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m-PREOPEN SETS IN BIMINIMAL SPACES

Abstract. The aim of this paper is to introduce and characterize the concepts of preopen sets and their related notions in biminimal spaces.

1. Introduction

In [4], Popa and Noiri introduced the notion of minimal structure which is a generalization of a topology on a given nonempty set. They also introduced the notion of m-continuous function as a function defined between a minimal structure and a topological space. They showed that the m-continuous functions have properties similar to those of continuous functions between topological spaces. Let X be a topological space and $A \subset X$. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subfamily m of the power set $P(X)$ of a nonempty set X is called a minimal structure [4] on X if $\varnothing$ and X belong to m. By (X, m), we denote a nonempty set X with a minimal structure m on X. The members of the minimal structure m are called m-open sets [4], and the pair (X, m) is called an m-space. The complement of m-open set is said to be m-closed [4]. In this paper we introduce and characterize the concepts of preopen sets in a biminimal space (X, m_1, m_2), which is a set X with two arbitrary minimal structures m_1 and m_2.

2. Preliminaries

In this section, we recall the m-structure and the m-operator notions. Also, we recall some important subsets associated to these concepts.

Definition 2.1. [1] Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the set of power of X. We say that m_X is an m-structure (or a minimal structure) on X, if $\emptyset$ and X belong to m_X.

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The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of an m_X-open set is said to be an m_X-closed set. Given $A \subseteq X$, we define m_X-interior of A abbreviate m_X-Int(A) as $\bigcup\{W/W \in m_X, W \subseteq A\}$ and the m_X-closure of A abbreviate m_X-Cl(A) as $\bigcap\{F/A \subseteq F, X \setminus F \in m_X\}$. An immediate consequence of the above definition is the following theorem.

Theorem 2.2. [1, 4] *Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X$-Cl(A) if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x, and satisfying the following properties:*

(1) $m_X-\text{Cl}(m_X-\text{Cl}(A)) = m_X-\text{Cl}(A)$.
(2) $m_X-\text{Int}(m_X-\text{Int}(A)) = m_X-\text{Int}(A)$.
(3) $m_X-\text{Int}(X \setminus A) = X \setminus (m_X-\text{Cl}(A)$.
(4) $m_X-\text{Cl}(X \setminus A) = X \setminus (m_X-\text{Int}(A)$.
(5) *If* $A \subseteq B$ *then* $m_X-\text{Cl}(A) \subseteq m_X-\text{Cl}(B)$.
(6) $m_X\text{-Cl}(A \cup B) \subseteq m_X\text{-Cl}(A) \cup m_X\text{-Cl}(B)$.
(7) $A \subseteq m_X\text{-Cl}(A)$ *and* $m_X\text{-Int}(A) \subseteq A$.

Observe that m_X-Cl(A) is not necessarily an m_X-closed set. At this point arises a natural question. Do there exist any conditions on the set X or in the m-structure of X which guarantee that the m_X-Cl(A) is an m_X-closed set. At this point we introduce the following property.

Definition 2.3. [1] Let (X, m_X) be an m-space. We say that m_X has the property of Maki, if the union of any family of elements of m_X belongs to m_X.

Observe that any collection $\emptyset \neq \mathcal{J} \subseteq P(X)$, is always contained in an m-structure that have the property of Maki, as we know, $m_X(\mathcal{J}) = \{\emptyset, X\} \cup \{\bigcup_{M \in J} M : \emptyset \neq J \subseteq \mathcal{J}\}$. In particular, when $\mathcal{J} = m_X$, we denote by $m'_X = m_X(\mathcal{J})$. Clearly $m_X = m'_X$, if m_X have the property of Maki. Note that if m_X is an m-structure and $Y \subseteq X$, then $\{M \cap Y : M \in m_X\}$ is an m-structure on Y, and is denoted by $m_{X|Y}$, and the pair $(Y, m_{X|Y})$ is called an m-subspace of (X, m_X).

In general, the m_X-open sets is not stable for the union. Nevertheless, for certain m_X-structure, the class of m_X-open sets is stable under union of sets, like it is demonstreated in the following lemma.

Lemma 2.4. [1, 4] *Let m_X be an m-structure which satisfy the property of Maki. If $\{A_i : i \in I\}$ is a collection of m_X-open sets (resp., m_X-closed sets), then $\bigcup_{i\in I} A_i$ (resp., $\bigcap_{i\in I} A_i$) is an m_X-open set (resp., m_X-closed set).*

Theorem 2.5. [1, 4] *Let (X, m_X) be an m-space and m_X satisfying the property of Maki. For a subset A of X, the following properties hold:*

(1) m_X-Int$(A) \in m_X$ *and* m_X-Cl(A) *is* m_X*-closed.*
(2) $A \in m_X$ *if and only if* m_X-Int$(A) = A$.
(3) A *is* m_X*-closed if and only if* m_X-Cl$(A) = A$.

Definition 2.6. Let (X, m) be a space with a minimal structure m on X and $A \subset X$. Then a set A is called an m-preopen [2] set in X if $A \subset m\,\mathrm{Int}(m\,\mathrm{Cl}(A))$.

A set A is called an m-preclosed set if the complement of A is m-preopen. The family of all m-preopen (resp. m-preclosed) subsets of (X, m) is denoted by $mPO(X)$ (resp. $mPC(X)$).

Definition 2.7. [2] Let $f : (X, m) \to (Y, \sigma)$ be a function between a space X with a minimal structure m and a topological space Y. Then f is said to be minimal precontinuous (briefly m-precontinuous) if for each x and each open set V containing $f(x)$, there exists an m-preopen set U containing x such that $f(U) \subset V$.

3. (i, j)-m-preopen sets

Definition 3.1. *A* set X equipped with two m-spaces is called a biminimal space.

Let A be a subset of a biminimal space (X, m_1, m_2). We denote the closure of A and the interior of A with respect to m_i by m_i-Cl(A) and m_i-Int(A), respectively.

Definition 3.2. A subset A of a biminimal space (X, m_1, m_2) is said to be (i, j)-m-preopen if and only if $A \subset m_i$-Int$(m_j$-Cl$(A))$, where $i, j = 1, 2$ and $i \neq j$.

The family of all (i, j)-m-preopen sets of (X, m_1, m_2) is denoted by $PO(X, m_1, m_2)$ or (i, j)-$mPO(X)$. Also, the family of all (i, j)-m-preopen sets of (X, m_1, m_2) containing x is denoted by (i, j)-$mPO(X, x)$.

It is clear that every m_i-open sets is (i, j)-m-preopen but the converse is not true in general as it can be seen from the following example.

Example 3.3. Let $X = \{a, b, c\}$, $m_1 = \{\emptyset, \{a\}, \{b\}, X\}$, $m_2 = \{\emptyset, \{a\}, X\}$. Then $\{a, b\}$ is $(1, 2)$-m-preopen but is neither m_1-open nor m_2-open.

Proposition 3.4. *Let* A *be a subset of a biminimal space* (X, m_1, m_2) *and* A *be an* (i, j)*-*m*-preopen set. Then we have the following:*

(1) m_j-Cl$(m_i$-Int$(m_j$-Cl$(A))) = m_j$-Cl(A).
(2) m_j-Cl$(m_i$-Int$(m_j$-Cl$(A))) = m_j$-Cl(A).

Proof. The proof is obvious. ∎

Remark 3.5. The intersection of two (i,j)-m-preopen sets need not be (i,j)-m-preopen as it can be seen from the following example.

Example 3.6. Let $X = \{a,b,c\}$, $m_1 = \{\emptyset, \{a\}, \{c\}, X\}$, $m_2 = \{\emptyset, \{a,c\}, X\}$. Then the sets $\{a,b\}$ and $\{b,c\}$ are $(1,2)$-m-preopen sets of (X,m_1,m_2) but their intersection $\{b\}$ is not an $(1,2)$-m-preopen set of (X,m_1,m_2).

Theorem 3.7. *If $\{A_\alpha\}_{\alpha\in\Omega}$ is a family of (i,j)-m-preopen sets in (X,m_1,m_2), then $\bigcup_{\alpha\in\Omega} A_\alpha$ is (i,j)-m-preopen in (X,m_1,m_2).*

Proof. Since $\{A_\alpha : \alpha \in \Omega\} \subset (i,j)\text{-}mPO(X)$, then $A_\alpha \subset m_i\text{-Int}(m_j\text{-Cl}(A_\alpha))$ for every $\alpha \in \Omega$. Thus, $\bigcup_{\alpha\in\Omega} A_\alpha \subset \bigcup_{\alpha\in\Omega} m_i\text{-Int}(m_j\text{-Cl}(A_\alpha)) \subset m_i\text{-Int}(\bigcup_{\alpha\in\Omega} m_j\text{-Cl}(A_\alpha)) = m_i\text{-Int}(\bigcup_{\alpha\in\Omega}(A_\alpha) \cup A_\alpha) = m_i\text{-Int}(m_j\text{-Cl}(\bigcup_{\alpha\in\Omega} A_\alpha))$. Therefore, we obtain $\bigcup_{\alpha\in\Omega} A_\alpha \subset m_i\text{-Int}(m_j\text{-Cl}(\bigcup_{\alpha\in\Omega} A_\alpha))$. Hence any union of (i,j)-m-preopen sets is (i,j)-m-preopen. ■

Definition 3.8. Let (X,m_1,m_2) be a biminimal space. $A \subset X$ is said to be (i,j)-m-preclosed if $X \backslash A$ is (i,j)-m-preopen in X, for $i,j = 1,2$ and $i \neq j$.

Theorem 3.9. *A is an (i,j)-m-preclosed set in a biminimal space (X,m_1,m_2) if and only if $m_i\text{-Cl}(m_j\text{-Int}(A)) \subset A$.*

Proof. The proof follows from the definitions. ■

Theorem 3.10. *Arbitrary intersection of (i,j)-m-preclosed sets is always (i,j)-m-preclosed.*

Proof. Follows from Theorems 3.7 and 3.9. ■

Definition 3.11. Let (X,m_1,m_2) be a biminimal space, S a subset of X and x be a point of X. Then

(i) x is called an (i,j)-m-preinterior point of S if there exists $V \in (i,j)\text{-}mPO(X,m_1,m_2)$ such that $x \in V \subset S$.
(ii) The set of all (i,j)-m-preinterior points of S is called (i,j)-m-preinterior of S and is denoted by $(i,j)\text{-}mp\,\text{Int}(S)$.

Theorem 3.12. *Let A and B be subsets of (X,m_1,m_2). Then the following properties hold:*

(i) $(i,j)\text{-}mp\,\text{Int}(A) = \cup\{T : T \subset A \text{ and } A \in (i,j)\text{-}mPO(X)\}$.
(ii) *$(i,j)\text{-}mp\,\text{Int}(A)$ is the largest (i,j)-m-preopen subset of X contained in A.*
(iii) *A is (i,j)-m-preopen if and only if $A = (i,j)\text{-}mp\,\text{Int}(A)$.*
(iv) $(i,j)\text{-}mp\,\text{Int}((i,j)\text{-}mp\,\text{Int}(A)) = (i,j)\text{-}mp\,\text{Int}(A)$.
(v) *If $A \subset B$, then $(i,j)\text{-}mp\,\text{Int}(A) \subset (i,j)\text{-}mp\,\text{Int}(B)$.*
(vi) $(i,j)\text{-}mp\,\text{Int}(A) \cup (i,j)\text{-}mp\,\text{Int}(B) \subset (i,j)\text{-}mp\,\text{Int}(A \cup B)$.
(vii) $(i,j)\text{-}mp\,\text{Int}(A \cap B) \subset (i,j)\text{-}mp\,\text{Int}(A) \cap (i,j)\text{-}mp\,\text{Int}(B)$.

Proof. (i). Let $x \in \cup\{T : T \subset A$ and $A \in (i,j)\text{-}mPO(X)\}$. Then, there exists $T \in (i,j)\text{-}mPO(X,x)$ such that $x \in T \subset A$ and hence $x \in (i,j)\text{-}mp\,\mathrm{Int}(A)$. This shows that $\cup\{T : T \subset A$ and $A \in (i,j)\text{-}mPO(X)\} \subset (i,j)\text{-}mp\,\mathrm{Int}(A)$. For the reverse inclusion, let $x \in (i,j)\text{-}mp\,\mathrm{Int}(A)$. Then there exists $T \in (i,j)\text{-}mPO(X,x)$ such that $x \in T \subset A$. We obtain $x \in \cup\{T : T \subset A$ and $A \in (i,j)\text{-}mPO(X)\}$. This shows that $(i,j)\text{-}mp\,\mathrm{Int}(A) \subset \cup\{T : T \subset A$ and $A \in (i,j)\text{-}mPO(X)\}$. Therefore, we obtain $(i,j)\text{-}mp\,\mathrm{Int}(A) = \cup\{T : T \subset A$ and $A \in (i,j)\text{-}mPO(X)\}$.

The proof of (ii)-(v) are obvious.

(vi). Clearly, $(i,j)\text{-}mp\,\mathrm{Int}(A) \subset (i,j)\text{-}mp\,\mathrm{Int}(A \cup B)$ and $(i,j)\text{-}mp\,\mathrm{Int}(B) \subset (i,j)\text{-}mp\,\mathrm{Int}(A \cup B)$. Then by (v) we obtain $(i,j)\text{-}mp\,\mathrm{Int}(A) \cup (i,j)\text{-}mp\,\mathrm{Int}(B) \subset (i,j)\text{-}mp\,\mathrm{Int}(A \cup B)$.

(vii). Since $A \cap B \subset A$ and $A \cap B \subset B$, by (v), we have $(i,j)\text{-}mp\,\mathrm{Int}(A \cap B) \subset (i,j)\text{-}mp\,\mathrm{Int}(A)$ and $(i,j)\text{-}mp\,\mathrm{Int}(A \cap B) \subset (i,j)\text{-}mp\,\mathrm{Int}(B)$. By (v) $(i,j)\text{-}mp\,\mathrm{Int}(A \cap B) \subset (i,j)\text{-}mp\,\mathrm{Int}(A) \cap (i,j)\text{-}mp\,\mathrm{Int}(B)$. ∎

Definition 3.13. Let (X, m_1, m_2) be a biminimal space, S a subset of X and x be a point of X. Then

(i) x is called an (i,j)-m-precluster point of S if $V \cap S \neq \emptyset$ for every $V \in (i,j)\text{-}mPO(X,x)$.

(ii) The set of all (i,j)-m-precluster points of S is called (i,j)-m-preclosure of S and is denoted by $(i,j)\text{-}mp\,\mathrm{Cl}(S)$.

Theorem 3.14. *Let A and B be subsets of (X, m_1, m_2). Then the following properties hold:*

(i) $(i,j)\text{-}mp\,\mathrm{Cl}(A) = \cap\{F : A \subset F$ *and* $F \in (i,j)\text{-}mPC(X)\}$.

(ii) $(i,j)\text{-}mp\,\mathrm{Cl}(A)$ *is the smallest* (i,j)-*m-preclosed subset of* X *containing* A.

(iii) A *is* (i,j)-*m-preclosed if and only if* $A = (i,j)\text{-}mp\,\mathrm{Cl}(A)$.

(iv) $(i,j)\text{-}mp\,\mathrm{Cl}((i,j)\text{-}mp\,\mathrm{Cl}(A) = (i,j)\text{-}mp\,\mathrm{Cl}(A)$.

(v) *If* $A \subset B$, *then* $(i,j)\text{-}mp\,\mathrm{Cl}(A) \subset (i,j)\text{-}mp\,\mathrm{Cl}(B)$.

(vi) $(i,j)\text{-}mp\,\mathrm{Cl}(A \cup B) = (i,j)\text{-}mp\,\mathrm{Cl}(A) \cup (i,j)\text{-}mp\,\mathrm{Cl}(B)$.

(vii) $(i,j)\text{-}mp\,\mathrm{Cl}(A \cap B) \subset (i,j)\text{-}mp\,\mathrm{Cl}(A) \cap (i,j)\text{-}mp\,\mathrm{Cl}(B)$.

Proof. (i). Suppose that $x \notin (i,j)\text{-}mp\,\mathrm{Cl}(A)$. Then there exists $V \in (i,j)\text{-}mPO(X,x)$ such that $V \cap A = \emptyset$. Since $X \backslash V$ is (i,j)-m-preclosed set containing A and $x \notin X \backslash V$, we obtain $x \notin \cap\{F : A \subset F$ and $F \in (i,j)\text{-}mPC(X)\}$. Suppose that $x \notin \cap\{F : A \subset F$ and $F \in (i,j)\text{-}mPC(X)\}$. Then there exists $F \in (i,j)\text{-}mPC(X)$ such that $A \subset F$ and $x \notin F$. Since $X \backslash F$ is (i,j)-m-preopen set containing x, we obtain $(X \backslash F) \cap A = \emptyset$. This shows that $x \notin (i,j)\text{-}mp\,\mathrm{Cl}(A)$. Therefore, we obtain $(i,j)\text{-}mp\,\mathrm{Cl}(A) = \cap\{F : A \subset F$ and $F \in (i,j)\text{-}PC(X)$.

The other proofs are obvious. ∎

Theorem 3.15. *Let (X, m_1, m_2) be a biminimal space and $A \subset X$. Then the following propeties hold:*

(i) (i,j)-$mp\,\mathrm{Int}(X\backslash A) = X\backslash(i,j)$-$mp\,\mathrm{Cl}(A)$;
(ii) (i,j)-$mp\,\mathrm{Cl}(X\backslash A) = X\backslash(i,j)$-$mp\,\mathrm{Int}(A)$.

Proof. (i). Let $x \notin (i,j)$-$mp\,\mathrm{Cl}(A)$. There exists $V \in (i,j)$-$mPO(X, x)$ such that $V \cap A = \emptyset$; hence we obtain $x \in (i,j)$-$mp\,\mathrm{Int}(X\backslash A)$. This shows that $X\backslash(i,j)$-$mp\,\mathrm{Cl}(A) \subset (i,j)$-$mp\,\mathrm{Int}(X\backslash A)$. Let $x \in (i,j)$-$mp\,\mathrm{Int}(X\backslash A)$. Since (i,j)-$mp\,\mathrm{Int}(X\backslash A) \cap A = \emptyset$, we obtain $x \notin (i,j)$-$mp\,\mathrm{Cl}(A)$; hence $x \in X\backslash(i,j)$-$mp\,\mathrm{Cl}(A)$. Therefore, we obtain (i,j)-$mp\,\mathrm{Int}(X\backslash A) = X\backslash(i,j)$-$mp\,\mathrm{Cl}(A)$.

(ii). Follows from (i). ∎

Definition 3.16. A subset B_x of a biminimal space (X, m_1, m_2) is said to be an (i,j)-m-pre neighbourhood of a point $x \in X$ if there exists an (i,j)-m-preopen set U such that $x \in U \subset B_x$.

Theorem 3.17. *A subset of a biminimal space (X, m_1, m_2) is (i,j)-m-preopen if and only if it is an (i,j)-m-pre neighbourhood of each of its points.*

Proof. Let G be an (i,j)-m-preopen set of X. Then by definition, it is clear that G is an (i,j)-m-pre neighbourhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and G is (i,j)-m-preopen. Conversely, suppose G is an (i,j)-m-pre neighbourhood of each of its points. Then for each $x \in G$, there exists $S_x \in (i,j)$-$mPO(X)$ such that $S_x \subset G$. Then $G = \bigcup\{S_x : x \in G\}$. Since each S_x is (i,j)-m-preopen and arbitrary union of (i,j)-m-preopen sets is (i,j)-m-preopen, G is (i,j)-m-preopen in (X, m_1, m_2). ∎

4. Pairwise m-precontinuous functions

Definition 4.1. A function $f : (X, m_1, m_2,) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-m-precontinuous if the inverse image of every σ_i-open set is (i,j)-m-preopen in (X, m_1, m_2), where $i \neq j$, i, j=1, 2.

Proposition 4.2. *Every m_i-continuous function is (i,j)-m-precontinuous.*

Proof. The proof follows from the definitions. ∎

However, the converse may be false.

Example 4.3. Let $X = \{a, b, c\}$, $m_1 = \{\emptyset, \{a\}, \{b\}, X\}$, $m_2 = \{\emptyset, \{a\}, X\}$, $\sigma_1 = \{\emptyset, \{a, b\}, X\}$, $\sigma_2 = \{\emptyset, \{a, c\}, X\}$. Then the identity function $f : (X, m_1, m_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-precontinuous but not m_i-precontinuous.

Theorem 4.4. *For a function $f:(X,m_1,m_2)\to(Y,\sigma_1,\sigma_2)$, the following statements are equivalent:*

(i) *f is (i,j)-m-precontinuous.*
(ii) *For each point x in X and each σ_i-open set F in Y such that $f(x)\in F$, there is a (i,j)-m-preopen set A in X such that $x\in A$, $f(A)\subset F$.*
(iii) *The inverse image of each σ_i-closed set in Y is (i,j)-m-preclosed in X.*
(iv) *For each subset A of X, $f((i,j)\text{-}mp\,\mathrm{Cl}(A))\subset\sigma_i\text{-}\mathrm{Cl}(f(A))$.*
(v) *For each subset B of Y, $(i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(B))$.*
(vi) *For each subset C of Y, $f^{-1}(\sigma_i\text{-}\mathrm{Int}(C))\subset(i,j)\text{-}mp\,\mathrm{Int}(f^{-1}(C))$.*

Proof. (i)⇒(ii): Let $x\in X$ and F be a σ_i-open set of Y containing $f(x)$. By (i), $f^{-1}(F)$ is (i,j)-m-preopen in X. Let $A=f^{-1}(F)$. Then $x\in A$ and $f(A)\subset F$.

(ii)⇒(i): Let F be σ_i-open in Y and let $x\in f^{-1}(F)$. Then $f(x)\in F$. By (ii), there is an (i,j)-m-preopen set U_x in X such that $x\in U_x$ and $f(U_x)\subset F$. Then $x\in U_x\subset f^{-1}(F)$. Hence $f^{-1}(F)$ is (i,j)-m-preopen in X.

(i)⇔(iii): This follows due to the fact that for any subset B of Y, $f^{-1}(Y\backslash B)=X\backslash f^{-1}(B)$.

(iii)⇒(iv): Let A be a subset of X. Since $A\subset f^{-1}(f(A))$ we have $A\subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(f(A)))$. Now, $(i,j)\text{-}mp\,\mathrm{Cl}(f(A))$ is σ_i-closed in Y and hence $(i,j)\text{-}mp\,\mathrm{Cl}(A))\subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(f(A)))$, for $(i,j)\text{-}mp\,\mathrm{Cl}(A)$ is the smallest (i,j)-m-preclosed set containing A. Then $f((i,j)\text{-}mp\,\mathrm{Cl}(A))\subset\sigma_i\text{-}\mathrm{Cl}(f(A))$.

(iv)⇒(iii): Let F be any σ_i-closed subset of Y. Then $f((i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(F)))\subset\sigma_i\text{-}\mathrm{Cl}(f(f^{-1}(F)))\subset\sigma_i\text{-}\mathrm{Cl}(F)=F$. Therefore, $(i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(F))\subset f^{-1}(F)$. Consequently, $f^{-1}(F)$ is (i,j)-m-preclosed in X.

(iv)⇒(v): Let B be any subset of Y. Now, $f((i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(B)))\subset\sigma_i\text{-}\mathrm{Cl}(f(f^{-1}(B)))\subset\sigma_i\text{-}\mathrm{Cl}(B)$. Consequently, $(i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(B))$.

(v)⇒(iv): Let $B=f(A)$ where A is a subset of X. Then, $(i,j)\text{-}mp\,\mathrm{Cl}(A)\subset(i,j)\text{-}mp\,\mathrm{Cl}(f^{-1}(B))\subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(B))=f^{-1}(\sigma_i\text{-}\mathrm{Cl}(f(A)))$. This shows that $f((i,j)\text{-}mp\,\mathrm{Cl}(A))\subset\sigma_i\text{-}\mathrm{Cl}(f(A))$.

(i)⇒(vi): Let B be any subset of Y. Clearly, $f^{-1}(\sigma_i\text{-}\mathrm{Int}(B))$ is (i,j)-m-preopen and we have $f^{-1}(\sigma_i\text{-}\mathrm{Int}(B))\subset(i,j)\text{-}mp\,\mathrm{Int}(f^{-1}\sigma_i\text{-}\mathrm{Int}(B))\subset(i,j)\text{-}mp\,\mathrm{Int}(f^{-1}B)$.

(vi)⇒(i): Let B be a σ_i-open set in Y. Then $\sigma_i\text{-}\mathrm{Int}(B)=B$ and $f^{-1}(B)\subset f^{-1}(\sigma_i\text{-}\mathrm{Int}(B))\subset(i,j)\text{-}mp\,\mathrm{Int}(f^{-1}(B))$. Hence we have $f^{-1}(B)=(i,j)\text{-}mp\,\mathrm{Int}(f^{-1}(B))$. This shows that $f^{-1}(B)$ is (i,j)-m-preopen in X. ■

Definition 4.5. The graph $G(f)$ of a function $f:(X,m_1,m_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (i,j)-m-preclosed in $X\times Y$ if for each $(x,y)\in$

$(X \times Y) \setminus G(f)$, there exists $U \in (i,j)$-$mPO(X,x)$ and a σ_i-open set V of Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 4.6. *The graph $f : (X, m_1, m_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-m-preclosed in $X \times Y$ if and only if for each $(x,y) \in (X \times Y) \setminus G(f)$, there exists $U \in (i,j)$-$mPO(X,x)$ and a σ_i-open set V of Y containing y such that $f(U) \cap V = \emptyset$.*

Proof. The proof is an immediate consequence of Definition 4.5. ∎

Theorem 4.7. *If $f : (X, m_1, m_2) \to (Y, \sigma_1, \sigma_2)$ is an (i,j)-m-precontinuous function and (Y, σ_i) is T_2, then $G(f)$ is (i,j)-m-preclosed.*

Proof. Let $(x,y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since (Y, σ_i) is T_2, there exists a σ_i-open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is (i,j)-m-precontinuous, there exists $U \in (i,j)$-$mPO(X,x)$ such that $f(U) \subset V$. Therefore, $f(U) \cap V = \emptyset$. Therefore, by Lemma 4.6, $G(f)$ is (i,j)-m-preclosed. ∎

Definition 4.8. A biminimal space (X, m_1, m_2) is said to be an (i,j)-m-pre-T_2 space if for each pair of distinct points $x, y \in X$, there exist $U, V \in (i,j)$-$mPO(X)$ containing x and y, respectively, such that $U \cap V = \emptyset$.

Theorem 4.9. *If $f : (X, m_1, m_2) \to (Y, \sigma_1, \sigma_2)$ is an (i,j)-m-precontinuous injective function and (Y, σ_i) is a T_2 space, then (X, m_1, m_2) is a m-pre-T_2 space.*

Proof. The proof follows from the definitions. ∎

Theorem 4.10. *If $f : (X, m_1, m_2) \to (Y, \sigma_1, \sigma_2)$ is an injective (i,j)-m-precontinuous function with an (i,j)-m-preclosed graph, then X is an (i,j)-m-pre-T_2 space.*

Proof. Let x_1 and x_2 be any distinct points of X. Then $f(x_1) \neq f(x_2)$, so $(x_1, f(x_2)) \in (X \times Y) \setminus G(f)$. Since the graph $G(f)$ is (i,j)-m-preclosed, there exist an (i,j)-m-preopen set U containing x_1 and $V \in \tau$ containing $f(x_2)$ such that $f(U) \cap V = \emptyset$. Since f is (i,j)-m-precontinuous, $f^{-1}(V)$ is an (i,j)-m-preopen set containing x_2 such that $U \cap f^{-1}(V) = \emptyset$. Hence X is (i,j)-m-pre-T_2. ∎

Definition 4.11. A biminimal space (X, m_1, m_2) is said to be (i,j)-m-preconnected if X cannot be expressed as the union of two nonempty disjoint (i,j)-m-preopen sets.

Definition 4.12. A bitopological space (X, τ_1, τ_2) is said to be pairwise connected [3] if it cannot be expressed as the union of two nonempty disjoint sets U and V such that U is τ_i-open and V is τ_j-open, where $i, j = 1, 2$ and $i \neq j$.

THEOREM 4.13. *An (i, j)-m-precontinuous image of an (i, j)-m-preconnected space is pairwise connected.*

Proof. The proof is clear. ■

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