Research Article

Anupam Das, Marija Paunović*, Vahid Parvaneh, Mohammad Mursaleen, and Zohreh Bagheri

Existence of a solution to an infinite system of weighted fractional integral equations of a function with respect to another function via a measure of noncompactness

https://doi.org/10.1515/dema-2022-0192
received December 31, 2021; accepted November 30, 2022

Abstract: In this article, some new generalizations of Darbo's fixed-point theorem are given and the solvability of an infinite system of weighted fractional integral equations of a function with respect to another function is studied. Also, with the help of a proper example, we illustrate our findings.

Keywords: fixed point, measure of noncompactness, integral equation

MSC 2020: 35K90, 47H10

1 Introduction

Fractional integral equations (FIEs) are extremely important in real-world problems. Because of the relevance of fractional order integral equations (IEs), it has become necessary to understand them. The idea of measure of noncompactness (MNC) plays a vital role in fixed point theory. The notion of MNC was initially created by Kuratowski, according to [1]. Darbo established a result demonstrating the presence of the fixed point for a condensing operator [2] in 1955, employing the idea of MNC. Fixed point theory and MNC have numerous applications in the study of various IEs that arise in a variety of real-world problems (see [3,4]).

Nashine et al. [5] have discussed fixed-point theorems (FPTs) for a new contraction condition in partially ordered Banach spaces and also discussed an application to the existence of a local FIE. Nashine et al. [6–8] also with the help of new contraction operators established new FPTs and applied them on fractional dynamic systems, fractional Cauchy problem etc.

This theory has proven to be extremely useful in determining the solvability of a wide range of differential equations and IEs (e.g., [3,9]). The goal of this article is to derive certain generalizations of Darbo's

* **Corresponding author: Marija Paunović,** University of Kragujevac, Faculty of Hotel Management and Tourism, Vojvodjanska bb, 36210 Vrnjacka Banja, Serbia, e-mail: majap@rcub.bg.ac.rs
Anupam Das: Department of Mathematics, Cotton University, Panbazar, Guwahati-781001, Assam, India, e-mail: math.anupam@gmail.com
Vahid Parvaneh: Department of Mathematics, Gilan-E-Gharb Branch, Islamic Azad University, Gilan-E-Gharb, Iran, e-mail: zam.dalahoo@gmail.com
Mohammad Mursaleen: Department of Medical Research, China Medical University Hospital, China Medical University (Taiwan), Taichung 40001, Taiwan, e-mail: mursaleenm@gmail.com
Zohreh Bagheri: Department of Mathematics, Azadshahr Branch, Islamic Azad University, Azadshahr, Iran, e-mail: zohrehbagheri@yahoo.com

fixed point theorem (DFPT) and use them to determine if an infinite system of weighted fractional integral equations (WFIEs) of a map w.r.t. another map is solvable.

Let $\mathcal{H}$ be a real Banach space with the norm $\|\cdot\|$. In $\mathcal{H}$, suppose that $B(\theta, r) = \{\iota \in \mathcal{H} : \|\iota - \theta\| \leq r\}$. If Q be a nonempty subset of $\mathcal{H}$, then by $\bar{Q}$ and the Conv$Q$ we denote the closure and convex closure of Q. Moreover, let $\mathcal{B}_{\mathcal{H}}$ be the collection of all nonempty and bounded subsets of $\mathcal{H}$ and $\mathfrak{N}_{\mathcal{H}}$ be its sub-collection made up of all relatively compact sets.

[10] provides the following definition of MNC:

Definition 1.1. A map $\mu : \mathcal{B}_{\mathcal{H}} \to [0, \infty)$ will be an MNC in $\mathcal{H}$ if:

(i) $\mu(Ł) = 0$ deduces Ł is relatively compact for all $Ł \in \mathcal{B}_{\mathcal{H}}$.
(ii) $\ker \mu = \{Ł \in \mathcal{B}_{\mathcal{H}} : \mu(Ł) = 0\} \neq \varnothing$ and $\ker \mu \subset \mathfrak{N}_{\mathcal{H}}$.
(iii) $Ł \subseteq Ł_1 \Rightarrow \mu(Ł) \leq \mu(Ł_1)$.
(iv) $\mu(\bar{Ł}) = \mu(Ł) = \mu(\text{Conv}Ł)$.
(v) $\mu(\delta Ł + (1-\delta)Ł) \leq \delta\mu(Ł) + (1-\delta)\mu(Ł)$ for all $\delta \in [0, 1]$.
(vi) $\mathcal{B}_{\mathcal{H}}$ has the closed decreasing limit infinite intersection property, i.e., the intersection over any infinite closed decreasing subcollection of $\mathcal{B}_{\mathcal{H}}$ is non-empty, where $\lim_{j\to\infty}\mu(Ł_j) = 0$.

kerμ is called the kernel of measure μ. The set $Ł_\infty = \bigcap_{j=1}^{\infty} Ł_j \in ker\mu$, as $\mu(Ł_\infty) \leq \mu(Ł_j)$ for any j, so, we conclude that $\mu(Ł_\infty) = 0$.

1.1 MNC on tempered sequence space

The tempering sequence and the space of tempered sequences were introduced as follows by Banaś and Krajewska [11].

A fix positive nonincreasing real sequence $\alpha = (\alpha_i)_{i=1}^{\infty}$ is called a tempering sequence.

Rabbani et al. [12] recently developed the set $\coprod$, which includes any real or complex sequence $\wp = (\wp_i)_{i=1}^{\infty}$ so that $\sum_{i=1}^{\infty}\alpha_i^p\,|\wp_i|^p < \infty$, $(1 \leq p < \infty)$. It is self-evident that $\coprod$ forms a linear space over $\mathbb{R}$ (or $\mathbb{C}$), which we label it as $\coprod := \ell_p^\alpha$, for $1 \leq p < \infty$.

Evidently, ℓ_p^α for $1 \leq p < \infty$ is a Banach space with the norm

$$\|\wp\|_{\ell_p^\alpha} = \left(\sum_{i=1}^{\infty}\alpha_i^p\,|\wp_i|^p\right)^{\frac{1}{p}}.$$

Choosing $\alpha_i = 1$ for all $i \in \mathbb{N}$, then $\ell_p^\alpha = \ell_p$ for, $1 \leq p < \infty$.

The Hausdorff MNC $\chi_{\ell_p^\alpha}$ for a nonempty bounded set B^α of ℓ_p^α, $(1 \leq p < \infty)$ can be given by (see [12]),

$$\chi_{\ell_p^\alpha}(B^\alpha) = \lim_{n\to\infty}\left[\sup_{\wp\in B^\alpha}\left(\sum_{k\geq n}\alpha_k^p|\wp_k\,|^p\right)^{\frac{1}{p}}\right]. \tag{1}$$

Let $C(\mathbb{J}, \ell_p^\alpha)$ be the space of all continuous maps on $\mathbb{J} = [0, a]$, $a > 0$ with the value in $\ell_p^\alpha (1 \leq p < \infty)$. It is also a Banach space with the norm

$$\|\wp\|_{C(\mathbb{J},\ell_p^\alpha)} = \sup_{t\in\mathbb{J}}\|\wp(t)\|_{\ell_p^\alpha},$$

where $\wp(t) = (\wp_i(t))_{i=1}^{\infty} \in C(\mathbb{J}, \ell_p^\alpha)$.

Let E^α be a nonempty bounded subset of $C(\mathbb{J}, \ell_p^\alpha)$ and $E^\alpha(t) = \{\wp(t) : \wp \in E^\alpha\}$, for all $t \in \mathbb{J}$. Thus, the MNC for $E^\alpha \subset C(\mathbb{J}, \ell_p^\alpha)$ can be defined by

$$\chi_{C(\mathbb{J},\ell_p^\alpha)}(E^\alpha) = \sup_{t\in\mathbb{J}}\chi_{\ell_p^\alpha}(E^\alpha(t)).$$

1.2 Some important theorems and definitions

The following are some key theorems to remember:

Theorem 1.2. (Schauder [13]) *Let* Ł *be a nonempty, bounded, closed, and convex subset* (*NBCC*) *of the Banach space* $\bar{\mathcal{H}}$. *There is at least one fixed point for each compact continuous map* $\mathcal{G} : Ł \to Ł$.

Theorem 1.3. (Darbo [2]) *Let* Ł *be an NBCC subset of a Banach Space* $\bar{\mathcal{H}}$. *Let* $\mathcal{S} : Ł \to Ł$ *be a continuous mapping and there is a constant* $\wp \in [0, 1)$ *such that*

$$\mu(\mathcal{S}\mathcal{B}) \leq \wp\mu(\mathcal{B}),$$

for all $\mathcal{B} \subseteq Ł$. *Then* $\mathcal{S}$ *possesses a fixed point*.

The following related notions are needed to establish an extension of DFPT:

Definition 1.4. [3] Let $\mathfrak{A}$ be the collection of all maps $\mathbb{A} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, satisfying:
(1) $\max\{\hbar, \wp\} \leq \mathbb{A}(\hbar, \wp)$ for all $\hbar, \wp \geq 0$,
(2) $\mathbb{A}$ is continuous and nondecreasing,
(3) $\mathbb{A}(\hbar_1 + \hbar_2, \wp_1 + \wp_2) \leq \mathbb{A}(\hbar_1, \wp_1) + \mathbb{A}(\hbar_2, \wp_2)$.

As an example, let $\mathbb{A}(\hbar, \wp) = \hbar + \wp$.

Definition 1.5. [14] Let $T : W \subseteq \bar{\mathcal{H}} \to \bar{\mathcal{H}}$ and $\alpha : \mathcal{B}_{\bar{\mathcal{H}}} : \to [0, \infty)$ be given mappings. T is called α-admissible provided that

$$\alpha(\pounds) \geq 1 \;\Rightarrow\; \alpha(\mathrm{Conv}T\pounds) \geq 1,$$

where $\pounds \subseteq W$ and $\pounds, T\pounds \in \mathcal{B}_{\bar{\mathcal{H}}}$.

As an example, we can choose $T = I$, the identity mapping and $\bar{\mathcal{H}} = \mathbb{R}$, also let
(1) $\alpha(a) = 2^a,\ a \geq 0$,
(2) $\alpha(a) = k,\ k \geq 1$.

Denote by Φ the class of all maps $\aleph : [0, \infty) \to [0, \infty)$ such that $\lim_{n\to\infty}\aleph(a_n) = 0$ if $\lim_{n\to\infty}a_n = 0$ where $\{a_n\}$ is a nonnegative sequence.

As an example, let
(1) $\aleph(t) = t^2$,
(2) $\aleph(t) = t^3$.

Let $\aleph \in \Phi$, and let $\hbar : [0, \infty) \to [0, \infty)$ satisfying:
(1) $\hbar$ is a continuous map with $\hbar(t) = 0$ if and only if $t = 0$.
(2) $\lim_{n\to\infty}\aleph(a_n) < \hbar(a)$ if $\lim_{n\to\infty}a_n = a > 0$.

The above class of all such maps is denoted by $\Psi_{\aleph}$.

As an example, let
(1) $\hbar(t) = 2t^2$,
(2) $\hbar(t) = 3t^3$.

2 Fixed point results

In this section, we collect some fixed point results that will aid us in the next section.

Theorem 2.1. *Let $\mathcal{L}$ be an NBCC subset of a Banach space $\bar{\mathcal{H}}$. Also, let $\mathcal{K} : \mathcal{L} \to \mathcal{L}$ be a continuous mapping which is α-admissible with $\alpha(\mathcal{L}) \geq 1$. Also,*

$$\alpha(\pounds)\hbar[\mathbb{A}(\mu(\mathcal{K}\pounds), \gamma(\mu(\mathcal{K}\pounds)))] \leq \aleph[\mathbb{A}(\mu(\pounds), \gamma(\mu(\pounds)))] \tag{2}$$

for all nonempty $\pounds \subseteq \mathcal{L}$, where μ is an arbitrary MNC, $\mathbb{A} \in \mathfrak{A}$, $\aleph \in \Phi$ and $\hbar \in \Psi_{\aleph}$. Also, assume that $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing and continuous mapping. Then $\mathcal{K}$ possesses at least one fixed point in $\mathcal{L}$.

Proof. Consider the sequence $\{\mathcal{L}_p\}_{p=1}^{\infty}$ with $\mathcal{L}_1 = \mathcal{L}$ and $\mathcal{L}_{p+1} = \mathrm{Conv}(\mathcal{K}\mathcal{L}_p)$ for all $p \in \mathbb{N}$. Also, $\mathcal{K}\mathcal{L}_1 = \mathcal{K}\mathcal{L} \subseteq \mathcal{L} = \mathcal{L}_1$, $\mathcal{L}_2 = \mathrm{Conv}(\mathcal{K}\mathcal{L}_1) \subseteq \mathcal{L} = \mathcal{L}_1$. Continuing in the similar manner gives $\mathcal{L}_1 \supseteq \mathcal{L}_2 \supseteq \mathcal{L}_3 \supseteq \ldots \supseteq \mathcal{L}_p \supseteq \mathcal{L}_{p+1} \supseteq \ldots$.

On the other hand, as $\mathcal{K}$ is α-admissible and $\alpha(\mathcal{L}) \geq 1$, so, $\alpha(\mathcal{L}_2) = \alpha(\mathrm{Conv}\mathcal{K}\mathcal{L}_1) \geq 1$.

Using mathematical induction, we obtain $\alpha(\mathcal{L}_p) \geq 1$ for all $p \geq 1$.

Now, for all $p \in \mathbb{N}$,

$$\begin{aligned}\hbar[\mathbb{A}(\mu(\mathcal{L}_{p+1}), \gamma(\mu(\mathcal{L}_{p+1})))] &= \hbar[\mathbb{A}(\mu(\mathrm{Conv}\mathcal{K}\mathcal{L}_p), \gamma(\mu(\mathrm{Conv}\mathcal{K}\mathcal{L}_p)))] \\ &= \hbar[\mathbb{A}(\mu(\mathcal{K}\mathcal{L}_p), \gamma(\mu(\mathcal{K}\mathcal{L}_p)))] \\ &\leq \alpha(\mathcal{L}_p)\hbar[\mathbb{A}(\mu(\mathcal{K}\mathcal{L}_p), \gamma(\mu(\mathcal{K}\mathcal{L}_p)))] \\ &\leq \aleph[\mathbb{A}(\mu(\mathcal{L}_p), \gamma(\mu(\mathcal{L}_p)))].\end{aligned}$$

If there exists $p_0 \in \mathbb{N}$ satisfying $\mu(\mathcal{L}_{p_0}) = 0$, then $\mathcal{L}_{p_0}$ is a compact set. In this case, the Schauder's FPT implies $\mathcal{K}$ has a fixed point in $\mathcal{L}$.

Now, let $\mu(\mathcal{L}_p) > 0$, for all $p \in \mathbb{N}$. Since $\mu(\mathcal{L}_{p+1}) \leq \mu(\mathcal{L}_p)$, for all $p \in \mathbb{N}$, consequently, there exists $l \geq 0$ such that $\lim_{p\to\infty}\mu(\mathcal{L}_p) = l$.

If $l = 0$, then $\mathcal{K}$ has a fixed point.

If $l > 0$ when $p \to \infty$ we obtain that

$$\hbar(\mathbb{A}(l, \gamma(l))) \leq \aleph(\mathbb{A}(l, \gamma(l))) < \hbar(\mathbb{A}(l, \gamma(l))),$$

which is a contradiction. Consequently, $l = 0$, i.e., $\lim_{p\to\infty}\mu(\mathcal{L}_p) = 0$.

Since $\mathcal{L}_p \supseteq \mathcal{L}_{p+1}$, by Definition 1.1, we obtain $\mathcal{L}_\infty = \bigcap_{p=1}^{\infty}\mathcal{L}_p$ is an NBCC subset of $\mathcal{L}$ and $\mathcal{L}_\infty$ is T invariant. Thus, according to theorem 1.2, $\mathcal{K}$ admits a fixed point in $\mathcal{L}$. □

Corollary 2.2. *For $\alpha(\pounds) = 2^a$, $a \geq 0$; $\hbar(t) = 2t^2$; $\aleph(t) = t^2$ and $\mathbb{A}(x, y) = x + y$ condition* (2) *reduces to*

$$2^{a+1}(\mu(\mathcal{K}\pounds) + \gamma(\mu(\mathcal{K}\pounds)))^2 \leq (\mu(\pounds) + \gamma(\mu(\pounds)))^2.$$

For $\alpha(\pounds) = K$, $K \geq 1$; $\hbar(t) = 3t^3$; $\aleph(t) = t^3$ and $\mathbb{A}(x, y) = x + y$ condition (2) *reduces to*

$$3K(\mu(\mathcal{K}\pounds) + \gamma(\mu(\mathcal{K}\pounds)))^3 \leq (\mu(\pounds) + \gamma(\mu(\pounds)))^3.$$

Theorem 2.3. *Let $\mathcal{L}$ be an NBCC subset of a Banach space $\mathcal{H}$. Also, let $\mathcal{K} : \mathcal{L} \to \mathcal{L}$ be a continuous and α-admissible mapping with $\alpha(\mathcal{L}) \geq 1$. Also,*

$$\alpha(\pounds)\hbar[\mu(\mathcal{K}\pounds) + \gamma(\mu(\mathcal{K}\pounds))] \leq \aleph[\mu(\pounds) + \gamma(\mu(\pounds))] \tag{3}$$

for all nonempty $\pounds \subseteq \mathcal{L}$, where μ is an arbitrary MNC, $\aleph \in \Phi$ and $\hbar \in \Psi_{\aleph}$. Also, let $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ be an increasing and continuous mapping. Then $\mathcal{K}$ possesses at least one fixed point in $\mathcal{L}$.

Proof. The result follows by taking $\mathbb{A}(\hbar, \wp) = \hbar + \wp$ in Theorem 2.1. □

Theorem 2.4. *Let $\mathcal{L}$ be an NBCC subset of a Banach space $\mathcal{H}$. Also, let $\mathcal{K} : \mathcal{L} \to \mathcal{L}$ be a continuous mapping and*

$$\hbar[\mathbb{A}(\mu(\mathcal{K}\pounds), \gamma(\mu(\mathcal{K}\pounds)))] \leq \aleph[\mathbb{A}(\mu(\pounds), \gamma(\mu(\pounds)))] \tag{4}$$

for all nonempty $\pounds \subseteq \mathcal{L}$, where μ is an arbitrary MNC, $\mathbb{A} \in \mathfrak{A}$, $\aleph \in \Phi$, and $\hbar \in \Psi_{\aleph}$. Also, let $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ be increasing and continuous. Then $\mathcal{K}$ admits at least one fixed point in $\mathcal{L}$.

Proof. The result follows by taking $\alpha(\pounds) = 1$ in Theorem 2.1. □

Remark 1. In Theorem 2.4, let $\mathbb{A}(a, b) = a + b$, $\gamma(t) = 0$, $\hbar(t) = t$ and $\aleph(t) = kt$ for all $t \geq 0$ where $k \in [0, 1)$. So,

$$\mu(\mathcal{K}\pounds) \leq k\mu(\pounds).$$

So, we obtain the DFPT. Hence, our results generalize the DFPT.

Definition 2.5. [15] An element $(p, q) \in \mathfrak{Q} \times Q$ is called a coupled fixed point (CFP) of a mapping $\mathcal{K} : Q \times Q \to Q$ if $\mathcal{K}(p, q) = p$ and $\mathcal{K}(q, p) = q$.

Theorem 2.6. [10] *Suppose that ω_1 be an MNC in $\mathcal{H}_1$, ω_2 be an MNC in $\mathcal{H}_2$,... and ω_n be an MNC in $\mathcal{H}_n$. Moreover, let the map $\mathfrak{A} : \mathbb{R}_+^n \to \mathbb{R}_+$ be convex and $\mathfrak{A}(\varsigma_1, \varsigma_2, \ldots, \varsigma_n) = 0$ if and only if $\varsigma_l = 0$ for $l = 1, 2, \ldots, n$. Then $\omega(Q) = \mathfrak{A}(\omega_1(Q_1), \omega_2(Q_2), \ldots, \omega_n(Q_n))$ defines an MNC in $\mathcal{H}_1 \times \mathcal{H}_2 \times \cdots \times \mathcal{H}_n$, where Q_l denotes the natural projection of Q into $\mathcal{H}_l$ for $l = 1, 2, \ldots, n$.*

Example 1. [10] Let ω be an MNC on $\mathcal{H}$. Define $\mathfrak{A}(p, q) = p + q$, $p, q \in \mathbb{R}_+$. Then $\mathfrak{A}$ has all the properties mentioned in Theorem 2.6. Hence, $\omega^{cf}(Q) = \omega(Q_1) + \omega(Q_2)$ is an MNC in the space $\mathcal{H} \times \mathcal{H}$, where Q_l, $l = 1, 2$ denote the natural projections of Q.

Theorem 2.7. *Let $\mathcal{L}$ be an NBCC subset of a Banach space $\mathcal{H}$, $\mathcal{K} : \mathcal{L} \times \mathcal{L} \to \mathcal{L}$ be a continuous mapping such that*

$$\hbar[\mathbb{A}(\mu(\mathcal{K}(Q_1 \times Q_2)), \gamma(\mu(\mathcal{K}(Q_1 \times Q_2))))] \leq \frac{1}{2}\aleph[\mathbb{A}(\mu(Q_1) + \mu(Q_2), \gamma(\mu(Q_1) + \mu(Q_2)))]$$

for any nonempty $Q_1, Q_2 \subseteq \mathcal{L}$, where μ be an arbitrary MNC and $\mathbb{A} \in \mathfrak{A}$. Also, let $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ be an increasing and continuous mapping, $\gamma(\hbar + \wp) \leq \gamma(\hbar) + \gamma(\wp)$ and $\hbar(\hbar + \wp) \leq \hbar(\hbar) + \hbar(\wp)$. Then $\mathcal{K}$ has at least one CFP in $\mathcal{L} \times \mathcal{L}$.

Proof. We observe that $\mu^{cf}(Q) = \mu(Q_1) + \mu(Q_2)$ is an MNC on $\mathcal{H} \times \mathcal{H}$ for any bounded subset $Q \subseteq \mathcal{H} \times \mathcal{H}$, where Q_1, Q_2 denote the natural projection of Q.

Consider a mapping $\mathcal{K}^{cf} : \mathcal{L} \times \mathcal{L} \to \mathcal{L} \times \mathcal{L}$ by $\mathcal{K}^{cf}(u, v) = (\mathcal{K}(u, v), \mathcal{K}(v, u))$.

It is trivial that $\mathcal{K}^{cf}$ is continuous. Let $Q \subseteq \mathcal{L} \times \mathcal{L}$. We obtain

$$\begin{aligned}
&\hbar[\mathbb{A}(\mu^{cf}(\mathcal{K}^{cf}(Q)), \gamma(\mu^{cf}(\mathcal{K}^{cf}(Q))))] \\
&\leq \hbar[\mathbb{A}(\mu^{cf}(\mathcal{K}(Q_1 \times Q_2)), \gamma(\mu^{cf}(\mathcal{K}(Q_1 \times Q_2))))] \\
&= \hbar[\mathbb{A}(\mu(\mathcal{K}(Q_1 \times Q_2)) + \mu(\mathcal{K}(Q_2 \times Q_1)), \gamma(\mu(\mathcal{K}(Q_1 \times Q_2)) + \mu(\mathcal{K}(Q_2 \times Q_1))))] \\
&\leq \hbar[\mathbb{A}(\mu(\mathcal{K}(Q_1 \times Q_2)) + \mu(\mathcal{K}(Q_2 \times Q_1)), \gamma(\mu(\mathcal{K}(Q_1 \times Q_2))) + \gamma(\mu(\mathcal{K}(Q_2 \times Q_1))))] \\
&\leq \hbar[\mathbb{A}(\mu(\mathcal{K}(Q_1 \times Q_2)), \gamma(\mu(\mathcal{K}(Q_1 \times Q_2))))] + \hbar[\mathbb{A}(\mu(\mathcal{K}(Q_2 \times Q_1)), \gamma(\mu(\mathcal{K}(Q_2 \times Q_1))))] \\
&\leq \aleph[\mathbb{A}(\mu(Q_1) + \mu(Q_2), \gamma(\mu(Q_1) + \mu(Q_2)))] \\
&= \aleph[\mathbb{A}(\mu^{cf}(Q), \gamma(\mu^{cf}(Q)))].
\end{aligned}$$

By Theorem 2.1, we conclude that $\mathcal{K}^{cf}$ admits at least one fixed point in $\mathcal{L} \times \mathcal{L}$, i.e., $\mathcal{K}$ has at least one CFP. □

Corollary 2.8. *For $\hbar(t) = 2t^2$; $\aleph(t) = t^2$ and $\mathbb{A}(x, y) = x + y$ the condition in* Theorem 2.7 *reduces to*

$$(\mu(\mathcal{K}(Q_1 \times Q_2)) + \gamma(\mu(\mathcal{K}(Q_1 \times Q_2))))^2 \leq \frac{1}{4}(\mu(Q_1) + \mu(Q_2) + \gamma(\mu(\mu(Q_1) + \mu(Q_2))))^2.$$

For $h(t) = 3t^3$;$\aleph(t) = t^3$ and $\mathbb{A}(x, y) = x + y$ the condition in Theorem 2.7 *reduces to*

$$(\mu(\mathcal{K}(Q_1 \times Q_2)) + \gamma(\mu(\mathcal{K}(Q_1 \times Q_2))))^3 \leq \frac{1}{6}(\mu(Q_1) + \mu(Q_2) + \gamma(\mu(\mu(Q_1) + \mu(Q_2))))^3.$$

3 Solvability of an infinite system of WFIEs of a map w.r.t. another map

Let $w(\hbar) \neq 0$, $w^{-1}(\hbar) = \frac{1}{w(\hbar)}$ and g be a strictly increasing differentiable map.

The weighted fractional integral of a continuous map f on $[a, \infty)$, $(a \in \mathbb{R})$ w.r.t. another map g of order α has been introduced by Jarad et al. in [16],

$$({}^{w}_{a+}J_{\alpha}f)(\hbar) = \frac{1}{w(\hbar)\Gamma(\alpha)} \int_{a}^{\hbar} \frac{w(t)g'(t)f(t)}{(g(\hbar) - g(t))^{1-\alpha}} \mathrm{d}t, \ \alpha > 0, \ \hbar > a.$$

In this part, the existence of a solution for the following FIE shall be investigated:

$$z_n(\hbar) = K_n\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)} \int_{a}^{\hbar} \frac{w(t)g'(t)H_n(\hbar, t, z(t))}{(g(\hbar) - g(t))^{1-\alpha}} \mathrm{d}t\right), \ n \in \mathbb{N}, \tag{5}$$

where $0 < \alpha < 1$, $\hbar \in \mathbb{J} = [a, T]$, $T > 0$, $a \geq 0$, $z(\hbar) = (z_n(\hbar))_{n=1}^{\infty} \in \mathcal{H}$ and $\mathcal{H}$ is a tempered sequence space.

Consider the following assumptions:

(1) The map $K_n : \mathbb{J} \times C(\mathbb{J}, \ell_p^{\alpha}) \times \mathbb{R} \to \mathbb{R}$ is continuous and

$$|K_n(\hbar, z(\hbar), l) - K_n(\hbar, \bar{z}(\hbar), m)|^p \leq \aleph_n(\hbar)|z_n(\hbar) - \bar{z}_n(\hbar)|^p + P_n(\hbar)|l - m|^p$$

for all $z(\hbar) = (z_n(\hbar))_{n=1}^{\infty}$, $\bar{z}(\hbar) = (\bar{z}_n(\hbar))_{n=1}^{\infty} \in C(\mathbb{J}, \ell_p^{\alpha})$ and $\aleph_n, \hbar_n : \mathbb{J} \to \mathbb{R}_+$ are continuous maps for all $n \in \mathbb{N}$.

Also,

$$\sum_{n=1}^{\infty} \alpha_n^p \, |K_n(\hbar, z^0, 0)|^p$$

converges to zero for all $\hbar \in \mathbb{J}$, where $z^0 = (z_n^0(\hbar))_{n=1}^{\infty} \in C(\mathbb{J}, \ell_p^{\alpha})$ such that $z_n^0(\hbar) = 0$ for all $\hbar \in \mathbb{J}$ and for all $n \in \mathbb{N}$.

(2) $H_n : \mathbb{J} \times \mathbb{J} \times C(\mathbb{J}, \ell_p^{\alpha}) \to \mathbb{R}$ $(n \in \mathbb{N})$ is continuous and there exists

$$\hat{H}_n = \sup\{|H_n(\hbar, t, z(t))| : \hbar, t \in \mathbb{J}; z(t) \in C(\mathbb{J}, \ell_p^{\alpha})\}.$$

Also,

$$Q_k = \sum_{n \geq k} \alpha_n^p \hat{H}_n^p$$

$\sup_{k \in \mathbb{N}} Q_k = \hat{Q}$ and $\lim_{k \to \infty} Q_k = 0$.

(3) $g : \mathbb{J} \to \mathbb{R}_+$ and $w : \mathbb{J} \to \mathbb{R}$ are C^1 and nondecreasing. Also, let

$$0 < l \leq |w(\hbar)| \leq L$$

for all $\hbar \in \mathbb{J}$.

(4) Define an operator $\mathcal{K}$ from $\mathbb{J} \times C(\mathbb{J}, \ell_p^{\alpha})$ to $C(\mathbb{J}, \ell_p^{\alpha})$ as follows:

$$(\hbar, z(\hbar)) \to (\mathcal{K}z)(\hbar),$$

where

$$(\mathcal{K}z)(\hbar) = \left(K_n\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)} \int_{a}^{\hbar} \frac{w(t)g'(t)H_n(\hbar, t, z(t))}{(g(\hbar) - g(t))^{1-\alpha}} \mathrm{d}t\right)\right)_{n=1}^{\infty}.$$

(5) Let

$$\sup_{\hbar\in\mathbb{J}} \aleph_n(\hbar) = \hat{\aleph}_n,$$

$$\sup_{n\in\mathbb{N}} \hat{\aleph}_n = \hat{\aleph},\ \ 0 < 2^{1-\frac{1}{p}}\hat{\aleph}^{\frac{1}{p}} < 1.$$

Also, for all $\hbar \in \mathbb{J}$,

$$\sum_{n\geq 1} \alpha_n^p P_n(\hbar) \leq \hat{\hbar}.$$

Let $B_{p,\alpha,\hat{r}} = \{z \in C(\mathbb{J}, \ell_p^\alpha) : \|z\|_{C(\mathbb{J},\ell_p^\alpha)} \leq \hat{r}\}$.

Theorem 3.1. *Under hypotheses* (1)–(5), *equation* (5) *has at least one solution in* $C(\mathbb{J}, \ell_p^\alpha)$ *where* $p > 1$.

Proof. For arbitrary fixed $\hbar \in \mathbb{J}$,

$$\begin{aligned}
\|z(\hbar)\|_{\ell_p^\alpha}^p &= \sum_{n\geq 1} \alpha_n^p \left| K_n\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)} \int_a^\hbar \frac{w(t)g'(t)H_n(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right)\right|^p \\
&\leq 2^{p-1} \sum_{n\geq 1} \alpha_n^p \left| K_n\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)} \int_a^\hbar \frac{w(t)g'(t)H_n(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right) - K_n(\hbar, z^0(\hbar), 0)\right|^p \\
&\quad + 2^{p-1} \sum_{n\geq 1} \alpha_n^p \left|K_n(\hbar, z^0(\hbar), 0)\right|^p \\
&\leq 2^{p-1} \sum_{n\geq 1} \alpha_n^p \left[\aleph_n(\hbar)|z_n(\hbar)|^p + P_n(\hbar)\left| \frac{1}{w(\hbar)\Gamma(\alpha)} \int_a^\hbar \frac{w(t)g'(t)H_n(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right|^p\right] \\
&\leq 2^{p-1}\hat{\aleph}\|z(\hbar)\|_{\ell_p^\alpha}^p + \frac{2^{p-1}\hat{\hbar}}{l^p\Gamma(\alpha)^p} \sum_{n\geq 1} \alpha_n^p \hat{H}_n^p L^p \left| \int_a^\hbar \frac{g'(t)}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right|^p \\
&\leq 2^{p-1}\hat{\aleph}\|z(\hbar)\|_{\ell_p^\alpha}^p + \frac{2^{p-1}\hat{\hbar}(g(T)-g(a))^{p\alpha}L^p}{l^p\Gamma(\alpha)^p\alpha^p} \sum_{n\geq 1} \alpha_n^p \hat{H}_n^p \\
&\leq 2^{p-1}\hat{\aleph}\|z(\hbar)\|_{\ell_p^\alpha}^p + \frac{2^{p-1}\hat{\hbar}(g(T)-g(a))^{p\alpha}L^p\hat{Q}}{l^p\Gamma(\alpha)^p\alpha^p}.
\end{aligned}$$

Therefore,

$$(1 - 2^{p-1}\hat{\aleph})\|z(\hbar)\|_{\ell_p^\alpha}^p \leq \frac{2^{p-1}\hat{\hbar}(g(T)-g(a))^{p\alpha}L^p\hat{Q}}{l^p\Gamma(\alpha)^p\alpha^p},$$

which implies

$$\|z(\hbar)\|_{\ell_p^\alpha}^p \leq \frac{2^{p-1}\hat{\hbar}(g(T)-g(a))^{p\alpha}L^p\hat{Q}}{l^p\Gamma(\alpha)^p\alpha^p(1-2^{p-1}\hat{\aleph})} = \hat{r}^p \text{ (say)}.$$

Hence, $\|z\|_{C(\mathbb{J},\ell_p^\alpha)} \leq \hat{r}$.

Let $\mathcal{K} : \mathbb{J} \times B_{p,\alpha,\hat{r}} \to B_{p,\alpha,\hat{r}}$ be an operator given by

$$(\mathcal{K}z)(\hbar) = \left(K_n\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)} \int_a^\hbar \frac{w(t)g'(t)H_n(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right)\right)_{n=1}^\infty = ((\mathcal{K}_n z)(\hbar))_{n=1}^\infty,$$

where $z(\hbar) \in B_{p,\alpha,\hat{r}}$ and $\hbar \in \mathbb{J}$.

By assumption (4),

$$\sum_{n\geq 1}(\mathcal{K}_n z)(\hbar)$$

is finite and unique. Hence, $(\mathcal{K}z)(\hbar) \in C(\mathbb{J}, \ell_p^\alpha)$.

Again,

$$\|\mathcal{K}z\|_{C(\mathbb{J},\ell_p^\alpha)} \leq \hat{r}.$$

So, $\mathcal{K}$ is a self-mapping on $B_{p,\alpha,\hat{r}}$.

Let $\bar{z}(\hbar) = (\bar{z}_n(\hbar))_{n=1}^{\infty} \in B_{p,\alpha,\hat{r}}$ and $\varepsilon > 0$ such that $\|z - \bar{z}\|_{C(\mathbb{J},\ell_p^\alpha)} < \frac{\varepsilon}{2\hat{\aleph}} = \delta$.

Again for arbitrary fixed $\hbar \in \mathbb{J}$,

$$\begin{aligned}
&|(\mathcal{K}_n z)(\hbar) - (\mathcal{K}_n \bar{z})(\hbar)|^p \\
&= \left| K_n\left(\hbar, z, \frac{1}{w(\hbar)\Gamma(\alpha)}\int_a^{\hbar}\frac{w(t)g'(t)H_n(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right) - K_n\left(\hbar, \bar{z}, \frac{1}{w(\hbar)\Gamma(\alpha)}\int_a^{\hbar}\frac{w(t)g'(t)H_n(\hbar,t,\bar{z}(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right)\right|^p \\
&\leq \aleph_n(\hbar)|z_n(\hbar) - \bar{z}_n(\hbar)|^p + \frac{L^p P_n(\hbar)}{l^p\Gamma(\alpha)^p}\left\{\int_a^{\hbar}\frac{g'(t)|H_n(\hbar,t,z(t)) - H_n(\hbar,t,\bar{z}(t))|}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right\}^p.
\end{aligned}$$

As H_n is continuous for all $n \in \mathbb{N}$, so, for $\|z - \bar{z}\|_{C(\mathbb{J},\ell_p^\alpha)}^p < \frac{\varepsilon^p}{2\hat{\aleph}}$ we have

$$\alpha_n|H_n(\hbar,t,z(t)) - H_n(\hbar,t,\bar{z}(t))| < \frac{\alpha l\Gamma(\alpha)\varepsilon}{2^{\frac{1}{p}}\hat{\hbar}^{\frac{1}{p}}(g(T)-g(a))^\alpha L},$$

for all $n \in \mathbb{N}$.

Therefore,

$$\begin{aligned}
\sum_{n\geq 1}\alpha_n^p\,|(\mathcal{K}_n z)(\hbar) - (\mathcal{K}_n\bar{z})(\hbar)|^p &\leq \hat{\aleph}\sum_{n\geq 1}\alpha_n^p\,|z_n(\hbar) - \bar{z}_n(\hbar)|^p + \frac{\hat{\hbar}L^p}{l^p\Gamma(\alpha)^p}\left\{\frac{\alpha l\Gamma(\alpha)\varepsilon}{2^{\frac{1}{p}}\hat{\hbar}^{\frac{1}{p}}(g(T)-g(a))^\alpha L}\cdot\frac{(g(T)-g(a))^\alpha}{\alpha}\right\}^p \\
&< \hat{\aleph}\|z - \bar{z}\|_{C(\mathbb{J},\ell_p^\alpha)}^p + \frac{\varepsilon^p}{2} \\
&< \hat{\aleph}\frac{\varepsilon^p}{2\hat{\aleph}} + \frac{\varepsilon^p}{2} = \varepsilon^p.
\end{aligned}$$

Therefore, $\|\mathcal{K}z - \mathcal{K}\bar{z}\|_{C(\mathbb{J},\ell_p^\alpha)}^p < \varepsilon^p$, when $\|z - \bar{z}\|_{C(\mathbb{J},\ell_p^\alpha)}^p < \frac{\varepsilon^p}{2\hat{\aleph}}$. Hence, $\mathcal{K}$ is continuous on $B_{p,\alpha,\hat{r}}$.

Finally,

$$\begin{aligned}
\chi_{\ell_p^\alpha}(\mathcal{K}B_{p,\alpha,\hat{r}}) &= \lim_{n\to\infty}\sup_{z\in B_{p,\alpha}}\left\{\sum_{k\geq n}\alpha_k^p\left|K_k\left(\hbar, z(\hbar), \frac{1}{w(\hbar)\Gamma(\alpha)}\int_a^{\hbar}\frac{w(t)g'(t)H_k(\hbar,t,z(t))}{(g(\hbar)-g(t))^{1-\alpha}}\mathrm{d}t\right)\right|^p\right\}^{\frac{1}{p}} \\
&\leq \lim_{n\to\infty}\sup_{z\in B_{p,\alpha,\hat{r}}}\left\{2^{p-1}\sum_{k\geq n}\alpha_k^p\left[\hat{\aleph}\,|z_k(\hbar)|^p + \frac{\hat{\hbar}(g(T)-g(a))^{p\alpha}L^p\hat{H}_k^p}{l^p\Gamma(\alpha)^p\alpha^p}\right]\right\}^{\frac{1}{p}} \\
&= 2^{1-\frac{1}{p}}\lim_{n\to\infty}\sup_{z\in B_{p,\alpha,\hat{r}}}\left\{\hat{\aleph}\sum_{k\geq n}\alpha_k^p\,|z_k(\hbar)|^p + \frac{\hat{\hbar}(g(T)-g(a))^{p\alpha}L^pQ_n}{l^p\Gamma(\alpha)^p\alpha^p}\right\}^{\frac{1}{p}},
\end{aligned}$$

i.e.,

$$\chi_{\ell_p^\alpha}(\mathcal{K}B_{p,\alpha,\hat{r}}) \leq 2^{1-\frac{1}{p}}\hat{\aleph}^{\frac{1}{p}}\chi_{\ell_p^\alpha}(B_{p,\alpha,\hat{r}}).$$

Therefore,

$$\chi_{C(\mathbb{J},\ell_p^\alpha)}(\mathcal{K}B_{p,\alpha,\hat{r}}) \leq 2^{1-\frac{1}{p}}\hat{\aleph}^{\frac{1}{p}}\chi_{C(\mathbb{J},\ell_p^\alpha)}(B_{p,\alpha,\hat{r}}).$$

Thus, assumption (5) and Corollary 1 yield that $\mathcal{K}$ possesses one fixed point in $B_{p,\alpha,\hat{r}} \subseteq C(\mathbb{J}, \ell_p^\alpha)$. Hence, system (5) possesses one solution in $C(\mathbb{J}, \ell_p^\alpha)$. □

Example 2. Let

$$z_n(\hbar) = \frac{z_n(\hbar)}{6n^2 + \hbar} + \frac{4}{n^2\Gamma(\frac{1}{2})}\int_1^{\hbar} \frac{t^3\cos(z_n(t))}{(\hbar^4 - t^4)^{\frac{1}{2}}(t + n^2)}\mathrm{d}t, \tag{6}$$

where $\hbar \in \mathbb{J} = [1, 2]$ and $n \in \mathbb{N}$.

Here,

$$K_n(\hbar, z(\hbar), l(z(\hbar))) = \frac{z_n(\hbar)}{6n^2 + \hbar} + \frac{l(z(\hbar))}{n^2},$$

$$H_n(\hbar, t, z(t)) = \frac{\cos(z_n(t))}{t + n^2},$$

$$l(z(\hbar)) = \frac{4}{\Gamma(\frac{1}{2})}\int_1^{\hbar} \frac{t^3\cos(z_n(t))}{(\hbar^4 - t^4)^{\frac{1}{2}}(t + n^2)}\mathrm{d}t,$$

$$g(\hbar) = \hbar^4,\ \ w(\hbar) = 1,\ \ \alpha = \frac{1}{2},\ \ \alpha_n = \frac{1}{n},$$

and

$$T = 2.$$

Let $z(\hbar) \in \ell_p^\alpha$ for some fixed $\hbar \in \mathbb{J}$. Then

$$\begin{aligned}\sum_{n\geq 1}\alpha_n^p\,|K_n(\hbar, z(\hbar), l(z(\hbar)))|^p &= \sum_{n\geq 1}\frac{1}{n^p}\left|\frac{z_n(\hbar)}{6n^2 + \hbar} + \frac{4}{n^2\Gamma(\frac{1}{2})}\int_1^{\hbar}\frac{t^3\cos(z_n(t))}{(\hbar^4 - t^4)^{\frac{1}{2}}(t + n^2)}\mathrm{d}t\right|^p \\ &\leq \frac{2^{p-1}}{7}\sum_{n\geq 1}\frac{1}{n^p}\,|z_n(\hbar)|^p + \frac{2^{p-1}}{\Gamma(\frac{1}{2})^p}\sum_{n\geq 1}\frac{1}{n^{2p}}\left\{\int_1^{\hbar}\frac{4t^3\mathrm{d}t}{(\hbar^4 - t^4)^{\frac{1}{2}}}\right\}^p \\ &\leq \frac{2^{p-1}}{7}\|z(\hbar)\|_{\ell_p^\alpha}^p + \frac{2^{2p-1}(2^4 - 1)^{\frac{p}{2}}}{\Gamma(\frac{1}{2})^p}\sum_{n\geq 1}\frac{1}{n^{2p}} < \infty,\end{aligned}$$

as both $\sum_{n\geq 1}\frac{1}{n^p}$ and $\sum_{n\geq 1}\frac{1}{n^{2p}}$ are convergent for $p > 1$.

Therefore, for fixed $\hbar \in \mathbb{J}$,

$$\{K_n(\hbar, z(\hbar), l(z(\hbar)))\}_{n=1}^{\infty} \in \ell_p^\alpha,$$

i.e.,

$$\{K_n(\hbar, z(\hbar), l(z(\hbar)))\}_{n=1}^{\infty} \in C(\mathbb{J}, \ell_p^\alpha).$$

It is obvious that K_n is continuous for all $n \in \mathbb{N}$ and

$$\begin{aligned}|K_n(\hbar, z(\hbar), l(z(\hbar))) - K_n(\hbar, \bar{z}(\hbar), l(\bar{z}(\hbar)))|^p &= \left|\frac{1}{6n^2 + \hbar}(z_n(\hbar) - \bar{z}(\hbar)) + \frac{1}{n^2}(l(z(\hbar)) - l(\bar{z}(\hbar)))\right|^p \\ &\leq \frac{2^{p-1}}{6^p n^{2p}}\,|z_n(\hbar) - \bar{z}(\hbar)|^p + \frac{2^{p-1}}{n^{2p}}\,|l(z(\hbar)) - l(\bar{z}(\hbar))|^p.\end{aligned}$$

Here, both $\aleph_n$ and P_n are continuous maps for all $n \in \mathbb{N}$ and

$$\aleph_n(\hbar) = \frac{2^{p-1}}{6^p n^{2p}}, \hat{\aleph}_n = \frac{2^{p-1}}{6^p n^{2p}}, \hat{\aleph} = \frac{2^{p-1}}{6^p}$$

and

$$P_n(\hbar) = \frac{2^{p-1}}{n^{2p}}, \sum_{n=1}^{\infty} \alpha_n^p P_n(\hbar) = \sum_{n=1}^{\infty} \frac{2^{p-1}}{n^{3p}} = 2^{p-1} \sum_{n=1}^{\infty} \frac{1}{n^{3p}} < \infty.$$

Also,

$$\sum_{n=1}^{\infty} \alpha_n^p \, |K_n(\hbar, z^0, 0)|^p = 0$$

and $2^{1-\frac{1}{p}} \hat{\aleph}^{\frac{1}{p}} = 2^{1-\frac{1}{p}} \cdot 2^{1-\frac{1}{p}} \cdot 6^{-1}$, i.e., $0 < 2^{1-\frac{1}{p}} \hat{\aleph}^{\frac{1}{p}} < 1$.

The map $g(\hbar) = \hbar^4$ is C^1 and nondecreasing.

Again, H_n is continuous for all $n \in \mathbb{N}$ and

$$|H_n(\hbar, t, z(t))| = \frac{1}{t + n^2}$$

and

$$\hat{H}_n = \frac{1}{n^2},$$

which gives $Q_k = \sum_{n \geq k} \frac{1}{n^{3p}} < \infty$ and $\lim_{k \to \infty} Q_k = 0$.

Thus, all the assumptions (1)–(5) of Theorem 3.1 are satisfied. Hence, system (6) has a solution in $C(\mathbb{J}, \ell_p^{\alpha})$.

Funding information: This research received no external funding.

Author contributions: All authors contributed equally to the writing of this article. All authors have accepted responsibility for entire content of the manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Ethical approval: The conducted research is not related to either human or animal use.

Data availability statement: Data sharing is not applicable to the article as no datasets were generated or analyzed during the current study.

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