in [5], the stability analysis and control synchronization of NNs are presented. In [6], the Riemannian geometry of deep NNs is shown. Then, in [7,8], an image classification application of convolutional NNs and prediction application is presented, respectively.

Quaternion NNs are crucial for this research study taking into consideration that these type of networks are implemented. The quaternion NNs are divided into a scalar quaternion part and a quaternion vector part. There are several research studies in which these quaternion-based NNs are mentioned: so, for example, in [9], quaternion memristive NNs are analyzed by means of the Lagrangian stability considering that this possessses mixed time delays. Then, in [10], the Mittag-Leffler stability analysis is presented. In [11], a quaternion NN with leakage and proportional delays is analyzed. Then, in [12], a novel synchronization strategy of quaternion-based NNs with time delays is presented. Then, in [13,14], an stochastic version of quaternion NN and the stabilization of quaternion delayed NNs are shown.

NNs have been extensively implemented nowadays in the control and stabilization of nonlinear and many kinds of complex dynamical systems. For example, in articles like [15], the neural control of discrete weak formulation is presented. Then, in [16], the neural control of a space vehicle with output constraints is presented. Meanwhile, in [17], a space manipulator is controlled by means of NN control designed by selecting barrier tan-type Lyapunov functions. Then, in [18], a review of different types of neural controllers for nuclear power plants is presented. Then, in [19,20], two interesting applications of NN control for an underwater vehicle and switched nonlinear interconnected systems are shown.

Dynamic control is important taking into consideration that the type of quaternion-based neural controller is dynamic. So the following references are mentioned taking into consideration that this kind of control strategy is worthy to mention. In references like [21], a dynamic controller for aerobic granular sludge reactor is presented. Meanwhile, in [22], the energy decay of the wave equation by a delay in the dynamic control is evinced. Then, in [23], the control of refrigeration cycle with single, dual, and triple effects is presented. Finally, in [24,25], the design of an adaptive backstepping and NN dynamic control with dynamic surface is presented, respectively.

As mentioned earlier, port-Hamiltonian systems are important to be studied and analyzed considering the energy properties of these kinds of systems. It is crucial to take into consideration that novel control strategies as proposed in this article are necessary considering the complexity of this kind of complex dynamic system. In the literature, there are some research studies regarding this topic such as [26], in which the control with output consensus for PH system is presented. Meanwhile, in [27], the discretization of PH systems by the finite elements is shown. Meanwhile, in [28], the exponential decay rate of port-Hamiltonian system is evinced. It is worthy to cite references like [29], and the port-Hamiltonian derivation for chemical processes is shown. Finally, in [30,31], the mixed geometric coupling of PH system is achieved and the model predictive control of PH systems is presented, respectively.

In this study, the design and synthesis of a quaternion-based NN controller for a PH system is evinced. First, in order to improve the tractability of the port-Hamiltonian mathematical formulation, the PH system is represented by means of the Poisson bracket. The use of this operator makes it easy in order to synthesize the controller for this kind of mathematical model. The NN quaternion-based controller is designed by dividing the quaternion NN into the scalar quaternion and the vector quaternion; so by selecting appropriate control law, the implementation of suitable Lyapunov functionals is done in order to obtain a feasible control law. In this study, two kinds of NN quaternion-based controllers are shown, for either stabilization and trajectory tracking purposes, so in this way, the controller is suitable to be implemented, in order to follow the trajectory of a reference variable or simply stabilizes the port-Hamiltonian system in its equilibrium point. This experiment consists by selecting the movement of a particle in order to stabilize its trajectory. Discussions and conclusions of the theoretical and experimental results are shown, respectively.

The contributions are shown as follows:

- Considering the complex dynamics of port-Hamiltonian systems, NN controllers are suitable to stabilize and for trajectory tracking control.
- Quaternion NNs are suitable considering that the dynamics of robots and unmanned aerial vehicles, for example, are established in quaternions.
- Quaternion NNs provide more degrees of freedom for control purposes in comparison with other types of NNs.
- The NN controller provided in this research study provides a fast and accurate closed-loop system response.
- This NN controller approach provides an easy-to-tune NNs parameters.

2 Related work

In this section, the comparison of the quaternion NNs with other type of NNs is presented considering the advantages and disadvantages. Besides, passivity-based and energy-based control for PH systems is presented taking into account that these control strategies are commonly found.

Quaternion NNs are fundamental in this article, so it is important to mention the following studies. For example, in [32], in which the power law synchronization of quaternion-valued NN is shown. Then, in [11], quaternion-based NNs stabilization with time delays is presented. It is important to consider time delays in quaternion NNs because it is a common phenomenon found in many real physical systems that can produce instability or poor closed-loop performance. Meanwhile, in [33], the stochastic synchronization of Markovian jump quaternion NN synchronization is presented. In [34], the dissipative and synchronization of quaternion-based NNs is shown. Then, in [35], the stability analysis and control of Markovian jump quaternion-based NNs is evinced.

Other types of neural controllers that are useful for this research study are in [36], in which the synchronization of fractional order fuzzy NN is achieved by the implementation of a nonlinear feedback control law. Meanwhile, in [37], a direct current motor is controlled by a fuzzy neural PID control, which is is presented in this article. In [38], the state estimation and NN control of a Markovian jump system is presented. Then, in [39], a fractional order NN is presented to control a rehabilitation robot. In [40], a sliding mode controller and a recurrent NN based on a robotic operated vehicle are presented. In [41], a neural controller is designed for the stabilization of a water reactor.

Diverse kinds of controllers have been developed during the past decades, considering that plenty of physical systems in which this controller can be implemented has been discovered. For example, in articles like [42], the observer design for a 1-D parameter-distributed port-Hamiltonian system is evinced. Then, in articles like [43], a fully actuated mechanical system in the PH formulation is controlled by virtual contractivity. In [44], an analysis of reduced model boundary control strategies for linear PH systems is presented. Meanwhile, in [45], a boundary-controlled port-Hamiltonian system is presented. Then, in [46,47], the thermodynamic PH system optimal control and the structure-preserving discretization of PH system are shown.

Energy control of PH systems is considered as one of the important control strategies for PH systems taking into consideration the energy properties of port-Hamiltonian systems; among these important papers found in the literature are papers like [48], in which the design of an energy controller and observer for infinite dimensional PH system is presented. Meanwhile, in [49], an energy controller design in domain control of infinite dimensional PH system is evinced. Then, in [50], the controller design of the PH system with minimal energy supply is evinced. In [51], the structural invariance and control of PH systems are presented. Then, in [52,53], the boundary control based in energy shaping of PH systems and energy-based feedback control of port-Hamiltonian systems is presented, respectively.

To finalize this literature review, consider the following references regarding the control of PH systems based on passivity, in which this dissipative-based control strategy is important to this study, taking into consideration that this strategy is widely used [54–59].

In this paragraph are remarked the differences, advantages, and disadvantages of quaternion NNs in comparison with other types of NNs. In this case, is important to mention the comparison of quaternion NNs with complex variable NNs and fuzzy NNs. In references like [60–62], complex-valued NNs are presented. It is proved that quaternion NNs provide more degrees of freedom for control purposes in comparison with complex variable NNs. So for this reason, the measurement of the performances is significantly superior with the quaternion NN in comparison with complex variable NNs. Besides, taking advantage that the kinematics and dynamics of many robotics and unmanned aerial vehicle systems are represented with the quaternion mathematical definition, it is advantageous to implement the quaternion NNs as presented in this research study.

On the other side in articles like [63–65], fuzzy NNs are presented, the advantages that these kinds of NNs possesss are that they are useful for dynamic system with uncertainties, something that the quaternion NNs do not possess these characteristics. It must be recalled that the computational effort of quaternion NNs is significantly low in comparison with other NN strategies, something that is an advantage in real-time applications of neural controllers.

3 Theoretical background

In this section, basically the fundamentals of the Poisson bracket and the port-Hamiltonian formulation are explained.

3.1 Poisson bracket

The Poisson bracket is given by [66,67]

$$\partial_t z^A = \{z^A, \mathcal{H}\}, \tag{1}$$

where $z^A = \{q_1, q_2, \ldots, q_N, p_1, p_2, \ldots, p_N\}$ is the state vector, $\mathcal{H}$ is the Hamiltonian, and $\{.,.\}$ denotes the Poisson bracket. Their properties are:

Property 1. The Poisson bracket properties, in which it is defined by $\{.,.\}$:
- Anti-symmetry $\{\mathcal{F}, \mathcal{G}\} = -\{\mathcal{G}, \mathcal{F}\}$.
- Leibniz rule $\{\mathcal{G}, \mathcal{F}\mathcal{E}\} = \mathcal{G}\{\mathcal{F}, \mathcal{E}\} + \{\mathcal{F}, \mathcal{G}\}\mathcal{E}$,

where $\mathcal{P}$ is an N-dimensional manifold.

The Poisson bracket is obtained by:

$$\{\mathcal{F}, \mathcal{G}\} = \frac{\partial \mathcal{F}^T}{\partial z}\mathbb{J}(z)\frac{\partial \mathcal{G}}{\partial z}, \tag{2}$$

where $\mathbb{J}(z)$ is the Poisson matrix.

3.2 Port-Hamiltonian dynamic system formulation

So in order to establish the port-Hamiltonian formulation, consider the state vector $x = [q, p]^T$ with the following partial derivatives:

$$\begin{aligned}\frac{\partial p}{\partial x} &= [0_n, I_n]^T, \\ \frac{\partial q}{\partial x} &= [I_n, 0_n]^T, \\ \frac{\partial H}{\partial x} &= \left[\frac{\partial H}{\partial q}, \frac{\partial H}{\partial p}\right]^T.\end{aligned} \tag{3}$$

So consider the following Poisson bracket:

$$\mathbb{J}(z) = \begin{bmatrix} 0 & I \\ -I & -D \end{bmatrix}, \tag{4}$$

where I is the identity matrix and $D \in \mathbb{R}^{n\times n}$ is an appropriate matrix. So by implementing the following Poisson matrix is obtained:

$$\begin{aligned} \{p, \mathcal{H}\} &= \frac{\partial p^T}{\partial x}\mathbb{J}(x)\frac{\partial \mathcal{H}}{\partial x} = -\frac{\partial \mathcal{H}}{\partial q} - D\frac{\partial \mathcal{H}}{\partial p} \\ \{q, \mathcal{H}\} &= \frac{\partial q^T}{\partial x}\mathbb{J}(x)\frac{\partial \mathcal{H}}{\partial x} = \frac{\partial \mathcal{H}}{\partial p}, \end{aligned} \tag{5}$$

where $\mathcal{H}$ is the Hamiltonian. So the PH formulation is:

$$\begin{aligned} \dot{q} &= \{q, \mathcal{H}\}, \\ \dot{p} &= \{p, \mathcal{H}\} + u, \end{aligned} \tag{6}$$

where $u \in \mathbb{R}^n$ is the control input.

4 Stabilization and trajectory tracking of port-Hamiltonian dynamic systems by quaternion-based controllers

Consider the following quaternion-based NN controller:

$$\dot{z} = -A_c z^r(t) + \sum_{r=1}^{n} d_{pr} f_c(z) + I_c, \tag{7}$$

for the quaternion variable $z = z^r + iz^i + jz^j + kz^k$ and the function $f_c(z) = f_c^r + if_c^i + jf_c^j + kf_c^k$; meanwhile, I_c is the neural controller input, and A_c is a positive definite matrix of appropriate dimension. In this section it is important to remark that the NN architecture for the quaternion NN controller is basically feedforward alike. This means that the quaternion NN controllers possess the input neurons and output neurons with their input and bias weights. This quaternion NN controller possess a nonlinear activation function very similar to the standard feedforward NN architecture. The quaternion operation follows the following properties:

$$\begin{aligned} i\cdot i &= 0, & j\cdot j &= 0, \\ i\cdot j &= k, & i\cdot k &= -j, \\ j\cdot k &= i, & k\cdot k &= 0, \end{aligned} \tag{8}$$

so by separating the scalar quaternion part and the vector quaternion part yields

$$\begin{aligned} \dot{z}^r &= -A_c z^r + \sum_{r=1}^{n} d_{pr} f_c^r(z) + I_c, \\ \dot{z}^i &= \sum_{r=1}^{n} d_{pr} f_c^i(z), \\ \dot{z}^j &= \sum_{r=1}^{n} d_{pr} f_c^j(z), \\ \dot{z}^k &= \sum_{r=1}^{n} d_{pr} f_c^k(z), \end{aligned} \tag{9}$$

in which the NN parameters are given by the positive definite matrix A_c and the NN weights d_{pr}; similar to the NN parameters as evinced in (7). It is important to remark that (7) and (8) are used to obtain (9). The reason to separate the variables of the quaternion NN is to facilitate the design of the quaternion NN controller in

Theorems 1 and 2 so in this way the control synthesis by the Lyapunov theorem is facilitated. The controller is obtained by implementing this important characteristic because in other way, the quaternion NN controller would not be implementable for real physical system applications.

4.1 Stabilization of a port-Hamiltonian system with quaternion-based NNs

The stabilization is achieved by selecting a suitable control law obtained by selecting an appropriate Lyapunov functional, as appears in the following theorem:

Theorem 1. *System* (6) *is stabilized by the following control law iff:*

$$\begin{aligned} u = -\{p, \mathcal{H}\} - \frac{1}{\|p\|^2} pq^T\{q, \mathcal{H}\} - \frac{1}{\|p\|^2} pz^{rT}\left[-A_c z^r + \sum_{r=1}^{n} d_{pr} f_c^r(z) + Kp\right] \\ - \frac{1}{\|p\|^2} pz^{iT} \sum_{r=1}^{n} d_{pr} f_c^i(z) - \frac{1}{\|p\|^2} pz^{jT} \sum_{r=1}^{n} d_{pr} f_c^j(z) - \frac{1}{\|p\|^2} pz^{k} \sum_{r=1}^{n} d_{pr} f^k(z) - Rp, \end{aligned} \tag{10}$$

where the NN controller input $I_c = Kp$ for $K \in \mathbb{R}^{n\times n}$ is the input gain. $z \in \mathbb{R}^2$ is the quaternion controller input variable.

Proof. Consider the following Lyapunov functional:

$$V = \frac{1}{2} z^{rT} z^r + \frac{1}{2} z^{iT} z^i + \frac{1}{2} z^{jT} z^j + \frac{1}{2} z^{kT} z^k + \frac{1}{2} q^T q + \frac{1}{2} p^T p. \tag{11}$$

The derivative of the previous Lyapunov functional is obtained as follows:

$$\dot{V} = z^{rT} \dot{z}^r + z^{iT} \dot{z}^i + z^{jT} \dot{z}^j + z^{kT} \dot{z}^k + q^T \dot{q} + p^T \dot{p}, \tag{12}$$

yielding

$$\begin{aligned} \dot{V} = z^{rT}\left[-A_c z^r + \sum_{r=1}^{n} d_{pr} f_c^r(z) + Kp\right] + z^{iT} \sum_{r=1}^{n} d_{pr} f_c^i(z) + z^{jT} \sum_{r=1}^{n} d_{pr} f_c^j(z) \\ + z^{kT} \sum_{r=1}^{n} d_{pr} f_c^k(z) + q^T\{q, \mathcal{H}\} + p^T\{p, \mathcal{H}\} + p^T u. \end{aligned} \tag{13}$$

Now, substituting (10) into the previous equation yields

$$\dot{V} = -p^T Rp. \tag{14}$$

So the system is globally asymptotically stable, and the proof is complete. □

4.2 Trajectory tracking of a port-Hamiltonian system by quaternion-based NN

Taking into consideration that the trajectory tracking control for PH systems is important in many real systems, in this subsection, it is evinced the synthesis of these kinds of controllers. First as explained earlier, the quaternion NN is divided into four parts in order to obtain a feasible mathematical model for controller synthesis. The error variables for the position q and momentum p reduce the tracking error to zero. In this way, variables such as position, angular velocity, and currents can be stabilized following a predefined trajectory. It is important to remark that the Lyapunov stability theory is implemented in this research study to synthesize the appropriate control law. For the trajectory tracking of a port-Hamiltonian dynamic system, consider the following two error variables:

$$\begin{aligned} e_1 &= q_r - q, \\ e_2 &= p_r - p \end{aligned} \tag{15}$$

where q_r and p_r are the respective references for the position and momentum. The following theorem evinces that the system is globally closed-loop stable.

Theorem 2. *The following control law makes the error dynamics* (15) *asymptotically stable iff*:

$$\begin{aligned} u = &-\{p, \mathcal{H}\} + \dot{p}_r + Q_2^{-1}\frac{1}{\|e_2\|^2}e_2 e_1^T Q_1 \dot{e}_1 + Q_2^{-1}\frac{1}{\|e_2\|^2}e_2 z^{kT}\sum_{r=1}^{n} d_{pr} f_c^k(z) + Q_2^{-1}\frac{1}{\|e_2\|^2}e_2 z^{jT}\sum_{r=1}^{n} d_{pr} f_c^j(z) \\ &+ Q_2^{-1}\frac{1}{\|e_2\|^2}e_2 z^{iT}\sum_{r=1}^{n} d_{pr} f_c^i(z) + Q_2^{-1}\frac{1}{\|e_2\|^2}e_2 z^{rT}\left[-A_c z^r + \sum_{r=1}^{n} d_{pr} f_c^r(z) + Kp\right] + Q_2^{-1}e_2, \end{aligned} \tag{16}$$

for the NN input $I_c = Kp$, in which I_c is the momentum state feedback, with gain matrices K, Q_1, Q_2; in which all these matrices are positive definite. The NN is embedded in the control law in order to select the required weight values in order to stabilize the closed-loop system by making it asymptotically globally stable.

Proof. Consider the following:

$$V = \frac{1}{2}z^{rT}z^r + \frac{1}{2}z^{iT}z^i + \frac{1}{2}z^{jT}z^j + \frac{1}{2}z^{kT}z^k + \frac{1}{2}e_1^T Q_1 e_1 + \frac{1}{2}e_2^T Q_2 e_2. \tag{17}$$

Now, the derivative of the previous Lyapunov functional is given by

$$\begin{aligned} \dot{V} = &z^{rT}\left[-A_c z_r + \sum_{r=1}^{n} d_{pr} f_c^r(z) + Kp\right] + z^{iT}\sum_{r=1}^{n} d_{pr} f_c^i(z) + z^{jT}\sum_{r=1}^{n} d_{pr} f_c^j(z) + z^{kT}\sum_{r=1}^{n} d_{pr} f_c^k(z) + e_1^T Q_1 \dot{e}_1 \\ &+ e_2^T Q_2[\dot{p}_r - \{p, \mathcal{H}\} - u]. \end{aligned} \tag{18}$$

Now, substituting the control law (16) into the previous equation yields

$$\dot{V} = -e_2^T e_2. \tag{19}$$

So the proof is complete, and the system is asymptotically stable. □

5 Experiments

For experimental purposes, the following Hamiltonian that represents a particle movement is presented as follows:

$$\mathcal{H} = \frac{1}{2m}p^T p + \frac{1}{2}kq^T q. \tag{20}$$

So by the implementation of the Poisson bracket, the representation is obtained as follows:

$$\begin{aligned} \{p, \mathcal{H}\} &= -\frac{\partial \mathcal{H}}{\partial q} - D\frac{\partial \mathcal{H}}{\partial p} = -\frac{1}{2}kq - D\frac{1}{2m}p, \\ \{q, \mathcal{H}\} &= \frac{1}{2m}p, \end{aligned} \tag{21}$$

where D is the identity matrix of appropriate dimensions.

5.1 Experiment 1: Stabilization

In the following experiment, the results of the stabilization of a PH system by quaternion-based NNs are presented. Consider the following value for the gain:

$$K = \begin{bmatrix} 100{,}000 & 0 \\ 0 & 100{,}000 \end{bmatrix}. \tag{22}$$

In Figures 1 and 2, the stabilized variables of the PH system for the momentum are evinced in order to observe the evolution in time for the momentum variables in the x and y axis. As can be noted the momentum reaches the origin in finite time. As it is verified later, it can be corroborated that the variables can be

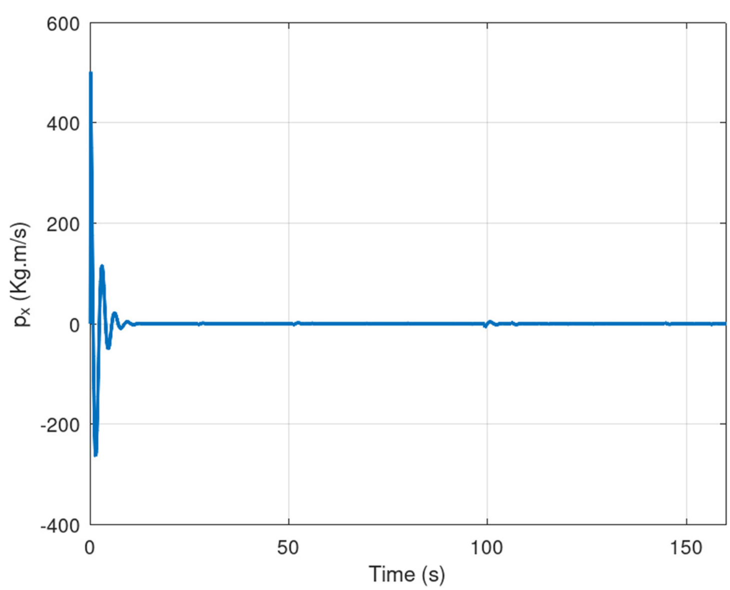

Figure 1: Momentum p_x for Experiment 1.

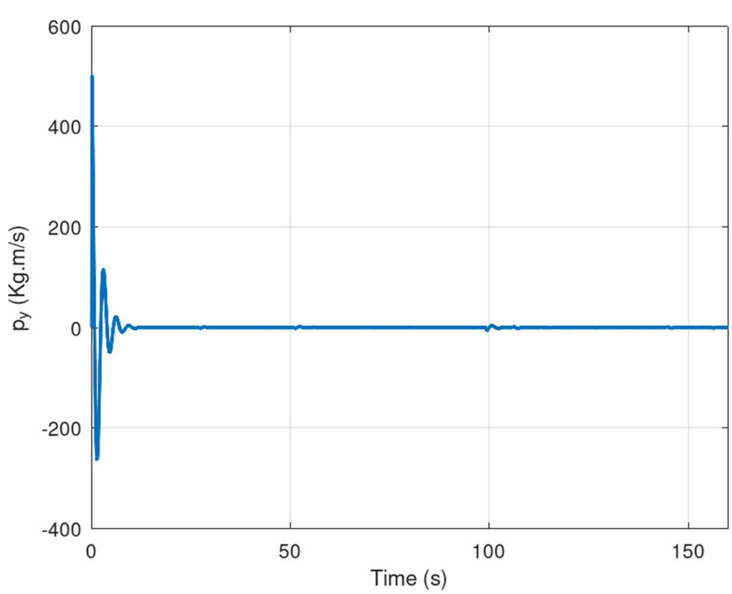

Figure 2: Momentum p_y for Experiment 1.

stabilized independently of the initial conditions. It is verified that there is some chattering between 50 and 100 s; the reason is as it is observed the gains ensure only global asymptotic stability due to the positive definiteness. In order to improve performance in the future, optimization algorithms will be implemented.

Meanwhile, in Figures 3 and 4, it is evinced the evolution in time of the position variables for the x and y position of the particle. Similar to the momentum variable, it is observed how the variables reach the origin in finite time, proving that the quaternion-based controller stabilizes the system variables.

Meanwhile, in Figures 5 and 6, the control effort input is shown, and as it is verified, the control effort drives the system variables to the origin in finite time. The action of the controller provides the sufficient control effort in order to drive the position and momentum variable to zero.

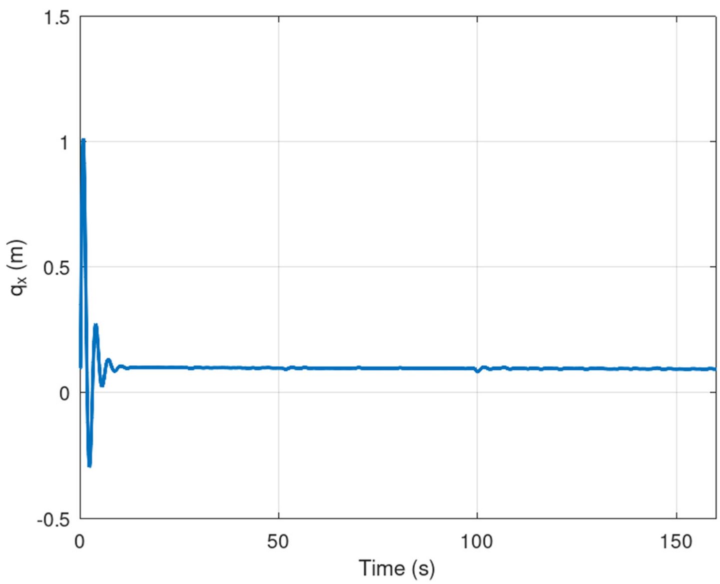

Figure 3: Position q_x for Experiment 1.

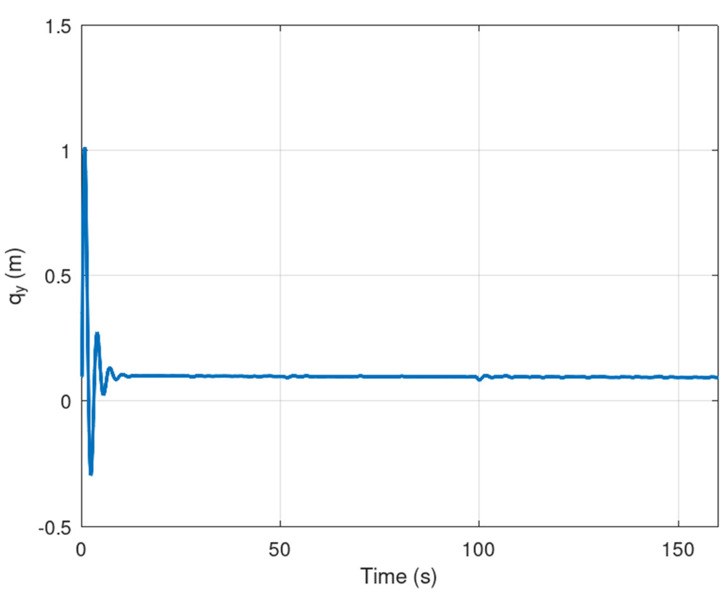

Figure 4: Position q_y for Experiment 1.

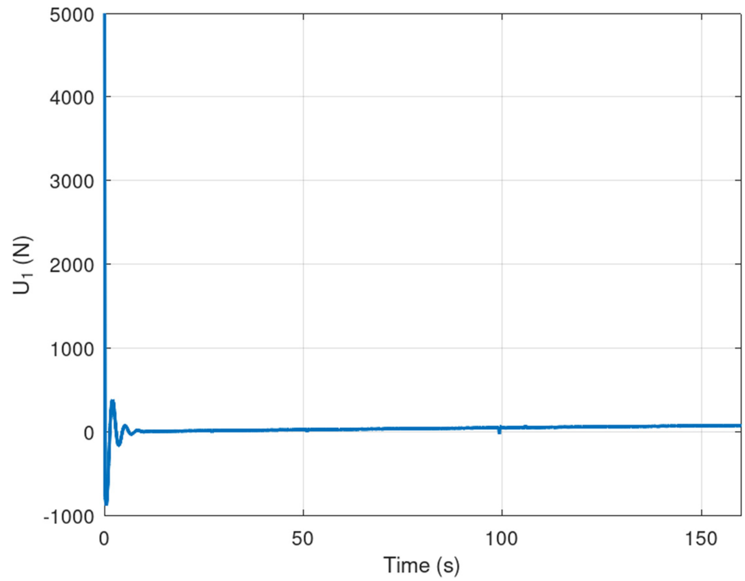

Figure 5: Input variable U_1.

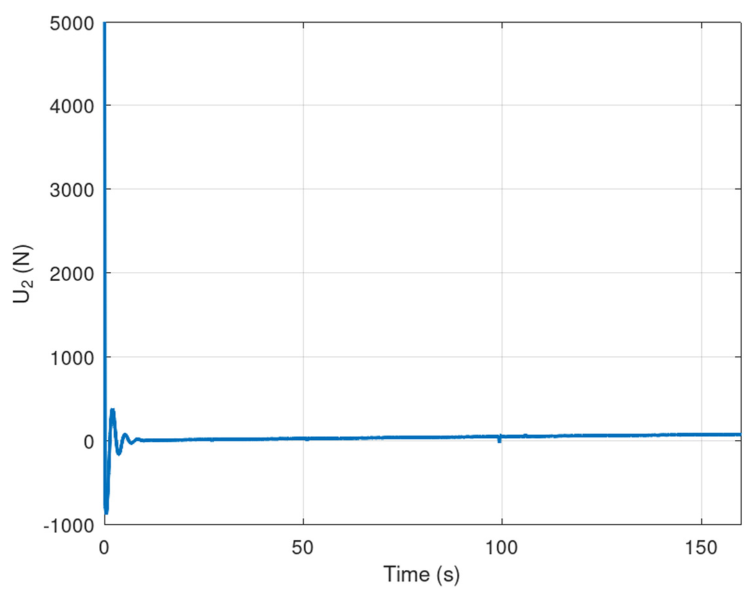

Figure 6: Input variable U_2.

5.2 Experiment 2: trajectory tracking

In this section, the trajectory tracking of a port-Hamiltonian system is presented in order to drive the reference variables to the final desired value in finite time and the results obtained with the proposed controller are compared with [58,68]. The parameter gains for this simulation setup are given by

$$K = \begin{bmatrix} 1\times10^{-8} & 0 \\ 0 & 1\times10^{-8} \end{bmatrix} \quad Q_1 = \begin{bmatrix} 1\times10^{-5} & 0 \\ 0 & 1\times10^{-5} \end{bmatrix} \quad Q_2 = \begin{bmatrix} 1\times10^{-1} & 0 \\ 0 & 1\times10^{-1} \end{bmatrix}. \tag{23}$$

In Figures 7 and 8, the evolution in time of the momentum in the x and y axis is presented. It is verified that the variables reach the desired value in finite time, despite the initial condition of the system. This final

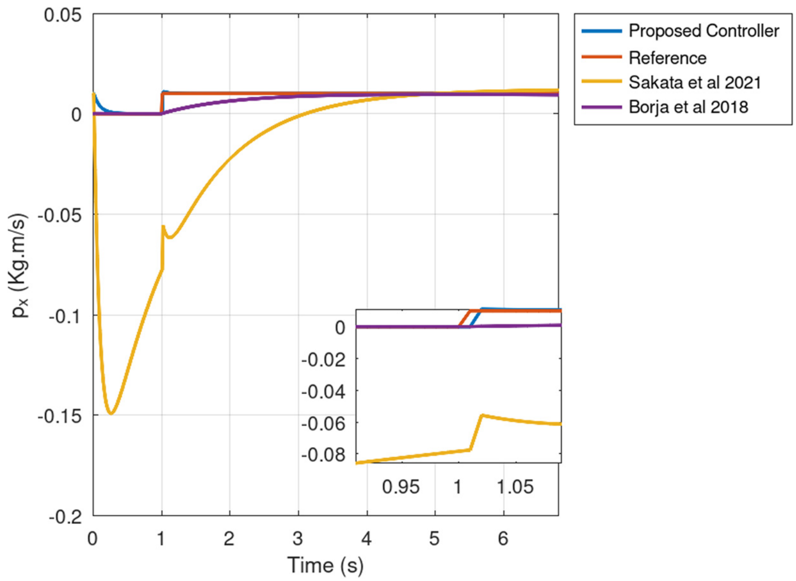

Figure 7: Momentum p_x for Experiment 2.

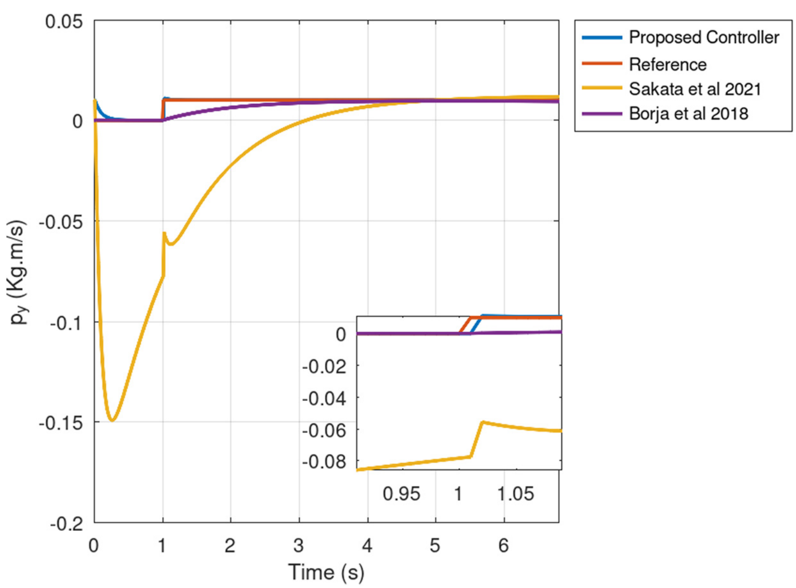

Figure 8: Momentum p_y for Experiment 2.

value is achieved by selecting appropriate gain matrices in order to make the system stable, so the variables reach the desired final value. It is verified that the time response yielded by the proposed controller is better in comparison with the strategies shown in [58,68]

Meanwhile, in Figures 9 and 10 are presented the results of the evolution in time of the control inputs U_1 and U_2 in order to evince how these control input variables vary only in the transition of the reference signal value, while keeping the value in zero when the desired final value is reached. It is also verified that the control effort is smaller with the proposed control strategy in comparison with the strategies [58,68].

Finally, in Figure 11, the evolution in time of the quaternion-valued NN controller are presented for the scalar and vectorial part of the quaternion. It is verified how this quaternion-valued NN reaches the final value in finite time.

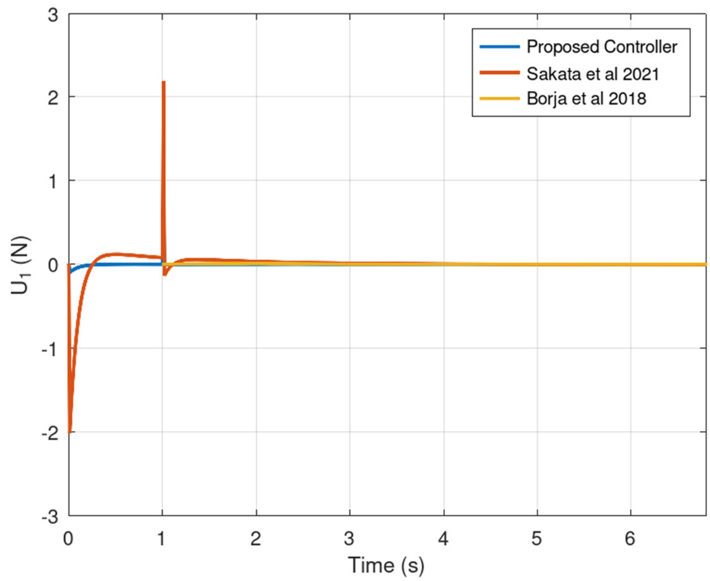

Figure 9: Input U_1 for Experiment 2.

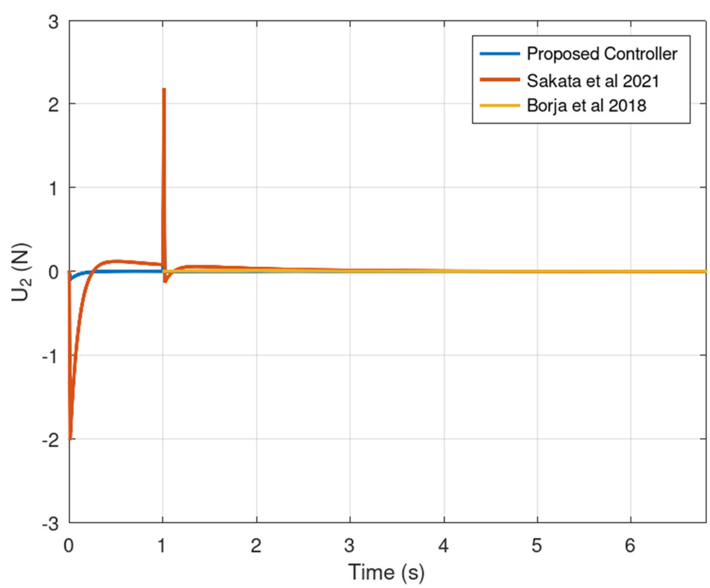

Figure 10: Input U_2 for Experiment 2.

5.3 Experiment 3: Angular velocity trajectory tracking of an induction machine

Consider the following dq transformation for a three-phase electric motor [69]:

$$T_{abc} = \frac{2}{3}\begin{bmatrix} \cos(\theta_s) & \cos\left(\theta_s - \frac{2*\pi}{3}\right) & \cos\left(\theta_s + \frac{2*\pi}{3}\right) \\ \sin(\theta_s) & \sin\left(\theta_s - \frac{2*\pi}{3}\right) & \sin\left(\theta_s + \frac{2*\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \tag{24}$$

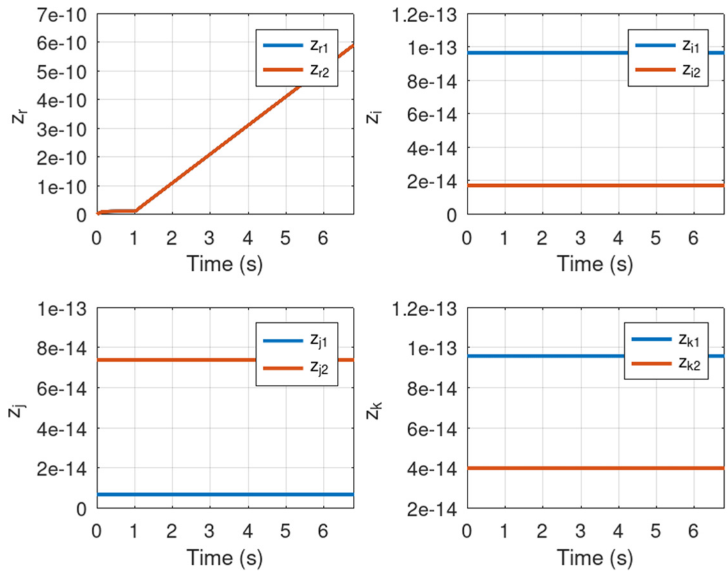

Figure 11: Quaternion-based variable z for Experiment 2.

in which in synchronous rotating reference frame is $\omega_c = \omega_s = 2\pi f$ in which $f = 60$ Hz. The voltages are converted to the d-q axis by the following:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_o \end{bmatrix} = T_{abc} \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix}, \tag{25}$$

in which the three-phase voltages used in the numerical experiment have an source amplitude of $V_A = \frac{200}{\sqrt{3}}\sqrt{2} = 163.3v$ [69]. The model in synchronous reference frame of this induction machine is given by [69]

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_s L_s & L_m p & \omega_s L_m \\ -\omega_s L_s & R_s + L_s p & -\omega_s L_m & L_m p \\ L_m p & (\omega_s - \omega_r) L_m & R_r + L_r p & (\omega_s - \omega_r) L_r \\ -(\omega_s - \omega_r) L_m & L_m p & -(\omega_s - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix}, \tag{26}$$

with the respective voltages and currents as specified in [69]. R, L, p, and ω_r indicate the resistances, inductances, number of poles, and rotor angular velocity, respectively. The parameter values for this numerical experiment are [69] 5 hp, 200 v, three-phases, 60 Hz, 4 poles, star connected, $R_s = 0.277\Omega$, $R_r = 0.183\Omega$, $L_m = 0.0538$ H, $L_s = 0.0553$ H, and $L_r = 0.056$ H.

Consider the following state vector $Q = [i_{qs}, i_{ds}, i_{qr}, i_{dr}, \omega_r]$ with the following inductance-inertia matrix:

$$L = \begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix}, \tag{27}$$

where J is the motor inertia and the matrix M is

$$M = \begin{bmatrix} L_s & 0 & 0 & 0 \\ 0 & L_s & 0 & 0 \\ 0 & 0 & L_r & 0 \\ 0 & 0 & 0 & L_r \end{bmatrix}. \tag{28}$$

Consider the following Hamiltonian

$$\mathcal{H} = \frac{1}{2} p^T L^{-1} p + \frac{1}{2} Q^T L Q, \tag{29}$$

in which $p \in \mathbb{R}^5$ is the momentum vector. The PH representation is given by:

$$\begin{aligned} \dot{q} &= \{q, \mathcal{H}\}, \\ \dot{p} &= \{p, \mathcal{H}\} + U_1, \end{aligned} \tag{30}$$

with the following operators:

$$\begin{aligned} \{p, \mathcal{H}\} &= -LQ - DL^{-1}P, \\ \{q, \mathcal{H}\} &= L^{-1}P, \end{aligned} \tag{31}$$

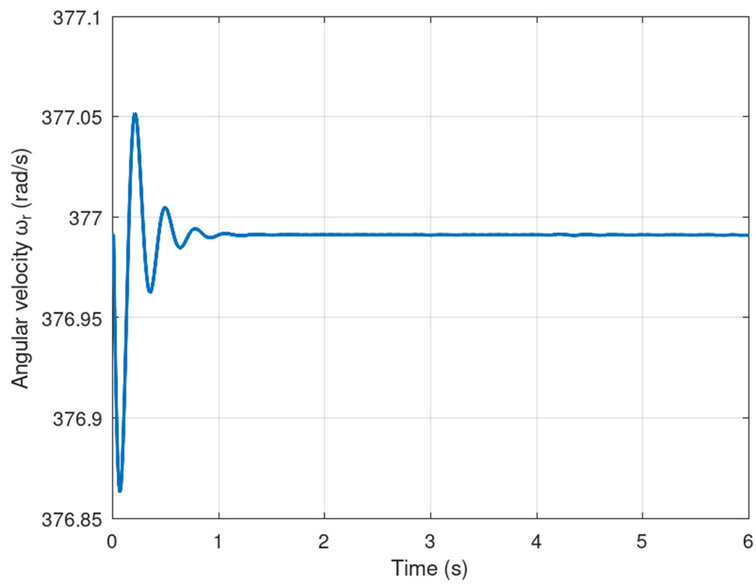

Figure 12: Rotor angular velocity.

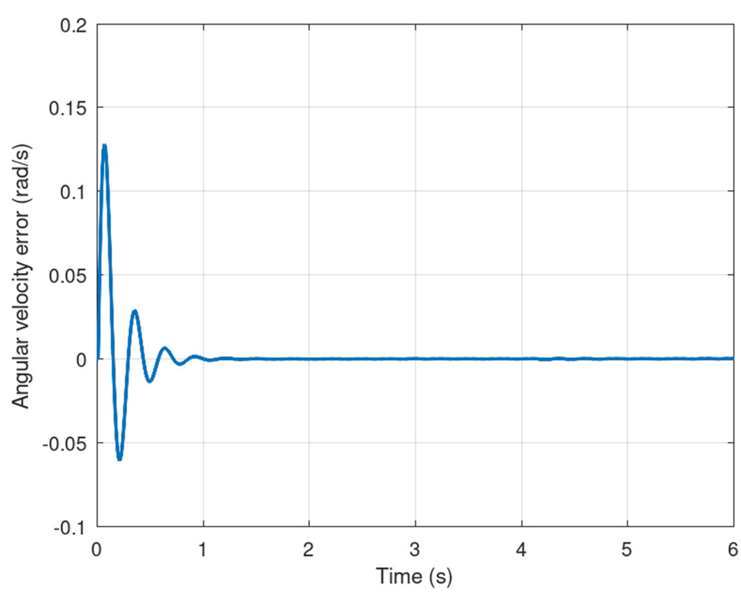

Figure 13: Rotor angular velocity error.

where $D = I_5$ is an identity matrix of appropriate dimensions with the input vector given by:

$$U_1 = \begin{bmatrix} U \\ T_e - T_L \end{bmatrix}, \tag{32}$$

where T_L is the torque generated by the motor load and T_e is given by [69]:

$$T_e = \frac{3}{2}\frac{p}{2}L_m(i_{qs}i_{dr} - i_{ds}i_{qr}). \tag{33}$$

For this numerical experiment, a torque load $T_L = 1$ N × m is applied at 0 s with a load inertia of $L = 200$ kg × m^2. The parameters of the neural-network controller are given by $K = 70 \times I_5$, $Q_1 = Q_2 = 0.1 \times I_5$.

In Figures 12 and 13, the angular velocity of the motor rotor is depicted with the respective error. It is noted how the angular velocity reaches the synchronous speed in finite time by driving the error to zero.

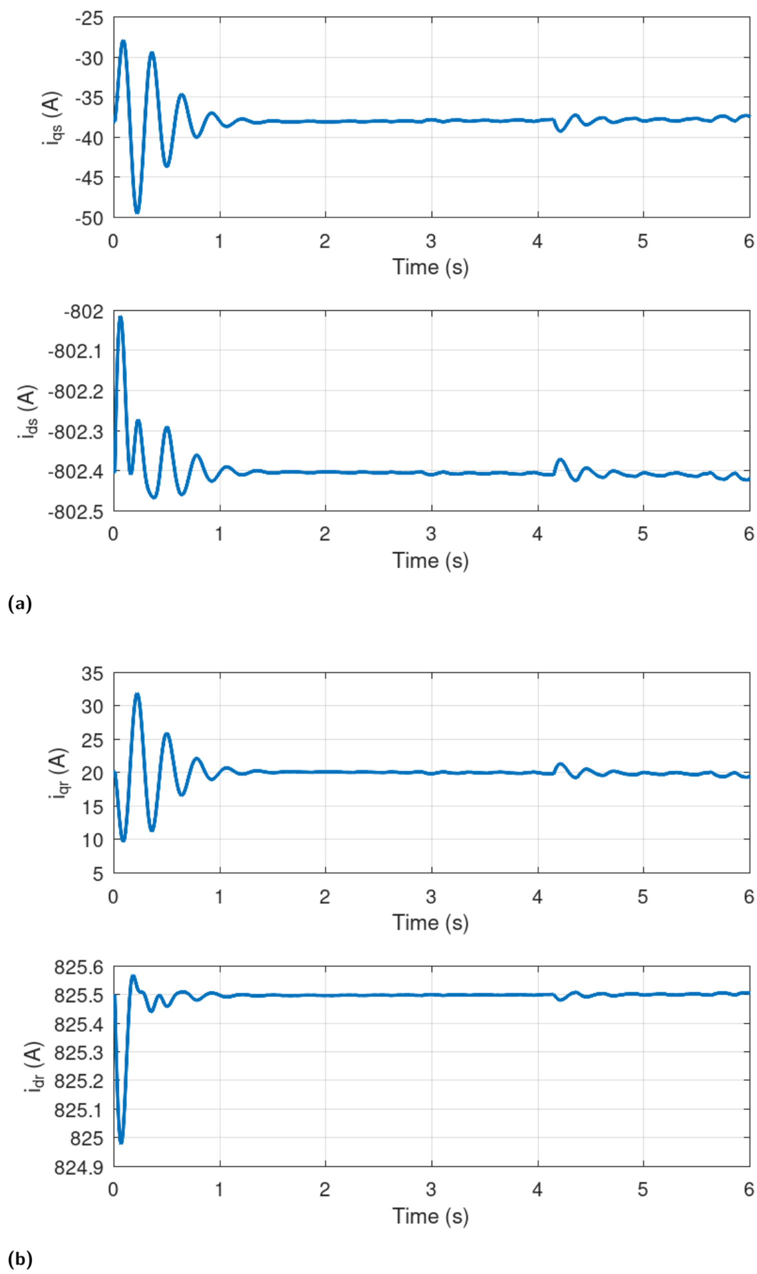

Figure 14: Stator and rotor currents. (a) i_s in the d-q axis and (b) i_r in the d-q axis.

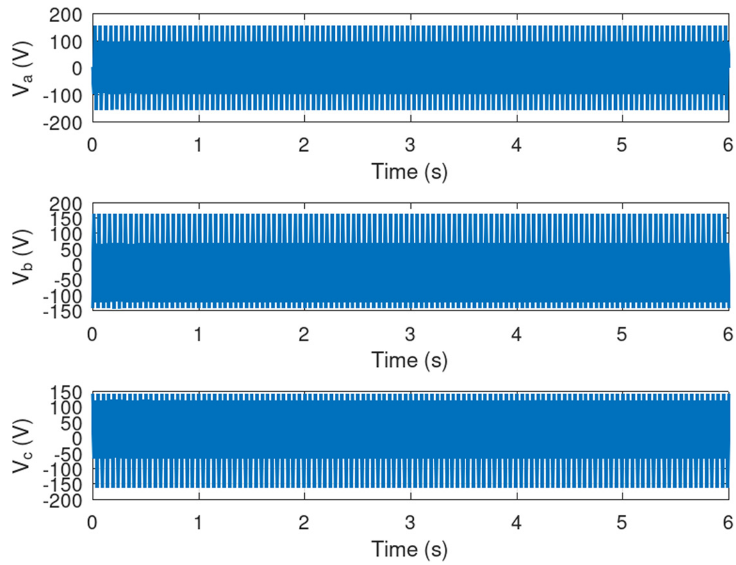

Figure 15: Input voltages for the a, b, and c phases.

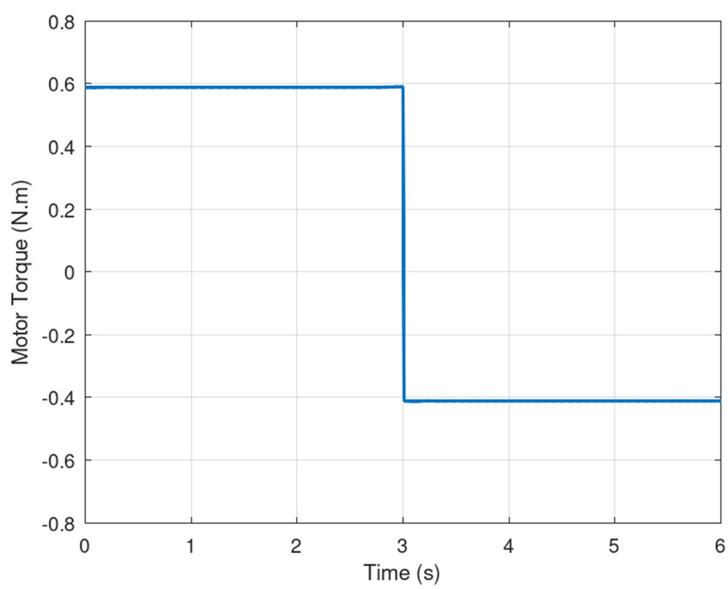

Figure 16: Motor torque.

Meanwhile, in Figure 14, the currents are shown evincing that the currents reach the steady-state value in finite time.

Finally, in Figures 15 and 16 are shown the three-phase input voltages along with the motor torque as they vary in time.

6 Results and discussion

In this section, the discussion of the theoretical and experimental results of this research study is presented in order to analyze its contributions. As shown in the theoretical part of this research study, first the PH system is simplified by the implementation of a Poisson bracket in order to synthesize the derivation of the NN controller. The Poisson brackets allow us to obtain the port-Hamiltonian formulation, and at the same time, it provides a compact representation of these kinds of systems. In this research study is only considered that the port-Hamiltonian system is for one and multi-particle systems; in other words, these can be implemented for the representation by port-Hamiltonian systems that model the dynamics of electrical, mechanical, molecular, and diverse types of physical systems.

In the case of the numerical experiment simulation, it is verified efficiently how the port-Hamiltonian system is stabilized taking into account the dynamics of a particle. It is important to note that the quaternion NN controller stabilizes the position and the momentum variable. The quaternion NN controller acts as a stabilizer for this system taking into consideration the dynamic properties of this port-Hamiltonian system. Apart, the proposed port-Hamiltonian system is driven to a final desired value using a reference trajectory taking into consideration the flexibility of a quaternion NN controller by selecting the appropriate gain matrices, the system position, and momentum variables to the final desired variables. With these numerical experiments, the derivation of NN control laws by the selection of appropriate Lyapunov functions is verified.

7 Conclusion

In this research study, the derivation of a stabilizer and a trajectory tracking control law is shown, based on a quaternion-based NN. It is important to consider that in both cases, the quaternion-based NN is divided into the scalar and vectorial quaternion part, in order to obtain the suitable control laws in order to drive the system variables, the momentum and position variables, to the final desired value in finite time. It is important to take into consideration that the Poisson brackets are implemented in order to simplify the mathematical deductions taking into consideration that the mathematical derivations of the port-Hamiltonian mathematical formalism are compact and simple. By means of two numerical examples, the theory shown in this article is verified and validated. The control laws are obtained by selecting suitable Lyapunov functionals in order to make the system stable. It is important to clarify that the theoretical results obtained in this research study were compared with passivity-based strategies, taking into consideration that energy-based controllers are commonly designed to control and stabilizes port-Hamiltonian systems. The experimental results prove that the proposed control strategy yields a better closed-loop performance in comparison with passivity-based control due to a better time response.

Funding information: This research work was funded by Institutional Fund Projects under Grant No. IFPIP: 144-611-1443. The authors gratefully acknowledge technical and financial support provided by the Ministry of Education and King Abdulaziz University, DSR, Jeddah, Saudi Arabia.

Author contributions: Fawaz E. Alsaadi: revision, edition, methods. Fernando E. Serrano: revision, edition, methods, simulation, experiments. Larissa M. Batrancea: revision, edition, methods.

Conflict of interest: Authors state no conflict of interest.

References

[1] L. M. Batrancea, A. Nichita, M. A. Balcı, and A. Akgüller, *Empirical investigation on how wellbeing-related infrastructure shapes economic growth: Evidence from the european union regions*, PLos One **18** (2023), 1–28.

[2] L. M. Batrancea, *The hard worker, the hard earner, the young and the educated: Empirical study on economic growth across 11 CEE countries*, Sustain. **15** (2023), no. 22, 15996.

[3] A. Khan, S. Hayat, Y. Zhong, A. Arif, L. Zada, and M. Fang, *Computational and topological properties of neural networks by means of graph-theoretic parameters*, Alexandria Eng. J. **66** (2022), 957–977.

[4] I. Famelis, A. Donas, and G. Galanis, *Comparative study of feedforward and radial basis function neural networks for solving an environmental boundary value problem*, Results Appl. Math. **16** (2022), 100344.
[5] Y. Chen, N. Zhang, and J. Yang, *A survey of recent advances on stability analysis, state estimation and synchronization control for neural networks*, Neurocomputing **515** (2023), 26–36.
[6] A. Benfenati and A. Marta, *A singular Riemannian geometry approach to deep neural networks ii. reconstruction of 1-d equivalence classes*, Neural Networks **158** (2023), 344–358.
[7] S. Coulibaly, B. Kamsu-Foguem, D. Kamissoko, and D. Traore, *Deep convolution neural network sharing for the multi-label images classification*, Machine Learn. Appl. **10** (2022), 100422.
[8] C. G. S. Capanema, G. S. de Oliveira, F. A. Silva, T. R. M. B. Silva, and A. A. F. Loureiro, *Combining recurrent and graph neural networks to predict the next place's category*, Ad Hoc Networks **138** (2023), 103016.
[9] Y. Chen, Y. Xue, X. Yang, and X. Zhang, *A direct analysis method to Lagrangian global exponential stability for quaternion memristive neural networks with mixed delays*, Appl. Math. Comput. **439** (2023), 127633.
[10] S. Chen, H.-L. Li, H. Bao, L. Zhang, H. Jiang, and Z. Li, *Global Mittage-Leffler stability and synchronization of discrete-time fractional-order delayed quaternion-valued neural networks*, Neurocomputing **511** (2022), 290–298.
[11] Q. Song, L. Yang, Y. Liu, and F. E. Alsaadi, *Stability of quaternion-valued neutral-type neural networks with leakage delay and proportional delays*, Neurocomputing **521** (2023), 191–198.
[12] W. Shang, W. Zhang, D. Chen, and J. Cao, *New criteria of finite time synchronization of fractional-order quaternion-valued neural networks with time delay*, Appl. Math. Comput. **436** (2023), 127484.
[13] Q. Song, R. Zeng, Z. Zhao, Y. Liu, and F. E. Alsaadi, *Mean-square stability of stochastic quaternion-valued neural networks with variable coefficients and neutral delays*, Neurocomputing **471** (2022), 130–138.
[14] T. Peng, J. Lu, Z. Tu, and J. Lou, *Finite-time stabilization of quaternion-valued neural networks with time delays: An implicit function method*, Inform. Sci. **613** (2022), 747–762.
[15] I. Brevis, I. Muga, and K. G. van der Zee, *Neural control of discrete weak formulations: Galerkin, least squares & minimal-residual methods with quasi-optimal weights*, Comput. Methods Appl. Mech. Eng. **402** (2022), 115716, A Special Issue in Honor of the Lifetime Achievements of J. Tinsley Oden.
[16] F. W. Alsaade, Q. Yao, M. S. Al-Zahrani, A. S. Alzahrani, and H. Jahanshahi, *Neural-based fixed-time attitude tracking control for space vehicle subject to constrained outputs*, Adv. Space Res. **71** (2023), no. 9, 3588–3599.
[17] H. Jahanshahi, Q. Yao, M. Ijaz Khan, and I. Moroz, *Unified neural output-constrained control for space manipulator using tan-type barrier lyapunov function*, Adv. Space Res. **71** (2022), 3712–3722.
[18] G. Zhou and D. Tan, *Review of nuclear power plant control research: Neural network-based methods*, Ann. Nuclear Energy **181** (2023), 109513.
[19] P. Nguyen, N. Thanh, and H. P. Huy Anh, *Advanced neural control technique for autonomous underwater vehicles using modified integral barrier lyapunov function*, Ocean Eng. **266** (2022), 112842.
[20] D. Zeng, Z. Liu, C. L. Philip Chen, Y. Zhang, and Z. Wu, *Decentralized adaptive neural asymptotic control of switched nonlinear interconnected systems with predefined tracking performance*, Neurocomputing **510** (2022), 37–47.
[21] F. De Vleeschauwer, M. Caluwé, T. Dobbeleers, H. Stes, L. Dockx, F. Kiekens, et al. *A dynamic control system for aerobic granular sludge reactors treating high cod/p wastewater, using ph and do sensors*, J. Water Process Eng. **33** (2020), 101065.
[22] G. Bayili, S. Nicaise, and R. Silga, *Rational energy decay rate for the wave equation with delay term on the dynamical control*, J. Math. Anal. Appl. **495** (2021), no. 1, 124693.
[23] A.-S. Kyriakides, A. I. Papadopoulos, P. Seferlis, and I. Hassan, *Dynamic modelling and control of single, double and triple effect absorption refrigeration cycles*, Energy **210** (2020), 118529.
[24] X. Shi, Y. Cheng, C. Yin, X. Huang, and S. Ming Zhong, *Design of adaptive backstepping dynamic surface control method with RBF neural network for uncertain nonlinear system*, Neurocomputing **330** (2019), 490–503.
[25] Z. Zhou, D. Tong, Q. Chen, W. Zhou, and Y. Xu, *Adaptive nn control for nonlinear systems with uncertainty based on dynamic surface control*, Neurocomputing **421** (2021), 161–172.
[26] S. Feng, Y. Kawano, M. Cucuzzella, and J. M. A. Scherpen, *Output consensus control for linear port-Hamiltonian systems*, IFAC-PapersOnLine **55** (2022), no. 30, 230–235, 25th International Symposium on Mathematical Theory of Networks and Systems MTNS 2022.
[27] A. Brugnoli, R. Rashad, and S. Stramigioli, *Dual field structure-preserving discretization of port-Hamiltonian systems using finite element exterior calculus*, J. Comput. Phys. **471** (2022), 111601.
[28] L. A. Mora and K. Morris, *Exponential decay rate of port-Hamiltonian systems with one side boundary damping*, IFAC-PapersOnLine **55** (2022), no. 30, 400–405, 25th International Symposium on Mathematical Theory of Networks and Systems MTNS 2022.
[29] D. Tamiru Tefera, S. Dubljevic, and V. Prasad, *A port Hamiltonian approach to dynamical chemical process systems network modeling and analysis*, Chem. Eng. Sci. **261** (2022), 117907.
[30] J. Jäschke, N. Skrepek, and M. Ehrhardt, *Mixed-dimensional geometric coupling of port-Hamiltonian systems*, Appl. Math. Lett. **137** (2023), 108508.
[31] T. H. Pham, N. M. T. Vu, I. Prodan, and L. Lefèvre, *A combined control by interconnection-model predictive control design for constrained port-Hamiltonian systems*, Syst. Control Lett. **167** (2022), 105336.
[32] W. Wei, J. Yu, L. Wang, C. Hu, and H. Jiang, *Fixed/preassigned-time synchronization of quaternion-valued neural networks via pure power-law control*, Neural Networks **146** (2022), 341–349.

[33] J. Shu, B. Wu, L. Xiong, T. Wu, and H. Zhang, *Stochastic stabilization of markov jump quaternion-valued neural network using sampled-data control*, Appl. Math. Comput. **400** (2021), 126041.
[34] R. Li and J. Cao, *Dissipativity and synchronization control of quaternion-valued fuzzy memristive neural networks: Lexicographical order method*, Fuzzy Sets Syst. **443** (2022), 70–89, From Learning to Modeling and Control.
[35] J. Shu, B. Wu, and L. Xiong, *Stochastic stability criteria and event-triggered control of delayed Markovian jump quaternion-valued neural networks*, Appl. Math. Comput. **420** (2022), 126904.
[36] H.-L. Li, C. Hu, L. Zhang, H. Jiang, and J. Cao, *Complete and finite-time synchronization of fractional-order fuzzy neural networks via nonlinear feedback control*, Fuzzy Sets Syst. **443** (2022), 50–69, From Learning to Modeling and Control.
[37] R. Zhang and L. Gao, *The brushless dc motor control system based on neural network fuzzy PID control of power electronics technology*, Optik **271** (2022), 169879.
[38] Z. Wu, B. Jiang, and Q. Gao, *State estimation and fuzzy sliding mode control of nonlinear Markovian jump systems via adaptive neural network*, J. Franklin Inst. **359** (2022), no. 16, 8974–8990.
[39] A. Razzaghian, *A fuzzy neural network-based fractional-order lyapunov-based robust control strategy for exoskeleton robots: Application in upper-limb rehabilitation*, Math. Comput. Simulat. **193** (2022), 567–583.
[40] M. Yang, Z. Sheng, G. Yin, and H. Wang, *A recurrent neural network based fuzzy sliding mode control for 4-dof rov movements*, Ocean Eng. **256** (2022), 111509.
[41] J. Wan, Q. Jiang, L. Liao, S. Wuuuuu, and P. Wang, *A neural-network based variable universe fuzzy control method for power and axial power distribution control of large pressurized water reactors*, Ann. Nuclear Energy **175** (2022), 109241.
[42] J. Toledo, Y. Wu, H. Ramirez, and Y. Le Gorrec, *Observer design for 1-d boundary controlled port-Hamiltonian systems with different boundary measurements*, IFAC-PapersOnLine **55** (2022), no. 26, 95–100, 4th IFAC Workshop on Control of Systems Governed by Partial Differential Equations CPDE 2022.
[43] R. Reyes-Báez, A. van der Schaft, and B. Jayawardhana, *Virtual contractivity-based control of fully-actuated mechanical systems in the port-Hamiltonian framework*, Automatica **141** (2022), 110275.
[44] C. Ponce, H. Ramirez, Y. L. Gorrec, and F. Vargas, *A comparative study of reduced model based boundary control design for linear port Hamiltonian systems*, IFAC-PapersOnLine **55** (2022), no. 26, 107–112, 4th IFAC Workshop on Control of Systems Governed by Partial Differential Equations CPDE 2022.
[45] H. Ramirez, Y. L. Gorrec, and B. Maschke, *Boundary controlled irreversible port-Hamiltonian systems*, Chem. Eng. Sci. **248** (2022), 117107.
[46] B. Maschke, F. Philipp, M. Schaller, K. Worthmann, and T. Faulwasser, *Optimal control of thermodynamic port-Hamiltonian systems*, IFAC-PapersOnLine **55** (2022), no. 30, 55–60, 25th International Symposium on Mathematical Theory of Networks and Systems MTNS 2022.
[47] A. Brugnoli, G. Haine, and D. Matignon, *Explicit structure-preserving discretization of port-Hamiltonian systems with mixed boundary control*, IFAC-PapersOnLine **55** (2022), no. 30, 418–423, 25th International Symposium on Mathematical Theory of Networks and Systems MTNS 2022.
[48] T. Malzer, L. Ecker, and M. Schöberl, *Energy-based control and observer design for higher-order infinite-dimensional port-Hamiltonian systems*, IFAC-PapersOnLine **54** (2021), no. 19, 44–51, 7th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control LHMNC 2021.
[49] T. Malzer, J. Toledo, Y. L. Gorrec, and M. Schöberl, *Energy-based in-domain control and observer design for infinite-dimensional port-Hamiltonian systems*, IFAC-PapersOnLine **54** (2021), no. 9, 468–475, 24th International Symposium on Mathematical Theory of Networks and Systems MTNS 2020.
[50] M. Schaller, F. Philipp, T. Faulwasser, K. Worthmann, and B. Maschke, *Control of port-Hamiltonian systems with minimal energy supply*, Europ. J. Control **62** (2021), 33–40, 2021 European Control Conference Special Issue.
[51] T. Malzer, H. Rams, and M. Schöberl, *On structural invariants in the energy-based in-domain control of infinite-dimensional port-Hamiltonian systems*, Syst. Control Lett. **145** (2020), 104778.
[52] A. Macchelli, Y. L. Gorrec, and H. Ramirez, *Asymptotic stabilisation of distributed port-Hamiltonian systems by boundary energy-shaping control*, IFAC-PapersOnLine **48** (2015), no. 1, 488–493, 8th Vienna International Conferenceon Mathematical Modelling.
[53] W. M. Haddad, T. Rajpurohit, and X. Jin, *Energy-based feedback control for stochastic port-controlled Hamiltonian systems*, Automatica **97** (2018), 134–142.
[54] F. Lamoline, *Passivity of boundary controlled and observed stochastic port-Hamiltonian systems subject to multiplicative and input noise*, Europ. J. Control **62** (2021), 41–46.
[55] K. Hamada, P. Borja, K. Fujimoto, I. Maruta, and J. M. A. Scherpen, *On passivity-based high-order compensators for mechanical port-Hamiltonian systems without velocity measurements the work was supported in part by mori manufacturing research and technology foundation*, IFAC-PapersOnLine **54** (2021), no. 14, 287–292, 3rd IFAC Conference on Modelling, Identification and Control of Nonlinear Systems MICNON 2021.
[56] H. Ramirez, Y. L. Gorrec, B. Maschke, and F. Couenne, *Passivity based control of irreversible port Hamiltonian systems*, IFAC Proc. Vol. **46** (2013), no. 14, 84–89, 1st IFAC Workshop on Thermodynamic Foundations of Mathematical Systems Theory.
[57] G. Nishida, K. Yamaguchi, and No. Sakamoto, *Optimality of passivity-based controls for distributed port-Hamiltonian systems*, IFAC Proc. Vol. **46** (2013), no. 23, 146–151, 9th IFAC Symposium on Nonlinear Control Systems.
[58] N. Sakata, K. Fujimoto, and I. Maruta, *On trajectory tracking control of simple port-Hamiltonian systems based on passivity based sliding mode control*, IFAC-PapersOnLine **54** (2021), no. 19, 38–43, 7th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control LHMNC 2021.

[59] F. Castannnos and D. Gromov, *Passivity-based control of implicit port-Hamiltonian systems with holonomic constraints*, Syst. Control Lett. **94** (2016), 11–18.
[60] J. Pan and Z. Zhang, *Finite-time synchronization for delayed complex-valued neural networks via the exponential-type controllers of time variable*, Chaos Solitons Fractals **146** (2021), 110897.
[61] J. Feng, Y. Chai, and C. Xu, *A novel neural network to nonlinear complex-variable constrained nonconvex optimization*, J. Franklin Inst. **358** (2021), no. 8, 4435–4457.
[62] Q. Song, Q. Yu, Z. Zhao, Y. Liu, and F. E. Alsaadi, *Dynamics of complex-valued neural networks with variable coefficients and proportional delays*, Neurocomputing **275** (2018), 2762–2768.
[63] W. Li, H. Xu, X. Liu, Y. Wang, Y. Zhu, X. Lin, et al., *Regenerative braking control strategy for pure electric vehicles based on fuzzy neural network*, Ain Shams Eng. J. **15** (2023), 102430.
[64] U. Kadak and L. Coroianu, *Integrating multivariate fuzzy neural networks into fuzzy inference system for enhanced decision making*, Fuzzy Sets Syst. **470** (2023), 108668.
[65] A. M. El-Nagar, M. El-Bardini, and A. Aziz Khater, *Recurrent general type-2 fuzzy neural networks for nonlinear dynamic systems identification*, ISA Trans. **140** (2023), 170–182.
[66] S. Nguyen and A. A. Turski, *Examples of the dirac approach to dynamics of systems with constraints*, Phys A Stat Mechanics Appl. **290** (2001), no. 3, 431–444.
[67] C. Chandre, *Incomplete dirac reduction of constrained Hamiltonian systems*, Ann. Phys. **361** (2015), 1–13.
[68] P. Borja, R. Ortega, and E. Nuño, *New results on PID passivity-based controllers for port-Hamiltonian systems*, IFAC-PapersOnLine **51** (2018), no. 3, 175–180, 6th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control LHMNC 2018.
[69] R. Krishnan, *Electric Motor Drives, Modeling, Analysis and Control*, Prentice Hall, New Delhi, 2006.