Research Article

Wael W. Mohammed*, Farah M. Al-Askar, and Mahmoud El-Morshedy

Impacts of Brownian motion and fractional derivative on the solutions of the stochastic fractional Davey-Stewartson equations

https://doi.org/10.1515/dema-2022-0233
received July 15, 2022; accepted April 17, 2023

Abstract: In this article, the stochastic fractional Davey-Stewartson equations (SFDSEs) that result from multiplicative Brownian motion in the Stratonovich sense are discussed. We use two different approaches, namely the Riccati-Bernoulli sub-ordinary differential equations and sine-cosine methods, to obtain novel elliptic, hyperbolic, trigonometric, and rational stochastic solutions. Due to the significance of the Davey-Stewartson equations in the theory of turbulence for plasma waves, the discovered solutions are useful in explaining a number of fascinating physical phenomena. Moreover, we illustrate how the fractional derivative and Brownian motion affect the exact solutions of the SFDSEs using MATLAB tools to plot our solutions and display a number of three-dimensional graphs. We demonstrate how the multiplicative Brownian motion stabilizes the SFDSE solutions at around zero.

Keywords: fractional Davey-Stewartson equations, stochastic Davey-Stewartson equations, Riccati-Bernoulli sub-ODE method

MSC 2020: 35Q51, 83C15, 60H10, 35A20, 60H15

1 Introduction

In 1974, Davey and Stewartson developed the Davey-Stewartson equation (DSE) [1]. DSE is used to illustrate how a three-dimensional wave-packet changes over time on water with a limited depth. It is a set of coupled partial differential equations for the complex field (wave amplitude) $u(x, y, t)$ and the real field (mean flow) $v(x, y, t)$:

$$iu_t + \frac{1}{2}\delta^2(u_{xx} + \delta^2 u_{yy}) + \kappa|u|^2u - v_x u = 0, \tag{1}$$

$$v_{xx} - \delta^2 v_{yy} - 2\kappa(|u|^2)_x = 0, \tag{2}$$

where $\kappa = \pm 1$ and $\delta^2 = \pm 1$. The constant κ measures the cubic nonlinearity. The case $\delta = 1$ is known as the DS-I equation, while $\delta = i$ is known as the DS-II equation. The DS-I and DS-II are two integrable equations in two space dimensions that originate from higher dimensional versions of the nonlinear Schrödinger

*** Corresponding author: Wael W. Mohammed,** Department of Mathematics, Faculty of Science, University of Ha'il 2440, Ha'il, Saudi Arabia; Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt, e-mail: wael.mohammed@mans.edu.eg
Farah M. Al-Askar: Department of Mathematical Science, Collage of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia, e-mail: famalaskar@pnu.edu.sa
Mahmoud El-Morshedy: Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia, e-mail: m.elmorshedy@psau.edu.sa

equation. They occur in a variety of applications, including the description of gravity-capillarity surface wave packets in shallow water.

The solutions to the DS equations (1)–(2) have been utilized in plasma physics, nonlinear optics, hydrodynamics, and other disciplines. For instance, the solutions of the DS equation might describe the interaction of a properly matched microwaves and spatiotemporal optical plus. Therefore, many authors have provided exact solutions for this equation utilizing different methods such as double exp-function [2], sine–cosine [3], (G'/G)-expansion [4], generalized (G'/G)-expansion [5], the uniform algebraic method [6], the extended Jacobi's elliptic function [7], trial equation method [8], and the first integral method [9].

Many important phenomena, such as image processing, electro magnetic, acoustic, anomalous diffusion, and electrochemical, are now represented by fractional derivatives [10–24]. One advantage of fractional models is that they may be expressed more explicitly than integer models, which motivates us to create a number of important and useful fractional models. Recently, Khalil et al. [25] proposed a novel concept of fractional derivative that expands the well-known limit definition of the derivatives of a function. This definition is known as the conformable fractional derivative. In contrast to earlier definitions, the new definition is clearly compatible with the classical derivative and appears to meet all of the standards of the standard derivative. It is crucial for fulfilling the product formula, the quotient formula, and it has a less complicated formula for the chain rule. The conformable fractional derivative of the fractional partial differential equations has been studied by a number of researchers, including [26–30] and references theirin.

Conversely, several disciplines, such as engineering, climate dynamics,chemistry, physics, geophysics, the atmosphere, biology, fluid mechanics, and others, have pointed out the benefits of adding random impacts in the statistical features, analysis, simulation, prediction, and modeling of complex processes [31–33]. Noise cannot be neglected because it has the potential to initiate interesting phenomena. Finding exact solutions to fractional PDEs with a stochastic term is typically more challenging than finding solutions to classical PDEs.

To reach a better degree of qualitative agreement, we consider DSEs (1) and (2) with fractional space and forced by multiplicative noise in the Stratonovich sense as follows:

$$iu_t + \frac{1}{2}\delta^2(\mathbb{T}^{\alpha}_{xx}u + \delta^2\mathbb{T}^{\alpha}_{yy}u) + \kappa|u|^2u - u\mathbb{T}^{\alpha}_{x}v + i\sigma u \circ B_t = 0, \tag{3}$$

$$\mathbb{T}^{\alpha}_{xx}v - \delta^2\mathbb{T}^{\alpha}_{yy}v - 2\kappa\mathbb{T}^{\alpha}_{x}(|u|^2) = 0, \tag{4}$$

where $\mathbb{T}^{\alpha}$ is the conformable fractional derivative [25], σ is the strength of noise, and $B(t)$ is the Brownian motion.

We know that there are several possible interpretations for the stochastic integral $\int_0^t X\mathrm{d}B$. A stochastic integral is often interpreted using the Stratonovich and Itô calculus [34]. A Stratonovich stochastic integral (written as $\int_0^t X \circ \mathrm{d}B$) is one that is computed at the middle, whereas the stochastic integral is Itô (written as $\int_0^t X\mathrm{d}B$) when it is investigated at the left end. Next equation is how the Stratonovich integral and Itô integral are connected:

$$\int_0^t X(\tau, Z_\tau)\mathrm{d}B(\tau) = \int_0^t X(\tau, Z_\tau) \circ \mathrm{d}B(\tau) - \frac{1}{2}\int_0^t X(\tau, Z_\tau)\frac{\partial X(\tau, Z_\tau)}{\partial z}\mathrm{d}\tau, \tag{5}$$

where X is supposed to be sufficiently regular and $\{Z_t, t \geq 0\}$ is a stochastic process.

The objective of this work is to determine the exact solutions to the stochastic fractional Davey-Stewarts equations (SFDSEs). This study is the first to successfully get the exact solutions of SFDSEs (3) and (4). We used two methods, the Riccati-Bernoulli sub-ordinary differential equation (sub-ODE) approach and the sine-cosine method, to acquire a broad range of solutions, such as those for trigonometric, hyperbolic, and rational functions. Additionally, we use MATLAB package to produce 3D figures for some of the created solutions in this work to examine the impact of the Brownian motion on the solutions of SFDSEs (3) and (4).

This article is set up as follows: In Section 2, we employ an appropriate wave transformation to provide the wave equation of SFDSEs. In Section 3, to create the analytical solutions of SFDSEs (3) and (4), we use two methods. In Section 4, we examine how the Brownian motion impacts on the produced solutions. Finally, we present conclusions of this article.

2 Wave equation for SFDSEs

The next wave transformation is conducted in order to acquire the wave equation for the SFDSEs (3) and (4):

$$u(x,y,t)=\varphi(\eta)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\quad v(x,y,t)=\psi(\eta)e^{(-2\sigma B(t)-2\sigma^2 t)}, \tag{6}$$

with

$$\eta=\frac{\eta_1}{\alpha}x^\alpha+\frac{\eta_2}{\alpha}y^\alpha-\eta_3 t \quad \text{and} \quad \omega=\frac{\omega_1}{\alpha}x^\alpha+\frac{\omega_2}{\alpha}y^\alpha+\omega_3 t,$$

where the functions ψ and φ are deterministic, $\{\eta_j\}_{j=1}^3$,$\{\omega_j\}_{j=1}^3$ are undefined constants. Putting equation (6) into equation (3) and utilizing

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t}&=(-\eta_3\varphi'+i\omega_3\varphi-\sigma\varphi B_t+\frac{\sigma^2}{2}\varphi-\sigma^2\varphi)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\\&=(-\eta_3\varphi'+i\omega_3\varphi-\sigma\varphi B_t-\frac{\sigma^2}{2}\varphi)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\\&=(-\eta_3\varphi'+i\omega_3\varphi-\sigma\varphi\circ B_t)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\end{aligned}$$

where we used (5), and

$$\begin{aligned}\mathbb{T}^\alpha_{xx}u&=(\eta_1^2\varphi''+2i\omega_1\eta_1\varphi'-\omega_1^2\varphi)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\\\mathbb{T}^\alpha_{yy}u&=(\eta_2^2\varphi''+2i\omega_2\eta_2\varphi'-\omega_2^2\varphi)e^{(i\omega-\sigma B(t)-\sigma^2 t)},\\\mathbb{T}^\alpha_{x}(|u|^2)&=\eta_1(\varphi^2)'e^{(-2\sigma B(t)-2\sigma^2 t)},\\\mathbb{T}^\alpha_{x}v&=\eta_1\psi'e^{(-2\sigma B(t)-2\sigma^2 t)},\quad \mathbb{T}^\alpha_{xx}v=\eta_1^2\psi''e^{(-2\sigma B(t)-2\sigma^2 t)},\end{aligned}$$

we obtain for real part

$$\left(\frac{1}{2}\eta_1^2\delta^2+\frac{1}{2}\eta_2^2\delta^4\right)\varphi''-\left(\omega_3+\frac{1}{2}\delta^2\omega_1^2+\frac{1}{2}\delta^4\omega_2^2\right)\varphi+(\kappa\varphi^3-\eta_1\varphi\psi')e^{(-2\sigma B(t)-2\sigma^2 t)}=0, \tag{7}$$

$$(\eta_1^2-\delta^2\eta_2^2)\psi''-2\eta_1\kappa(\varphi^2)'=0, \tag{8}$$

and for imaginary part

$$(-\eta_3+2\eta_1\omega_1+2\eta_2\omega_2)\varphi'=0. \tag{9}$$

From equation (9), we obtain

$$\eta_3=2\eta_1\omega_1+2\eta_2\omega_2. \tag{10}$$

Now, integrating equation (8) once and putting the constant of integral equal zero, we have

$$\psi'=\frac{2\eta_1\kappa}{(\eta_1^2-\delta^2\eta_2^2)}\varphi^2. \tag{11}$$

Plugging equation (11) into equation (7), we obtain

$$\varphi''-\gamma_2\varphi+\gamma_1\varphi^3e^{(-2\sigma B(t)-2\sigma^2 t)}=0, \tag{12}$$

where

$$\gamma_1 = \frac{2\kappa}{\delta^2(\eta_1^2 - \delta^2\eta_2^2)} \quad \text{and} \quad \gamma_2 = \frac{2\omega_3 + \delta^2\omega_1^2 + \delta^4\omega_2^2}{\eta_1^2\delta^2 + \eta_2^2\delta^4}. \tag{13}$$

Taking expectation $\mathbb{E}(\cdot)$, yields

$$\varphi'' - \gamma_2\varphi - \gamma_1\varphi^3 e^{-2\sigma^2 t}\mathbb{E}(e^{-2\sigma B(t)}) = 0. \tag{14}$$

Since $B(t)$ is the standard normal process, hence $\mathbb{E}(e^{-2\sigma B(t)}) = e^{2\sigma^2 t}$. Hence, equation (14) becomes

$$\varphi'' - \gamma_1\varphi^3 - \gamma_2\varphi = 0. \tag{15}$$

3 Analytical solutions of the SFDSEs

We utilize two distinct methods including the Riccati-Bernoulli sub-ODE [35] and sine-cosine [36,37] to obtain the solutions of SFDSEs (equations (3) and (4)). The methods that have been proposed are useful tools that can solve a wide variety of other nonlinear PDEs. In addition to that, this procedure can provide a brand new infinite sequence of solutions.

3.1 Riccati-Bernoulli sub-ODE method

First, let us define the Riccati-Bernoulli equation (RBE) as follows:

$$\varphi' = \ell_1\varphi^2 + \ell_2\varphi + \ell_3, \tag{16}$$

where ℓ_1, ℓ_2, and ℓ_3 are unknown constants and $\varphi = \varphi(\eta)$.

Differentiating equation (16) and using equation (16), we have

$$\varphi'' = 2\ell_1^2\varphi^3 + 3\ell_1\ell_2\varphi^2 + (2\ell_1\ell_3 + \ell_2^2)\varphi + \ell_2\ell_3. \tag{17}$$

We obtain by putting (17) into (15)

$$(2\ell_1^2 - \gamma_1)\varphi^3 + 3\ell_1\ell_2\varphi^2 + (2\ell_1\ell_3 + \ell_2^2 - \gamma_2)\varphi + \ell_2\ell_3 = 0.$$

Making all $\varphi^i (i = 0, 1, 2, 3)$ coefficients null, we attain the following system:

$$\ell_2\ell_3 = 0, \quad (2\ell_1\ell_3 + \ell_2^2 - \gamma_2) = 0, \quad 3\ell_1\ell_2 = 0, \quad 2\ell_1^2 - \gamma_1 = 0.$$

The solution of the aforementioned system is

$$\ell_1 = \pm\sqrt{\frac{\gamma_1}{2}}, \quad \ell_2 = 0, \quad \ell_3 = \frac{\gamma_2}{2\ell_1}. \tag{18}$$

There are different solutions to the RBE (16) relying on ℓ_1 and ℓ_3.

First case: If $\ell_3 = 0$ (i.e., $\gamma_2 = 0$), hence the solution of the RBE (16) is

$$\varphi(\eta) = -\sqrt{\frac{2}{\gamma_1}}\frac{1}{\eta} \quad \text{for } \gamma_1 > 0. \tag{19}$$

Substituting equation (19) into equation (11) and integrating we obtain

$$\psi(\eta) = \frac{2\eta_1\kappa}{(\eta_1^2 - \delta^2\eta_2^2)}\int\varphi^2 d\eta = \frac{2\eta_1\kappa}{(\eta_1^2 - \delta^2\eta_2^2)}\int\frac{1}{(\ell_1\eta)^2}d\eta = \frac{2\eta_1\kappa}{(\eta_1^2 - \delta^2\eta_2^2)}\frac{-1}{\ell_1^2\eta}.$$

Therefore, the solution of SFDSEs (3) and (4), for $\gamma_1 > 0$, is

$$u(x,y,t) = \frac{-\sqrt{2}}{\sqrt{\gamma_1}\eta} e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{20}$$

$$v(x,y,t) = \frac{-4\kappa\eta_1}{(\eta_1^2 - \delta^2\eta_2^2)(\gamma_1\eta)} e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{21}$$

where $\eta = \frac{\eta_1}{\alpha}x^\alpha + \frac{\eta_2}{\alpha}y^\alpha - (2\eta_1\omega_1 + 2\eta_2\omega_2)t$.

Second case: If $\gamma_2 > 0$ and $\gamma_1 > 0$, hence the solution of RBE (16) is

$$\varphi(\eta) = \sqrt{\frac{\gamma_2}{\gamma_1}} \tan\left(\sqrt{\frac{\gamma_2}{2}}\eta\right)$$

or

$$\varphi(\eta) = -\sqrt{\frac{\gamma_2}{\gamma_1}} \cot\left(\sqrt{\frac{\gamma_2}{2}}\eta\right).$$

Substituting into equation (11) and integrating, we obtain

$$\psi(\eta) = \frac{2\eta_1\kappa\gamma_2}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\left[\sqrt{\frac{2}{\gamma_2}} \tan\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) - \eta\right]$$

or

$$\psi(\eta) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\left[\sqrt{\frac{2}{\gamma_2}} \cot\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) + \eta\right],$$

respectively.

Therefore, the solutions of SFDSEs (3) and (4) are as follows:

$$u(x,y,t) = \sqrt{\frac{\gamma_2}{\gamma_1}} \tan\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{22}$$

$$v(x,y,t) = \frac{2\eta_1\kappa\gamma_2}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\left[\sqrt{\frac{2}{\gamma_2}} \tan\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) - \eta\right] e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{23}$$

or

$$u(x,y,t) = -\sqrt{\frac{\gamma_2}{\gamma_1}} \cot\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{24}$$

$$v(x,y,t) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\left[\sqrt{\frac{2}{\gamma_2}} \cot\left(\sqrt{\frac{\gamma_2}{2}}\eta\right) + \eta\right] e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{25}$$

where $\eta = \frac{\eta_1}{\alpha}x^\alpha + \frac{\eta_2}{\alpha}y^\alpha - (2\eta_1\omega_1 + 2\eta_2\omega_2)t$.

Third case: If $\gamma_2 < 0$, $\gamma_1 > 0$, and $|\varphi| < \sqrt{-\frac{\gamma_2}{\gamma_1}}$, hence the solution of RBE (16) is

$$\varphi(\eta) = -\sqrt{\frac{-\gamma_2}{\gamma_1}} \tanh\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right).$$

Substituting into equation (11) and integrating, we obtain

$$\psi(\eta) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\left[\eta - \sqrt{\frac{-2}{\gamma_2}} \tanh\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)\right].$$

Thus, the solution of the SFDSEs (3) and (4) is

$$u(x, y, t) = -\sqrt{\frac{-\gamma_2}{\gamma_1}}\tanh\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)e^{(i\omega-\sigma B(t)-\sigma^2 t)}, \tag{26}$$

$$v(x, y, t) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2-\delta^2\eta_2^2)\gamma_1}\left[\eta-\sqrt{\frac{-2}{\gamma_2}}\tanh\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)\right]e^{(-2\sigma B(t)-2\sigma^2 t)}, \tag{27}$$

where $\eta = \frac{\eta_1}{\alpha}x^\alpha + \frac{\eta_2}{\alpha}y^\alpha - (2\eta_1\omega_1 + 2\eta_2\omega_2)t$.

Fourth case: If $\gamma_2 < 0$, $\gamma_1 > 0$ and $\varphi^2 > \frac{-\gamma_2}{\gamma_1}$, then the solution of the RBE (16) is

$$\varphi(\eta) = -\sqrt{\frac{-\gamma_2}{\gamma_1}}\coth\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right).$$

Substituting into equation (11) and integrating, we obtain

$$\psi(\eta) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2-\delta^2\eta_2^2)\gamma_1}\left[\eta-\sqrt{\frac{-2}{\gamma_2}}\coth\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)\right].$$

Consequently, the solution of the SFDSEs (3) and (4) is

$$u(x, y, t) = -\sqrt{\frac{-\gamma_2}{\gamma_1}}\coth\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)e^{(i\omega-\sigma B(t)-\sigma^2 t)}, \tag{28}$$

$$v(x, y, t) = \frac{-2\eta_1\kappa\gamma_2}{(\eta_1^2-\delta^2\eta_2^2)\gamma_1}\left[\eta-\sqrt{\frac{-2}{\gamma_2}}\coth\left(\sqrt{\frac{-\gamma_2}{2}}\eta\right)\right]e^{(-2\sigma B(t)-2\sigma^2 t)}, \tag{29}$$

where γ_1 and γ_2 defined in equation (13) and $\eta = \frac{\eta_1}{\alpha}x^\alpha + \frac{\eta_2}{\alpha}y^\alpha - (2\eta_1\omega_1 + 2\eta_2\omega_2)t$.

3.2 Sine-cosine method

Let the solutions φ of equation (15) have the form

$$\varphi(\eta) = AH^n, \tag{30}$$

where

$$H = \cos(B\eta) \quad \text{or} \quad H = \sin(B\eta). \tag{31}$$

Putting equation (30) into equation (15) we obtain

$$(\gamma_2 A - AB^2n^2)H^n + n(n-1)AB^2H^{n-2} + \gamma_1A^3H^{3n} = 0. \tag{32}$$

Equalizing the term of H in equation (32), we have

$$n - 2 = 3n \Rightarrow n = -1. \tag{33}$$

Setting equation (33) into equation (32)

$$(\gamma_2 A - AB^2)H^{-1} + (\gamma_1A^3 + 2AB^2)H^{-3} = 0. \tag{34}$$

Balancing each coefficient of H^{-3} and H^{-1} to zero, we have

$$\gamma_2 A - AB^2 = 0 \tag{35}$$

and

$$\gamma_1 A^3 + 2AB^2 = 0. \tag{36}$$

We obtain by solving these equations

$$B = \sqrt{\gamma_2} \quad \text{and} \quad A = \sqrt{\frac{-2\gamma_2}{\gamma_1}}. \tag{37}$$

Hence, the solution of equation (15) is

$$\varphi(\eta) = \sqrt{\frac{-2\gamma_2}{\gamma_1}} \sec(\sqrt{\gamma_2}\eta) \quad \text{or} \quad \varphi(\eta) = \sqrt{\frac{-2\gamma_2}{\gamma_1}} \csc(\sqrt{\gamma_2}\eta).$$

Substituting into equation (11) and integrating, we obtain

$$\psi(\eta) = \frac{-4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \tan(\sqrt{\gamma_2}\eta) \quad \text{or} \quad \psi(\eta) = \frac{4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \cot(\sqrt{\gamma_2}\eta).$$

There are various cases relying on the sign of γ_1 and γ_2.

Case 1: If $\gamma_2 > 0$ and $\gamma_1 < 0$, then the SFDSEs (3)–(4) have the exact solutions:

$$u(x, y, t) = \sqrt{\frac{-2\gamma_2}{\gamma_1}} \sec[\sqrt{\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{38}$$

$$v(x, y, t) = \frac{-4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \tan(\sqrt{\gamma_2}\eta) e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{39}$$

or

$$u(x, y, t) = \sqrt{\frac{-2\gamma_2}{\gamma_1}} \csc[\sqrt{\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{40}$$

$$v(x, y, t) = \frac{4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \cot(\sqrt{\gamma_2}\eta) e^{(-2\sigma B(t) - 2\sigma^2 t)}. \tag{41}$$

Case 2: If $\gamma_2 < 0$ and $\gamma_1 < 0$, then the SFDSEs (3) and (4) have the exact solutions:

$$u(x, y, t) = i\sqrt{\frac{2\gamma_2}{\gamma_1}} \operatorname{sech}[\sqrt{-\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{42}$$

$$v(x, y, t) = \frac{-4\eta_1\kappa\sqrt{-\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \tanh[\sqrt{-\gamma_2}\eta] e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{43}$$

or

$$u(x, y, t) = \sqrt{\frac{2\gamma_2}{\gamma_1}} \operatorname{csch}[\sqrt{-\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{44}$$

$$v(x, y, t) = \frac{4\eta_1\kappa\sqrt{-\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \coth[\sqrt{-\gamma_2}\eta] e^{(-2\sigma B(t) - 2\sigma^2 t)}. \tag{45}$$

Case 3: If $\gamma_2 < 0$ and $\gamma_1 > 0$, then the SFDSEs (3) and (4) have the exact solutions:

$$u(x, y, t) = \sqrt{\frac{-2\gamma_2}{\gamma_1}} \operatorname{sech}[\sqrt{-\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{46}$$

$$v(x, y, t) = \frac{-4\eta_1\kappa\sqrt{-\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1} \tanh[\sqrt{-\gamma_2}\eta] e^{(-2\sigma B(t) - 2\sigma^2 t)}, \tag{47}$$

or

$$u(x, y, t) = -i\sqrt{\frac{-2\gamma_2}{\gamma_1}} \operatorname{csch}[\sqrt{-\gamma_2}\eta] e^{(i\omega - \sigma B(t) - \sigma^2 t)}, \tag{48}$$

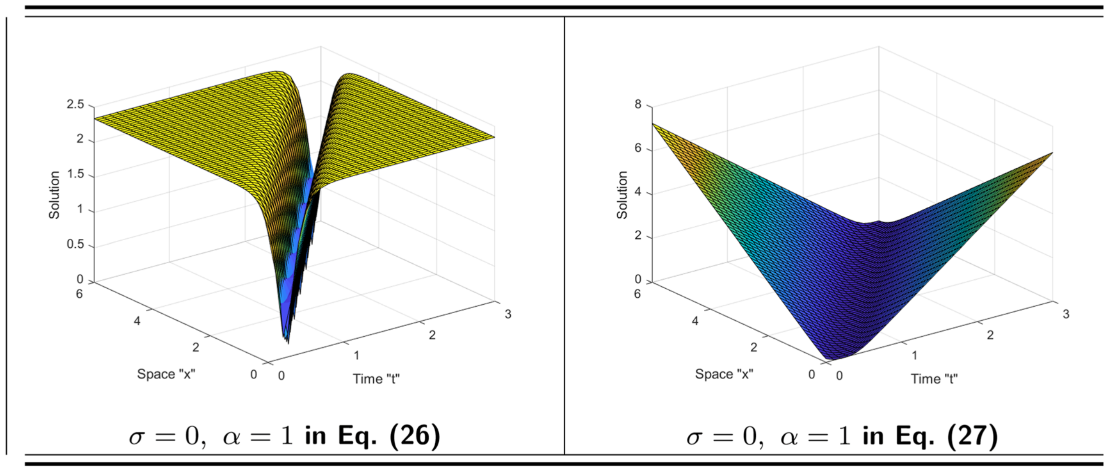

Figure 1: 3D plot of equations (26) and (27) with $\sigma = 0$ and $\alpha = 1$.

$$v(x,y,t) = \frac{-4\eta_1\kappa\sqrt{-\gamma_2}}{(\eta_1^2 - \delta^2\eta_2^2)\gamma_1}\coth[\sqrt{-\gamma_2}\eta]e^{(-2\sigma B(t)-2\sigma^2 t)}. \tag{49}$$

Case 4: If $\gamma_2 > 0$ and $\gamma_1 > 0$, then the SFDSEs (3) and (4) have the exact solutions:

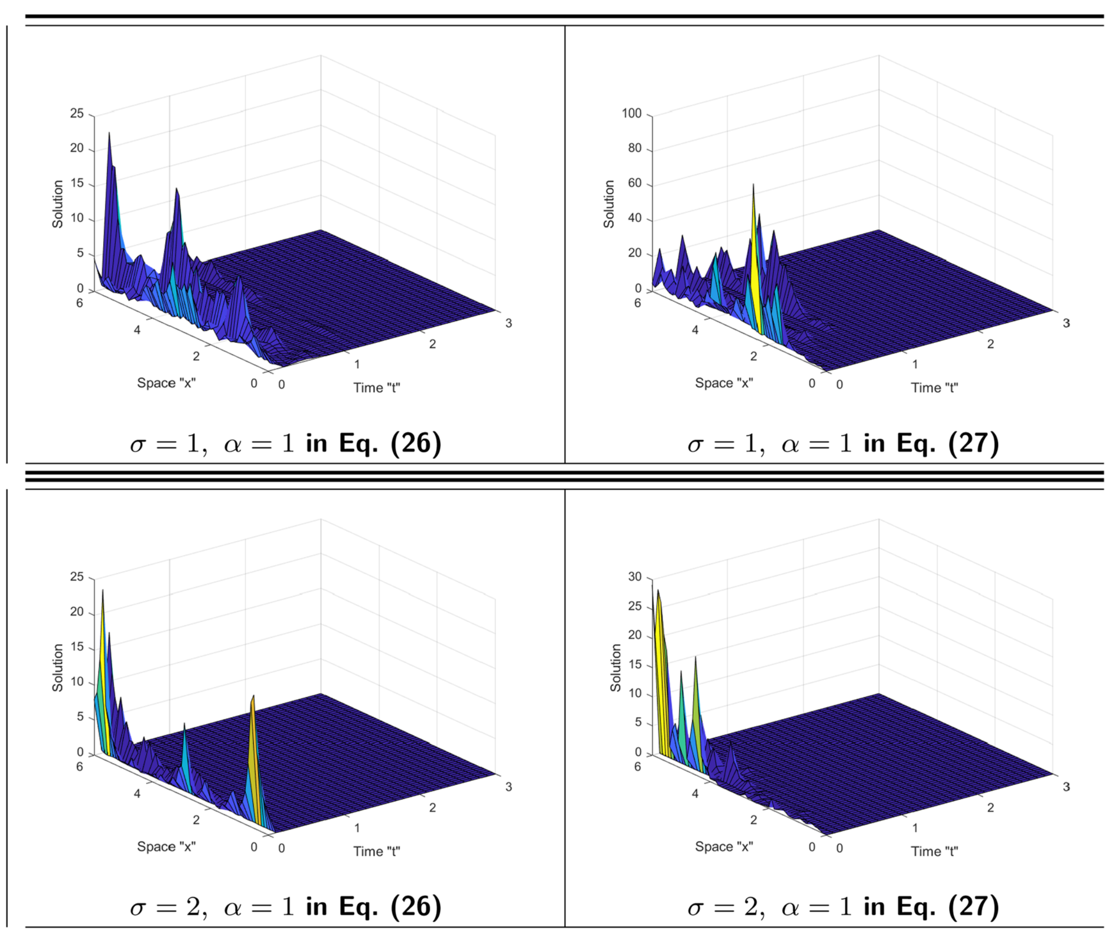

Figure 2: 3D plot of equations (26) and (27) with $\sigma = 1, 2$ and $\alpha = 1$.

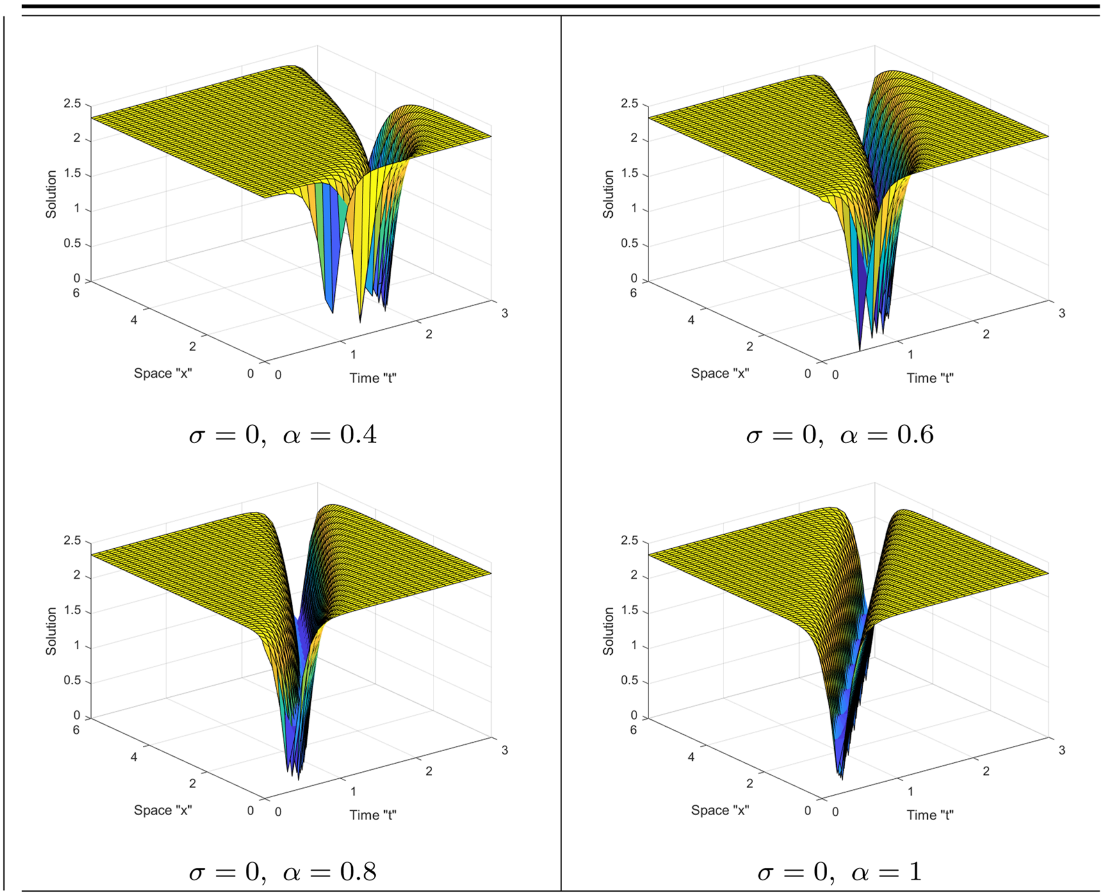

Figure 3: 3D plot of equation (26) with $\sigma = 0$ and different α.

$$u(x,y,t) = i\sqrt{\frac{2\gamma_2}{\gamma_1}}\sec[\sqrt{\gamma_2}\eta]e^{(i\omega-\sigma B(t)-\sigma^2 t)}, \tag{50}$$

$$v(x,y,t) = \frac{-4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2-\delta^2\eta_2^2)\gamma_1}\tan[\sqrt{\gamma_2}\eta]e^{(-2\sigma B(t)-2\sigma^2 t)}, \tag{51}$$

or

$$u(x,y,t) = i\sqrt{\frac{2\gamma_2}{\gamma_1}}\csc[\sqrt{\gamma_2}\eta]e^{(i\omega-\sigma B(t)-\sigma^2 t)}, \tag{52}$$

$$v(x,y,t) = \frac{4\eta_1\kappa\sqrt{\gamma_2}}{(\eta_1^2-\delta^2\eta_2^2)\gamma_1}\cot[\sqrt{\gamma_2}\eta]e^{(-2\sigma B(t)-2\sigma^2 t)}, \tag{53}$$

where γ_1 and γ_2 are defined in equations (13) and $\eta = \frac{\eta_1}{\alpha}x^\alpha + \frac{\eta_2}{\alpha}y^\alpha - (2\eta_1\omega_1 + 2\eta_2\omega_2)t$.

4 Effect of fractional derivative and noise on solutions

Effects of noise and fractional derivative on the exact solutions of the SFDSEs are examined in this article. If $\kappa = -1$, $\delta = i$, $\omega_1 = \eta_1 = 0.3$, $\omega_2 = \eta_2 = 1$, and $\omega_3 = -1.5$, then $\eta_3 = 2.18$, $\gamma_1 = \frac{2}{1.09}$, and $\gamma_2 = \frac{391}{91}$. We use the

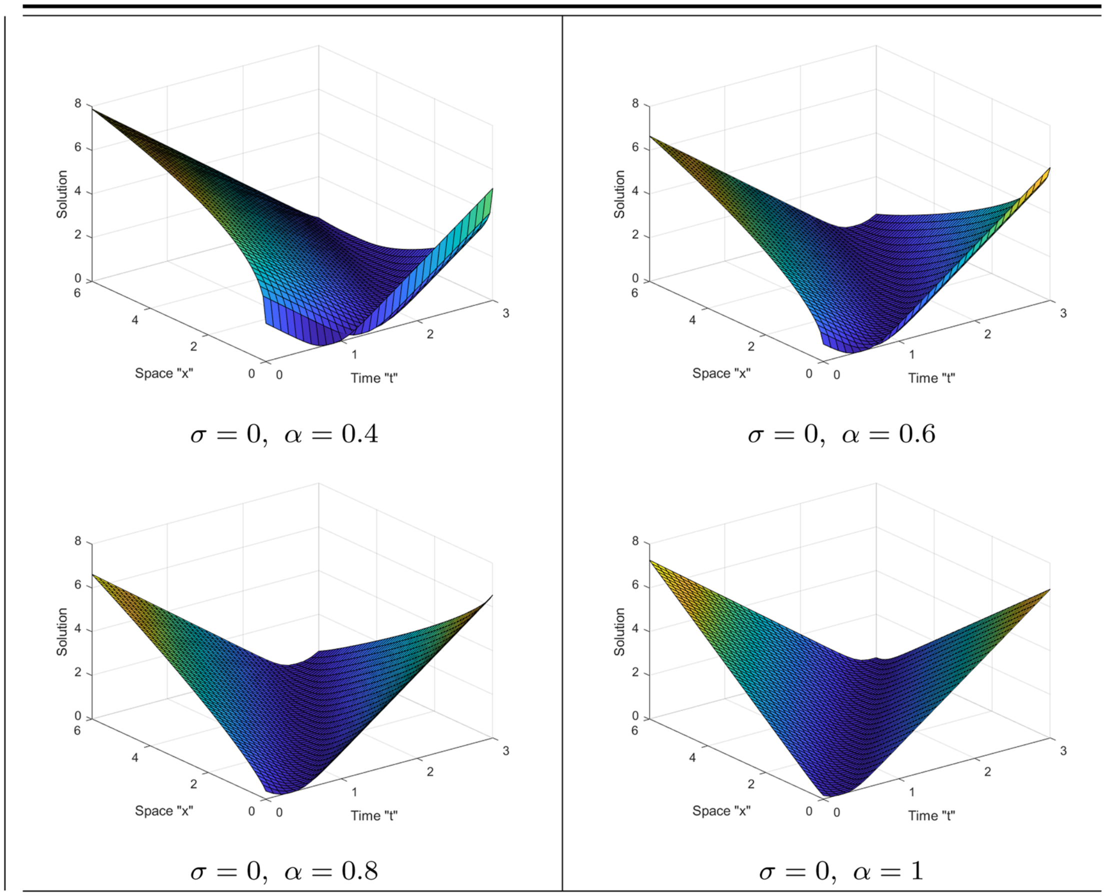

Figure 4: 3D plot of equation (27) with $\sigma = 0$ and different α.

MATLAB package to plot many figures for different noise intensity σ and for different value of fractional derivative order σ.

First, the impact of noise:

In Figure 1, for various values of α, we observe that the surface fluctuates when $\sigma = 0$.

Figure 2 shows that when the noise intensity increases, after these short periods of transit, the surface becomes more flat.

Figures 1 and 2 allow us to draw the following conclusion: when the noise is not taken into account (i.e., $\sigma = 0$), there are several distinct kinds of solutions, including periodic solution, kink solution, and others. When noise is added, and its intensity is increased by a factor of $\sigma = 1, 2$, the surface, which before had only modest transit patterns, becomes substantially flatter. This illustrates that the SFDSE solutions are affected by the white noise and that it stabilizes them around zero.

Second the impact of fractional derivative: In Figures 3 and 4, if we set $\sigma = 0$, we see that the surface grows with increasing α.

We inferred from Figures 3 and 4 that the solution curves do not overlap. Additionally, the surface gets smaller as the order of the fractional derivative gets smaller.

5 Conclusion

The stochastic-fractional (2 + 1)-dimensional DSEs (3) and (4) were taken into consideration in this article. To acquire stochastic fractional hyperbolic, rational, trigonometric solutions, we applied the Riccati-

Bernoulli sub-ODE and sine-cosine methods. These solutions are essential for comprehending many fundamentally complex basic phenomena. Additionally, the obtained answers will be very helpful for further research in subjects like hydrodynamics, nonlinear optics, plasma physics, and other areas. Finally, the influence of multiplicative Brownian motion on the exact solutions of the SFDSEs (3) and (4) is illustrated. We came to the conclusion that the noise makes the solutions stable around zero. In the future work, we might think about solving the $(2 + 1)$-dimensional DSEs equations using either additive noise or color noise.

Acknowledgments: This study was supported by Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2023R273), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Funding information: This research received no external funding.

Author contributions: All the authors contributed equally and significantly in writing this article.

Conflict of interest: The authors declare no conflict of interest.

Data availability statement: All data are available in this article.

References

[1] A. Davey and K. Stewartson, *On three-dimensional packets of surface waves*, Proc. Royal. Soc. Lond. Ser. A **338** (1974), 101–110, DOI: https://doi.org/10.1098/rspa.1974.0076.

[2] H. M. Fu and Z. D. Dai, *Double exp-function method and application*, Int. J. Nonlinear Sci. Numer. Simul. **10** (2009), 927–933, DOI: https://doi.org/10.1515/IJNSNS.2009.10.7.927.

[3] H. A. Zedan and S. J. Monaquel, *The sine-cosine method for the Davey-Stewartson equations*, Appl. Math. E-Notes **10** (2010), 103–111. http://www.math.nthu.edu.tw/amen/.

[4] G. Ebadi and A. Biswas, *The (G'/G) method and 1-soliton solution of the Davey-Stewartson equation*, Math. Comput. Modelling **53** (2011), no. 5–6, 694–698, DOI: https://doi.org/10.1016/j.mcm.2010.10.005.

[5] M. A. M. Abdelaziz, A. E. Moussa, and D. M. Alrahal, *Exact Solutions for the nonlinear $(2+1)$-dimensional Davey-Stewartson equation using the generalized (G'/G)-expansion method*, J. Math. Res. **6** (2014), 91–99, DOI: https://doi.org/10.5539/jmr.v6n2p91.

[6] A. ElAchab, *Constructing new wave solutions to the $(2+1)$-dimensional Davey-Stewartson equation (DSE) which arises in fluid dynamics*, JMST Adv. **1** (2019), 227–232, DOI: https://doi.org/10.1007/s42791-019-00025-0.

[7] A. H. Bhrawy, M. A. Abdelkawy, and A. Biswas, *Cnoidal and snoidal wave solutions to coupled nonlinear wave equations by the extended Jacobi's elliptic function method*, Commun. Nonlinear Sci. Numer. Simul. **18** (2013), no. 4, 915–925, DOI: https://doi.org/10.1016/j.cnsns.2012.08.034.

[8] M. Mirzazadeh, *Soliton solutions of Davey-Stewartson equation by trial equation method and ansatz approach*, Nonlinear Dyn. **82** (2015), no. 4, 1775–1780, DOI: https://doi.org/10.1007/s11071-015-2276-x.

[9] H. Jafari, A. Sooraki, Y. Talebi, and A. Biswas, *The first integral method and traveling wave solutions to Davey-Stewartson equation*, Nonlinear Anal. Model Control **17** (2012), no. 2, 182–193, DOI: https://doi.org/10.15388/NA.17.2.14067.

[10] S. B. Yuste, L. Acedo, and K. Lindenberg, *Reaction front in an $A + B \to C$ reaction-subdiffusion process*, Phys. Rev. E **69** (2004), 036126, DOI: https://doi.org/10.1103/PhysRevE.69.036126.

[11] W. W. Mohammed, N. Iqbal, and T. Botmart, *Additive noise effects on the stabilization of fractional-space diffusion equation solutions*, Mathematics **10** (2022), no. 1, 130, DOI: https://doi.org/10.3390/math10010130.

[12] D. A. Benson, S. W. Wheatcraft, and M. M. Meerschaert, *The fractional-order governing equation of Lévy motion*, Water Resour. Res. **36**, (2000), no. 6, 1413–1423, DOI: https://doi.org/10.1029/2000WR900032.

[13] S. B. Yuste and K. Lindenberg, *Subdiffusion-limited $A + A$ reactions*, Phys. Rev. Lett. **87** (2001), no. 1–2, 118301, DOI: https://doi.org/10.1016/S0301-0104(02)00546-3.

[14] W. W. Mohammed, *Fast-diffusion limit for reaction-diffusion equations with degenerate multiplicative and additive noise*, J. Dynam. Differential Equations. **33** (2021), no. 1, 577–592, DOI: https://doi.org/10.1007/s10884-020-09821-y.

[15] E. Barkai, R. Metzler, and J. Klafter, *From continuous time random walks to the fractional Fokker-Planck equation*, Phys Rev. **61** (2000), 132–138, DOI: https://doi.org/10.1103/PhysRevE.61.132.

[16] M. Alshammari, W. W. Mohammed, and M. Yar, *Novel analysis of fuzzy fractional Klein-Gordon model via semianalytical method*, J. Funct. Spaces **2022** (2022), 4020269, DOI: https://doi.org/10.1155/2022/4020269.
[17] W. W. Mohammed, C. Cesarano, and F. M. Al-Askar, *Solutions to the (4+1)-dimensional time-fractional Fokas equation with M-Truncated derivative*, Mathematics **11** (2022), no. 1, 194, DOI: https://doi.org/10.3390/math11010194.
[18] M. Mouy, H. Boulares, S. Alshammari, M. Alshammar, Y. Laskri, and W. W. Mohammed, *On Averaging Principle for Caputo-Hadamard fractional stochastic differential Pantograph equation*, Fractal Fract. **7** (2023), no. 1, 31, DOI: https://doi.org/10.3390/fractalfract7010031.
[19] N. Iqbal, A. M. Albalahi, M. S. Abdo, and W. W. Mohammed, *Analytical analysis of fractional-order Newell-Whitehead-Segel equation: A modified homotopy perturbation transform method*, J. Funct. Spaces **2022** (2022), 3298472, DOI: https://doi.org/10.1155/2022/3298472.
[20] O. J. Peter, F. A. Oguntolu, M. M. Ojo, A. O. Oyeniyi, R. Jan, and I. Khan, *Fractional order mathematical model of monkeypox transmission dynamics*, Phys. Scr. **97** (2022), 084005, DOI: https://doi.org/10.1088/1402-4896/ac7ebc.
[21] R. Jan and S. Boulaaras, *Analysis of fractional-order dynamics of dengue infection with non-linear incidence functions*, Trans. Inst. Meas. Control **44** (2022), no. 13, 2630–2641, DOI: https://doi.org/10.1177/01423312221085049.
[22] S. Boulaaras, R. Jan, A. Khan, and M. Ahsan, *Dynamical analysis of the transmission of dengue fever via Caputo-Fabrizio fractional derivative*, Chaos Solit. Fractals **8** (2022), 100072, DOI: https://doi.org/10.1016/j.csfx.2022.100072.
[23] F. M. Al-Askar, C. Cesarano, and W. W. Mohammed, *The influence of white noise and the beta derivative on the solutions of the BBM equation*, Axioms **12** (2023), 447, DOI: https://doi.org/10.3390/axioms12050447.
[24] F. M. Al-Askar, C. Cesarano, and W. W. Mohammed, *Abundant solitary wave solutions for the Boiti-Leon-Manna-Pempinelli equation with M-Truncated derivative*, Axioms **12** (2023), 466, DOI: https://doi.org/10.3390/axioms12050466.
[25] R. Khalil, M. AlHorani, A. Yousef, and M. Sababheh, *A new definition of fractional derivative*, J. Comput. Appl. Math. **264** (2014), 65–70, DOI: https://doi.org/10.1016/j.cam.2014.01.002.
[26] M. Eslami and H. Rezazadeh, *The first integral method for Wu-Zhang system with conformable time-fractional derivative*, Calcolo **53** (2016), no. 3, 475–485, DOI: https://doi.org/10.1007/s10092-015-0158-8.
[27] M. Eslami, *Exact traveling wave solutions to the fractional coupled nonlinear Schrodinger equations*, Appl. Math. Comput. **285** (2016), 141–148, DOI: https://doi.org/10.1016/j.amc.2016.03.032.
[28] A. K. Ozlem, E. Hepson, K. Hosseini, H. Rezazadeh, and M. Eslami, *Sine-Gordon expansion method for exact solutions to conformable time fractional equations in RLW-class*, J. King Saud Univ. Sci. **32** (2020), no. 1, 567–574, DOI: https://doi.org/10.1016/j.jksus.2018.08.013.
[29] F. M. Al-Askar, W. W. Mohammed, S. K. Samura, and M. El-Morshedy, *The exact solutions for fractional-stochastic Drinfelad-Sokolov-Wilson equations using a conformable operator*, J. Funct. Spaces **2022** (2022), 7133824, DOI: https://doi.org/10.1155/2022/7133824.
[30] F. M. Al-Askar, W. W. Mohammed, and M. Alshammari, *Impact of Brownian motion on the analytical solutions of the space-fractional stochastic approximate long water wave equation*, Symmetry **14** (2022), no. 4, 740, DOI: https://doi.org/10.3390/sym14040740.
[31] E. Weinan, X. Li, and E. Vanden-Eijnden, *Some recent progress in multiscale modeling, multiscale modeling and simulation*, Lecture Notes in Computer Science Engineering, Springer-Verlag, Berlin, vol. 39, 2004, pp. 3–21, DOI: https://doi.org/10.1007/978-3-642-18756-8_1.
[32] P. Imkeller and A. H. Monahan, *Conceptual stochastic climate models*, Stoch. Dynam. **2** (2002), no. 3, 311–326, DOI: https://doi.org/10.1142/S0219493702000443.
[33] W. W. Mohammed, *Stochastic amplitude equation for the stochastic generalized Swift-Hohenberg equation*, J. Egypt. Math. Soc. **23** (2015), 482–489, DOI: https://doi.org/10.1016/j.joems.2014.10.005.
[34] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*, Springer Verlag, New York, 1995.
[35] X. F. Yang, Z. C. Deng, and Y. Wei, *A Riccati-Bernoulli sub-ODE method for nonlinear partial differential equations and its application*, Adv. Differential Equations **117** (2015), no. 2015, 117–133, DOI: https://doi.org/10.1186/s13662-015-0452-4.
[36] A. M. Wazwaz, *A sine-cosine method for handling nonlinear wave equations*, Math. Comput. Modelling **40** (2004), no. 5–6, 499–508, DOI: https://doi.org/10.1016/j.mcm.2003.12.010.
[37] C. Yan, *A simple transformation for nonlinear waves*, Phys. Lett. A. **224** (1996), no. 1–2, 77–84, DOI: https://doi.org/10.1016/S0375-9601(96)00770-0.