Research Article

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The cross-border e-commerce platform selection based on the probabilistic dual hesitant fuzzy generalized dice similarity measures

https://doi.org/10.1515/dema-2022-0239
received September 5, 2022; accepted April 28, 2023

Abstract: Cross-border e-commerce platform (CBECP) plays a very important role in the development of a cross-border e-commerce (CBEC). How to select the best CBECP scientifically and reasonably is a very critical multi-attribute group decision-making (MAGDM) issue. With the uncertainty of people's cognition of the objective world, the decision-making process is full of a lot of fuzzy information. In view of the great advantages of probabilistic dual hesitation fuzzy set (FS) in expressing decision-making information, and in combination with the very extensive use of the Dice similarity measure (DSM), a new MAGDM method is proposed for the optimal CBECP selection (CBECPS) under the probabilistic dual hesitation fuzzy (PDHF) environment. First, on the basis of reviewing a large number of documents on the CBECPS for CBEC, the evaluation index system for the CBECPS is constructed; second, several new DSMs are proposed in the PDHF environment; third, based on the two newly proposed probabilistic dual hesitant weighted generalized Dice similarity measures, two novel MAGDM methods are provided for CBECPS, which are used for CBECPS; finally, the two established MAGDM techniques are compared with the existing decision-making methods, and the parameter analysis is carried out to illustrate the effectiveness and superiority of the two established MAGDM techniques. The two established techniques can not only be used for CBECPS of CBEC, but also be extended to similar related research.

Keywords: multi-attribute group decision-making, probabilistic dual hesitant FSs, Dice similarity measure, cross-border e-commerce platform selection

MSC 2020: 03E72

1 Introduction

In order to better implement the research work proposed in this article, we will systematically review the cross-border e-commerce platform selection (CBECPS) evaluation index system, the development process of fuzzy sets (FSs), and the research work of some MADM/multi-attribute group decision-making (MAGDM) methods. Through a large number of literature reviews, we will propose the research motivation, innovative work, and contributions of this article.

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Despite the rapid development of China's cross-border e-commerce (CBEC) in recent years, the application level of China's CBECs is still not high enough, or copy other successful enterprise cases, or blindly follow the trend, lacking certain scientificity. Therefore, the research of this article becomes very meaningful. By analyzing the relevant factors affecting the CBECPS of CBECs, and starting from its platform characteristics, this study establishes a comprehensive assessment index system, which mainly serves the CBECPS for CBECs. As a high-tech field, there are some relevant theories and literature about CBECPS. We will extract useful information from the limited studies and finally establish the evaluation index system of CBECPS. Zhang et al. [1] analyzed the influencing factors of intelligent perception system of big data decision in CBEC. Zhang et al. [2] designed and studied a visitor information system of the cross-border e-commerce platform (CBECP) based on mobile edge computing. Wang and Li [3] studied the supply chain in detail under the term of centralized and decentralized decisions. Sun and Gu [4] proposed building a CBEC logistics supervision system and gave an evaluation index system. Rui [5] thought e-commerce platform (ECP) is the carrier of CBEC and developed a novel classification technology. Lu et al. [6] analyzed the influencing factors of consumers' intention in CBEC shopping by fusing documents and empirical studies. Li et al. [7] analyzed the research status of content-based image retrieval algorithms, and the hierarchical commodity classification and retrieval system are designed. Chen [8] improved the technological and logical problems of CBEC in the operational procedure. Li [9] discussed the influence factors of the artificial intelligence system for CBEC. Ma et al. [10] proposed the framework that uses electronic word of mouth, perceived value, website design quality, trust, perceived risk, and the uncertainty avoidance index as the exogenous variables of cross-border shopping. From the perspective of literature analysis, there is little research on the CBECPS. This article establishes the evaluation index system of CBECPS by combining the existing literature and the common sense of relevant e-commerce platform selection. The details are as follows. By reviewing the literature at home and abroad, by analyzing the relevant factors affecting the CBECPS of CBECs, and starting from its platform characteristics, this article establishes a comprehensive evaluation index system, which mainly serves the CBECPS of CBECs. The index covers four dimensions, namely, platform and information quality, service quality, system function, and operating cost.

Most of the aforementioned problems and related MADM/MAGDM methods are researched under the condition of definite information, but in practical problems, when experts study related MAGDM problems, their opinions are often not very clear and uncertain. With the increasing frequency of similar issues, to tackle these uncertain issues, the famous concept of FS was built by Zadeh [11], which was then used in many practical issues and to play some good roles. In order to better meet people's problems in dealing with some different situations in real life, some new FSs have been proposed one after another [12,13], such as multidimensional FSs, which have been fully developed and applied. Eminli and Guler [14] extended the multidimensional fuzzy subset to the premise fuzzy rule of Takagi and Sugeno's model and used the model in the prediction performance, robustness, and initialization capabilities to examine the validity of the model. Husek [15] used the multidimensional FS to describe the parameter space and gave a novel algorithm. Manuel Martinez-Jimenez et al. [16] proposed several algorithms to overcome some defects in the perceptual properties of texture model and gave a novel approach that can transform the attributes with multidimensional FS into particular perception for a new user. Lima et al. [17] introduced the multidimensional FS for overcoming the defects of n-dimensional FS and the hesitant FS (HFS) and proposed two class of aggregation functions, and two MCDM approaches were built on these two aggregation functions. Although the FS has developed rapidly since it was put forward, it still has some disadvantages, that is, it can well express the approval attitude of decision-maker (DM), but cannot reflect the DM's opposition attitude and cannot reflect the DM's hesitation at the same time. Hence, the well-known intuitionistic FS (IFS) with the membership degree (MD) and non-membership degree (NMD) was proposed by Atanassov [18] and applied in various fields. Al-Kenani et al. [19] built the intuitionistic fuzzy prioritized aggregation and intuitionistic fuzzy prioritized geometric operators through considering the priority levels between two decision attributes and applied to the best choice problem. Pandey et al. [20] proposed a novel intuitionistic fuzzy (IF) entropy and used it to feature selection. Chen and Liu [21] proposed some novel similarity measures and a group of axioms, finally, and some pattern recognition examples were performed to testify the validity of these similarity measures. Ecer [22] built the intuitionistic fuzzy multi-attribute ideal-real comparative analysis approach by merging the IFS and multi-attribute ideal-real comparative analysis method and applied to evaluate five coronavirus vaccines for corona virus disease 2019. Gupta and Kumar [23] built the novel SMs in IF setting by analyzing the defects of the existing SMs and used in the pattern recognition and clustering analysis to testify the effectiveness of these novel SMs. To meet the needs of reality, the interval IFS (IVIFS) was built by Atanassov and Gargov [24] and applied in many fields. Wang et al. [25] put forward the Jenson–Shannon divergence under interval valued intuitionistic fuzzy (IVIF) setting and studied some valuable properties; a novel MADM technique was constructed and used in the medical diagnosis and network system selection. Wang and Li [26] defined a new MAGDM approach by merging the IVIFS and VIse kriterijumski optimizacioni racun approach and applied it in the project investment decision. Salimian et al. [27] used the IVIF–Shannon entropy to weight for decision attributes and constructed a new MCDM approach by merging the extended VIse kriterijumski optimizacioni racun and measurement alternatives and ranking according to the compromise solution methods; finally, the MCDM approach was used in the assessment of the sustainable suppliers and the parameters and comparative analysis were implemented to testify the validity of the MCDM technique. Ohlan [28] studied the entropy and distance measure under IVIF setting and constructed a new MCGDM approach that depends on the VIKOR and technique for order of preference by similarity to ideal solution approaches; finally, the proposed MCGDM technique was applied in the evaluation of four firms. Although IFS can reflect the MD, NMD, and hesitant degree of DM and although IFS can express the DM's MD, NMD, and hesitation degree, it still cannot express the DM's hesitation on multiple values. To better deliver the demands of DMs, HFS was constructed by Torra [29] and was used in various fields. Senapati et al. [30] defined the operations of Aczel-Alsina under HF setting and some novel aggregation operators, such as hesitant fuzzy aczel-alsina weighted averaging, hesitant fuzzy aczel-alsina ordered weighted averaging, hesitant fuzzy Aczel-Alsina hybrid averaging, hesitant fuzzy Aczel-Alsina weighted geometric, hesitant fuzzy Aczel-Alsina ordered weighted geometric, hesitant fuzzy aczel-alsina hybrid geometric, and hesitant fuzzy aczel-alsina weighted bonferroni mean operators, and investigated some precious properties of these operators, and a fresh MADM approach was constructed that depends on these operators and used in the cyclone disaster evaluation. Li et al. [31] proposed a novel MADM technique by merging the gray relational analysis technique and HFS and applied it in the selection of two equipment systems. Krishankumar et al. [32] built a novel entropy measure to weight for decision attributes and a novel decision-making model for CESs. Although HFS can express the MD of the DM on multiple values, it cannot express the opposition of the DM. The dual HFS (DHFS) with the MD and NMD for expressing approval and disapproval was developed by Zhu et al. [33] for tackling the situation and has been used in many MADM fields. Ni et al. [34] extended the projection technique to DHF environment, and a fresh MADM approach was built; then, the novel MADM technique was used in the practical problem and compared with four existing decision-making techniques. Wei and Wang [35] defined some novel distance measures and applied them in the clustering analysis. There are also some fuzzy MADM methods applied in many fields [36–38]. Although the MD and NMD of DHFS can allow DMs to choose from multiple values, it is still unable to express the size of each value; for tackling the issue, the probabilistic DHFS (PDHFS) [39] and q-rung PDHFS (q-RPDHFS) [40] were built; PDHFS perfectly solves the aforementioned problems and can better meet the needs of reality and has been used in various practical problems. Ren et al. [41] extended the tomada de decisaointerativa e multi-critévio technique to probabilistic dual hesitation fuzzy (PDHF) setting and built a novel MADM technique; finally, the PDHF–TODIM method was used in the enterprise strategic evaluation; the biggest advantage of this method is that it can fully reflect the psychological behavior of DMs during the decision-making process. Garg and Kaur [42] defined some novel distance measures and operators, such as probabilistic dual hesitant fuzzy weighted Einstein average (PDHFWEA), probabilistic dual hesitant fuzzy ordered weighted Einstein average, probabilistic dual hesitant fuzzy weighted Einstein geometric (PDHFWEG), and probabilistic dual hesitant fuzzy ordered weighted Einstein geometric operators; finally, these operators were used in the evaluation of consumer's buying behavior; the advantage of this method is that it can fully consider the supporting relationship between decision attributes during the decision-making process. Garg and Kaur [43] developed some new correlation coefficients under PDHF setting, which were used to select best candidate for a company; although this method can measure the correlation between two PDHFEs, the calculation process of the northwest corner method for determining probability proposed in this article is too complex and inconvenient to use in practical applications. Garg and Kaur [44] merged the maclaurin symmetric mean operator to PDHF setting and proposed PDHFMSMA and PDHFMSMG operators, and these operators were used in the medical diagnosis problem; the proposed operators can capture the correlation between decision attributes. Zhao et al. [45] defined the MADM approach by extending preference ranking organization method for enrichment of evaluations-II based on bivariate almost stochastic dominance technique to PDHF environment and used in the assessment of arctic geopolitics risk; this method can also reflect the psychological behavior of DMs. Garg and Kaur [46] built some distance measures and investigated some precious properties, a decision-making approach by merging the bipartite graph theory to PDHF setting, which was used in screening task of travelers. Wang et al. [47] constructed a new weighting method for decision attributes by merging the BWM and superiority and inferiority ranking method to PDHF environment and a fresh MAGDM technique was proposed; finally, the decision-making method was used in the selection of green suppliers. Li et al. [48] defined a Hamming distance measure for PDHFSs and extended the TODIM method to the PDHF setting; a fresh MADM approach was constructed and used in the assessment of supply chain credit risk. Ning et al. [49] developed some novel distance and entropy measures and extended the combinative distance-based assessment method to PDHF setting for MADM issue. Ning et al. [50] systematically proposed some distance measures in discrete and continuous cases under PDHF setting. Ning et al. [51] extended the power Maclaurin symmetric mean operator to PDHF setting and used it in the MADM issue. Ning et al. [52] extended the evaluation based on distance from average solution method to PDHF setting and developed a new MAGDM method, which was used in supplier selection.

When reviewing the studies in the PDHF environment, we found that there have been some studies on the score function, entropy measure, distance measure, decision-making method, and information aggregation operator of PDHFEs, but compared with other studies in other fuzzy environments, it is still slightly insufficient. At the same time, we also found that there is currently no measurement method for the similarity between two PDHFEs in the PDHF environment; therefore, studying similarity measurement methods in the PDHF environment is very meaningful research work.

The aforementioned CBECPS is obviously an MAGDM problem, so it is very critical to act a scientific and reasonable MAGDM technique to study the CBECPS problem. At present, there are many well-known methods for the research of MADM/MAGDM methods. One of the most important methods is to determine the optimal alternative from a limited number of alternatives by studying the similarity measure between the alternative and the positive ideal alternative. The Dice similarity measure (DSM) is a very important method for calculating the similarity between two variables. DSM was first proposed by Dice [53]; so far, it has been extended to many fuzzy environments. Garg et al. [54] proposed the generalized DSMs for complex q-rung orthopair fuzzy set and applied it to the medical diagnoses and pattern recognition. Jan et al. [55] extended the DSM to q-rung orthopair fuzzy set and applied in the election of the best company for dealing of business. Singh and Kumar [56] proposed a novel DSM for IFS, and it was applied to pattern and face recognition. Wang et al. [57] extended the generalized DSM to PyFS, an MAGDM method that was proposed and applied in the evaluation of potential enterprise resource planning systems. Wei and Gao [58] extended the generalized Dice similarity measure (GDSM) to PFS, an MAGDM method that was proposed and applied in building material recognition. Zhang et al. [59] proposed the generalized DSMs for picture 2-tuple linguistic set, an MAGDM approach that was proposed and used in practical MADM issue. Lei et al. [60] proposed an MAGDM method for PLS, which depends on the generalized DSMs to the MADM problem. Zhang et al. [61] proposed an evaluation method that depends on the generalized DSMs under 2-TLS information. Wei et al. [62] extended the DSM to PL setting and built some novel PLDSMs and applied them to location planning of electric vehicle charging station. There are too many applications of the DSM, which will not be listed here.

From the review of CBECPS, MAGDM/MADM method, and DSM in various fuzzy settings, the research targets of this article are as follows:

(1) Through a review of the studies on the CBECPS of CBEC, we can find that although there have been some studies on the CBECPS of CBEC, there are different standards for the evaluation indicator system. After analyzing the evaluation indicators on the CBECPS of CBEC in a large number of studies, we find that the evaluation indicator system on the CBECPS of CBEC is not scientific; constructing a scientific and reasonable evaluation index system for CBECPS of CBEC is a crucial prerequisite;
(2) The issue of CBECPS is a very important component in the development process of CBEC. Scientific and reasonable evaluation methods are crucial to the development of CBEC, and it is crucial to propose some novel evaluation methods for CBECPS.
(3) We found that the concept of entropy of PDHFE proposed in document [33] requires the use of auxiliary functions during the use process, which adds artificial subjective factors, resulting in situations where the

calculated results are inconsistent with the actual situation. It is a key scientific issue to propose a new PDHF entropy to scientifically and reasonably weight for indicators;

(4) Since DSM was proposed, it has been extended to many fuzzy environments, and such excellent methods have not been involved in the PDHF environment. At the same time, in view of the lack of similarity research in the PDHF environment, it is obvious that both in enriching the decision-making methods of PDHFS and in the new extension research of DSM, it is very necessary to study DSM in the PDHF environment and the decision-making methods based on it.

The final witness of this article is to propose new MAGDM technique for CBECPS of CBEC. The innovative works of this study are as follows:

(1) Through reviewing a large number of evaluation index systems of CBECPS, the evaluation index system of CBECPS has been systematically sorted out, and the evaluation index system of CBECPS has been reconstructed;
(2) Since PDHFS can not only reflect the approval, disapproval, and hesitation attitudes of DMs, but also measure the degree of the three components, it has greater advantages in describing fuzzy decision information. PDHFS is first used in CBECPS issue;
(3) Due to the advantages of PDHFS in describing decision information and the successful application of DSM in various fuzzy environments, DSM and PDHFS were integrated for the first time, and a DSM decision method in the PDHF environment was proposed. In the process of MAGDM, the main advantage of the proposed method is that it is more general and flexible than existing decision-making methods with PDHF information to meet practical needs;
(4) The newly proposed PDHF entropy does not need to be calculated with the aid of auxiliary functions, which largely avoids the problem of artificially determining auxiliary functions that are inconsistent with the actual situation.

On the basis of the aforementioned innovative works, the main contributions of such study are as follows:

(1) A geometric weighting operator (PDHFWG operator) for PDHFS is proposed, which provides a new operator for the aggregation of PDHF information;
(2) Six forms of DSM in PDHF environment are proposed, providing a new method for measuring the similarity between two PDHFSs;
(3) Two PDHF MAGDM models based on the generalized DSMs are proposed, which enriches MAGDM methods in PDHF environment and lays a theoretical foundation for the further development of PDHFS.

The whole article is mainly constituted of the following sections except the aforementioned section. Section 2 reviews the basic content of PDHFS, and the PDHFWG aggregation operator was first built; the traditional DSM and merged DSM to PDHF setting were reviewed in Section 3; some new PDHF DSMs are proposed; in Section 4, a new objective weight approach is constructed, which depends on the PDHF entropy; based on the PMII, the combined weight calculation approach was constructed, and a new MAGDM method was built, which depends on the PDHF DSMs; in Section 5, the new PDHF MAGDM approach is applied to the CBECPS and compared with several existing aggregation operators and MADM methods, and the effectiveness of the new PDHF MAGDM approach built in the article was investigated; Section 6 gives the conclusions and prospects of some next studies.

2 Preliminaries

In the next section, we review some classical FSs and the basic operations for PDHFSs.

2.1 Several classes of FSs

In order to better tackle some practical decision-making problems, in 1965, the famous FS was built by Zadeh [11].

Definition 1. [11]. An FS $\mathfrak{I}$ on X is shown as:

$$\mathfrak{I} = \{\ell, \mu_{\mathfrak{I}}(\ell) | \ell \in X\}, \tag{1}$$

where $\mu_{\mathfrak{I}} : X \to [0, 1]$ is the MD function of X.

Definition 2. [18]. An IFS $\mathfrak{I}$ on X is shown as:

$$\mathfrak{I} = \{\ell_i, \mu_{\mathfrak{I}}(\ell_i), \nu_{\mathfrak{I}}(\ell_i) | \ell_i \in X\}, \tag{2}$$

where $\mu_{\mathfrak{I}} : X \to [0, 1]$ is the MD, $\nu_{\mathfrak{I}} : X \to [0, 1]$ is the NMD, and for $\ell_i \in X$, and $0 \le \mu_{\mathfrak{I}}(\ell_i) + \nu_{\mathfrak{I}}(\ell_i) \le 1$, $\pi_{\mathfrak{I}}(\ell_i) = 1 - \mu_{\mathfrak{I}}(\ell_i) - \nu_{\mathfrak{I}}(\ell_i)$ is called the hesitant degree.

The HFS was constructed by Torra [29], which can better express the DM's hesitation in multiple values.

Definition 3. [29]. An HFS $\mathfrak{I} : X \to [0, 1]$ is recorded as:

$$\mathfrak{I} = \{(\ell, \hbar_{\mathfrak{I}}(\ell)) | \ell \in X\}, \tag{3}$$

where $\hbar_{\mathfrak{I}}(\ell) \in [0, 1]$.

Definition 4. [33]. A DHFS $\mathfrak{I}$ on X is shown as:

$$\mathfrak{I} = \{(x, \hbar_{\mathfrak{I}}(\ell), \lambda_{\mathfrak{I}}(\ell)) | \ell \in X\}, \tag{4}$$

where $\hbar_{\mathfrak{I}}(\ell)$ and $\lambda_{\mathfrak{I}}(\ell)$ are two group of discrete values in [0,1], called as the MD and NMD, and satisfy $0 \le \gamma_{\mathfrak{I}}, \eta_{\mathfrak{I}} \le 1$ and $0 \le \gamma_{\mathfrak{I}}^+ + \eta_{\mathfrak{I}}^+ \le 1$, where $\gamma_{\mathfrak{I}} \in \hbar_{\mathfrak{I}}, \eta_{\mathfrak{I}} \in \lambda_{\mathfrak{I}}, \gamma_{\mathfrak{I}}^+ \in \hbar_{\mathfrak{I}}^+ = \cup_{\gamma_{\mathfrak{I}} \in \hbar_{\mathfrak{I}}} \max\{\gamma_{\mathfrak{I}}\}, \eta_{\mathfrak{I}}^+ \in \lambda_{\mathfrak{I}}^+ = \cup_{\eta_{\mathfrak{I}} \in \lambda_{\mathfrak{I}}} \max\{\eta_{\mathfrak{I}}\}$. Zhu et al. [33] called $\mathfrak{I}=\langle \hbar_{\mathfrak{I}}, \lambda_{\mathfrak{I}} \rangle = \cup_{\gamma_{\mathfrak{I}} \in \hbar_{\mathfrak{I}}, \eta_{\mathfrak{I}} \in \lambda_{\mathfrak{I}}} \langle \{\gamma_{\mathfrak{I}}\}, \{\eta_{\mathfrak{I}}\} \rangle$ as DHFE.

Definition 5. [63]. A PHFS $\mathfrak{I}$ on X is recorded as:

$$\mathfrak{I} = \{(\ell, \hbar_{\ell} | \tau_{\ell}) | \ell \in X\}, \tag{5}$$

where $\hbar_{\ell}| \in [0, 1]$ be the MD values in the set $\mathfrak{I}$, and the probability is $\tau_{\ell} \in [0, 1]$.

Xu and Zhou [63] called $\hbar_{\ell} | \tau_{\ell}$ as a PHFE and is shown as:

$$\hbar(\mathfrak{I}) = \{(\gamma_i | \tau_i) | i = 1, 2, \cdots, \#\hbar(\mathfrak{I})\}, \tag{6}$$

where $\gamma_i \in \hbar_i$ and τ_i is the probability of γ_i, and $\sum_{i=1}^{\#\hbar(\mathfrak{I})} \tau_i = 1$, $\#\hbar(\mathfrak{I})$ indicates the number of $\gamma_i | \tau_i$.

2.2 PDHFS

Definition 6. [39]. A PDHFS on X is defined as:

$$\mathfrak{I} = \{\langle \ell, \hbar(\ell) | \tau(\ell), \lambda(\ell) | \upsilon(\ell) \rangle | \ell \in X\}. \tag{7}$$

The components $\hbar(\ell) | \tau(\ell)$ and $\lambda(\ell) | \upsilon(\ell)$ are called as some possible MD and NMD, where $\hbar(\ell)$ is MD and $\lambda(\ell)$ is NMD. $\tau(\ell)$ and $\upsilon(\ell)$ indicate the probability of $\hbar(\ell)$ and $\lambda(\ell)$, respectively, and,

$$0 \le \gamma, \eta \le 1, \; 0 \le \gamma^+ + \eta^+ \le 1, \tag{8}$$

and

$$\tau_i \in [0, 1], \; \upsilon_j \in [0, 1], \; \sum_{i=1}^{\#\hbar} \tau_i = 1, \; \sum_{j=1}^{\#\lambda} \upsilon_j = 1, \tag{9}$$

where $\gamma \in \hbar(\ell)$, $\eta \in \lambda(\ell)$, $\gamma^+ \in \hbar^+(x) = \cup_{\gamma \in \hbar(x)} \max\{\gamma\}$, $\eta^+ \in \lambda^+(x) = \cup_{\gamma \in \lambda(x)} \max\{\eta\}$, $\tau_i \in \tau(\ell)$, and $\upsilon_i \in \upsilon(\ell)$. The symbol $\#\hbar$ is the number of elements in $\hbar(\ell) | \tau(\ell)$, and the symbol $\#\lambda$ is the number of elements in $\lambda(\ell) | \upsilon(\ell)$.

$\mathfrak{I} = \langle \hbar(\ell) | \tau(\ell), \lambda(\ell) | \upsilon(\ell) \rangle$ is named as the PDHFE, shown as $\mathfrak{I} = \langle \hbar | \tau, \lambda | \upsilon \rangle$ [39].

If $\sum_{i=1}^{\#\hbar}\tau_i < 1$ and $\sum_{j=1}^{\#\lambda}\upsilon_j < 1$, we normalize the PDHFS by the following formula:

$$\bar{\mathfrak{I}} = \{\langle \ell, \hbar(\ell)|\tau(\ell), \lambda(\ell)|\upsilon(\ell)\rangle | \ell \in X\}, \tag{10}$$

where $\tau(\ell) = \tau_i/\sum_{i=1}^{\#\hbar}\tau_i$, $\upsilon(\ell) = \upsilon_i/\sum_{i=1}^{\#\lambda}\upsilon_i$.

Let $\mathfrak{I}, \mathfrak{I}_1$, and $\mathfrak{I}_2$ be three PDHFEs, $\mathfrak{I} = \langle \hbar|\tau, \lambda|\upsilon\rangle$, $\mathfrak{I}_1 = \langle \hbar_1|\tau_{\hbar_1}, \lambda_1|\upsilon_{\lambda_1}\rangle$, and $\mathfrak{I}_2 = \langle \hbar_2|\tau_{\hbar_2}, \lambda_2|\upsilon_{\lambda_2}\rangle$, then operations of PDHFEs were shown as [39]:

(1) $\mathfrak{I}_1 \oplus \mathfrak{I}_2 = \cup_{\gamma_1\in\hbar_1,\eta_1\in\lambda_1,\gamma_2\in\hbar_2,\eta_2\in\lambda_2}\{\langle\{\gamma_1 + \gamma_2 - \gamma_1\gamma_2|p_{\gamma_1}p_{\gamma_2}\}, \{\eta_1\eta_2|q_{\eta_1}q_{\eta_2}\}\rangle\}$;

(2) $\mathfrak{I}_1 \otimes \mathfrak{I}_2 = \cup_{\gamma_1\in\hbar_1,\eta_1\in\lambda_1,\gamma_2\in\hbar_2,\eta_2\in\lambda_2}\{\langle\{\gamma_1\gamma_2|p_{\gamma_1}p_{\gamma_2}\}, \{\eta_1 + \eta_2 - \eta_1\eta_2|q_{\eta_1}q_{\eta_2}\}\rangle\}$;

(3) $\lambda\mathfrak{I} = \cup_{\gamma\in\hbar,\eta\in\lambda}\{\langle\{(1 - (1 - \gamma)^{\lambda})|p_\gamma\}, \{\eta^{\lambda}|q_\eta\}\rangle\}$;

(4) $\mathfrak{I}^{\lambda} = \cup_{\gamma\in\hbar,\eta\in\lambda}\{\langle\{\gamma^{\lambda}|p_\gamma\}, \{(1 - (1 - \eta)^{\lambda})|q_\eta\}\rangle\}$;

(5) $\mathfrak{I}^c = \begin{cases} \cup_{\gamma_j\in\hbar_{\mathfrak{I}},\eta_k\in\lambda_{\mathfrak{I}}}\{\langle\{\eta_k|q_k\}, \{\gamma_j|p_j\}\rangle\}, & \text{if } \hbar_{\mathfrak{I}} \neq \phi \text{ and } \lambda_{\mathfrak{I}} \neq \phi, \\ \cup_{\gamma_j\in\hbar_{\mathfrak{I}}}\{\langle\{1 - \gamma_j|p_j\}, \{\phi\}\rangle\}, & \text{if } \hbar_{\mathfrak{I}} \neq \phi \text{ and } \lambda_{\mathfrak{I}} = \phi, \\ \cup_{\eta_k\in\lambda_{\mathfrak{I}}}\{\langle\{\phi\}, \{1 - \eta_k|q_k\}\rangle\}, & \text{if } \hbar_{\mathfrak{I}} = \phi \text{ and } \lambda_{\mathfrak{I}} \neq \phi. \end{cases}$

Definition 7. [39]. $\mathfrak{I} = \langle \hbar|\tau, \lambda|\upsilon\rangle$ is a PDHFE, then the score function and deviation degree of the PDHFE are recorded as:

$$s(\mathfrak{I}) = \sum_{i=1}^{\#\hbar}\sum_{\gamma\in\hbar}\gamma_i \cdot p_i - \sum_{j=1}^{\#\lambda}\sum_{\gamma\in\lambda}\eta_j \cdot q_j, \tag{11}$$

$$\sigma(\mathfrak{I}) = \left(\sum_{i=1}^{\#\hbar}\sum_{\gamma\in\hbar}(\gamma_i - s)^2\cdot \tau_i + \sum_{j=1}^{\#\lambda}\sum_{\gamma\in\lambda}(\eta_j - s)^2\cdot \upsilon_j\right)^{1/2}. \tag{12}$$

The comparative approach for two PDHFEs is shown as [39]:

If $s(\mathfrak{I}_1) > s(\mathfrak{I}_2)$, then $\mathfrak{I}_1$ is superior to $\mathfrak{I}_2$, denoted by $\mathfrak{I}_1 > \mathfrak{I}_2$; on the contrary, there is $\mathfrak{I}_1 < \mathfrak{I}_2$.

If $s(\mathfrak{I}_1) = s(\mathfrak{I}_2)$, then

(1) If $\sigma(\mathfrak{I}_1) < \sigma(\mathfrak{I}_2)$, the PDHFE $\mathfrak{I}_1$ is superior to $\mathfrak{I}_2$, denoted by $\mathfrak{I}_1 > \mathfrak{I}_2$;

(2) If $\sigma(\mathfrak{I}_1) > \sigma(\mathfrak{I}_2)$, the PDHFE $\mathfrak{I}_1$ is inferior to $\mathfrak{I}_2$, denoted by $\mathfrak{I}_1 < \mathfrak{I}_2$;

(3) If $\sigma(\mathfrak{I}_1) = \sigma(\mathfrak{I}_2)$, the PDHFE $\mathfrak{I}_1$ is equal to $\mathfrak{I}_2$, denoted by $\mathfrak{I}_1 = \mathfrak{I}_2$.

In order to measure the non-fuzziness of an FS, Dumitrescu [64] first introduced the concept of informational energy of FS; it can be regarded as a measure of the degree of non-fuzziness; since the concept was proposed, it was extended to other fuzzy settings.

Definition 8. [43]. Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\}, \{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\}, \{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1, 2,\cdots, M_j$; $t = 1, 2,\cdots, N_j$; $s' = 1, 2,\cdots, M'_j$; $t' = 1, 2,\cdots, N'_j$; $j = 1, 2,\cdots, n$, then the informational energy of the two PDHFSs is recorded as:

$$I(X) = \sum_{j=1}^{n}\left(\sum_{s=1}^{M_j}(\gamma_{s,j})^2 p_{s,j} + \sum_{t=1}^{N_i}(\eta_{t,j})^2 q_{t,j}\right), \tag{13}$$

and

$$I(Y) = \sum_{j=1}^{n}\left(\sum_{s'=1}^{M'_j}(\gamma'_{s',j})^2 p'_{s',j} + \sum_{t'=1}^{N'_j}(\eta'_{t',j})^2 q'_{t',j}\right). \tag{14}$$

2.3 Probabilistic dual hesitant fuzzy aggregation operators

In the subsection, we introduce two classes of operators for PDHFE.

Definition 9. [39]. Let $\Im_i = \langle h_i|p_i, g_i|q_i\rangle (i = 1, 2, \cdots, n)$ be n PDHFEs and their weight be $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ and $\omega_j \in [0, 1]$, $\sum_{j=1}^{n}\omega_j = 1$. Then, the PDHF weighted averaging (PDHFWA) operator is shown as:

$$\text{PDHFWA}(\Im_1, \Im_2, \cdots, \Im_n,) = \bigoplus_{j=1}^{n} \omega_j \Im_j. \tag{15}$$

Depending on the operations of PDHFEs, we can obtain the following result:

$$\begin{aligned} \text{PDHFWA}(\Im_1, \Im_2, \cdots, \Im_n,) &= \bigoplus_{j=1}^{n} \omega_j \Im_j \\ &= \bigcup_{\substack{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n \\ \eta_1 \in g_1, \eta_2 \in g_2, \cdots, \eta_n \in g_n}} \left\{ \left\{ \left(1 - \prod_{j=1}^{n} (1-\gamma_j)^{\omega_j} \right) \middle| \prod_{j=1}^{n} p_{\gamma_j} \right\}, \left\{ \left(\prod_{j=1}^{n} \eta_j^{\omega_j} \right) \middle| \prod_{j=1}^{n} q_{\eta_j} \right\} \right\}. \end{aligned} \tag{16}$$

In view of practical need, we give the definition of PDHFWG operator as follows:

Definition 10. Let $\Im_i = \langle h_i|p_i, g_i|q_i\rangle (i = 1, 2, \cdots, n)$ be n PDHFEs and their weight be $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n}\omega_j = 1$. Then, the PDHFWG operator is recorded as:

$$\text{PDHFWG}(\Im_1, \Im_2, \cdots, \Im_n,) = \bigotimes_{j=1}^{n} \Im_j^{\omega_j}. \tag{17}$$

Depending on the operations of PDHFEs, the aggregated value of PDHFWG operator is a PDHFE as well, shown as:

$$\begin{aligned} \text{PDHFWG}(\Im_1, \Im_2, \cdots, \Im_n,) &= \bigotimes_{j=1}^{n} \Im_j^{\omega_j} \\ &= \bigcup_{\substack{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n \\ \eta_1 \in g_1, \eta_2 \in g_2, \cdots, \eta_n \in g_n}} \left\{ \left\{ \left(\prod_{j=1}^{n} \gamma_j^{\omega_j} \right) \middle| \prod_{j=1}^{n} p_{\gamma_j} \right\}, \left\{ \left(1 - \prod_{j=1}^{n} (1-\eta_j)^{\omega_j} \right) \middle| \prod_{j=1}^{n} q_{\eta_j} \right\} \right\}. \end{aligned} \tag{18}$$

The probabilistic dual hesitant fuzzy weighted geometric (PDHFWG) operator has similar properties with the PDHFWA operator has in reference [39].

3 Some DSMs for PDHFSs

In the MADM problem, many researchers studied the similarity measures, but the more studied similarity measures are DSM and cosine similarity measure (CSM). However, DSM can better describe the similarity relationship between the two variables and overcome the defects of CSM. Therefore, in the next subsection, we will introduce the DSM probabilistic dual hesitant fuzzy environment.

Definition 8. [53]. Let $\wp = (\wp_1, \wp_2, \cdots, \wp_n)$ and $\partial = (\partial_1, \partial_2, \cdots, \partial_n)$, where $\wp_i > 0$ and $\partial_i > 0$, then the DSM $\rho_D(\wp, \partial)$ is recorded as:

$$\rho_D(\wp, \partial) = \frac{2\wp \cdot \partial}{\|\wp\|_2^2 + \|\partial\|_2^2} = \frac{2\sum_{j=1}^{n}\wp_j\partial_j}{\sum_{j=1}^{n}\wp_j^2 + \sum_{j=1}^{n}\partial_j^2}, \tag{19}$$

where $\wp \cdot \partial = \sum_{j=1}^{n}\wp_j\partial_j$ is the inner product between $\wp$ and ∂, $\|\wp\|_2 = \sqrt{\sum_{j=1}^{n}\wp_j^2}$ and $\|\partial\|_2 = \sqrt{\sum_{j=1}^{n}\partial_j^2}$ are the Euclidean norms of $\wp$ and ∂.

The value $\rho_D(\wp, \partial) \in [0, 1]$. Again, $\rho_D(\wp, \partial) = 0$ when $\wp_j = \partial_j = 0$ for all j.

3.1 DSMs for PDHFSs

In the subsection, we extend the DSM to PDHF setting; some DSM and weighted DSM (WDSM) for two PDHFSs depend on the definition of DSM. We reviewed the importance of informational energy in some fuzzy settings; it is obvious that informational energy of FS can measure the non-fuzziness of an FS; this study is inspired by several important studies [54,56–58,62], in the next subsection, we will propose the PDHF DSMs based on the informational energy in PDHF setting.

Definition 9. Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\},\{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1, 2,\cdots, M_j$; $t = 1, 2,\cdots, N_j$; $s' = 1, 2,\cdots, M'_j$; $t' = 1, 2,\cdots, N'_j$; $j = 1, 2,\cdots, n$, then the PDHFDSM between X and Y is recorded as:

$$\text{PDHFDSM}^1(X, Y) = \frac{1}{n}\sum_{j=1}^{n}\frac{2I(X)\cdot I(Y)}{I^2(X) + I^2(Y)}, \tag{20}$$

where $I(X)$ is the informational energy of X in Definition 8.

Example 1. Let $A = \left\{\begin{matrix}\left\langle\begin{matrix}\{0.6224|0.2, 0.6|0.2, 0.5817|0.6\},\\ \{0.3194|1\}\end{matrix}\right\rangle, \left\langle\begin{matrix}\{0.4777|1\},\\ \{0.6|1\}\end{matrix}\right\rangle,\\ \left\langle\begin{matrix}\{0.4338|0.4, 0.4373|0.6\},\\ \left\{\begin{matrix}0.3571|0.28, 0.3912|0.42,\\ 0.3612|0.12, 0.3464|0.18\end{matrix}\right\}\end{matrix}\right\rangle, \left\langle\begin{matrix}\left\{\begin{matrix}0.6224|0.35, 0.6536|0.15,\\ 0.6|0.35, 0.6331|0.15\end{matrix}\right\},\\ \{0.284|1\}\end{matrix}\right\rangle\end{matrix}\right\}$ and

$B = \left\{\begin{matrix}\left\langle\begin{matrix}\left\{\begin{matrix}0.4467|0.06, 0.4024|0.04,\\ 0.4205|0.54, 0.3741|0.36,\end{matrix}\right\},\\ \{0.2199|1\}\end{matrix}\right\rangle, \left\langle\begin{matrix}\{0.4338|0.5, 0.4609|0.5\},\\ \{0.2259|0.5, 0.2|0.5\}\end{matrix}\right\rangle,\\ \left\langle\begin{matrix}\{0.1312|1\},\\ \{0.7483|0.6, 0.6765|0.4\}\end{matrix}\right\rangle, \left\langle\begin{matrix}\left\{\begin{matrix}0.2314|0.42, 0.3344|0.18,\\ 0.2744|0.28, 0.3716|0.12\end{matrix}\right\},\\ \{0.2|1\}\end{matrix}\right\rangle\end{matrix}\right\}$ be two PDHFSs, then we can calculate the PDHFDSM$^1(A, B)$ between A and B as follows:

$$\begin{aligned}\text{PDHFDSM}^1(A, B) &= \frac{1}{n}\sum_{j=1}^{n}\frac{2I(A)\cdot I(B)}{I^2(A) + I^2(B)} = \frac{1}{4}\sum_{j=1}^{n}\frac{2I(A)\cdot I(B)}{I^2(A) + I^2(B)}\\
&= \frac{1}{4}\left(\begin{aligned}&\frac{2\times(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)\times(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)}{(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)^2+(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)^2}\\
&+\frac{2\times(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)\times(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)}{(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)^2+(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)^2}\\
&+\frac{2\times(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)\times(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)}{(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)^2+(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)^2}\\
&+\frac{2\times(0.6224^2\times 0.35+\cdots+0.284^2\times 1)\times(0.2314^2\times 0.42+\cdots+0.2^2\times 1)}{(0.6224^2\times 0.35+\cdots+0.284^2\times 1)^2+(0.2314^2\times 0.42+\cdots+0.2^2\times 1)^2}\end{aligned}\right)\\
&= \frac{1}{4}\left(\frac{2\times 0.4546\times 0.2127}{0.4546^2+0.2127^2}+\frac{2\times 0.2631\times 0.248}{0.2631^2+0.248^2}+\frac{2\times 0.3419\times 0.5363}{0.3419^2+0.5363^2}\right.\\
&\quad\left.+\frac{2\times 0.4664\times 0.1203}{0.4664^2+0.1203^2}\right)\\
&= 0.789.\end{aligned}$$

The PDHFDSM between PDHFSs fulfills the following three properties:

(1) $0 \le \text{PDHFDSM}^1(X, Y) \le 1$;

(2) $\text{DHFDSM}^1(X, Y) = \text{DHFDSM}^1(Y, X)$;

(3) $\text{DHFDSM}^1(X, Y) = 1$, if $X = Y$.

If we consider the importance of the weight of each PDHFE in PDHFDSM, then the probabilistic dual hesitant fuzzy weighted DSM (PDHFWDSM) between X and Y is recorded as follows.

Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\},\{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1,2,\cdots,M_j$; $t = 1,2,\cdots,N_j$; $s' = 1,2,\cdots,M'_j$; $t' = 1,2,\cdots,N'_j$; $j = 1,2,\cdots,n$, ω_j is the weight of jth PDHFE with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$, then the PDHFWDSM between X and Y is recorded as:

$$\mathrm{PDHFWDSM}^1(X,Y) = \sum_{j=1}^{n}\omega_j\frac{2I(X)\cdot I(Y)}{I^2(X)+I^2(Y)}. \tag{21}$$

Particularly, when $\omega = \left(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n}\right)^T$, then the PDHFWDSM^1 reduced to the PDHFDSM^1, and there is PDHFWDSM$^1(X,Y)$ = PDHFDSM$^1(X,Y)$.

Example 2. We take the data in Example 1, where $\omega = (0.1, 0.3, 0.2, 0.4)^T$, then the PDHFWDSM$^1(A,B)$ between A and B is computed as follows:

$$\begin{aligned}
\mathrm{PDHFWDSM}^1(A,B) &= \sum_{j=1}^{n}\omega_j\frac{2I(A)\cdots(B)}{I^2(A)+I^2(B)}\\
&= \left(\begin{array}{l}
0.1\times\frac{2\times(0.6224^2\times0.2+\cdots+0.3194^2\times1)\times(0.4467^2\times0.06+\cdots+0.2199^2\times1)}{(0.6224^2\times0.2+\cdots+0.3194^2\times1)^2+(0.4467^2\times0.06+\cdots+0.2199^2\times1)^2}\\
+0.3\times\frac{2\times(0.4777^2\times1+\cdots+0.2056^2\times0.2)\times(0.4338^2\times0.5+\cdots+0.2^2\times0.5)}{(0.4777^2\times1+\cdots+0.2056^2\times0.2)^2+(0.4338^2\times0.5+\cdots+0.2^2\times0.5)^2}\\
+0.2\times\frac{2\times(0.4838^2\times0.4+\cdots+0.3464^2\times0.18)\times(0.1312^2\times1+\cdots+0.6765^2\times0.4)}{(0.4838^2\times0.4+\cdots+0.3464^2\times0.18)^2+(0.1312^2\times1+\cdots+0.6765^2\times0.4)^2}\\
+0.4\times\frac{2\times(0.6224^2\times0.35+\cdots+0.284^2\times1)\times(0.2314^2\times0.42+\cdots+0.2^2\times1)}{(0.6224^2\times0.35+\cdots+0.284^2\times1)^2+(0.2314^2\times0.42+\cdots+0.2^2\times1)^2}
\end{array}\right)\\
&= \left(\begin{array}{l}
0.1\times\frac{2\times0.4546\times0.2127}{0.4546^2+0.2127^2}+0.3\times\frac{2\times0.2631\times0.248}{0.2631^2+0.248^2}\\
+0.2\times\frac{2\times0.3419\times0.5363}{0.3419^2+0.5363^2}+0.4\times\frac{2\times0.4664\times0.1203}{0.4664^2+0.1203^2}
\end{array}\right)\\
&= 0.751.
\end{aligned}$$

It is obvious that the PDHFWDSM$^1(X,Y)$ also fulfills the following three properties:

(1) $0 \le$ PDHFWDSM$^1(X,Y) \le 1$;

(2) DHFWDSM$^1(X,Y)$ = DHFWDSM$^1(Y,X)$;

(3) DHFWDSM$^1(X,Y) = 1$, if $X = Y$.

3.2 Another form of DSM for PDHFSs

In the subsection, another form of DSM for PDHFSs is designed as follows.

Definition 10. Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\},\{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1,2,\cdots,M_j$; $t = 1,2,\cdots,N_j$; $s' = 1,2,\cdots,M'_j$; $t' = 1,2,\cdots,N'_j$; $j = 1,2,\cdots,n$, then another form of PDHF DSM (PDHFDSM) between X and Y is defined as:

$$\mathrm{PDHFDSM}^2(X,Y) = \frac{2\sum_{j=1}^{n}I(X)\cdot I(Y)}{\sum_{j=1}^{n}(I^2(X)+I^2(Y))}. \tag{22}$$

Example 3. We take the data in Example 1, then the PDHFDSM$^2(A,B)$ can be calculated as:

$$\mathrm{PDHFDSM}^2(A,B) = \frac{2\sum_{j=1}^{n} I(A)\cdot I(B)}{\sum_{j=1}^{n}(I^2(A)+I^2(B))}$$

$$= \frac{2\times\begin{pmatrix}(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)\times(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)\\ +(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)\times(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)\\ +(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)\times(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)\\ +(0.6224^2\times 0.35+\cdots+0.284^2\times 1)\times(0.2314^2\times 0.42+\cdots+0.2^2\times 1)\end{pmatrix}}{\begin{pmatrix}(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)^2+(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)^2\\ +(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)^2+(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)^2\\ +(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)^2+(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)^2\\ +(0.6224^2\times 0.35+\cdots+0.284^2\times 1)^2+(0.2314^2\times 0.42+\cdots+0.2^2\times 1)^2\end{pmatrix}}$$

$$= 0.7877.$$

Another form of PDHFDSM between X and Y also fulfills the following three properties:

(1) $0 \le \mathrm{PDHFDSM}^2(X,Y) \le 1$;
(2) $\mathrm{DHFDSM}^2(X,Y) = \mathrm{DHFDSM}^2(Y,X)$;
(3) $\mathrm{DHFDSM}^2(X,Y) = 1$, if $X = Y$.

If we consider the importance of the weight of each PDHFE in PDHFDSM, then the PDHFWDSM between X and Y is recorded as follows.

Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s'}\in h_Y,\eta'_{s'}\in g_Y}\langle\{\gamma'_{s'}|p'_{s'}\},\{\eta'_{t'}|q'_{t'}\}\rangle$ be two PDHFSs, where $s = 1,2,\cdots,M_j$; $t = 1,2,\cdots,N_j$; $s' = 1,2,\cdots,M'_j$; $t' = 1,2,\cdots,N'_j$; $j = 1,2,\cdots,n$, ω_j is the weight of jth PDHFE with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$, then the PDHFWDSM between X and Y is recorded as:

$$\mathrm{PDHFWDSM}^2(X,Y) = \frac{2\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\sum_{j=1}^{n}\omega_j^2(I^2(X)+I^2(Y))}. \tag{23}$$

Particularly, when $\omega = \left(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n}\right)^T$, then the $\mathrm{PDHFWDSM}^2$ reduced to the $\mathrm{PDHFDSM}^2$, and there is $\mathrm{PDHFWDSM}^2(X,Y) = \mathrm{PDHFDSM}^2(X,Y)$.

Example 4. We take the data in Example 1, where $\omega = (0.1, 0.3, 0.2, 0.4)^T$, then the $\mathrm{PDHFDSM}^2(A,B)$ between A and B is calculated as follows.

$$\mathrm{PDHFWDSM}^2(A,B) = \frac{2\sum_{j=1}^{n}\omega_j^2 I(A)\cdot I(B)}{\sum_{j=1}^{n}\omega_j^2(I^2(A)+I^2(B))}$$

$$= \frac{2\times\begin{pmatrix}0.1^2\times(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)\times(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)\\ +0.3^2\times(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)\times(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)\\ +0.2^2\times(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)\times(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)\\ +0.4^2\times(0.6224^2\times 0.35+\cdots+0.284^2\times 1)\times(0.2314^2\times 0.42+\cdots+0.2^2\times 1)\end{pmatrix}}{\begin{pmatrix}0.1^2\times((0.6224^2\times 0.2+\cdots+0.3194^2\times 1)^2+(0.4467^2\times 0.06+\cdots+0.2199^2\times 1))\\ +0.3^2\times((0.4777^2\times 1+\cdots+0.2056^2\times 0.2)^2+(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)^2)\\ +0.2^2\times((0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)^2+(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)^2)\\ +0.4^2\times((0.6224^2\times 0.35+\cdots+0.284^2\times 1)^2+(0.2314^2\times 0.42+\cdots+0.2^2\times 1)^2)\end{pmatrix}}$$

$$= 0.685.$$

It is obvious that $\mathrm{PDHFWDSM}^2(X,Y)$ has the following three properties:

(1) $0 \le \mathrm{PDHFWDSM}^2(X,Y) \le 1$;
(2) $\mathrm{DHFWDSM}^2(X,Y) = \mathrm{DHFWDSM}^2(Y,X)$;
(3) $\mathrm{DHFWDSM}^2(X,Y) = 1$, if $X = Y$.

3.3 The generalized DSM for PDHFSs

In such subsection, the probabilistic dual hesitant fuzzy generalized DSM (PDHFGDSM) between X and Y as the generalization of DSM is shown as follows.

Definition 11. Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\},\{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1, 2,\cdots, M_j$; $t = 1, 2,\cdots, N_j$; $s' = 1, 2,\cdots, M'_j$; $t' = 1, 2,\cdots, N'_j$; $j = 1, 2,\cdots, n$, then the PDHF generalized DSM (PDHFGDSM) between X and Y is recorded as:

$$\text{PDHFGDSM}^1(X, Y) = \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)}, \tag{24}$$

$$\text{PDHFGDSM}^2(X, Y) = \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n} I^2(X) + (1-\lambda)\sum_{j=1}^{n} I^2(Y)}, \tag{25}$$

where λ is a nonnegative real parameter for $0 \le \lambda \le 1$.

Example 5. We take the data in Example 1, where $\lambda = 0.4$, then the PDHFDSM$^2(A, B)$ between A and B is computed as follows:

$$\begin{aligned}\text{PDHFGDSM}^2(A, B) &= \frac{\sum_{j=1}^{n} I(A)\cdot I(B)}{\lambda\sum_{j=1}^{n} I^2(A) + (1-\lambda)\sum_{j=1}^{n} I^2(B)}\\ &= \frac{\left(\begin{array}{l}(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)\times(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)\\ \quad+(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)\times(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)\\ \quad+(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)\times(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)\\ \quad+(0.6224^2\times 0.35+\cdots+0.284^2\times 1)\times(0.2314^2\times 0.42+\cdots+0.2^2\times 1)\end{array}\right)}{\left(\begin{array}{l}0.4\times(0.6224^2\times 0.2+\cdots+0.3194^2\times 1)^2+0.6\times(0.4467^2\times 0.06+\cdots+0.2199^2\times 1)^2\\ \quad+0.4\times(0.4777^2\times 1+\cdots+0.2056^2\times 0.2)^2+0.6\times(0.4338^2\times 0.5+\cdots+0.2^2\times 0.5)^2\\ \quad+0.4\times(0.4838^2\times 0.4+\cdots+0.3464^2\times 0.18)^2+0.6\times(0.1312^2\times 1+\cdots+0.6765^2\times 0.4)^2\\ \quad+0.4\times(0.6224^2\times 0.35+\cdots+0.284^2\times 1)^2+0.6\times(0.2314^2\times 0.42+\cdots+0.2^2\times 1)^2\end{array}\right)}\\ &= 0.8202.\end{aligned}$$

Then, the PDHFGDSM embodies some particular cases by adjusting the nonnegative real parameter λ.

(1) If $\lambda = 0.5$, then the two PDHFGDSM^1 (24) and PDHFGDSM^2 (25) reduced to PDHFDSM^1 (20) and PDHFDSM^2 (22), respectively.

$$\begin{aligned}\text{PDHFGDSM}^1(X, Y) &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{0.5I^2(X) + (1-0.5)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{2I(X)\cdot I(Y)}{I^2(X) + I^2(Y)} = \text{PDHFDSM}^1(X, Y),\end{aligned} \tag{26}$$

$$\begin{aligned}\text{PDHFGDSM}^2(X,Y) &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n} I^2(X) + (1-\lambda)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{0.5\sum_{j=1}^{n} I^2(X) + (1-0.5)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{2\sum_{j=1}^{n} I(X)\cdot I(Y)}{\sum_{j=1}^{n} I^2(X) + \sum_{j=1}^{n} I^2(Y)} = \text{PDHFDSM}^2(X,Y).\end{aligned} \tag{27}$$

(2) If $\lambda = 0$, then the two PDHFGDSM^1 (24) and PDHFGDSM^2 (25) reduced to the following asymmetric DSMs, respectively.

$$\begin{aligned}\text{PDHFGDSM}^1(X,Y) &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{0I^2(X) + (1-0)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{I^2(Y)},\end{aligned} \tag{28}$$

$$\begin{aligned}\text{PDHFGDSM}^2(X,Y) &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n} I^2(X) + (1-\lambda)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{0\sum_{j=1}^{n} I^2(X) + (1-0)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{2\sum_{j=1}^{n} I(X)\cdot I(Y)}{\sum_{j=1}^{n} I^2(Y)} = \text{PDHFDSM}^2(X,Y).\end{aligned} \tag{29}$$

(3) If $\lambda = 1$, then the two PDHFGDSM^1 (24) and PDHFGDSM^2 (25) reduced to the following asymmetric DSMs, respectively.

$$\begin{aligned}\text{PDHFGDSM}^1(X,Y) &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{I(X)\cdot I(Y)}{1I^2(X) + (1-1)I^2(Y)}\\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{2I(X)\cdot I(Y)}{I^2(X)},\end{aligned} \tag{30}$$

$$\begin{aligned}\text{PDHFGDSM}^2(X,Y) &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n} I^2(X) + (1-\lambda)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{\sum_{j=1}^{n} I(X)\cdot I(Y)}{1\sum_{j=1}^{n} I^2(X) + (1-1)\sum_{j=1}^{n} I^2(Y)}\\ &= \frac{2\sum_{j=1}^{n} I(X)\cdot I(Y)}{\sum_{j=1}^{n} I^2(X)}.\end{aligned} \tag{31}$$

In light of the aforementioned detailed analysis, we can find that four proposed asymmetrical DSMs are the homologous expansion of the PDHF relative projection measure.

If we consider the importance of the weight of each PDHFE in PDHFGDSM, then the probabilistic dual hesitant fuzzy weighted generalized Dice similarity measure (PDHFWGDSM) between X and Y is shown as follows:

Let $X = \langle h_X|p_X, g_X|q_X\rangle = \cup_{\gamma_{s,j}\in h_X,\eta_{t,j}\in g_X}\langle\{\gamma_{s,j}|p_{s,j}\},\{\eta_{t,j}|q_{t,j}\}\rangle$ and $Y = \langle h_Y|p_Y, g_Y|q_Y\rangle = \cup_{\gamma'_{s',j}\in h_Y,\eta'_{t',j}\in g_Y}\langle\{\gamma'_{s',j}|p'_{s',j}\},\{\eta'_{t',j}|q'_{t',j}\}\rangle$ be two PDHFSs, where $s = 1,2,\cdots,M_j$; $t = 1,2,\cdots,N_j$; $s' = 1,2,\cdots,M'_j$; $t' = 1,2,\cdots,N'_j$; $j = 1,2,\cdots,n$, ω_j is the weight of jth PDHFE with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$, then the PDHFWGDSM between X and Y is recorded as:

$$\text{PDHFWGDSM}^1(X, Y) = \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)}, \tag{32}$$

$$\text{PDHFWGDSM}^2(X, Y) = \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-\lambda)\sum_{j=1}^{n}\omega_j^2 I^2(Y)}. \tag{33}$$

Particularly, when $\omega = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)^T$, then the PDHFWGDSM reduced to the PDHFGDSM, and there is $\text{PDHFWGDSM}^\kappa(X, Y) = \text{PDHFGDSM}^\kappa(X, Y)(\kappa = 1, 2)$.

Example 6. We take the data in Example 1, where $\lambda = 0.4$ and $\omega = (0.1, 0.3, 0.2, 0.4)^T$, then the $\text{PDHFDSM}^2(A, B)$ between A and B is calculated as follows:

$$\begin{aligned}
\text{PDHFWGDSM}^2(A, B) &= \frac{\sum_{j=1}^{n}\omega_j^2 I(A)\cdot I(B)}{\lambda\sum_{j=1}^{n}\omega_j^2 I^2(A) + (1-\lambda)\sum_{j=1}^{n}\omega_j^2 I^2(B)} \\
&= \frac{\left(\begin{array}{l} 0.1^2 \times ((0.6224^2 \times 0.2 + \cdots + 0.3194^2 \times 1) \times (0.4467^2 \times 0.06 + \cdots + 0.2199^2 \times 1)) \\ \quad +0.3^2 \times ((0.4777^2 \times 1 + \cdots + 0.2056^2 \times 0.2) \times (0.4338^2 \times 0.5 + \cdots + 0.2^2 \times 0.5)) \\ \quad +0.2^2 \times ((0.4838^2 \times 0.4 + \cdots + 0.3464^2 \times 0.18) \times (0.1312^2 \times 1 + \cdots + 0.6765^2 \times 0.4)) \\ \quad +0.4^2 \times ((0.6224^2 \times 0.35 + \cdots + 0.284^2 \times 1) \times (0.2314^2 \times 0.42 + \cdots + 0.2^2 \times 1)) \end{array}\right)}{\left(\begin{array}{l} 0.4 \times 0.1^2 \times (0.6224^2 \times 0.2 + \cdots + 0.3194^2 \times 1)^2 + 0.6 \times 0.1^2 \\ \quad \times (0.4467^2 \times 0.06 + \cdots + 0.2199^2 \times 1)^2 \\ \quad +0.4 \times 0.3^2 \times (0.4777^2 \times 1 + \cdots + 0.2056^2 \times 0.2)^2 + 0.6 \times 0.3^2 \\ \quad \times (0.4338^2 \times 0.5 + \cdots + 0.2^2 \times 0.5)^2 \\ \quad +0.4 \times 0.2^2 \times (0.4838^2 \times 0.4 + \cdots + 0.3464^2 \times 0.18)^2 \\ \quad + 0.6 \times 0.2^2 \times (0.1312^2 \times 1 + \cdots + 0.6765^2 \times 0.4)^2 \\ \quad +0.4 \times 0.4^2 \times (0.6224^2 \times 0.35 + \cdots + 0.284^2 \times 1)^2 + 0.6 \times 0.4^2 \\ \quad \times (0.2314^2 \times 0.42 + \cdots + 0.2^2 \times 1)^2 \end{array}\right)} \\
&= 0.8172.
\end{aligned}$$

Then, the PDHFGDSM embodies some particular cases by adjusting the nonnegative real parameter λ.

(1) If $\lambda = 0.5$, then the two PDHFWGDSM^1 (32) and PDHFWGDSM^2 (33) reduced to PDHFWDSM^1 (21) and PDHFWDSM^2 (23), respectively.

$$\begin{aligned}
\text{PDHFWGDSM}^1(X, Y) &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)} \\
&= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{0.5I^2(X) + (1-0.5)I^2(Y)} \\
&= \sum_{j=1}^{n} \omega_j \frac{2I(X)\cdot I(Y)}{I^2(X) + I^2(Y)} = \text{PDHFWDSM}^1(X, Y),
\end{aligned} \tag{34}$$

$$\begin{aligned}
\text{PDHFWGDSM}^2(X, Y) &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-\lambda)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\
&= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{0.5\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-0.5)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\
&= \frac{2\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\sum_{j=1}^{n}\omega_j^2 (I^2(X) + I^2(Y))} = \text{PDHFWDSM}^2(X, Y).
\end{aligned} \tag{35}$$

(2) If $\lambda = 0$, then the two PDHFGDSM[1] (24) and PDHFGDSM[2] (25) reduced to the following asymmetric DSMs, respectively.

$$\begin{aligned} \mathrm{PDHFWGDSM}^1(X,Y) &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)} \\ &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{0I^2(X) + (1-0)I^2(Y)} \\ &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{I^2(Y)}, \end{aligned} \tag{36}$$

$$\begin{aligned} \mathrm{PDHFWGDSM}^2(X,Y) &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-\lambda)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\ &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{0\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-0)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\ &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\sum_{j=1}^{n}\omega_j^2 I^2(Y)}. \end{aligned} \tag{37}$$

(3) If $\lambda = 1$, then the two PDHFGDSM[1] (24) and PDHFGDSM[2] (25) reduced to the following asymmetric DSMs, respectively.

$$\begin{aligned} \mathrm{PDHFWGDSM}^1(X,Y) &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{\lambda I^2(X) + (1-\lambda)I^2(Y)} \\ &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{1I^2(X) + (1-1)I^2(Y)} \\ &= \sum_{j=1}^{n} \omega_j \frac{I(X)\cdot I(Y)}{I^2(X)}, \end{aligned} \tag{36}$$

$$\begin{aligned} \mathrm{PDHFWGDSM}^2(X,Y) &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\lambda\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-\lambda)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\ &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{1\sum_{j=1}^{n}\omega_j^2 I^2(X) + (1-1)\sum_{j=1}^{n}\omega_j^2 I^2(Y)} \\ &= \frac{\sum_{j=1}^{n}\omega_j^2 I(X)\cdot I(Y)}{\sum_{j=1}^{n}\omega_j^2 I^2(X)}. \end{aligned} \tag{37}$$

In light of the aforementioned detailed analysis, we can find that four proposed asymmetrical DSMs are the homologous expansion of the PDHF relative projection measure.

4 The PDHF MAGDM technique based on weighted GDSM

Assume an MAGDM issue with m alternatives and n attributes $C = \{C_1, C_2, \cdots, C_n\}$, whose weight is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ and satisfies $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n}\omega_j = 1$, and $e = \{e_1, e_2, \cdots, e_p\}$ be a group of experts, whose weight is $\theta = \{\theta_1, \theta_2, \cdots, \theta_p\}$, where $\theta_k \in [0,1]$, $k = 1, 2, \cdots, p$, $\sum_{k=1}^{p}\theta_k = 1$. Hence, we can obtain the matrix $\mathfrak{I}^k = [\mathfrak{I}_{ij}^k]_{m\times n} = [\langle h_{ij}^k | p_{ij}^k, g_{ij}^k | q_{ij}^k \rangle]_{m\times n}$, which is given by the expert e_k, and where $h_{ij}^k \in [0,1]$, $p_{ij}^k \in [0,1]$, $g_{ij}^k \in [0,1]$, and $q_{ij}^k \in [0,1]$. There is $0 \le \gamma_{ij}^k, \eta_{ij}^k \le 1$, $0 \le (\gamma_{ij}^k)^+ + (\eta_{ij}^k)^+ \le 1$, in which $\gamma_{ij}^k \in h_{ij}^k, \eta_{ij}^k \in g_{ij}^k$, $(\gamma_{ij}^k)^+ \in (h_{ij}^k)^+ = \cup_{\gamma_{ij}^k \in h_{ij}^k} \max \gamma_{ij}^k$, and $(\eta_{ij}^k)^+ \in (g_{ij}^k)^+ = \cup_{\eta_{ij}^k \in g_{ij}^k} \max \eta_{ij}^k$.

4.1 Phase 1. Normalize and aggregate decision information matrices from different experts

***Step* 1.** We use equations (38) and (39) to normalize attributes, then a new matrix can be obtained and shown as: $R^k = [r_{ij}^k]_{m\times n}(i = 1, 2, \cdots, m; \quad i = 1, 2, \cdots, n; \quad k = 1, 2, \cdots, p)$.

$$r_{ij}^k = \langle \hbar_{ij}^k | \iota_{ij}^k, \lambda_{ij}^k | \sigma_{ij}^k \rangle = \begin{cases} d_{ij}^k = \langle h_{ij}^k | p_{ij}^k, g_{ij}^k | q_{ij}^k \rangle & \text{if } C_j \in N_b \\ (d_{ij}^k)^c = \langle g_{ij}^k | q_{ij}^k, h_{ij}^k | p_{ij}^k \rangle & \text{if } C_j \in N_c, \end{cases} \tag{38}$$

where N_b indicates the set of beneficial attributes and N_b indicates the set of cost attributes.

$$R^k = [r_{ij}^k]_{m\times n} = \begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^k & r_{m2}^k & \cdots & r_{mn}^k \end{bmatrix}. \tag{39}$$

***Step* 2.** Aggregate the PDHF decision information matrices from several experts to construct a fresh decision information matrix by equation (40). The aggregated PDHF decision information matrix is recorded as $V = [v_{ij}]_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$.

$$V = [v_{ij}]_{m\times n} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix},$$

$$\begin{aligned} v_{ij} &= \langle \overline{h}_{ij} | \overline{p}_{ij}, \overline{g}_{ij} | \overline{q}_{ij} \rangle = \mathrm{PDHFWA}_\theta(r_{ij}^1, r_{ij}^2, \cdots, r_{ij}^p) = \mathop{\oplus}_{k=1}^{n} \theta_k r_{ij}^k \\ &= \bigcup_{\substack{\gamma_{ij}^1 \in \hbar_{ij}^1, \gamma_{ij}^2 \in \hbar_{ij}^2, \cdots, \gamma_{ij}^p \in \hbar_{ij}^p \\ \eta_{ij}^1 \in \lambda_{ij}^1, \eta_{ij}^1 \in \lambda_{ij}^1, \cdots, \eta_{ij}^1 \in \lambda_{ij}^1}} \left\{ \left\{ \left(1 - \prod_{k=1}^{p} (1 - \gamma_{ij}^k)^{\theta_k} \right) \middle| \prod_{k=1}^{p} p_{ij}^k \right\}, \left\{ \left(\prod_{k=1}^{p} (\eta_{ij}^k)^{\theta_k} \right) \middle| \prod_{j=1}^{p} q_{ij}^k \right\} \right\}. \\ & i = 1, 2, \cdots, m; \quad j = 1, 2, \cdots, n \end{aligned} \tag{40}$$

4.2 Phase 2. Determine the weight

***Step* 3.** Obtain $\omega_j(j = 1, 2, \cdots, n)$ by using equations (41)–(45).

(1) Determine the information energy $I(v_{ij})$ by the following equation:

$$I(v_{ij}) = \sum_{k=1}^{\#\overline{h}_{ij}} (\overline{\gamma}_{ijk})^2 \overline{p}_{ijk} + \sum_{k=1}^{\#\overline{g}_{ij}} (\overline{\eta}_{ijk})^2 \overline{q}_{ijk}. \tag{41}$$

(2) Calculate the total entropy by the following equation:

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} \pi_{ij} \ln \pi_{ij}, \tag{42}$$

where $\pi_{ij} = \frac{I(v_{ij}) - \min_{1\le i\le m}(I(v_{ij}))}{\max_{1\le i\le m}(I(v_{ij})) - \min_{1\le i\le m}(I(v_{ij}))}$.

(3) Calculate the weight η_j by the following equation:

$$\eta_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)}. \tag{43}$$

(4) Calculate combined weights by the MIIP.

$w = (w_1, w_2, \cdots, w_n)^T$ is subjective weight obtained by experts, where $\sum_{j=1}^{n} w_j = 1, 0 \le w_j \le 1$. The objective weight $\eta = (\eta_1, \eta_2, \cdots, \eta_n)^T$ is obtained by equations (41)–(43), where $\sum_{j=1}^{n} \eta_j = 1, 0 \le \eta_j \le 1$. To let the combined weight reflect each weighting method, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$, $\eta = (\eta_1, \eta_2, \cdots, \eta_n)^T$, and $w = (w_1, w_2, \cdots, w_n)^T$ should be as close as possible. According to the MIIP, we can obtain

$$\begin{cases} \min F = \sum_{j=1}^{n} \omega_j \ln \frac{\omega_j}{\eta_j} + \sum_{j=1}^{n} \omega_j \ln \frac{\omega_j}{w_j} \\ s.t. \quad \sum_{j=1}^{n} \omega_j = 1, \omega_j \ge 0. \end{cases} \tag{44}$$

(5) According to the Lagrange multiplier method, the combined weight is calculated as follows:

$$\omega_j = \frac{\sqrt{\eta_j w_j}}{\sum_{j=1}^{n} \sqrt{\eta_j w_j}}. \tag{45}$$

4.3 Phase 3. By the maximum DSM for acquiring the final ranking

Step 4. Obtain the PDHF positive-ideal solution (PIS) by equations (46) and (47).

$$X^{+} = [X_j^{+}]_{1\times n}, \tag{46}$$

$$X_j^{+} = \max_i v_{ij}, \tag{47}$$

where $\max_i v_{ij} = \max_i s(v_{ij})$.

Step 6: Calculate the similarity measure between each alternative and the PIS by equations (48) and (49).

$$\text{PDHFWGDSM}^1(X_i, X^{+}) = \sum_{j=1}^{n} \omega_j \frac{I(X_i) \cdot I(X^{+})}{\lambda I^2(X_i) + (1-\lambda) I^2(X^{+})} \tag{48}$$

or

$$\text{PDHFWGDSM}^2(X_i, X^{+}) = \frac{\sum_{j=1}^{n} \omega_j^2 I(X_i) \cdot I(X^{+})}{\lambda \sum_{j=1}^{n} \omega_j^2 I^2(X_i) + (1-\lambda) \sum_{j=1}^{n} \omega_j^2 I^2(X^{+})}. \tag{49}$$

Step 7: Obtain the ranking of all alternatives by $\text{PDHFWGDSM}^1(X_i, X^{+})$ or $\text{PDHFWGDSM}^2(X_i, X^{+})$ ($i = 1, 2, 3, 4$) from big to small, the alternative with the maximum DSM is the optimal alternative (Figure 1).

5 A case study and comparative analyses

5.1 Background

Since the end of the twentieth century, the development of China's foreign trade has experienced from the 1.0 era of exhibition channel business development, the 2.0 era of Internet B2B (enterprise to enterprise) platform interaction, the 3.0 era of digital marketing channel, to today's 4.0 era of Internet big data and cloud computing. At the same time, affected by the international financial crisis, the global supply chain tends to be flat and the world trade pattern is unpredictable. In this context, China's traditional foreign trade model has been

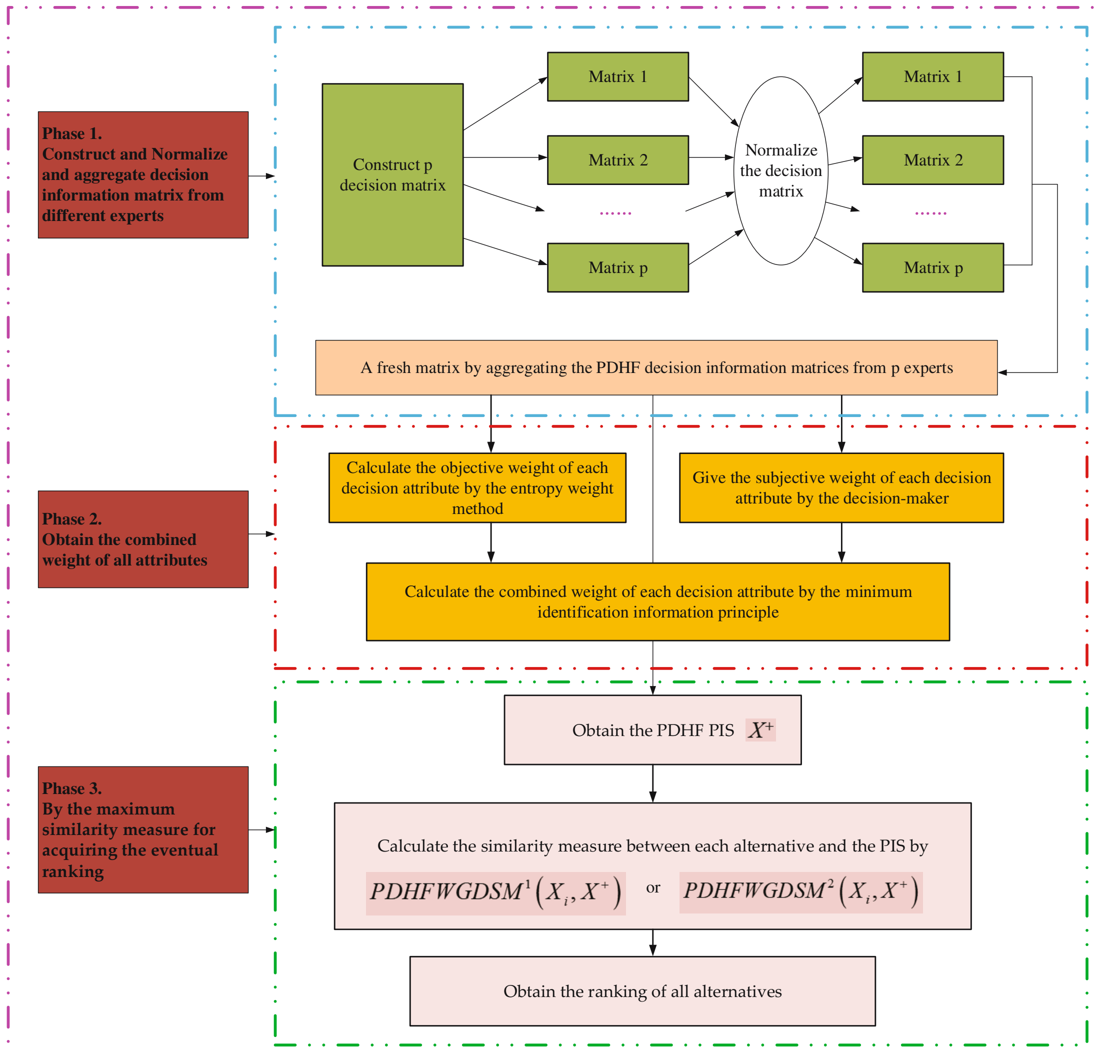

Figure 1: Flowchart of the built PDHF MAGDM method.

seriously challenged. If CBECs want to survive and develop, it is urgent to transform CBEC. However, despite the rapid development of China's CBEC in recent years, the application level of China's CBECs is still not high enough, or copy other successful enterprise cases, or blindly follow the trend, lacking certain scientificity. Therefore, the research of this article becomes very meaningful.

5.2 Decision process

We use the MAGDM method proposed in this article to demonstrate the effectiveness of the MAGDM method by using CBECP as an example for Company W in Guiyang City, Guizhou Province. Since Company W issued a bidding announcement requiring the CBECP, four companies with CBECP from Beijing, Wuhan, Chengdu, and Shenzhen have participated in the bidding. In order to evaluate the four companies more scientifically and reasonably, Company W selected three experts in the industry to rate the four companies from five aspects: system functions, service quality, platform, information quality, and cost. The data are in the form of PDHFEs, and four evaluation indexes are explained as follows:

(1) System functions. The system function is to measure the characteristics required by the e-commerce system, such as ease of learning, safety and reliability, data analysis function, whether the system supports

ERP, and flexibility is the system function quality that users pay more attention to. System function is an important function of e-commerce system.

(2) Service quality. Platform service is the basis for the successful exchange of products and funds between buyers and sellers, and the most important function of CBECP. It determines whether business flow, capital flow, information flow, and logistics can flow smoothly between buyers and sellers; the service quality of the platform plays a critical role in elevating consumers to use the platform again. The integrity, convenience, reliability, security of platform services and consumers' own shopping experience will indirectly affect the order conversion rate by affecting consumers' satisfaction. This is also a key factor for enterprises to consider when choosing a platform.

(3) Platform and information quality. Platform quality is the positioning characteristic of the platform. Whether the visits of the platform, the customer groups, and market distribution of the platform match the enterprise's target market, as well as the main hot selling product types, platform marketing methods and platform operation management methods of the platform, can promote the successful promotion and trading of enterprise products. Information quality is the content of e-commerce. Intelligibility, accuracy, completeness, and timeliness are the determinants of high-quality information. If the buyer and the seller conduct transactions through the e-commerce platform, the platform must effectively transmit the latest and comprehensive product information and industry information between the supplier and the demander, so as to achieve the purpose of smooth information flow between the supplier and the demander, so as to realize the platform transaction, which is the basis of the characteristics of the platform.

(4) Operating cost. The platform cost is based on the purpose of the buyer and the seller to realize the transaction with the platform as the medium, and the platform collects a certain proportion of operation and management expenses from both parties or one of them, such as platform entry fee, transaction commission, transaction handling fee, and marketing promotion fee. The charging items of different platforms are also different. Platform charging is not only related to the early investment of the enterprise's operating cost, but also one of the important benchmarks to measure the enterprise's final profit.

There are four CBECPs $X_i(i = 1, 2, \cdots, 4)$, and the following four decision attributes are selected for assessment: C_1 − system functions, C_2 − service quality, C_3 − platform and information quality, and C_4 − cost, whose subject weight is $w = (0.1, 0.2, 0.3, 0.4)^T$ and given by DMs. The three experts $e_i(i = 1, 2, 3)$ made corresponding evaluation on the four CBECPs, whose weight is $\theta = (0.2, 0.3, 0.5)^T$. Using the method given in this article, the four CBECPs are ranked and optimized, and the optimal platform with the best assessment value is selected. All assessment values are given in Tables 1–3.

Next, we shall give some detailed steps to show the process of PDHF MAGDM technique that is used to select the best CBECP for Company W.

Table 1: PDHF decision information matrix given by e_1

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.7\|0.2, 0.6\|0.2, 0.5\|0.6\}, \{0.2\|1\}\rangle$	$\langle\{0.7\|1\}, \{0.25\|1\}\rangle$	$\langle\{0.2\|1\}, \{0.2\|1\}\rangle$	$\langle\{0.7\|0.5, 0.6\|0.5\}, \{0.3\|1\}\rangle$
X_2	$\langle\{0.6\|1\}, \{0.35\|1\}\rangle$	$\langle\{0.56\|1\}, \{0.2\|1\}\rangle$	$\langle\{0.1\|1\}, \{0.7\|1\}\rangle$	$\langle\{0.2\|0.6, 0.4\|0.4\}, \{0.4\|1\}\rangle$
X_3	$\langle\{0.05\|0.7, 0.2\|0.3\}, \{0.5\|1\}\rangle$	$\langle\{0.3\|0.5, 0.2\|0.5\}, \{0.6\|0.5, 0.5\|0.5\}\rangle$	$\langle\{0.8\|1\}, \{0.15\|1\}\rangle$	$\langle\{0.2\|1\}, \{0.6\|1\}\rangle$
X_4	$\langle\{0.08\|1\}, \{0.6\|1\}\rangle$	$\langle\{0.2\|0.6, 0.1\|0.4\}, \{0.5\|1\}\rangle$	$\langle\{0.3\|1\}, \{0.4\|1\}\rangle$	$\langle\{0.1\|1\}, \{0.2\|0.3, 0.4\|0.7\}\rangle$

Table 2: PDHF decision information matrix given by e_2

Alternatives	C_1	C_2	C_3	C_4
X_1	⟨{0.6|1}, {0.3|1}⟩	⟨{0.4|1}, {0.4|0.8, 0.6|0.2}⟩	⟨{0.7|0.4, 0.6|0.6}, {0.3|0.7, 0.2|0.3}⟩	⟨{0.6|0.7, 0.7|0.3}, {0.25|1}⟩
X_2	⟨{0.4|0.1, 0.3|0.9}, {0.6|1}⟩	⟨{0.2|0.5, 0.3|0.5}, {0.3|0.5, 0.2|0.5}⟩	⟨{0.2|1}, {0.7|0.6, 0.5|0.4}⟩	⟨{0.3|1}, {0.4|1}⟩
X_3	⟨{0.2|1}, {0.4|0.9, 0.3|0.1}⟩	⟨{0.1|1}, {0.7|0.5}⟩	⟨{0.2|1}, {0.6|1}⟩	⟨{0.1|0.2, 0.2|0.8}, {0.2|0.6, 0.3|0.4}⟩
X_4	⟨{0.4|0.4, 0.5|0.6}, {0.3|1}⟩	⟨{0.3|0.4, 0.2|0.6}, {0.3|1}⟩	⟨{0.3|1}, {0.4|1}⟩	⟨{0.4|1}, {0.2|1}⟩

Table 3: PDHF decision information matrix given by e_3

Alternatives	C_1	C_2	C_3	C_4
X_1	⟨{0.4|1}, {0.5|0.4, 0.6|0.6}⟩	⟨{0.4|1}, {0.1|1}⟩	⟨{0.4|1}, {0.5|0.4, 0.6|0.6}⟩	⟨{0.6|1}, {0.3|1}⟩
X_2	⟨{0.1|1}, {0.8|1}⟩	⟨{0.5|1}, {0.2|1}⟩	⟨{0.1|1}, {0.8|1}⟩	⟨{0.2|0.7, 0.4|0.3}, {0.1|1}⟩
X_3	⟨{0.3|0.3, 0.5|0.7}, {0.2|0.5, 0.5|0.5}⟩	⟨{0.5|0.6, 0.7|0.4}, {0.1|1}⟩	⟨{0.3|0.3, 0.5|0.7}, {0.2|0.5, 0.5|0.5}⟩	⟨{0.1|0.6, 0.3|0.4}, {0.6|1}⟩
X_4	⟨{0.3|0.8, 0.3|0.2}, {0.5|1}⟩	⟨{0.3|1}, {0.3|1}⟩	⟨{0.3|0.8, 0.3|0.2}, {0.5|1}⟩	⟨{0.3|1}, {0.2|0.5, 0.1|0.5}⟩

Step 1: Normalize information matrices. Among the four decision attributes given in this study, C_4 is the cost decision attribute, and C_1, C_2, and C_3 are the benefit decision attributes. We use equation (38) to normalize attributes, which are shown in Tables 4–6.

Step 2: Aggregate the PDHF decision information matrices from three experts to build a fresh decision information matrix by equation (40). The aggregated PDHF decision information matrix V is recorded in Table 7.

Step 3: Calculate ω_j by equations (41)–(45), which are shown in Table 9.

(1) Calculate $I(v_{ij})$, which is recorded in Table 8.

(2) The calculation results are recorded in Table 9.

Table 4: Normalized PDHF decision information matrix given by e_1

Alternatives	C_1	C_2	C_3	C_4
X_1	⟨{0.7|0.2, 0.6|0.2, 0.5|0.6}, {0.2|1}⟩	⟨{0.7|1}, {0.25|1}⟩	⟨{0.2|1}, {0.2|1}⟩	⟨{0.3|1}, {0.7|0.5, 0.6|0.5}⟩
X_2	⟨{0.6|1}, {0.35|1}⟩	⟨{0.56|1}, {0.2|1}⟩	⟨{0.1|1}, {0.7|1}⟩	⟨{0.4|1}, {0.2|0.6, 0.4|0.4}⟩
X_3	⟨{0.05|0.7, 0.2|0.3}, {0.5|1}⟩	⟨{0.3|0.5, 0.2|0.5}, {0.6|0.5, 0.5|0.5}⟩	⟨{0.8|1}, {0.15|1}⟩	⟨{0.6|1}, {0.2|1}⟩
X_4	⟨{0.08|1}, {0.6|1}⟩	⟨{0.2|0.6, 0.1|0.4}, {0.5|1}⟩	⟨{0.3|1}, {0.4|1}⟩	⟨{0.2|0.3, 0.4|0.7}, {0.1|1}⟩

Table 5: The normalized PDHF decision information matrix given by e_2

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.6\|1\},\{0.3\|1\}\rangle$	$\langle\{0.4\|1\},\{0.4\|0.8, 0.6\|0.2\}\rangle$	$\langle\{0.7\|0.4, 0.6\|0.6\},\{0.3\|0.7, 0.2\|0.3\}\rangle$	$\langle\{0.25\|1\},\{0.6\|0.7, 0.7\|0.3\}\rangle$
X_2	$\langle\{0.4\|0.1, 0.3\|0.9\},\{0.6\|1\}\rangle$	$\langle\{0.2\|0.5, 0.3\|0.5\},\{0.3\|0.5, 0.2\|0.5\}\rangle$	$\langle\{0.2\|1\},\{0.7\|0.6, 0.5\|0.4\}\rangle$	$\langle\{0.4\|1\},\{0.3\|1\}\rangle$
X_3	$\langle\{0.2\|1\},\{0.4\|0.9, 0.3\|0.1\}\rangle$	$\langle\{0.1\|1\},\{0.7\|0.5\}\rangle$	$\langle\{0.2\|1\},\{0.6\|1\}\rangle$	$\langle\{0.2\|0.6, 0.3\|0.4\},\{0.1\|0.2, 0.2\|0.8\}\rangle$
X_4	$\langle\{0.4\|0.4, 0.5\|0.6\},\{0.3\|1\}\rangle$	$\langle\{0.3\|0.4, 0.2\|0.6\},\{0.3\|1\}\rangle$	$\langle\{0.3\|1\},\{0.4\|1\}\rangle$	$\langle\{0.2\|1\},\{0.4\|1\}\rangle$

Table 6: Normalized PDHF decision information matrix given by e_3

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.4\|1\},\{0.5\|0.4, 0.6\|0.6\}\rangle$	$\langle\{0.4\|1\},\{0.1\|1\}\rangle$	$\langle\{0.4\|1\},\{0.5\|0.4, 0.6\|0.6\}\rangle$	$\langle\{0.3\|1\},\{0.6\|1\}\rangle$
X_2	$\langle\{0.1\|1\},\{0.8\|1\}\rangle$	$\langle\{0.5\|1\},\{0.2\|1\}\rangle$	$\langle\{0.1\|1\},\{0.8\|1\}\rangle$	$\langle\{0.1\|1\},\{0.2\|0.7, 0.4\|0.3\}\rangle$
X_3	$\langle\{0.3\|0.3, 0.5\|0.7\},\{0.2\|0.5, 0.5\|0.5\}\rangle$	$\langle\{0.5\|0.6, 0.7\|0.4\},\{0.1\|1\}\rangle$	$\langle\{0.3\|0.3, 0.5\|0.7\},\{0.2\|0.5, 0.5\|0.5\}\rangle$	$\langle\{0.6\|1\},\{0.1\|0.6, 0.3\|0.4\}\rangle$
X_4	$\langle\{0.3\|0.8, 0.3\|0.2\},\{0.5\|1\}\rangle$	$\langle\{0.3\|1\},\{0.3\|1\}\rangle$	$\langle\{0.3\|0.8, 0.3\|0.2\},\{0.5\|1\}\rangle$	$\langle\{0.2\|0.5, 0.1\|0.5\},\{0.3\|1\}\rangle$

Step 4: Determine the fuzzy PIS by equations (46) and (47), which are recorded in Table 10.

Step 5: Calculate the similarity measure (SM) between each alternative and the PIS by equations (48) and (49), which are shown in Table 11.

Step 6: Obtain the ranking of all alternatives by $\text{PDHFWGDSM}^1(X_i, X^+)$ or $\text{PDHFWGDSM}^2(X_i, X^+)$ from big to small, the alternative with the maximum similarity measure is the optimal alternative, we can give the ranking of four alternatives, which is $X_1 > X_3 > X_2 > X_4$ (">" means "super to"), and then, the optimal CBECP for company W is X_1.

5.3 Sensitivity analysis of parameters

This study proposes an MAGDM technique in PDHF environment based on the newly proposed PDHFWGDSMs. It is obvious that the ranking of four CBECPs will be affected by the changes of several parameters:

(1) The sequencing of the four CBECPs will be affected by the parameter λ in $\text{PDHFWGDSM}^1(X_i, X^+)$ and $\text{PDHFWGDSM}^2(X_i, X^+)$;

(2) The ranking of the four CBECPs will be affected by w.

5.3.1 The sequencing of the four CBECPs changes with the change of parameter λ

(1) When $w = (0.1, 0.2, 0.3, 0.4)^T$, we observe that the ranking of the four CBECPs changes with the change of λ in $\text{PDHFWGDSM}^1(X_i, X^+)$ (abbreviated as $\text{SM}^1(X_i)$), and all computing results are listed in Table 12 and Figure 2.

Table 7: Aggregated PDHF decision information matrix V

Alternatives	C_1	C_2
X_1	$\langle\{0.6224 \mid 0.2, 0.6 \mid 0.2, 0.5817 \mid 0.6\}, \{0.3194 \mid 1\}\rangle$	$\langle\{0.4777 \mid 1\}, \{0.1821 \mid 0.8, 0.2056 \mid 0.2\}\rangle$
X_2	$\left\langle\begin{Bmatrix}0.4467 \mid 0.06, 0.4024 \mid 0.04,\\ 0.4205 \mid 0.54, 0.3741 \mid 0.36,\end{Bmatrix}, \{0.2199 \mid 1\}\right\rangle$	$\langle\{0.4338 \mid 0.5, 0.4609 \mid 0.5\}, \{0.2259 \mid 0.5, 0.2 \mid 0.5\}\rangle$
X_3	$\langle\{0.172 \mid 0.7, 0.2 \mid 0.3\}, \{0.5533 \mid 0.9, 0.50766 \mid 0.1\}\rangle$	$\left\langle\begin{Bmatrix}0.3621 \mid 0.3, 0.5059 \mid 0.2,\\ 0.3448 \mid 0.3, 0.4925 \mid 0.2\end{Bmatrix}, \{0.2565 \mid 0.5, 0.2474 \mid 0.5\}\right\rangle$
X_4	$\left\langle\begin{Bmatrix}0.2941 \mid 0.08, 0.2453 \mid 0.32,\\ 0.3317 \mid 0.12, 0.2855 \mid 0.48\end{Bmatrix}, \{0.3446 \mid 1\}\right\rangle$	$\left\langle\begin{Bmatrix}0.2881 \mid 0.24, 0.2517 \mid 0.36,\\ 0.2639 \mid 0.16, 0.2338 \mid 0.24\end{Bmatrix}, \{0.3323 \mid 1\}\right\rangle$

Alternatives	C_3	C_4
X_1	$\left\langle\{0.4838 \mid 0.4, 0.4373 \mid 0.6\}, \begin{Bmatrix}0.3571 \mid 0.28, 0.3912 \mid 0.42,\\ 0.3162 \mid 0.12, 0.3464 \mid 0.18\end{Bmatrix}\right\rangle$	$\left\langle\begin{Bmatrix}0.6224 \mid 0.35, 0.6536 \mid 0.15,\\ 0.6 \mid 0.35, 0.6331 \mid 0.15\end{Bmatrix}, \{0.284 \mid 1\}\right\rangle$
X_2	$\langle\{0.1312 \mid 1\}, \{0.7483 \mid 0.6, 0.6765 \mid 0.4\}\rangle$	$\left\langle\begin{Bmatrix}0.2314 \mid 0.42, 0.3344 \mid 0.18,\\ 0.2744 \mid 0.28, 0.3716 \mid 0.12\end{Bmatrix}, \{0.2 \mid 1\}\right\rangle$
X_3	$\langle\{0.4329 \mid 0.3, 0.5207 \mid 0.7\}, \{0.2625 \mid 0.5, 0.4151 \mid 0.5\}\rangle$	$\left\langle\begin{Bmatrix}0.121 \mid 0.12, 0.2248 \mid 0.08,\\ 0.1515 \mid 0.48, 0.2517 \mid 0.32\end{Bmatrix}, \{0.4315 \mid 0.6, 0.4874 \mid 0.4\}\right\rangle$
X_4	$\langle\{0.3 \mid 0.8, 0.3 \mid 0.2\}, \{0.4472 \mid 1\}\rangle$	$\left\langle\{0.2972 \mid 1\}, \begin{Bmatrix}0.2 \mid 0.15, 0.1414 \mid 0.15,\\ 0.2297 \mid 0.35, 0.1625 \mid 0.35\end{Bmatrix}\right\rangle$

Table 8: The information energy matrix I

Alternatives	C_1	C_2	C_3	C_4
X_1	0.4546	0.2631	0.3419	0.4664
X_2	0.2127	0.2480	0.5363	0.1203
X_3	0.3340	0.2382	0.3666	0.2438
X_4	0.1973	0.1764	0.2900	0.1250

Table 9: Combined weight ω_j

Attribute	C_1	C_2	C_3	C_4
Entropy weight η_j	0.2503	0.28	0.1973	0.2725
Subjective weight w_j	0.1	0.3	0.4	0.2
Combined weight ω_j	0.1644	0.3011	0.2919	0.2426

Table 10: The PDHF PIS X^+

Attributes	C_1	C_2	C_3	C_4
X^+	$\langle\{0.6224\|0.2, 0.6\|0.2, 0.5817\|0.6\}, \{0.3194\|1\}\rangle$	$\langle\{0.4777\|1\}, \{0.1821\|0.8, 0.2056\|0.2\}\rangle$	$\langle\{0.4329\|0.3, 0.5207\|0.7\}, \{0.2625\|0.5, 0.4151\|0.5\}\rangle$	$\langle\{0.6224\|0.35, 0.6536\|0.15, 0.6\|0.35, 0.6331\|0.15\}, \{0.284\|1\}\rangle$

Table 11: The similarity measures with λ = 0.5

Alternatives	X_1	X_2	X_3	X_4	ranking
$PDHFWGDSM^1(X_i, X^+)$	0.9727	0.8360	0.9277	0.7642	$X_1 > X_3 > X_2 > X_4$
$PDHFWGDSM^2(X_i, X^+)$	0.9616	0.8936	0.9195	0.7628	$X_1 > X_3 > X_2 > X_4$

Table 12: Ranking of the four CBECPs with different λ in $PDHFWGDSM^1(X_i, X^+)$

λ	$SM^1(X_1)$	$SM^1(X_2)$	$SM^1(X_3)$	$SM^1(X_4)$	Ranking
0	0.9089	0.6459	0.6913	0.4620	$X_1 > X_3 > X_2 > X_4$
0.1	0.9191	0.6641	0.7247	0.4975	$X_1 > X_3 > X_2 > X_4$
0.2	0.9305	0.6856	0.7625	0.5393	$X_1 > X_3 > X_2 > X_4$
0.3	0.9436	0.7115	0.8055	0.5891	$X_1 > X_3 > X_2 > X_4$
0.4	0.9588	0.7439	0.8551	0.6500	$X_1 > X_3 > X_2 > X_4$
0.5	0.9765	0.7855	0.9132	0.7262	$X_1 > X_3 > X_2 > X_4$
0.6	0.9975	0.8419	0.9824	0.8253	$X_1 > X_3 > X_2 > X_4$
0.7	1.0228	0.9235	1.0670	0.9609	$X_3 > X_1 > X_4 > X_2$
0.8	1.0538	1.0552	1.1737	1.1631	$X_3 > X_4 > X_2 > X_1$
0.9	1.0926	1.3148	1.3143	1.5170	$X_4 > X_2 > X_3 > X_1$
1	1.1428	2.1821	1.5122	2.4777	$X_4 > X_2 > X_3 > X_1$

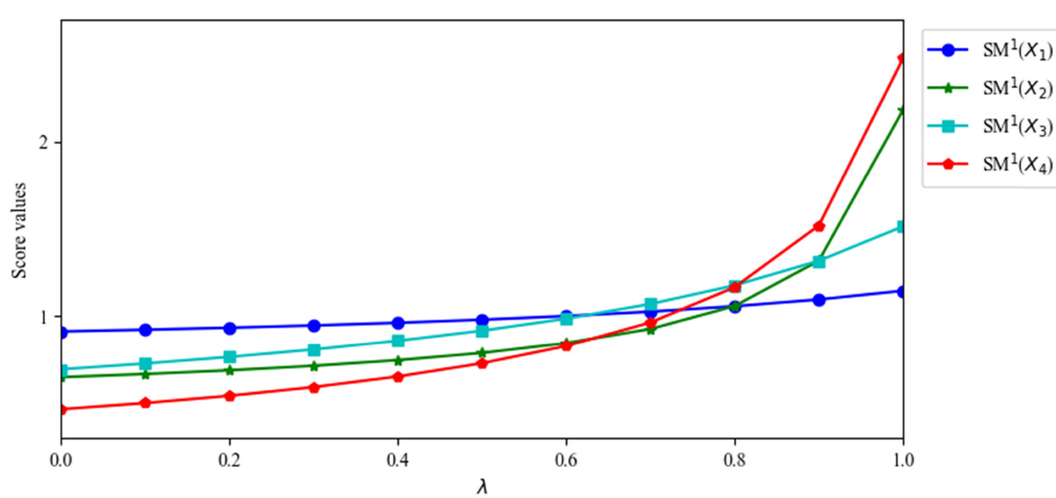

Figure 2: Changes of four similarity degrees with parameter λ in $PDHFWGDSM^1$.

Table 13: Ranking of the four CBECPs with different λ in PDHFWGDSM$^1(X_i, X^+)$

λ	$SM^2(X_1)$	$SM^2(X_2)$	$SM^2(X_3)$	$SM^2(X_4)$	Ranking
0	0.8761	0.5867	0.6295	0.4099	$X_1>X_3>X_2>X_4$
0.1	0.8942	0.6199	0.6690	0.4461	$X_1>X_3>X_2>X_4$
0.2	0.9131	0.6570	0.7138	0.4892	$X_1>X_3>X_2>X_4$
0.3	0.9329	0.6988	0.7650	0.5417	$X_1>X_3>X_2>X_4$
0.4	0.9535	0.7463	0.8241	0.6068	$X_1>X_3>X_2>X_4$
0.5	0.9750	0.8007	0.8931	0.6896	$X_1>X_3>X_2>X_4$
0.6	0.9975	0.8637	0.9747	0.7985	$X_1>X_3>X_2>X_4$
0.7	1.0211	0.9375	1.0727	0.9485	$X_3>X_1>X_4>X_2$
0.8	1.0459	1.0251	1.1926	1.1676	$X_3>X_4>X_1>X_2$
0.9	1.0718	1.1306	1.3428	1.5186	$X_4>X_3>X_2>X_1$
1	1.0991	1.2605	1.5361	2.1711	$X_4>X_3>X_2>X_1$

When w remains unchanged, SMs of the four CBECPs are increasing with the increase of λ. Meanwhile, the ranking of the four CBECPs is also changing, and the best CBECP is also changing. When $\lambda \in [0, 0.6]$, the best CBECP is X_1. When $\lambda \in [0.7, 0.8]$, the best CBECP is X_3. When $\lambda \in [0.9, 1]$, the best CBECP is X_4.

(2) When $w = (0.1, 0.2, 0.3, 0.4)^T$, the ranking of four CBECPs changes with λ in PDHFWGDSM$^2(X_i, X^+)$ (abbreviated as $SM^2(X_i)$), which are shown in Table 13.

We can obtain from Table 13 and Figure 3 that when w remains unchanged, the values of SMs of the four CBECPs are increasing with the increase of λ. Meanwhile, the ranking of the four CBECPs is also changing, and the best CBECP is also changing. When $\lambda \in [0, 0.6]$, the best CBECP is X_1. When $\lambda \in [0.7, 0.8]$, the best CBECP is X_3. When $\lambda \in [0.9, 1]$, the best CBECP is X_4.

5.3.2 The sequencing of the four CBECPs changes with the change of parameter w

(1) When $\lambda = 0.5$, we study the effect of parameter w in PDHFWGDSM$^1(X_i, X^+)$, which can be obtained and is shown in Table 14 and Figure 4.

We can see from Table 14 and Figure 4 that when w changes, the ranking of the four CBECPs unchanged and the best CBECP is X_1 and the worst CBECP is X_4.

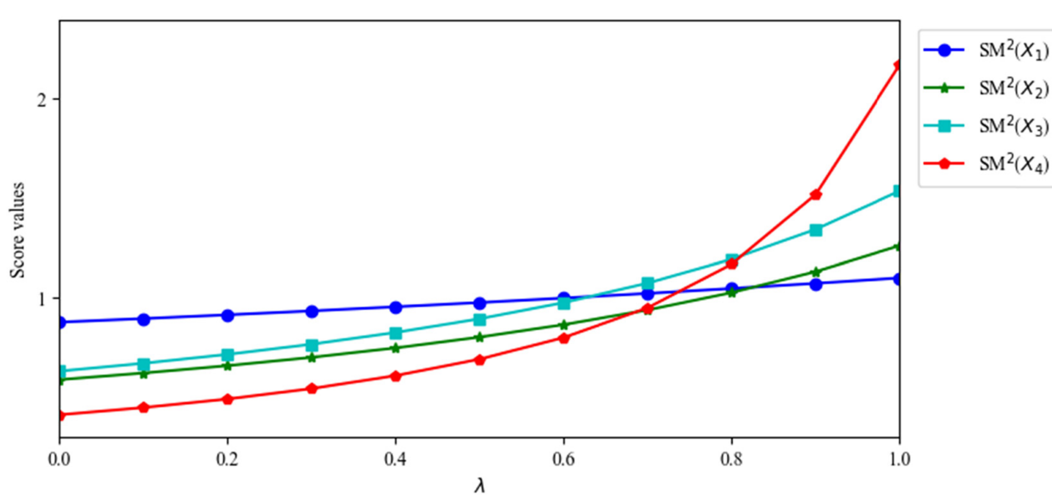

Figure 3: Changes of four similarity degrees with parameter λ in PDHFWGDSM^2.

Table 14: Ranking of the four CBECPs with different w in $PDHFWGDSM^1(X_i, X^+)$

w	$s(X_1)$	$s(X_2)$	$s(X_3)$	$s(X_4)$	Ranking
$w = (0.2, 0.5, 0.1, 0.2)^T$	0.9865	0.8218	0.9347	0.7654	$X_1 > X_3 > X_2 > X_4$
$w = (0.5, 0.1, 0.3, 0.1)^T$	0.9756	0.8196	0.9321	0.7524	$X_1 > X_3 > X_2 > X_4$
$w = (0.3, 0.4, 0.2, 0.1)^T$	0.9809	0.8463	0.9411	0.7801	$X_1 > X_3 > X_2 > X_4$
$w = (0.1, 0.2, 0.3, 0.4)^T$	0.9765	0.7855	0.9132	0.7262	$X_1 > X_3 > X_2 > X_4$
$w = (0.6, 0.1, 0.1, 0.2)^T$	0.9859	0.773	0.9248	0.7237	$X_1 > X_3 > X_2 > X_4$

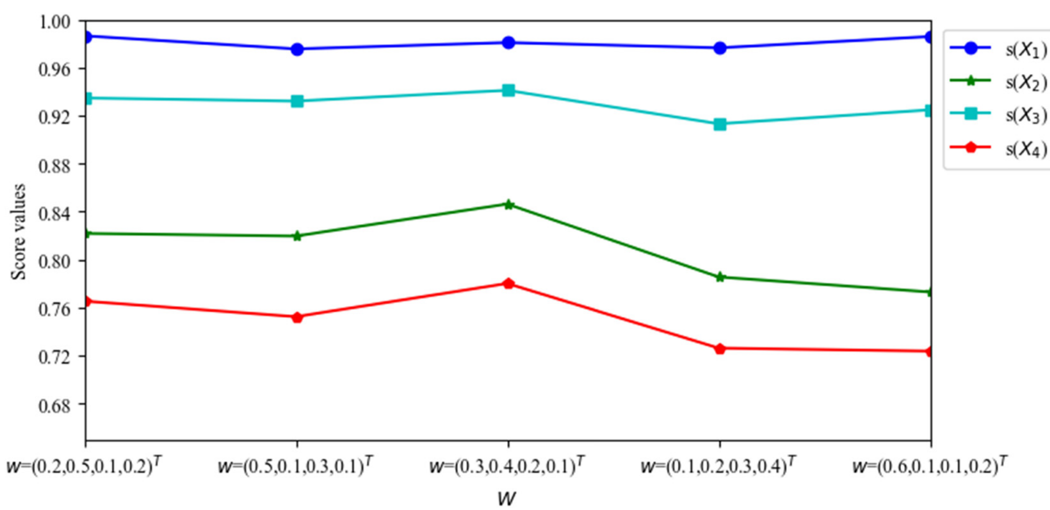

Figure 4: Changes of four similarity degrees with weights w in $PDHFWGDSM^1$.

(2) When $\lambda = 0.5$, we study the effect of parameter w in $PDHFWGDSM^2(X_i, X^+)$, which can be obtained and are shown in Table 15 and Figure 5.

We can see from Table 15 and Figure 5 that when w changes, the ranking remains unchanged, and the best CBECP is X_1 and the worst CBECP is X_4.

5.4 Validity analysis of the proposed MAGDM technique

In such section, we shall compare the MAGDM technique with several decision-making methods to investigate the validity of the proposed MAGDM technique. And the results are shown in Tables 16–19.

Table 15: Ranking of the four CBECPs with different w in $PDHFWGDSM^2(X_i, X^+)$

w	$s(X_1)$	$s(X_2)$	$s(X_3)$	$s(X_4)$	Ranking
$w = (0.2, 0.5, 0.1, 0.2)^T$	0.9896	0.82714	0.92834	0.7431	$X_1 > X_3 > X_2 > X_4$
$w = (0.5, 0.1, 0.3, 0.1)^T$	0.9755	0.85980	0.93554	0.7534	$X_1 > X_3 > X_2 > X_4$
$w = (0.3, 0.4, 0.2, 0.1)^T$	0.98	0.87721	0.94136	0.7755	$X_1 > X_3 > X_2 > X_4$
$w = (0.1, 0.2, 0.3, 0.4)^T$	0.975	0.80074	0.89308	0.6896	$X_1 > X_3 > X_2 > X_4$
$w = (0.6, 0.1, 0.1, 0.2)^T$	0.9924	0.76601	0.92682	0.7036	$X_1 > X_3 > X_2 > X_4$

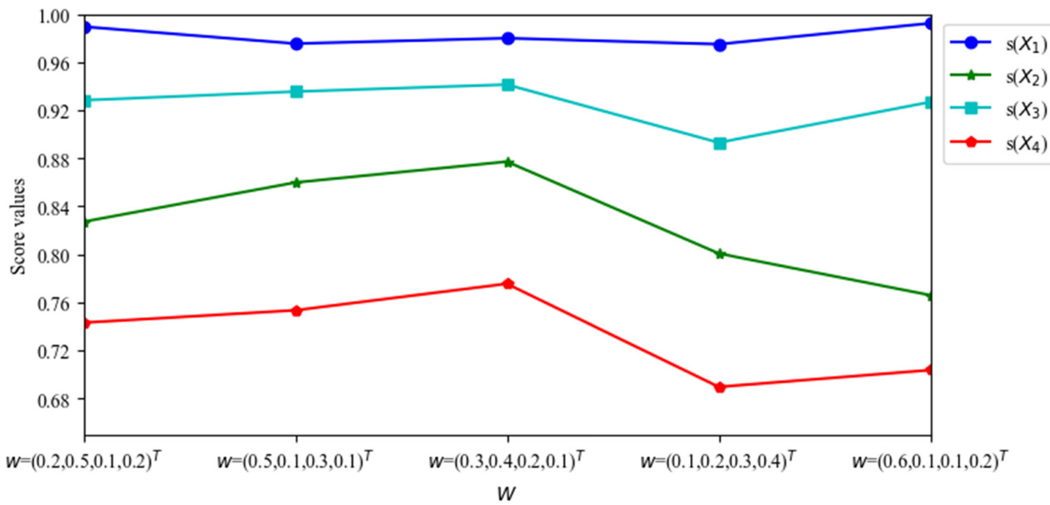

Figure 5: Changes of four similarity degrees with weights w in PDHFWGDSM[2].

Table 16: Outcome of four CBECPs

Alternatives	PDHFWEA operator	Score values	The ranking
X_1	$\left\langle \begin{Bmatrix} 0.5422\|0.028, & \cdots\cdots \\ \cdots\cdots, & 0.5255\|0.054 \end{Bmatrix}, \begin{Bmatrix} 0.2723\|0.224, & \cdots\cdots \\ \cdots\cdots, & 0.2794\|0.036 \end{Bmatrix} \right\rangle$	0.2533	$X_1 > X_2 > X_4 > X_3$
X_2	$\left\langle \begin{Bmatrix} 0.3063\|0.0126, & \cdots\cdots \\ \cdots\cdots, & 0.3345\|0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.3229\|0.3, & \cdots\cdots \\ \cdots\cdots, & 0.3001\|0.2 \end{Bmatrix} \right\rangle$	0.1415	
X_3	$\left\langle \begin{Bmatrix} 0.2982\|0.0076, & \cdots\cdots \\ \cdots\cdots, & 0.4025\|0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.336\|0.135, & \cdots\cdots \\ \cdots\cdots, & 0.3853\|0.01 \end{Bmatrix} \right\rangle$	0.0114	
X_4	$\left\langle \begin{Bmatrix} 0.2927\|0.0154, & \cdots\cdots \\ \cdots\cdots, & 0.2772\|0.023 \end{Bmatrix}, \begin{Bmatrix} 0.3249\|0.15, & \cdots\cdots \\ \cdots\cdots, & 0.3101\|0.35 \end{Bmatrix} \right\rangle$	0.0368	

5.4.1 Comparison with decision-making model of Garg and Kaur [42]

The data in Table 7 are substituted into equations (50) and (51), and $\omega = (0.1644, 0.3011, 0.2919, 0.2426)^T$, which are shown in Tables 16 and 17, and the best CBECP is X_2.

$$\mathrm{PDHFWEA}_{\omega}(\wp_1, \wp_2, \cdots, \wp_n) = \mathop{\oplus}_{i=1}^{n} \omega_i \wp_i$$
$$= \bigcup_{\substack{\lambda_{i_j} \in h_{i_j} \\ \eta_{i_j} \in g_{i_j}}} \left\langle \left\{ \frac{\prod_{i=1}^{n}(1+\gamma_i)^{\omega_i} - \prod_{i=1}^{n}(1-\gamma_i)^{\omega_i}}{\prod_{i=1}^{n}(1+\gamma_i)^{\omega_i} + \prod_{i=1}^{n}(1-\gamma_i)^{\omega_i}} \,\middle|\, \prod_{i=1}^{n} p_i \right\}, \left\{ \frac{2\prod_{i=1}^{n}(\eta_i)^{\omega_i}}{\prod_{i=1}^{n}(2-\eta_i)^{\omega_i} + \prod_{i=1}^{n}(\eta_i)^{\omega_i}} \,\middle|\, \prod_{i=1}^{k} q_i \right\} \right\rangle \tag{50}$$

Table 17: Outcome of four CBECPs

Alternatives	PDHFWEG operator	Score values	The ranking
X_1	$\left\langle \begin{Bmatrix} 05356\|0.028, & \cdots\cdots \\ \cdots\cdots, 0.5169\|0.054 \end{Bmatrix}, \begin{Bmatrix} 0.2819\|0.224, & \cdots\cdots \\ \cdots\cdots, 0.2854\|0.036 \end{Bmatrix} \right\rangle$	0.2363	$X_1 > X_4 > X_3 > X_2$
X_2	$\left\langle \begin{Bmatrix} 0.2704\|0.0126, & \cdots\cdots \\ \cdots\cdots, 0.299\|0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.412\|0.3, \cdots\cdots \\ \cdots\cdots, 0.3689\|0.2 \end{Bmatrix} \right\rangle$	−0.2414	
X_3	$\left\langle \begin{Bmatrix} 0.2639\|0.0076, & \cdots\cdots \\ \cdots\cdots, 0.373\|0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.3557\|0.135, \cdots\cdots \\ \cdots\cdots, 0.4021\|0.01 \end{Bmatrix} \right\rangle$	−0.068	
X_4	$\left\langle \begin{Bmatrix} 0.2926\|0.0154, & \cdots\cdots \\ \cdots\cdots, 0.2757\|0.023 \end{Bmatrix}, \begin{Bmatrix} 0.338\|0.15, \cdots\cdots \\ \cdots\cdots, 0.3304\|0.35 \end{Bmatrix} \right\rangle$	−0.0543	

$$\mathrm{PDHFWEG}_{\omega}(\wp_1, \wp_2, \cdots, \wp_n) = \bigoplus_{i=1}^{n} \omega_i \wp_i$$

$$\bigcup_{\substack{\lambda_{i_j} \in h_{i_j} \\ \eta_{i_j} \in g_{i_j}}} \left\langle \begin{array}{l} \left\{ \dfrac{2\prod_{i=1}^{n}(\gamma_i)^{\omega_i}}{\prod_{i=1}^{n}(2-\gamma_i)^{\omega_i} + \prod_{i=1}^{n}(\gamma_i)^{\omega_i}} \middle| \prod_{i=1}^{k} p_i \right\}, \\ \left\{ \dfrac{\prod_{i=1}^{n}(1+\eta_i)^{\omega_i} - \prod_{i=1}^{n}(1-\eta_i)^{\omega_i}}{\prod_{i=1}^{n}(1+\eta_i)^{\omega_i} + \prod_{i=1}^{n}(1-\eta_i)^{\omega_i}} \middle| \prod_{i=1}^{n} q_i \right\} \end{array} \right\rangle. \tag{51}$$

5.4.2 Comparison with decision-making model in the study

The data in Table 7 are substituted into PDHFWA and PDHFWG operators, and $\omega = (0.1644, 0.3011, 0.2919, 0.2426)^T$, which are shown in Tables 18 and 19, and the optimal CBECP is X_2.

5.4.3 Comparison with decision-making model of Ren et al. [41]

***Step* 1.** Calculate the relative weight.

$$\varpi_{jr} = \frac{\omega_j}{\max_j \omega_j}. \tag{52}$$

***Step* 2.** Calculate the dominance of alternative X_i over X_k.

$$\rho(X_i, X_k) = \sum_{j=1}^{n} \varphi_j(X_i, X_k), \tag{53}$$

where

Table 18: The outcome of four CBECPs

Alternatives	PDHFWA operator	Score values	The ranking
X_1	$\left\langle \begin{Bmatrix} 0.5438\vert 0.028, & \cdots\cdots \\ \cdots\cdots, 0.5276\vert 0.054 \end{Bmatrix}, \begin{Bmatrix} 0.2708\vert 0.224, & \cdots\cdots \\ \cdots\cdots, 0.2784\vert 0.036 \end{Bmatrix} \right\rangle$	0.2566	$X_1 > X_3 > X_4 > X_2$
X_2	$\left\langle \begin{Bmatrix} 0.3135\vert 0.0126, & \cdots\cdots \\ \cdots\cdots, 0.3409\vert 0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.3097\vert 0.3, & \cdots\cdots \\ \cdots\cdots, 0.2899\vert 0.2 \end{Bmatrix} \right\rangle$	−0.1261	
X_3	$\left\langle \begin{Bmatrix} 0.3046\vert 0.0076, & \cdots\cdots \\ \cdots\cdots, 0.409\vert 0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.3325\vert 0.135, & \cdots\cdots \\ \cdots\cdots, 0.3817\vert 0.01 \end{Bmatrix} \right\rangle$	0.0005	
X_4	$\left\langle \begin{Bmatrix} 0.2927\vert 0.0154, & \cdots\cdots \\ \cdots\cdots, 0.2776\vert 0.023 \end{Bmatrix}, \begin{Bmatrix} 0.3223\vert 0.15, & \cdots\cdots \\ \cdots\cdots, 0.3065\vert 0.35 \end{Bmatrix} \right\rangle$	−0.0337	

Table 19: Outcome of four CBECPs

Alternatives	PDHFWG operator	Score values	The ranking
X_1	$\left\langle \begin{Bmatrix} 0.4431\vert 0.028, & \cdots\cdots \\ \cdots\cdots, 0.5175\vert 0.054 \end{Bmatrix}, \begin{Bmatrix} 0.3828\vert 0.224, & \cdots\cdots \\ \cdots\cdots, 0.3853\vert 0.036 \end{Bmatrix} \right\rangle$	0.1006	$X_1 > X_3 > X_4 > X_2$
X_2	$\left\langle \begin{Bmatrix} 0.1905\vert 0.126, & \cdots\cdots \\ \cdots\cdots, 0.2911\vert 0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.554\vert 0.3, & \cdots\cdots \\ \cdots\cdots, 0.5153\vert 0.2 \end{Bmatrix} \right\rangle$	−0.3937	
X_3	$\left\langle \begin{Bmatrix} 0.3258\vert 0.0076, & \cdots\cdots \\ \cdots\cdots, 0.392\vert 0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.4786\vert 0.135, & \cdots\cdots \\ \cdots\cdots, 0.3325\vert 0.01 \end{Bmatrix} \right\rangle$	−0.1083	
X_4	$\left\langle \begin{Bmatrix} 0.2362\vert 0.0154, & \cdots\cdots \\ \cdots\cdots, 0.3\vert 0.023 \end{Bmatrix}, \begin{Bmatrix} 0.5191\vert 0.15, & \cdots\cdots \\ \cdots\cdots, 0.5137\vert 0.35 \end{Bmatrix} \right\rangle$	−0.2294	

$$\varphi_{j,i\neq k}(X_i, X_k) = \begin{cases} \sqrt{\dfrac{d(X_i, X_k)\cdot\ \varpi_{ir}}{\sum_{j=1}^{n}\varpi_{ir}}}, & \text{if} \quad s(v_{ij}) > s(v_{kj}), \\ 0, & \text{if} \quad s(v_{ij}) = s(v_{kj}), \\ -\psi \cdot \sqrt{\dfrac{d(X_i, X_k)\cdot \sum_{j=1}^{n}\varpi_{ir}}{\varpi_{ir}}}, & \text{if} \quad s(v_{ij}) < s(v_{kj}), \end{cases} \tag{54}$$

Table 20: Outcome of four CBECPs

Alternatives	$\varphi_{j,i\neq k}(X_i, X_k)$	$\pi(X_i)$	The ranking
X_1	−0.5651	1	$X_1>X_2>X_3>X_4$
X_2	−6.332	0.6918	
X_3	−8.1487	0.5941	
X_4	−19.247	0	

where $d(\tau, \upsilon) = \frac{1}{2}\left(\frac{\gamma+\eta}{2} + \frac{\Phi+\Psi}{2}\right)$. $\gamma = |\gamma(\tau) - \gamma(\upsilon)|$, $\gamma(d) = \sum_{j=1}^{\#h} h_j p_j$, $\eta(d) = \sum_{j=1}^{\#g} g_j q_j$, $\Phi(d) = \sqrt{\sum_{j=1}^{\#h}(h_j p_j - \gamma(d))^2}$, $\Psi(d) = \sqrt{\sum_{j=1}^{\#g}(g_j q_j - \eta(d))^2}$.

***Step* 3.** Calculate the prospect value of $X_i (i = 1, 2, \cdots, m)$.

$$\pi(X_i) = \frac{\sum_{k=1}^{m}\rho(X_i, X_k) - \min_i \sum_{k=1}^{m}\rho(X_i, X_k)}{\max_i \sum_{k=1}^{m}\rho(X_i, X_k) - \min_i \sum_{k=1}^{m}\rho(X_i, X_k)}. \tag{55}$$

***Step* 4.** The ranking can be obtained by $\pi(X_i)(i = 1, 2, \cdots, m)$, and the alternative with the maximum $\pi(X_i)$ is the optimal alternative.

We substitute $\omega = (0.1644, 0.3011, 0.2919, 0.2426)^T$ and data in Table 7 into decision-making model, and the results are shown in Table 20.

5.4.4 Comparison with decision-making model of Ren et al. [65]

***Step* 1:** Calculate v_j^+, v_j^-, S_i, and R_i.

$$S_i = \sum_{j=1}^{n} \omega_j \frac{d(v_j^+, v_{ij})}{d(v_j^+, v_j^-)}, \tag{56}$$

$$R_i = \max_j \frac{\omega_j d(v_j^+, v_{ij})}{d(v_j^+, v_j^-)}, \tag{57}$$

where $v_j^+ = \max_i s(v_{ij})$, $v_j^- = \min_i s(v_{ij})$ and $d(\tau, \upsilon) = \frac{1}{2}\left(\frac{\gamma+\eta}{2} + \frac{\Phi+\Psi}{2}\right)$. $\gamma = |\gamma(\tau) - \gamma(\upsilon)|$, $\gamma(d) = \sum_{j=1}^{\#h} h_j p_j$, $\eta(d) = \sum_{j=1}^{\#g} g_j q_j$, $\Phi(d) = \sqrt{\sum_{j=1}^{\#h}(h_j p_j - \gamma(d))^2}$, $\Psi(d) = \sqrt{\sum_{j=1}^{\#g}(g_j q_j - \eta(d))^2}$.

***Step* 2**: Determine Q_i. Four CBECPs are ranked based on S_i, R_i, and Q_i of four CBECPs, and the alternative is best with biggest Q_i.

$$Q_i = \gamma \frac{S_i - S^-}{S^+ - S^-} + (1-\gamma)\frac{R_i - R^-}{R^+ - R^-}, \tag{58}$$

where $S^- = \min_i\{S_i\}$, $S^+ = \max_i\{S_i\}$, $R^- = \min_i\{R_i\}$, $R^+ = \max_i\{R_i\}$, $\gamma \in (0, 1)$.

Table 21: Values of S_i, R_i, and Q_i of four CBECPs with $\gamma = 0.5$

Alternatives	S_i	R_i	Q_i	The ranking
X_1	0.0900	0.0900	0.0000	$X_1>X_3>X_2>X_4$
X_2	0.5959	0.2919	0.8584	
X_3	0.6652	0.2583	0.8308	
X_4	0.7552	0.3011	1.0000	

Table 22: Ranking of four CBECPs by using different methods

Decision-making methods	The ranking
PDHFWEA operator	$X_1 > X_2 > X_4 > X_3$
PDHFWEG operator	$X_1 > X_4 > X_3 > X_2$
PDHFWA operator	$X_1 > X_3 > X_4 > X_2$
PDHFWG operator	$X_1 > X_3 > X_4 > X_2$
Traditional PDHF TODIM method	$X_1 > X_2 > X_3 > X_4$
PDHF VIKOR method	$X_1 > X_3 > X_2 > X_4$
Our proposed MAGDM technique by PDHFWGDSM$^1(X_i, X^+)$	$X_1 > X_3 > X_2 > X_4$
Our proposed MAGDM technique by PDHFWGDSM$^2(X_i, X^+)$	$X_1 > X_3 > X_2 > X_4$

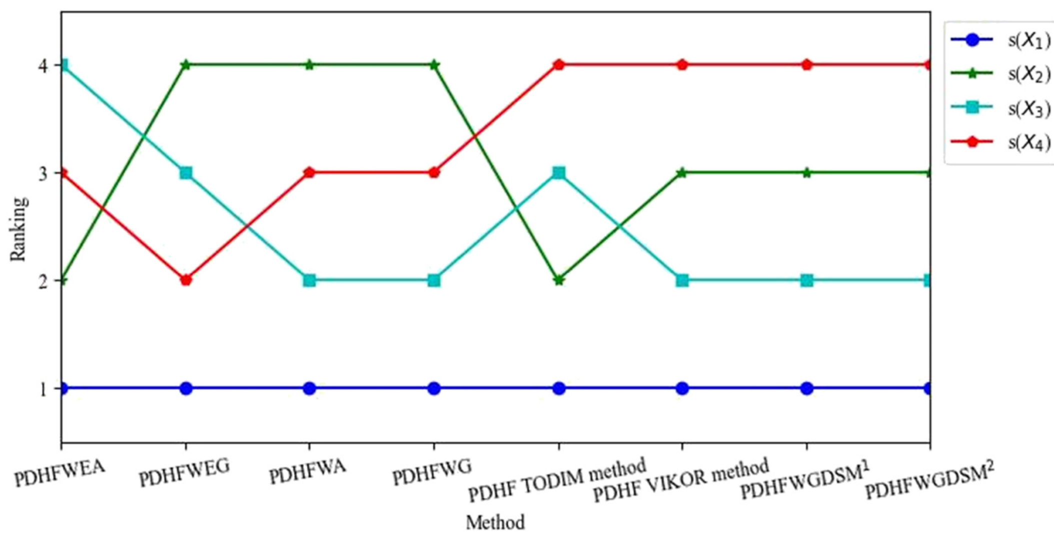

Figure 6: The ranking obtained by different decision-making methods.

S_i and R_i in Table 21 are substituted into equation (58); we can obtain Q_i, are shown in Table 21.

***Step* 3.** Finally, we can obtain the ranking according to Q_i.

(1) $Q(A^{R_2}) - Q(A^{R_1}) \geq \frac{1}{m-1}$, where A^{R_1} stands the first one in the ranking of Q_i and A^{R_2} is the second largest one; if only the condition is real, then A^{R_1} and A^{R_2} are compromise solution.

(2) A^{R_1} is still first while alternatives are ranked according to S_i or R_i, that is, result of decision is stable, if only the condition is real with $Q(A^{R_Y}) - Q(A^{R_1}) < \frac{1}{m-1}$, then all $A^{R_i}(i = 1, 2, \cdots, Y)$ are ideal solutions.

The data in Table 7 and $\omega = (0.1644, 0.3011, 0.2919, 0.2426)^T$ are substituted into equations (56)–(58) to obtain S_i, R_i, and Q_i of four CBECPs, which are shown in Table 21.

We can obtain the ranking, which is $X_1 > X_3 > X_2 > X_4$, and the best CBECP is X_1.

5.5 Comprehensive analysis

According to the aforementioned comparative analysis and the research results of this study (Table 22 and Figure 6), we can obtain the best CBECP, which is always X_1.

6 Conclusions

Reasonable CBECPS is very important for the sustainable development of CBECs, and the design of scientific and reasonable CBECPS decision-making technique is the premise. This study designs an MAGDM technique for CBECPS. The contributions and limitations of MAGDM technique proposed in this study are as follows. The MAGDM problem is facing more and more complex situations. In some unpredictable emergencies or changing social environment, there are many incomplete, fuzzy, and inconsistent information. The MAGDM methods based on PDHFWGDSMs in the PDHF environment can provide DMs with suggestions for making the most appropriate choice in the complex reality environment. Specifically, this study made the following main contributions to solve the limitations of the existing literature and overcome the challenges of research gaps: (1) this study constructs a scientific and reasonable evaluation index system for CBECPS of CBEC, which is a crucial prerequisite; (2) we propose a new PDHF entropy without auxiliary functions to scientifically and reasonably weight for decision attributes; (3) a novel aggregation operator named as PDHFWG operator is proposed and used in the aggregation of PDHF information; (4) since DSM has been extended to many fuzzy environments, the DSM is extended to PDHF environment and six forms of DSM in PDHF environment are proposed, providing a new method for measuring the similarity between two PDHFSs; (5) this study applies the newly developed MAGDM methods to the CBECPS. Through comparative analysis and parameter analysis, it proves the applicability of the new technique and its advantages over the existing methods; (6) the newly proposed PDHF entropy, aggregation operator, and PDHFWGDSM of PDHFEs can provide some theoretical supplements to the in-depth study of PDHFS to a certain extent, and will play a very fundamental role in the rapid development of PDHFS.

However, this study may have some limitations: first, the MAGDM technique cannot solve the MAGDM problems in the IVIFS, q-ROF, q-RPDHF, and IVPDHF environments. Second, in this study, we only considered the combined weighting method of entropy weighting method and subjective weighting method, but did not consider the application of CRITIC method [66] and FUCOM method in decision-making methods. Therefore, the scientific and reasonable weighting method is also the prerequisite for scientific application of this method.

In the future research, we will focus on the research of new decision methods that combine PDHFS and current classic decision methods under the PDHF environment, continue to focus on the research of aggregation operators, and propose some new MAGDM methods. It is also committed to applying the decision-making methods proposed in this study to uncertain MAGDM problems, such as strategy selection, site selection, green supplier selection, clean energy selection, and optimal selection of talents,.

Acknowledgments: This work was supported by the The Scientific Research and Cultivation Project of Liupanshui Normal University (LPSSY2023KJZDPY08), Discipline Cultivation Team of Liupanshui Normal University(LPSSY2023XKPYTD04), Young Scientific and Technological Talents Growth Project of Guizhou Provincial Department of Education (Qian jiao he KY [2018]369, Qian jiao he KY [2018]385, and Qian jiao he KY [2017]274), and Science and Technology Innovation Team of Liupanshui Normal University (LPSSYKJTD201702). There is no individual participant included in the study.

Funding information: This work was supported by the Young Scientific and Technological Talents Growth Project of Guizhou Provincial Department of Education (Qian jiao he KY [2018]369, Qian jiao he KY [2018]385, and Qian jiao he KY [2017]274), and Science and Technology Innovation Team of Liupanshui Normal University (LPSSYKJTD201702). There is no individual participant included in the study.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Ethical approval: The conducted research is not related to either human or animal use.

Data availability statement: All data generated or analyzed during this study are included in this published article.

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