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SOME NEW RESULTS ON INFORMATION TRANSMISSION OVER NOISY CHANNELS

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Abstract. Information transmission over communication channels has been characterized by weighted information schemes involving probabilities and weights. Binary erasure channel has been used as an example for determination of constants in the proposed measure.

1. Introduction

Increasing utilization of wireless packet erasure networks in different applications and aspects of real world has grabbed attentions of various researchers in a variety of streams. For the analysis and optimal design of such networks, the major challenge is the complexity and robustness of such systems and the distributed nature of their setup. Many researchers model these networks by directed cyclic graphs in which each edge represents a binary erasure channel with a constant erasure probability. Such networks play an important role in the design of unmanned aerial vehicles, underwater vehicles, remotely operated vehicles and many more. Information transmission over such networks is another challenging task due to the presence of noise in the channel. The term noise designates unwanted waves that disturb the transmission and processing of wanted signals in communication systems. The source of noise may be external or internal to the system. External source of noise includes atmospheric noise, man generated noise etc. and internal source of noise includes thermal noise, shot noise and so on.

In this present work, we have considered that the messages at the input and output of a communication channel are characterized by weighted information schemes, i.e. they are functions of probabilities with which they appear

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and of some costs which reflect the weight, importance or utility of the message from the communication and application point of view. From the communication point of view, parameters affecting the overall probability of correct transmission can be taken as costs and from the application point of view, external parameters (network specific) affecting the transmission of messages can be incorporated in the design of communication systems. By identifying the transmitted messages as functions of probabilities and costs, we have established expressions for weighted entropies of the communication channel. It is also shown that the relationships between these entropies are preserved. A non additive information theoretic approach based on suitable generalization of Shannon entropy would give a better characterization of such networks rather than an additive model (Shannon entropy approach).

We introduce a new non additive entropy measure, which is a generalization of Shannon entropy and Havrda Charvat entropy. We start by introducing the Shannon entropy and its generalization and then will model our channel using these entropy measures. Shannon [5] firstly introduced the entropy to measure the uncertainty associated with the random variable X with the corresponding probabilities $P = \{p_i,\, i = 1, 2, \ldots, n\}$, given by

$$H_S(P) = -\sum_{i=1}^{n} p_i \log p_i, \tag{1}$$

when the logarithm is to base 2, the unit of entropy is in bits and in nats, when the logarithm has base e.

The entropy measure proposed by Havrda-Charvat [4] which, in contrast to Shannon entropy [5] is non-additive, is called structural α entropy, and is given by

$$H^{\alpha}(P) = \frac{1}{(2^{1-\alpha} - 1)}\Big(\sum_{i=1}^{n} p_i^{\alpha} - 1\Big); \qquad \alpha > 0,\, \alpha \neq 1. \tag{2}$$

When $\alpha \to 1$, this measure reduces Shannon's measure.

The rest of the paper is organized as follows. Section 2 gives the main result. Some properties and bound of the marginal entropy function are introduced in Section 3. Section 4 concludes the paper.

2. Main result

Let us assume that the field at the input of a noisy channel is defined as

$$X = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ p(x_1) & p(x_2) & \cdots & p(x_n) \\ c(x_1) & c(x_2) & \cdots & c(x_n) \end{pmatrix}, \tag{3}$$

where x_k represents the symbols at the channel input; $p(x_k)$ denotes the probability with which x_k is applied at the channel input and $c(x_k)$ denotes the symbol cost at the channel input. Let Y be the field at the channel output, characterized by the distribution, given by

$$Y = \begin{pmatrix} y_1 & y_2 & \cdots & y_m \\ p(y_1) & p(y_2) & \cdots & p(y_m) \\ c(y_1) & c(y_2) & \cdots & c(y_m) \end{pmatrix}, \tag{4}$$

where y_j represents the symbols at the channel output; $p(y_j)$ denotes the probability with which y_j is received at the channel output and $c(y_j)$ denotes the symbol cost at the channel output.

The noise or channel matrix is defined as

$$P(Y|X) = \begin{pmatrix} p(y_1\,|x_1\,) & p(y_2\,|x_1\,) & \cdots & p(y_m\,|x_1\,) \\ p(y_1\,|x_2\,) & p(y_2\,|x_2\,) & \cdots & p(y_m\,|x_2\,) \\ \vdots & \vdots & \vdots & \vdots \\ p(y_1\,|x_n\,) & p(y_2\,|x_n\,) & \cdots & p(y_m\,|x_n\,) \end{pmatrix}. \tag{5}$$

We denote by $c(x_k, y_j)$, the cost corresponding to the symbol x_k at the input and y_j at the output, by $c(x_k|y_j)$, the cost of input symbol x_k, given the received symbol y_j, and by $c(y_j|x_k)$, the cost of output symbol y_j, given the input symbol x_k, $k = 1, \ldots, n$, $j = 1, \ldots, m$.

To characterize the new quantitative-qualitative mutual information $i_{pc}(x_{k,}y_j)$, we assume the following axioms [2].

Axiom 1. When the disturbances on the channel are very strong, consider

$$c(x_k, y_j) = c(x_k) + c(\,y_j|\,x_k) = c(y_j) + c(\,x_k|\,y_j). \tag{6}$$

Axiom 2. When the disturbances on the channel are very noisy, consider

$$c(x_k, y_j) = c(x_k) + c(y_j). \tag{7}$$

Using (7) in (6), we get

$$\left.\begin{aligned} c(\,y_j|\,x_k) &= c(y_j) \\ c(x_k\,|y_j\,) &= c(x_k) \end{aligned}\right\}. \tag{8}$$

Axiom 3. When no disturbances appear on the channel, i.e. for noiseless channel, consider

$$c(x_k, y_j) = c(x_k) = c(y_j). \tag{9}$$

Using (9) in (6), we get

$$c(y_j|x_k) = c(x_k|y_j) = 0. \tag{10}$$

Axiom 4. Let $i_{pc}(x_k)$ and $i_{pc}(y_j)$ be the quantitative-qualitative information of x_k and y_j, respectively. Let $i_{pc}(y_j|x_k)$ and $i_{pc}(x_k|y_j)$ be the quantitative-qualitative information of y_j given x_k and the quantitative-qualitative information of x_k given y_j, respectively. Let $p(x_k, y_j)$ and $c(x_k, y_j)$ denote the probability and the cost respectively, of the presence of x_k and y_j at the input and at the output of the transmission channel, respectively. Let $i_{pc}(x_{k,}y_j)$ be the mutual information, given by

$$i_{pc}(x_k, y_j) = i_{pc}(x_k) + i_{pc}(y_j|x_k) + \tau i_{pc}(x_k) i_{pc}(y_j|x_k), \tag{11}$$

where $\tau = 2^{1-\mu} - 1$.

In the following theorem, we obtain the quantitative-qualitative mutual information $i_{pc}(x_{k,}y_j)$ under the above axioms.

Theorem 2.1. *The quantitative-qualitative mutual information $i_{pc}(x_{k,}y_j)$ is given by*

$$i_{pc}(x_k, y_j) = \frac{(p(x_{k,}y_j))^{\mu} 2^{\lambda c(x_k, y_j)} - 1}{\tau}, \tag{12}$$

where λ, μ and τ are arbitrary constants to be determined by some suitable boundary conditions.

Proof. The mutual information $i_{pc}(x_{k,}y_j)$ is a function of $p(x_k, y_j)$ and $c(x_k, y_j)$, defined as

$$i_{pc}(x_k, y_j) = F[p(x_k, y_j), c(x_k, y_j)]. \tag{13}$$

Considering (13), we can similarly write

$$\left.\begin{aligned} i_{pc}(x_k) &= F[p(x_k), c(x_k)] \\ i_{pc}(y_j) &= F[p(y_j), c(y_j)] \\ i_{pc}(y_j|x_k) &= F[p(y_j|x_k), c(y_j|x_k)] \\ i_{pc}(x_k|y_j) &= F[p(x_k|y_j), c(x_k|y_j)] \end{aligned}\right\}. \tag{14}$$

For evaluating the function F, we substitute (13) and (14) in (11), thereby getting the functional equation, given by

$$\begin{aligned} F[p(x_{k,}y_j), c(x_{k,}y_j)] = {} & F[p(x_k), c(x_k)] + F[p(y_j|x_k), c(y_j|x_k)] \\ & + \tau F[p(x_k), c(x_k)] F[p(y_j|x_k), c(y_j|x_k)]. \end{aligned} \tag{15}$$

We also know that

$$p(x_{k,}y_j) = p(x_k) p(y_j\,|x_k\,) = p(y_j) p(x_k\,|y_j)\,. \tag{16}$$

We denote

$$\left.\begin{aligned} p(x_k) &= 2^{z_k} \\ p(y_j|x_k) &= 2^{z_{j|k}} \end{aligned}\right\}. \tag{17}$$

Using (6), (16) and (17) in (15), we get

$$F[2^{z_k+z_{j|k}}, c(x_k)+c(y_j|x_k)] = F[2^{z_k}, c(x_k)] + F[2^{z_{j|k}}, c(y_j|x_k)] + \tau F[2^{z_k}, c(x_k)]F[2^{z_{j|k}}, c(y_j|x_k)]. \tag{18}$$

Next, we assume

$$F(2^z, c) = G(z, c). \tag{19}$$

Using (19), (18) becomes

$$G[z_k + z_{j|k}, c(x_k)+c(y_j|x_k)] = G[z_k, c(x_k)] + G[z_{j|k}, c(y_j|x_k)] + \tau\, G[z_k, c(x_k)]G[z_{j|k}, c(y_j|x_k)]. \tag{20}$$

Substituting $z_k = z_{j|k} = 0$ in (20), we obtain

$$G[0, c(x_k)+c(y_j|x_k)] = G[0, c(x_k)] + G[0, c(y_j|x_k)] + \tau\, G\,[0, c(x_k)]\, G\,[0, c(y_j\,|x_k)]$$

or

$$[1+\tau\, G(0, c(x_k)+c(y_j|x_k))] = [1+\tau\, G(0, c(x_k))][1+\tau\, G(0, c(y_j|x_k))]. \tag{21}$$

Let us assume that

$$1+\tau\, G(0, c(x_k)) = f(c(x_k).$$

Then (21) becomes

$$f[c(x_k)+c(y_j|x_k)] = f(c(x_k))f(c(y_j|x_k)). \tag{22}$$

The solution of the functional equation (22) is given by [1, 3]

$$f(c(x_k)) = 2^{\lambda c(x_k)},$$

which further gives

$$G(0, c(x_k)) = \frac{2^{\lambda c(x_k)}-1}{\tau}, \tag{23}$$

where λ is some arbitrary constant.

Again taking $c(x_k) = c(y_j\,|x_k\,) = 0$ in (20), we obtain

$$G[z_k + z_{j|k}, 0] = G[z_k, 0] + G[z_{j|k}, 0] + \tau\, G[z_k, 0]G[z_{j|k}, 0],$$

which gives

$$[1+\tau\, G(z_k + z_{j|k}, 0)] = [1+\tau\, G(z_k, 0)][1+\tau\, G(z_{j|k}, 0)].$$

Assuming $1+\tau\, G(z_k, 0) = M(z_k)$ in the above equation, we obtain

$$M(z_k + z_{j|k}) = M(z_k)M(z_{j|k}). \tag{24}$$

The solution of the functional equation (24) is given by [1, 3]

$$M(z_k) = 2^{\mu z_k}, \tag{25}$$

which further gives

$$G(z_k, 0) = \frac{2^{\mu z_k} - 1}{\tau}, \tag{26}$$

where μ is some arbitrary constant.

Again taking $c(x_k) = z_{j|k} = 0$ in (20), we obtain

$$G[z_k, c(y_j|x_k)] = G[z_k, 0] + G[0, c(y_j|x_k)] + \tau\, G[z_k, 0]G[0, c(y_j|x_k)]. \tag{27}$$

Using (23) and (26) in (27), we obtain

$$G[z_k, c(y_j|x_k)] = \frac{2^{\mu z_k + \lambda c(y_j|x_k)} - 1}{\tau},$$

which further gives

$$F[2^{z_k}, c(y_j|x_k)] = \frac{2^{\mu z_k + \lambda c(y_j|x_k)} - 1}{\tau},$$

i.e.

$$F[p(x_k), c(y_j|x_k)] = \frac{(p(x_k))^{\mu} 2^{\lambda c(y_j|x_k)} - 1}{\tau}.$$

Therefore, we finally obtain

$$i_{pc}(x_k, y_j) = F[(p(x_k, y_j), c(x_k, y_j)] = \frac{(p(x_k, y_j))^{\mu} 2^{\lambda c(x_k, y_j)} - 1}{\tau}.$$

This completes the proof. ∎

The above relation express the weighted mutual information for the weighted information schemes defined by (3) and (4). The average of this information gives the joint quantitative qualitative entropy of the transmission channel defined by (3) and (4), given by

$$\begin{aligned} H_{pc}(X, Y) &= \sum_{k=1}^{n} \sum_{j=1}^{m} i_{pc}(x_k, y_j), p(x_k, y_j) \\ &= \sum_{k=1}^{n} \sum_{j=1}^{m} p(x_k, y_j) \frac{(p(x_k, y_j))^{\mu}\, 2^{\lambda c(x_k y_j)} - 1}{\tau}. \end{aligned} \tag{28}$$

The following cases would explain the joint quantitative qualitative entropy for noiseless channel and very noisy channel, respectively.

Case I. When no disturbances appear on the channel, i.e. for noiseless channels given by

$$p(x_k|y_j) = p(y_j|x_k) = \begin{cases} 1, & \text{if } x_k = y_j, \\ 0, & \text{if } x_k \neq y_j. \end{cases} \tag{29}$$

Using (9), (15) and (29) in (28), we obtain

$$\begin{aligned} H_{pc}(X,Y) &= \sum_{k=1}^{n} p(x_k)\frac{(p(x_k))^{\mu}2^{\lambda c(x_k)}-1}{\tau} \\ &= H_{pc}(X) = H_{pc}(Y), \end{aligned}$$

i.e. the input field coincides with the output field.

Case II. When strong disturbances arise on the channel, i.e. for very noisy channels given by

$$p(x_k,y_j) = p(x_k)p(y_j). \tag{30}$$

Using (7) and (30), (28) takes the form given by

$$\begin{aligned} H_{pc}(X,Y) = {} & \sum_{k=1}^{n} p(x_k)\frac{(p(x_k))^{\mu}2^{\lambda c(x_k)}-1}{\tau} + \sum_{j=1}^{m} p(y_j)\frac{(p(y_j))^{\mu}2^{\lambda c(y_j)}-1}{\tau} \\ & +\tau\left\{\sum_{k=1}^{n} p(x_k)\left\{\frac{(p(x_k))^{\mu}2^{\lambda c(x_k)}-1}{\tau}\right\}\right\}\left\{\sum_{j=1}^{m} p(y_j)\left\{\frac{(p(y_j))^{\mu}2^{\lambda c(y_j)}-1}{\tau}\right\}\right\}, \end{aligned} \tag{31}$$

$$H_{pc}(X,Y) = H_{pc}(X) + H_{pc}(Y) + \tau H_{pc}(X)H_{pc}(Y),$$

where the marginal entropies for X and Y are given by

$$H_{pc}(X) = \sum_{k=1}^{n} p(x_k)\frac{(p(x_k))^{\mu}2^{\lambda c(x_k)}-1}{\tau}, \tag{32}$$

and

$$H_{pc}(Y) = \sum_{j=1}^{m} p(y_j)\left\{\frac{(p(y_j))^{\mu}2^{\lambda c(y_j)}-1}{\tau}\right\}. \tag{33}$$

The above entropy measures reduces to Havrda-Charvat measure [4] at $\lambda = 0$ and Shannon's measure [5] at $\lambda = 0$, $\mu \to 1$.

We now consider an example of binary erasure channel for calculating the values of λ, μ and τ.

Example. Binary Erasure Channel. The binary erasure channels (BEC) model situations where information may be lost but is never corrupted, i.e. single bits are transmitted and either received correctly or known to be lost. The decoding problem is to find the values of the bits given the locations of the erasures and the non-erased part of the codeword. It either preserves the input or erases it. Let X be the transmitted random variable with alphabet (0, 1). Let Y be the received variable with alphabet (0, 1, e). The symbol e

is the erasure symbol which indicates that the input message is lost or erased. The channel matrix of binary erasure channel is given by

$$P[Y|X] = \begin{bmatrix} p & 0 & 1-p \\ 0 & p & 1-p \end{bmatrix},$$

which indicates that the source at the input is either correctly received or erased in the form of output represented by e. Let the input probability matrix be

$$P(X) = [\,p(x_1) \quad p(x_2)\,] = [\,\alpha \quad 1-\alpha\,].$$

Then the joint probability matrix is given by

$$P[X,Y] = \begin{bmatrix} \alpha p & 0 & \alpha(1-p) \\ 0 & (1-\alpha)p & (1-p)(1-\alpha) \end{bmatrix}.$$

The output probability matrix is given by

$$P(Y) = [\,p(y_1) \quad p(y_2) \quad p(y_3)\,] = [\,p\alpha \quad p(1-\alpha) \quad 1-p\,].$$

Considering (31), the joint quantitative qualitative entropy of binary erasure channel is given by

$$\begin{aligned} H_{pc}(X,Y) = \alpha p &\left\{ \frac{(\alpha p)^{\mu}\, 2^{\lambda(c(x_1)+c(y_1))} - 1}{\tau} \right\} \\ &+ \alpha(1-p) \left\{ \frac{(\alpha(1-p))^{\mu}\, 2^{\lambda(c(x_1)+c(y_3))} - 1}{\tau} \right\} \\ &+ (1-\alpha)p \left\{ \frac{((1-\alpha)p)^{\mu}\, 2^{\lambda(c(x_2)+c(y_2))} - 1}{\tau} \right\} \\ &+ (1-\alpha)(1-p) \left\{ \frac{((1-\alpha)(1-p))^{\mu}\, 2^{\lambda(c(x_2)+c(y_3))} - 1}{\tau} \right\}, \end{aligned}$$

where $\tau = 2^{1-\mu} - 1$.

Substituting the value of τ in the above expression, we get

$$\text{(34)} \quad \begin{aligned} H_{pc}(X,Y) = \alpha p &\left\{ \frac{((\alpha p))^{\mu} 2^{\lambda(c(x_1)+c(y_1))} - 1}{2^{1-\mu} - 1} \right\} \\ &+ \alpha(1-p) \left\{ \frac{(\alpha(1-p))^{\mu} 2^{\lambda(c(x_1)+c(y_3))} - 1}{2^{1-\mu} - 1} \right\} \end{aligned}$$

$$+ (1-\alpha)p \left\{ \frac{((1-\alpha)p)^{\mu} \, 2^{\lambda(c(x_2)+c(y_2))} - 1}{2^{1-\mu}-1} \right\}$$
$$+ (1-\alpha)(1-p) \left\{ \frac{((1-\alpha)(1-p))^{\mu} \, 2^{\lambda(c(x_2)+c(y_3))} - 1}{2^{1-\mu}-1} \right\}.$$

Using (32) and (33), the marginal entropies for X and Y of binary erasure channel are given by

$$H_{pc}(X) = \alpha\left\{\frac{(\alpha)^{\mu}2^{\lambda c(x_1)}-1}{2^{1-\mu}-1}\right\} + (1-\alpha)\left\{\frac{(1-\alpha)^{\mu}2^{\lambda c(x_2)}-1}{2^{1-\mu}-1}\right\}, \tag{35}$$

$$H_{pc}(Y) = \alpha p\left\{\frac{(\alpha p)^{\mu}2^{\lambda c(y_1)}-1}{2^{1-\mu}-1}\right\} + (1-\alpha)p\left\{\frac{((1-\alpha)p)^{\mu}2^{\lambda c(y_2)}-1}{2^{1-\mu}-1}\right\} \tag{36}$$
$$+ (1-p)\left\{\frac{(1-p)^{\mu}2^{\lambda c(y_3)}-1}{2^{1-\mu}-1}\right\}.$$

When the events are equiprobable with unit cost, then the entropy of the transmitted and received variable should attain the maximum value, i.e.

$$H_{pc}\left(\frac{1}{2},\frac{1}{2};1\right) = 1, \tag{37}$$

$$H_{pc}\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};1\right) = \log_2 3. \tag{38}$$

Using equation (37) in (35), we have

$$\left(\frac{1}{2}\right)\left\{\frac{\left(\frac{1}{2}\right)^{\mu}2^{\lambda}-1}{2^{1-\mu}-1}\right\} + \frac{1}{2}\left\{\frac{\left(\frac{1}{2}\right)^{\mu}2^{\lambda}-1}{2^{1-\mu}-1}\right\} = 1$$
$$\Rightarrow \left[\frac{2^{\lambda-\mu}-1}{2^{1-\mu}-1}\right] = 1 \Rightarrow \lambda = 1.$$

Using equation (38) in (36), we have

$$\left(\frac{1}{3}\right)\left\{\frac{\left(\frac{1}{3}\right)^{\mu}2^{\lambda}-1}{2^{1-\mu}-1}\right\} + \frac{1}{3}\left\{\frac{\left(\frac{1}{3}\right)^{\mu}2^{\lambda}-1}{2^{1-\mu}-1}\right\} + \frac{1}{3}\left\{\frac{\left(\frac{1}{3}\right)^{\mu}2^{\lambda}-1}{2^{1-\mu}-1}\right\} = \log_2 3$$
$$\Rightarrow \left[\frac{2^{\lambda}3^{-\mu}-1}{2^{1-\mu}-1}\right] = \log_2 3 \Rightarrow \mu = -0.875$$

and $\tau = 2^{1-\mu} - 1 = 2.668$.

In the next section, we list some properties and look for the upper bound of the marginal entropy function $H_{pc}(X)$.

3. Properties and bound of marginal entropy function

The proposed marginal entropy measure

$$H_{pc}(X) = \sum_{k=1}^{n} p(x_k)\left\{\frac{p(x_k)^{\mu} 2^{\lambda c(x_k)} - 1}{\tau}\right\};$$

$$0 \le p(x_k) \le 1,\ \sum_{k=1}^{n} p(x_k) = 1,\ c(x_k) \ge 0,$$

has the following properties:

1. $H(p(x_k); c(x_k))$ is continuous in the region $0 \le p(x_k) \le 1$, $\sum_{k=1}^{n} p(x_k) = 1$, $c(x_k) \ge 0$.
2. $H(p(x_1), p(x_2), \ldots, p(x_{k-1}), 0, p(x_{k+1}), \ldots p(x_n); c(x_1), c(x_2), \ldots, c(x_n))$ $= H(p(x_1), p(x_2), \ldots, p(x_{k-1}), p(x_{k+1}), \ldots p(x_n); c(x_1), c(x_2), \ldots, c(x_n))$, for all $k = 1, 2, \ldots, n$.
3. $H\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}; 1\right) = \frac{(\frac{1}{n})^{\mu} 2^{\lambda} - 1}{\tau}$, $H(1; 0) = 0$.
4. $H(p(x_k); c(x_k))$ and $H(p(y_j); c(x_j))$ is symmetric with respect to the pair $(p(x_k); c(x_k))$; $\forall k = 1, 2, \ldots, n$.
5. $H(p(x_k); c(x_k))$ is concave in nature.

The above properties also hold for the marginal entropy function $H_{pc}(Y)$.

In the next theorem, we determine the upper bound of the marginal entropy function $H_{pc}(X)$.

Theorem 3.1. *The upper bound for the entropy measure* (28) *is given by*

$$H_{pc}(X) \le \frac{\mu \log n \ln 2}{(1 - 2^{1-\mu})}. \tag{39}$$

Proof. For the mapping $f : (0, \infty) \to R$, which is differentiable and concave on$(0, \infty)$, we have the following inequality [6]

$$f(x) - f(y) \le f'(y)(x - y); \qquad \forall x, y > 0.$$

If we take $f(x) = \log_a x$ in the above relation, we get

$$\log_a(x) - \log_a(y) \le \frac{1}{\ln a}\frac{(x - y)}{y}; \qquad \forall x, y > 0.$$

Let $x = p(x_k)^{\mu} 2^{\lambda c(x_k)}$, $y = 1$ and $a = 2$ in the above inequality, we obtain

$$\log_2\left(p(x_k)^{\mu} 2^{\lambda c(x_k)}\right) - \log_2(1) \le \frac{1}{\ln\ 2}\left(p(x_k)^{\mu} 2^{\lambda c(x_k)} - 1\right)$$

$$\Rightarrow \mu \log_2(p(x_k)) \le \frac{1}{\ln\ 2}\frac{\left(p(x_k)^{\mu} 2^{\lambda c(x_k)} - 1\right)}{(2^{1-\mu} - 1)}\left(2^{1-\mu} - 1\right) - \lambda c(x_k)$$

$$\Rightarrow -\sum_{k=1}^{n} \mu p(x_k)\log_2\left(p(x_k)\right)$$
$$\geq \sum_{k=1}^{n}\left[\lambda p(x_k)c(x_k) - \frac{1}{\ln\, 2}p(x_k)\frac{\left(p(x_k)^{\mu}2^{\lambda c(x_k)}-1\right)}{(2^{1-\mu}-1)}\left(2^{1-\mu}-1\right)\right]$$
$$\Rightarrow \mu H_S(P)\ln 2 \geq \lambda \ln 2\sum_{k=1}^{n} p(x_k)c(x_k) + H_{pc}(X)\left(1-2^{1-\mu}\right).$$

For $\mu \geq 1$, the above inequality becomes

$$H_{pc}(X)(1-2^{1-\mu}) \leq \mu H_S(P)\ln 2$$
$$\Rightarrow H_{pc}(X)(1-2^{1-\mu}) \leq \mu \log n \ln 2 \Rightarrow H_{pc}(X) \leq \frac{\mu \log n \ln 2}{(1-2^{1-\mu})}. \blacksquare$$

4. Conclusion

In the present work, the weighted two dimensional entropies for the information schemes defined by (3) and (4) have been obtained. Binary Erasure channel has been used as an example for determining the constants in proposed measures. The additive counterpart of measure (12) can be seen in [2]. Further work on paramteric generalization of obtained results is in progress and will be reported elsewhere.

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