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SOME CLASSES OF ALMOST CONVERGENT PARANORMED SEQUENCE SPACES DEFINED BY ORLICZ FUNCTIONS

Abstract. In this paper we define $\ell_\infty(M,p,\phi,q,s)$, $c(M,p,\phi,q,s)$ and $c_0(M,p,\phi,q,s)$, the sequence spaces on a seminormed complex linear space, using an Orlicz function. We give various properties and some inclusion relations on this space.

1. Introduction

Let ℓ_∞ and c denote the Banach spaces of real bounded and convergent sequences $x=(x_n)$ respectively, normed by $\|x\| = \sup_n |x_n|$.

Let σ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^k(n) = \sigma(\sigma^{k-1}(n))$, $k=1,2,\ldots$. A continuous linear functional φ on ℓ_∞ is said to be an invariant mean or a σ-mean if and only if

(i) $\varphi(x) \geq 0$ when the sequence $x=(x_n)$ has $x_n \geq 0$ for all n,
(ii) $\varphi(e) = 1$, where $e=(1,1,1,\ldots)$ and
(iii) $\varphi\left(\left\{x_{\sigma(n)}\right\}\right) = \varphi(\{x_n\})$ for all $x \in \ell_\infty$.

If σ is the translation mapping $n \to n+1$, a σ-mean is often called a Banach limit [4], and V_σ, the set of σ-convergent sequences, that is, the set of bounded sequences all of whose invariant means are equal, is the set $\hat{f}$ of almost convergent sequences [11].

If $x=(x_n)$, set $Tx = (Tx_n) = \left(x_{\sigma(n)}\right)$. It can be shown (see Schaefer [18]) that

$$V_\sigma = \left\{x=(x_n) : \lim_k t_{kn}(x) = Le \text{ uniformly in } n,\ L = \sigma - \lim x\right\}, \tag{1.1}$$

where

$$t_{kn}(x) = \frac{1}{k+1}\sum_{j=0}^{k} T^j x_n.$$

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The special case of (1.1), in which $\sigma(n) = n + 1$ was given by Lorentz [11].

Subsequently invariant means have been studied by Ahmad and Mursaleen [1], Mursaleen [14], Raimi [17] and many others.

The space

$$BV_\sigma = \left\{ x \in \ell_\infty : \sum_k |\phi_{k,n}(x)| < \infty, \quad \text{uniformly in } n \right\}$$

was defined by Mursaleen [13], where

$$\phi_{k,n}(x) = t_{kn}(x) - t_{k-1,n}(x)$$

assuming that $t_{kn}(x) = 0$, for $k = -1$.

A straightforward calculation shows that

$$\phi_{k,n}(x) = \begin{cases} \frac{1}{k(k+1)} \sum_{j=1}^{k} j \left(x_{\sigma^j(n)} - x_{\sigma^{j-1}(n)} \right) & (k \geq 1), \\ x_n & (k = 0). \end{cases}$$

Note that for any sequences x, y and scalar λ, we have

$$\phi_{k,n}(x+y) = \phi_{k,n}(x) + \phi_{k,n}(y) \quad \text{and} \quad \phi_{k,n}(\lambda x) = \lambda \phi_{k,n}(x).$$

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$, which is continuous, non decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$ (for detail see Krasnoselskii and Rutickii [9]).

It is well known that if M is a convex function and $M(0) = 0$, then $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda < 1$.

Let (Ω, Σ, μ) be a finite measure space. We denote by $E(\mu)$ the space of all (equivalence classes of) Σ-measurable functions x from Ω into $[0, \infty)$. Given an Orlicz function M, we define on $E(\mu)$ a convex functional I_M by

$$I_M(x) = \int_\Omega M(x(t))\, d\mu,$$

and an Orlicz space $L^M(\mu)$ by $L^M(\mu) = \{x \in E(\mu) : I_M(\lambda x) < +\infty$ for some $\lambda > 0\}$ (for detail see [16], [9]).

Lindenstrauss and Tzafriri [10] used the idea of Orlicz function to construct the sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) < \infty, \quad \text{for some } \rho > 0 \right\}.$$

The space ℓ_M is a Banach space with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) \leq 1 \right\}$$

and this space is called an Orlicz sequence space. For $M(t) = t^p$, $1 \leq p < \infty$, the space ℓ_M coincides with the classical sequence space ℓ_p.

Definition 1.1. Any two Orlicz functions M_1 and M_2 are said to be equivalent if there are positive constants α and β, and x_0 such that $M_1(\alpha x) \leq M_2(x) \leq M_1(\beta x)$ for all x with $0 \leq x \leq x_0$ (see Kamthan and Gupta [8]).

Later on, different types of sequence spaces were introduced by using an Orlicz function by Mursaleen *et al.* [15], Choudhary and Parashar [5], Et *et al.* [6], Tripathy and Mahanta ([20], [21]), Tripathy and Sarma ([24], [26]), Tripathy and Sen [22], Tripathy and Dutta [25], Tripathy *et al.* [23], Srivastava and Ghosh [19], Khan[7], Altin ([3], [2]) and many others.

A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$.

It is well known that a sequence space E is normal implies that E is monotone (see for instance Kamthan and Gupta [8]).

Definition 1.2. Let q_1, q_2 be seminorms on a vector space X. Then q_1 is said to be stronger than q_2 if whenever (x_n) is a sequence such that $q_1(x_n) \to 0$, then also $q_2(x_n) \to 0$. If each is stronger than the others, q_1 and q_2 are said to be equivalent (one may refer to Wilansky [27]).

Lemma 1.3. *Let q_1 and q_2 be seminorms on a linear space X. Then q_1 is stronger than q_2 if and only if there exists a constant T such that $q_2(x) \leq T q_1(x)$ for all $x \in X$ (see for instance Wilansky* [27]).

The following inequalities will be used throughout the paper. Let $p = (p_k)$ be a bounded sequence of positive real numbers with $0 < p_k \leq \sup p_k = H$, $D = \max\left(1, 2^{H-1}\right)$, then

$$|a_k + b_k|^{p_k} \leq D\left\{|a_k|^{p_k} + |b_k|^{p_k}\right\}, \tag{1.2}$$

where a_k, $b_k \in \mathbb{C}$.

Throughout the article $p = (p_k)$ will represent a sequence of strictly positive real numbers and (X, q) a seminormed space over the field $\mathbb{C}$ of complex numbers with the seminorm q. The symbol $\ell_\infty(X)$ denotes the spaces of all bounded sequences defined over X. We define the following sequence spaces:

$$\ell_\infty(M, p, \phi, q, s) = \left\{ \begin{array}{c} x \in \ell_\infty(X) : \sup_{n,k} k^{-s}\left[M\left(q\left(\dfrac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k} < \infty \\ \text{for some } \rho > 0,\ s \geq 0 \end{array} \right\},$$

$$c\left(M,p,\phi,q,s\right)=\left\{\begin{array}{c} x\in\ell_{\infty}\left(X\right):\lim\limits_{k\to\infty}k^{-s}\Big[M\Big(q\Big(\dfrac{\phi_{k,n}(x)-\ell}{\rho}\Big)\Big)\Big]^{p_k}=0\\ \text{uniformly in } n, \text{ for some } \rho>0,\ell\in X \text{ and } s\geq 0\end{array}\right\},$$

$$c_0\left(M,p,\phi,q,s\right)=\left\{\begin{array}{c} x\in\ell_{\infty}\left(X\right):\lim\limits_{k\to\infty}k^{-s}\Big[M\Big(q\Big(\dfrac{\phi_{k,n}(x)}{\rho}\Big)\Big)\Big]^{p_k}=0\\ \text{uniformly in } n, \text{ for some } \rho>0,\ s\geq 0\end{array}\right\}.$$

2. Main results

Theorem 2.1. *Let $p=(p_k)$ be a bounded sequence, then $Z\left(M,p,\phi,q,s\right)$ are linear spaces over the set of complex numbers, for $Z=\ell_{\infty},c$ and c_0.*

Proof. We give the proof for $Z=c_0$ only. The other cases can be treated similarly. Let $x,y\in c_0\left(M,p,\phi,q,s\right)$ and $\alpha,\beta\in\mathbb{C}$. There exist positive numbers ρ_1 and ρ_2 such that

$$k^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right)\right]^{p_k}\to 0 \text{ as } k\to\infty, \text{ uniformly in } n,\ s\geq 0$$

and

$$k^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right)\right]^{p_k}\to 0 \text{ as } k\to\infty, \text{ uniformly in } n,\ s\geq 0.$$

Define $\rho_3=\max\left(2\left|\alpha\right|\rho_1,2\left|\beta\right|\rho_2\right)$. Since M is non-decreasing and convex

$$\begin{aligned} k^{-s}&\left[M\left(q\left(\frac{\alpha\phi_{k,n}(x)+\beta\phi_{k,n}(y)}{\rho_3}\right)\right)\right]^{p_k}\\ &\leq k^{-s}\left[M\left(q\left(\frac{\alpha\phi_{k,n}(x)}{\rho_3}\right)+q\left(\frac{\beta\phi_{k,n}(y)}{\rho_3}\right)\right)\right]^{p_k}\\ &\leq k^{-s}\frac{1}{2^{p_k}}\left[M\left(q\left(\frac{\phi_{k,n}\left(x\right)}{\rho_1}\right)+M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right)\right)\right]^{p_k}\\ &\leq Dk^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right)\right]^{p_k}+Dk^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right)\right]^{p_k}\\ &\to 0, \text{ as } k\to\infty, \text{ uniformly in } n,\ s\geq 0. \end{aligned}$$

This proves that $c_0\left(M,p,\phi,q,s\right)$ is linear. ∎

Theorem 2.2. *The spaces $Z\left(M,p,\phi,q,s\right)$ (for $Z=\ell_{\infty},c$ and c_0) are paranormed space (not necessarily totally paranormed), paranormed by*

$$g(x)=\inf\left\{\rho^{p_m/H}:\sup_{k\geq 1}k^{-s}M\Big(q\Big(\frac{\phi_{k,n}(x)}{\rho}\Big)\Big)\leq 1,\ \rho>0,\ \textit{uniformly in } n\right\},$$

where $H=\max\left(1,\sup_k p_k\right)$.

Proof. Clearly $g(x) = g(-x)$ and $g(\bar{\theta}) = 0$, where $\bar{\theta}$ is the zero sequence of X. Let $(x_k), (y_k) \in c_0\,(M, p, \phi, q, s)$.

Then there exist ρ_1, ρ_2 such that

$$\sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right) \leq 1, \quad \text{uniformly in } n$$

and

$$\sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right) \leq 1, \quad \text{uniformly in } n.$$

Let $\rho = \rho_1 + \rho_2$, then we have

$$\sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x) + \phi_{k,n}(y)}{\rho}\right)\right) \leq \left(\frac{\rho_1}{\rho_1 + \rho_2}\right) \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right) + \left(\frac{\rho_2}{\rho_1 + \rho_2}\right) \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right) \leq 1, \text{ uniformly in } n.$$

Hence

$$\begin{aligned} & g(x+y) \\ &= \inf\left\{ (\rho_1+\rho_2)^{p_m/H} : \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x)+\phi_{k,n}(y)}{\rho}\right)\right) \leq 1,\ \rho > 0, \text{ uniformly in } n \right\} \\ &\leq \inf\left\{ (\rho_1)^{p_m/H} : \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right) \leq 1,\ \rho_1 > 0, \text{ uniformly in } n \right\} \\ &\quad + \inf\left\{ (\rho_2)^{p_m/H} : \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(y)}{\rho_2}\right)\right) \leq 1,\ \rho_2 > 0, \text{ uniformly in } n \right\} \\ &= g(x)+g(y). \end{aligned}$$

Hence g satisfies the triangle inequality.

The continuity of product follows from the following equality:

$$\begin{aligned} & g(\lambda x) \\ &= \inf\left\{ \rho^{p_m/H} : \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\lambda\phi_{k,n}(x)}{\rho}\right)\right) \leq 1,\ \rho > 0, \text{ uniformly in } n \right\} \\ &= \inf\left\{ (|\lambda| t)^{p_m/H} : \sup_{k\geq 1} k^{-s} M\left(q\left(\frac{\phi_{k,n}(x)}{t}\right)\right) \leq 1,\ t > 0, \text{ uniformly in } n \right\}, \end{aligned}$$

where $t = \rho / |\lambda|$.

Hence the space $c_0\,(M, p, \phi, q, s)$ is a paranormed space, paranormed by g. The other cases can be proved in a similar way. ∎

Theorem 2.3. *Let M_1 and M_2 be two Orlicz functions. Then*

$$Z(M_1, p, \phi, q, s) \cap Z(M_2, p, \phi, q, s) \subseteq Z(M_1 + M_2, p, \phi, q, s),$$

for $Z = \ell_\infty, c$ and c_0.

Proof. We prove the result for $Z = c_0$ and for other spaces it will follow on applying similar arguments. Let $x \in c_0(M_1, p, \phi, q, s) \cap c_0(M_2, p, \phi, q, s)$. Then there exist ρ_1 and ρ_2 such that

$$\lim_{k\to\infty} k^{-s}\left[M_1\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right)\right]^{p_k} = 0 \text{ uniformly in } n,$$

and

$$\lim_{k\to\infty} k^{-s}\left[M_2\left(q\left(\frac{\phi_{k,n}(x)}{\rho_2}\right)\right)\right]^{p_k} = 0 \text{ uniformly in } n.$$

Let $\rho = \max(\rho_1, \rho_2)$. Then we have

$$k^{-s}\left[(M_1 + M_2)\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k} \le k^{-s}D\left[M_1\left(q\left(\frac{\phi_{k,n}(x)}{\rho_1}\right)\right)\right]^{p_k} + k^{-s}D\left[M_2\left(q\left(\frac{\phi_{k,n}(x)}{\rho_2}\right)\right)\right]^{p_k} \to 0 \text{ uniformly in } n.$$

We have $x \in c_0(M_1 + M_2, p, \phi, q, s)$. ∎

Theorem 2.4. *Let M be an Orlicz function then $c_0(M, p, \phi, q, s) \subset c(M, p, \phi, q, s) \subset \ell_\infty(M, p, \phi, q, s)$.*

Proof. Let $x \in c(M, p, \phi, q, s)$. Then we have

$$\begin{aligned} &k^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k} \\ &\le Dk^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x) - L}{\rho}\right)\right)\right]^{p_k} + Dk^{-s}\left[M\left(\left(\frac{q(L)}{\rho}\right)\right)\right]^{p_k} \\ &\le Dk^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x) - L}{\rho}\right)\right)\right]^{p_k} + Dk^{-s}\max\left[1, \left(M\left(\frac{q(L)}{\rho}\right)\right)^H\right]. \end{aligned}$$

Thus we get $x \in \ell_\infty(M, p, \phi, q, s)$. The inclusion $c_0(M, p, \phi, q, s) \subset c(M, p, \phi, q, s)$ is obvious. ∎

Theorem 2.5. *For any two sequences $p = (p_k)$ and $t = (t_k)$ of positive* real *numbers and for any two seminorms q_1 and q_2 on X we have*

$$Z(M, p, \phi, q_1, s) \cap Z(M, t, \phi, q_2, s) \neq \emptyset.$$

Proof. The proof follows from the fact that the zero element $\bar{\theta}$ belongs to each of the classes of sequences involved in the intersection. ∎

Theorem 2.6. *Let M be an Orlicz function, q_1 and q_2 be two seminorms on X. Then*

i) $Z(M,p,\phi,q_1,s) \cap Z(M,p,\phi,q_2,s) \subseteq Z(M,p,\phi,q_1+q_2,s)$,

ii) *if q_1 is stronger than q_2, then* $Z(M,p,\phi,q_1,s) \subseteq Z(M,p,\phi,q_2,s)$,

iii) *if $q_1 \simeq$ (equivalent to) q_2, then* $Z(M,p,\phi,q_1,s) = Z(M,p,\phi,q_2,s)$.

Proof. Straightforward and hence omitted. ∎

Theorem 2.7. i) *Let $0 < p_k \le r_k$ and (r_k/p_k) be bounded. Then* $Z(M,r,\phi,q,s) \subset Z(M,p,\phi,q,s)$,

ii) $s_1 \le s_2$ *implies* $Z(M,p,\phi,q,s_1) \subset Z(M,p,\phi,q,s_2)$.

Proof. i) Let us take $w_{kn} = k^{-s}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{r_k}$ for all k. Following the technique applied for establishing Theorem 5 of Maddox [12], we can easily prove the theorem.

ii) Since $k^{-s_2}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k} \le k^{-s_1}\left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k}$ for all k and n, so we have $Z(M,p,\phi,q,s_1) \subset Z(M,p,\phi,q,s_2)$. ∎

Theorem 2.8. *The spaces $\ell_\infty(M,p,\phi,q,s)$ and $c_0(M,p,\phi,q,s)$ are solid and as such are monotone, but $c(M,p,\phi,q,s)$ is not monotone and hence is not solid.*

Proof. Let $(x_k) \in \ell_\infty(M,p,\phi,q,s)$ or $c_0(M,p,\phi,q,s)$ and (α_k) be a sequence of scalars such that $|\alpha_k| \le 1$ for all $k \in \mathbb{N}$. Then the result follows from the following inequality:

$$\left[M\left(q\left(\frac{\alpha_k\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k} \le \left[M\left(q\left(\frac{\phi_{k,n}(x)}{\rho}\right)\right)\right]^{p_k}, \text{ for all } k \in \mathbb{N}.$$ ∎

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