DEMONSTRATIO MATHEMATICA
Vol. XLV No 3 2012

10.1515/dema-2013-0393

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QUASI cl-SUPERCONTINUOUS FUNCTIONS AND THEIR FUNCTION SPACES

Abstract. A new class of functions called 'quasi cl-supercontinuous functions' is introduced. Basic properties of quasi cl-supercontinuous functions are studied and their place in the hierarchy of variants of continuity that already exist in the mathematical literature is elaborated. The notion of quasi cl-supercontinuity, in general, is independent of continuity but coincides with cl-supercontinuity ($\equiv$ clopen continuity) (Applied General Topology 8(2) (2007), 293–300; Indian J. Pure Appl. Math. 14(6) (1983), 767–772), a significantly strong form of continuity, if range is a regular space. The class of quasi cl-supercontinuous functions properly contains each of the classes of (i) quasi perfectly continuous functions and (ii) almost cl-supercontinuous functions; and is strictly contained in the class of quasi z-supercontinuous functions. Moreover, it is shown that if X is sum connected (e.g. connected or locally connected) and Y is Hausdorff, then the function space $L_q(X, Y)$ of all quasi cl-supercontinuous functions as well as the function space $L_\delta(X, Y)$ of all almost cl-supercontinuous functions from X to Y is closed in Y^X in the topology of pointwise convergence.

1. Introduction

Several variants of continuity occur in the lore of mathematical literature. Certain of these variants of continuity are stronger than continuity while others are weaker than continuity and yet others, although analogous to but, are independent of continuity.

The main purpose of this paper is to introduce a new class of functions called 'quasi cl-supercontinuous functions' and to elaborate on their basic properties and discuss their interplay and interrelations with other variants of continuity that already exist in the lore of mathematical literature. It turns out that in general the notion of quasi cl-supercontinuity is inde-

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2000 *Mathematics Subject Classification*: Primary 54C08, 54C10, 54C35; Secondary 54D10.

Key words and phrases: almost cl-supercontinuous function, δ-perfectly continuous function, almost perfectly continuous function, quasi hyperconnected space, quasi zero dimensional space, sum connected space, topology of pointwise convergence.

pendent of continuity but coincides with cl-supercontinuity [45] (≡ clopen continuity [41]) a significantly strong form of continuity, if the range space is a regular space. The class of quasi cl-supercontinuous functions properly contains the class of almost cl-supercontinuous (≡ almost clopen continuous) functions introduced by Ekici [5] and further studied in [21], and so includes all almost perfectly continuous (≡ regular set connected) functions introduced by Dontchev, Ganster and Reilly [3] and further studied in [46] which in their turn include all δ-perfectly continuous functions studied by Kohli and Singh [20]. Thus the class of quasi cl-supercontinuous functions contains all perfectly continuous functions due to Noiri [38] and so all strongly continuous functions of Levine [29].

Paper is organized as follows: Section 2 is devoted to preliminaries and basic definitions. In Section 3, we introduce the notion of 'quasi cl-supercontinuous function' and elaborate on its place in the hierarchy of variants of continuity that already exist in the mathematical literature. Examples are included to reflect upon the distinctiveness of the notions so introduced with the ones which already exist in the mathematical literature. Characterizations of quasi cl-supercontinuous functions are given in Section 4. In Section 5, we study the basic properties of quasi cl-supercontinuous functions wherein it is shown that quasi cl-supercontinuity is preserved under compositions and expansion of range. Moreover, sufficient conditions are formulated for the preservation of quasi cl-supercontinuity under restrictions and shrinking of range. The interplay between quasi cl-supercontinuous functions and topological properties is considered in Section 6, while properties of graph of quasi cl-supercontinuous functions are studied in Section 7. In Section 8, we consider the retopologization of the domain and/or range of quasi cl-supercontinuous function, wherein it is shown that quasi cl-supercontinuity is transformed into certain other variants of continuity if its domain/range is retopologized in an appropriate way. The function spaces $L_q(X,Y)$ and $L_\delta(X,Y)$ of quasi cl-supercontinuous functions and almost cl-supercontinuous functions, respectively, with the topology of pointwise convergence are considered in Section 9. Correlation between connectedness and existence/non-existence of certain functions is considered in Section 10. In Section 11, we discuss minimal structures and M-continuous functions due to Popa and Noiri [40] and conclude with alternative proofs of certain results of the preceding sections.

In the course of our presentation we shall omit proofs of certain results which are similar to the corresponding results for almost cl-supercontinuous functions [21] and include others which are necessary for the clarity and continuity of presentation.

2. Preliminaries and basic definitions

A collection β of subsets of a space X is called an **open complementary system** [6] if β consists of open sets such that for every $B \in \beta$, there exist $B_1, B_2, \ldots \in \beta$ with $B = \bigcup\{X \backslash B_i : i \in \mathbb{N}\}$. A subset U of a space X is called **strongly open F_σ-set** [6] if there exists a countable open complimentary system $\beta(U)$ with $U \in \beta(U)$. The complement of a strongly open F_σ-set is referred to as a **strongly closed G_δ-set**. A subset H of a space X is called a **regular G_δ-set** [33] if H is an intersection of a sequence of closed sets whose interiors contains H, i.e. $H = \bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} F_n^0$, where each F_n is a closed subset of X. The complement of a regular G_δ-set is called a **regular F_σ-set**. A subset A of a space X is said to be **regular open** if it is the interior of its closure, i.e., $A = \overline{A}^0$. The complement of a **regular open** set is called **regular closed**. A point $x \in X$ is called a **θ-adherent point** [51] of a set $A \subset X$ if every closed neighbourhood of x intersects A. Let $cl_\theta A$ denote the set of all θ-adherent points of A. The set A is called **θ-closed** if $A = cl_\theta A$. The complement of a θ-closed set is referred to as **θ-open**. A point $x \in X$ is called a **$u\theta$-adherent point** ([12], [13]) of $A \subset X$ if every θ-open set U containing x intersects A. The set of all $u\theta$-adherent points of A is denoted by $\mathbf{cl_{u\theta}A}$. The set A is called **$u\theta$-closed** if $A = A_{u\theta}$. A subset G of a space X is said to be **cl-open** [45] (1) if for each $x \in G$ there exists a clopen set H such that $x \in H \subset G$, or equivalently, G is expressible as a union of clopen sets. The complement of a cl-open set will be referred to as **cl-closed**. A set G is said to be **δ-open** [51] if for each $x \in G$ there exists a regular open set H such that $x \in H \subset G$, or equivalently, G can be expressed as an arbitrary union of regular open sets. The complement of a δ-open set will be referred to as a **δ-closed set**.

Lemma 2.1. [11, 14] *A set U in a space X is θ-open if and only if for each $x \in U$ there exists an open set V containing x such that $\overline{V} \subset U$.*

Definitions 2.2. A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) **strongly continuous** [29] if $f(\overline{A}) \subset f(A)$ for each subset A of X.
(b) **perfectly continuous** [24, 38] if $f^{-1}(V)$ is clopen in X for every open set $V \subset Y$.
(c) **cl-supercontinuous** [45] ($\equiv$ **clopen continuous** [41]) if for each $x \in X$ and each open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.
(d) **z-supercontinuous** [15] if for each $x \in X$ and each open set V containing $f(x)$, there exists a cozero set U containing x such that $f(U) \subset V$.

(1) Staum calls 'cl-open' sets as 'quasi-open' in [49].

(e) **$\mathbf{D}_\delta$-supercontinuous** [17] if for each $x \in X$ and each open set V containing $f(x)$, there exists a regular F_σ-set U containing x such that $f(U) \subset V$.
(f) **strongly θ-continuous** [37] if for each $x \in X$ and each open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.
(g) **δ-perfectly continuous** [20] if $f^{-1}(V)$ is clopen set in X for every δ-open set $V \subset Y$.
(h) **quasi perfectly continuous** [28] if $f^{-1}(V)$ is clopen in X for every θ-open set $V \subset Y$.
(i) **almost perfectly continuous** [46] ($\equiv$ **regular set connected** [3]) if $f^{-1}(V)$ is clopen in X for every regular open set V in Y.
(j) **quasi z-supercontinuous** [25] if for each $x \in X$ and each θ-open set V containing $f(x)$, there exists a cozero set U containing x such that $f(U) \subset V$.
(k) **quasi $\mathbf{D}_\delta$-supercontinuous** [27] if for each $x \in X$ and each θ-open set V containing $f(x)$, there exists a regular F_σ-set U containing x such that $f(U) \subset V$.
(l) **almost z-supercontinuous** [27] if for each $x \in X$ and each regular open set V containing $f(x)$ there exists a cozero set U containing x such that $f(U) \subset V$.
(m) **almost cl-supercontinuous** [21] ($\equiv$ **almost clopen** [5]) if for each $x \in X$ and each regular open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.
(n) **almost $\mathbf{D}_\delta$-supercontinuous** [27] if for each $x \in X$ and each regular open set V containing $f(x)$ there exists a regular F_σ-set U containing x such that $f(U) \subset V$.

Definitions 2.3. A function $f : X \to Y$ from a topological space X into topological space Y is said to be

(a) **almost continuous** [42] if for each $x \in X$ and each open set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset (\overline{V})^0$.
(b) **D-continuous** [9] if for each $x \in X$ and each open F_σ-set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.
(c) **$\mathbf{D}^*$-continuous** [44] if for each point $x \in X$ and each strongly open F_σ-set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.
(d) **$\mathbf{D}_\delta$-continuous** [18] if for each point $x \in X$ and each regular F_σ-set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.
(e) **z-continuous** [43] if for each $x \in X$ and each cozero set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.

(f) **θ-continuous** [4] if for each $x \in X$ and each open set V containing $f(x)$ there is an open set U containing x such that $f(\overline{U}) \subset \overline{V}$.
(g) **weakly continuous** [30] if for each $x \in X$ and each open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset \overline{V}$.
(h) **faintly continuous** [32] if for each $x \in X$ and each θ-open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset V$.
(i) **quasi θ-continuous** [39] if for each $x \in X$ and each θ-open set V containing $f(x)$ there exists an θ-open set U containing x such that $f(U) \subset V$.
(j) **cl-continuous** [18] if $f^{-1}(V)$ is open in X for every clopen set $V \subset Y$.

Definitions 2.4. A topological space X is said to be

(a) **weakly cl-normal** [23] if every pair of disjoint *cl*-closed subsets of X can be separated by disjoint open sets in X.
(b) **weakly θ-normal** [11] if every pair of disjoint θ-closed subsets of X can be separated by disjoint open sets in X.
(c) **ultra Hausdorff** [49] if for each pair of distinct points in X there is a clopen set containing one but not the other.
(d) **θ-compact** ([7], [13]) if every θ-open cover of X has a finite subcover.
(e) **mildly compact** (2) [49] if every clopen cover of X has a finite subcover.

Definition 2.5. [45] Let $p : X \to Y$ be a surjection from a topological space X onto a set Y. The collection of all subsets A of Y, such that $p^{-1}(A)$ is cl-open in X, is a topology on Y and is called the **cl-quotient topology**. The map p is called **cl-quotient map**.

In general cl-quotient topology is coarser than quotient topology. However, if X is a zero dimensional space, then the two are identical. Several other variants of quotient topology occur in the literature which in general are coarser than quotient topology. Interrelations and interplay among these variants of quotient topology are well elaborated in ([22], [26]).

3. Quasi cl-supercontinuous functions

We call a function $f : X \to Y$ **quasi cl-supercontinuous** if for each $x \in X$ and each θ-open set V containing $f(x)$ there exists a clopen set U containing x such that $f(U) \subset V$. The class of quasi cl-supercontinuous functions properly contains the class of quasi perfectly functions which in turn include all δ-perfectly continuous functions and so contain all perfectly continuous functions due to Noiri [38] which are further studied in [24].

The following diagram enlarges the diagram already existing in the literature and well illustrates the place of quasi cl-supercontinuity in the hierarchy

(2) Sostak calls mildly compact spaces as clustered spaces in [48].

of variants of continuity that already exist in mathematical literature and are related to the theme of the present paper.

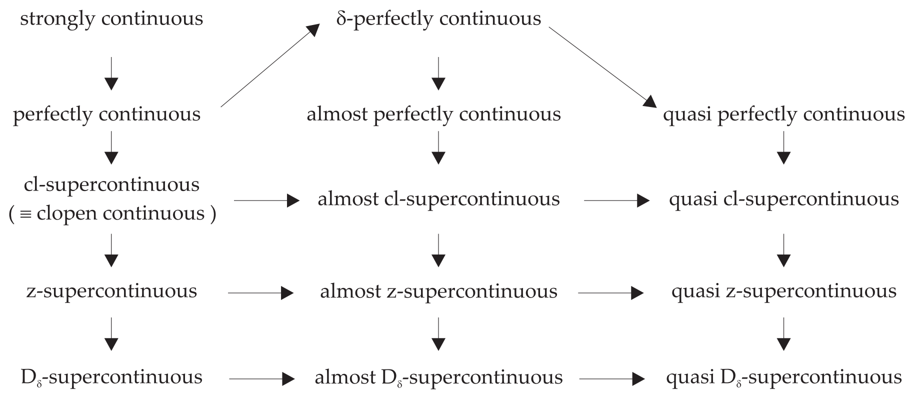

Fig. 1.

However, none of the above implications is reversible as is either well known or is exhibited by the following examples/observations and examples in ([20], [21], [25], [27], [46]).

Observations and examples

3.1 Let X be the real line endowed with lower limit topology and Y be the real line equipped with Smirnov's deleted sequence topology [50]. Then the identity function from X onto Y is quasi cl-supercontinuous but not continuous. On the other hand, let $X = Y$ denote the real line with usual topology and let f be the identity function defined on X. Then f is continuous but not quasi cl-supercontinuous.

3.2 Let $X = Y = \mathbb{R}$ be the set of all real numbers. Let X be endowed with usual topology and Y be equipped with Smirnov's deleted sequence topology [50]. Then the identity function from X onto Y is quasi z-supercontinuous function but not quasi cl-supercontinuous.

3.3 Let X be the real line endowed with lower limit topology and Y be the real line with usual topology. Then the identity function from X onto Y is quasi cl-supercontinuous (indeed cl-supercontinuous) but not quasi perfectly continuous.

3.4 Let $X = \{a, b, c\}$ be endowed with indiscrete topology. Let Y be the same set equipped with topology $\tau = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. Then the identity function from X to Y is quasi perfectly continuous and so quasi cl-supercontinuous but not almost cl-supercontinuous.

Proposition 3.5. *Let $f : X \to Y$ be a quasi cl-supercontinuous function. If Y is a regular space, then f is cl-supercontinuous.*

Proof. In view of Lemma 2.1, it follows that every open set in Y is θ-open. ■

Proposition 3.6. *If $f : X \to Y$ is a faintly continuous function defined on a zero dimensional space X, then f is a quasi cl-supercontinuous.*

Proof. Let V be a θ-open set in Y. Since f is faintly continuous, $f^{-1}(V)$ is open in X and so cl-open in X. Thus f is quasi cl-supercontinuous. ■

4. Characterizations

Definitions 4.1. A filter base $\mathcal{F}$ in a space X is said to

(i) **cl-converge** [45] to a point $x \in X$, written as $\mathcal{F} \overset{cl}{\to} x$, if every clopen set containing x contains a member of $\mathcal{F}$; and
(ii) **$u\theta$-converge** [13] to a point $x \in X$, written as $\mathcal{F} \overset{u\theta}{\to} x$, if every θ-open set containing x contains a member of $\mathcal{F}$.

Definitions 4.2. A net (x_λ) in a space X is said to

(i) **cl-converge** [45] to a point $x \in X$, written as $x_\lambda \overset{cl}{\to} x$, if it is eventually in every clopen set containing x; and
(ii) **$u\theta$-converge** [13] to a point $x \in X$, written as $x_\lambda \overset{u\theta}{\to} x$, if it is eventually in every θ-open set containing x.

Theorem 4.3. *For a function $f : X \to Y$ the following statements are equivalent.*

(a) *f is quasi cl-supercontinuous.*
(b) *$f^{-1}(V)$ is cl-open for each θ-open set $V \subset Y$.*
(c) *$f(\mathcal{F}) \overset{u\theta}{\to} f(x)$, for every filter base $\mathcal{F}$ in X, which cl-converges to x.*
(d) *$f(x_\lambda) \overset{u\theta}{\to} f(x)$, for every net (x_λ) in X, which cl-converges to x.*
(e) *$f([A]_{cl}) \subset [f(A)]_{u\theta}$ for every set $A \subset X$.*
(f) *$[f^{-1}(B)]_{cl} \subset f^{-1}([B]_{u\theta})$ for every set $B \subset Y$.*
(g) *$f^{-1}(B)$ is cl-closed for each θ-closed set $B \subset Y$.*

We omit the proof of Theorem 4.3.

5. Basic properties

Theorem 5.1. *If $f : X \to Y$ is z-continuous and $g : Y \to Z$ is quasi cl-supercontinuous, then $g \circ f$ is quasi cl-supercontinuous.*

Proof. Let W be a θ-open set in Z. Since g is quasi cl-supercontinuous, $g^{-1}(W)$ is cl-open set in Y. Let $g^{-1}(W) = \bigcup V_\alpha$, where each V_α is a clopen set in Y. Since f is z-continuous and since a clopen set is both a zero set and a cozero set, in view of [43, Theorem 2.2] each $f^{-1}(V_\alpha)$ is a clopen set. Thus $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W)) = \bigcup f^{-1}(V_\alpha)$ is cl-open. ■

Corollary 5.2. *If $f : X \to Y$ is continuous and $g : Y \to Z$ is quasi cl-supercontinuous, then $g \circ f$ is quasi cl-supercontinuous.*

Remark 5.3. In Theorem 5.1, z-continuity of f can be replaced by any one of the following weak variants of continuity listed in the following diagram, since each one of them implies z-continuity.

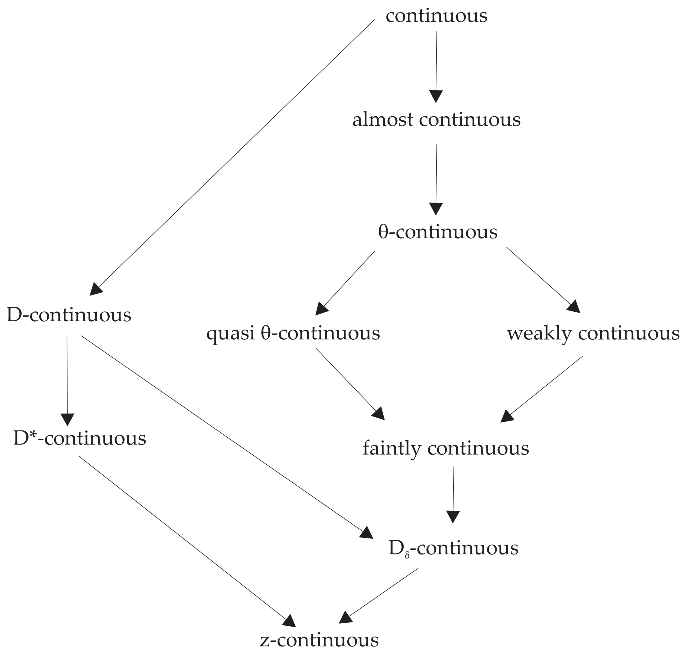

Fig. 2.

It is well known that none of the above implications is reversible. In particular for an example of a faintly continuous function which is not quasi θ-continuous we refer the reader to [18, Example 2.7].

Theorem 5.4. *If $f : X \to Y$ is quasi cl-supercontinuous and $g : Y \to Z$ is quasi θ-continuous, then $g \circ f$ is quasi cl-supercontinuous. In particular, composition of two quasi cl-supercontinuous functions is quasi cl-supercontinuous.*

Proof. Let W be a θ-open set in Z. Since g is quasi θ-continuous, $g^{-1}(W)$ is a θ-open subset of Y and so $f^{-1}(g^{-1}(W))$ is cl-open in X. Since $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W)), g \circ f$ is quasi cl-supercontinuous. ∎

We call a function $f : X \to Y$ **cl-open** if f maps cl-open sets in X to open sets in Y.

Theorem 5.5. *Let $f : X \to Y$ be a cl-supercontinuous, cl-open surjection and let $g : Y \to Z$ be any function. Then $g \circ f$ is quasi cl-supercontinuous if and only if g is faintly continuous.*

Proof. Suppose $g \circ f$ is quasi cl-supercontinuous and let G be a θ-open subset of Z. Then $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is cl-open in X. Since f is a cl-open surjection, $f(f^{-1}(g^{-1}(G))) = g^{-1}(G)$ is open in Y. Hence g is faintly continuous. Conversely, suppose that g is faintly continuous and let $V \subset Z$ be a θ-open set. Then $g^{-1}(V)$ is an open set in Y and so $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open in X and thus $g \circ f$ is quasi cl-supercontinuous. ∎

Theorem 5.6. *Let $f : X \to Y$ be a cl-continuous function and let $g : Y \to Z$ be a quasi cl-supercontinuous function. Then $g \circ f : X \to Z$ is faintly continuous.*

Proof. Let V be a θ-open set in Z. Then $g^{-1}(V)$ is a cl-open set in Y. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ and since f is cl-continuous $f^{-1}(g^{-1}(V))$ is open in X. Thus $g \circ f$ is faintly continuous. ∎

Theorem 5.7. *Let $f : X \to Y$ be a quasi cl-supercontinuous function and let $g : Y \to Z$ be a strongly θ-continuous function. Then $g \circ f : X \to Z$ is cl-supercontinuous.*

Proof. Let W be an open subset of Z. In view of strong θ-continuity of g, $g^{-1}(W)$ is a θ-open subset of Y. Again, since f is quasi cl-supercontinuous, $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is a cl-open set in X. Hence $g \circ f$ is cl-supercontinuous. ∎

Theorem 5.8. *Let $f : X \to Y$ be a cl-quotient map. Then $g : Y \to Z$ is faintly continuous if and only if $g \circ f$ is quasi cl-supercontinuous.*

Proof. Suppose g is faintly continuous and let V be a θ-open set in Z. Then $g^{-1}(V)$ is open in Y. Since f is a cl-quotient map, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open. Thus $g \circ f$ is quasi cl-supercontinuous. Conversely, let V be a θ-open set in Z. Then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a cl-open set in X. Since f is a cl-quotient map, $g^{-1}(V)$ is open in Y and so g is faintly continuous. ∎

Theorem 5.9. *Let $f : X \to Y$ be surjection which maps clopen sets to clopen sets and let $g : Y \to Z$ be any function such that $g \circ f : X \to Z$ is quasi cl-supercontinuous. Then g is quasi cl-supercontinuous.*

Proof. Let $V \subset Z$ be a θ-open set. Since $g \circ f$ is quasi cl-supercontinuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open. Let $f^{-1}(g^{-1}(V)) = \bigcup_{\alpha \in \Lambda} F_\alpha$, where each F_α is a clopen set in X. In view of hypothesis on $f, g^{-1}(V) = f(f^{-1}(g^{-1}(V))) = f(\bigcup F_\alpha) = \bigcup f(F_\alpha)$, where each $f(F_\alpha)$ is clopen and hence $g^{-1}(V)$ is cl-open in Y. Thus g is quasi cl-supercontinuous. ∎

Definition 5.10. [12, 13] A subspace S of a space X is said to be ***θ*-embedded** in X if every θ-open set in S is the intersection of a θ-open set in X with S, or equivalently every θ-closed set in S is the intersection of a θ-closed set in X with S.

The following theorem elaborates on the behaviour of quasi cl-supercontinuity under restrictions, shrinking and expansion of range.

Theorem 5.11. *Let $f : X \to Y$ be a function from a topological space X into a topological space Y. Then*

(i) *if f is quasi cl-supercontinuous and $f(X)$ is θ-embedded in Y, then $f : X \to f(X)$ is quasi cl-supercontinuous,*

(ii) *if f is quasi cl-supercontinuous and Y is a subspace of Z, then $g : X \to Z$ defined by $g(x) = f(x)$ for each $x \in X$ is quasi cl-supercontinuous,*

(iii) *if f is quasi cl-supercontinuous and $A \in X$, then $f_{|A} : A \to Y$ is quasi cl-supercontinuous. Further, if $f(A)$ is θ-embedded in Y, then $f_{|A} : A \to f(A)$ is also quasi cl-supercontinuous.*

Proof. (i) Let V_1 be a θ-open set in $f(X)$. Since $f(X)$ is θ-embedded in Y, there exists a θ-open set V in Y such that $V_1 = V \cap f(X)$. Again, since $f : X \to Y$ is quasi cl-supercontinuous, $f^{-1}(V)$ is cl-open in X. Now, $f^{-1}(V_1) = f^{-1}(V \cap f(X)) = f^{-1}(V)$ and so $f : X \to f(X)$ is quasi cl-supercontinuous.

(ii) Let W be a θ-open set in Z. Then $W \cap Y$ is a θ-open set in Y. Since f is quasi cl-supercontinuous, $f^{-1}(W \cap Y)$ is cl-open in X. Now, since $g^{-1}(W) = g^{-1}(W \cap Y) = f^{-1}(W \cap Y)$, it follows that g is quasi cl-supercontinuous.

(iii) Let V be a θ-open set in Y. Then $(f_{|A})^{-1}(V) = f^{-1}(V) \cap A$. Since f is quasi cl-supercontinuous, $f^{-1}(V)$ is cl-open in X. Consequently, $f^{-1}(V) \cap A$ is cl-open in A and so $f_{|A}$ is quasi cl-supercontinuous. The last assertion in (iii) is immediate in view of (i). ∎

Theorem 5.12. *Let $f : X \to Y$ be any function. If $\{U_\alpha : \alpha \in \Lambda\}$ is a cl-open cover of X and if for each $\alpha \in \Lambda, f_\alpha = f_{|U_\alpha} : U_\alpha \to Y$ is quasi cl-supercontinuous, then f is quasi cl-supercontinuous.*

Theorem 5.13. *Let $\{f_\alpha : X \to Y_\alpha : \alpha \in \Lambda\}$ be a family of functions and let $f : X \to \Pi Y_\alpha$ be defined by $f(x) = (f_\alpha(x))$ for each $x \in X$. If f is quasi cl-supercontinuous, then each f_α is quasi cl-supercontinuous.*

Proof. Let $f : X \to \Pi Y_\alpha$ be quasi cl-supercontinuous. For each $\alpha \in \Lambda$, $f_\alpha = p_\alpha \circ f$, where p_α denotes the projection map $p_\alpha : \Pi Y_\alpha \to Y_\alpha$. For each $\beta \in \Lambda$, it suffices to prove that f_β is quasi cl-supercontinuous. To this end, let V_β be a θ-open set in Y_β. Then $p_\beta^{-1}(V_\beta) = V_\beta \times \prod_{\alpha \neq \beta} Y_\alpha$ is a θ-open set in the product space ΠY_α. Since f is quasi cl-supercontinuous, in view of Theorem 4.3, $f^{-1}(V_\beta \times \prod_{\alpha\neq\beta} Y_\alpha) = f^{-1}(p_\beta^{-1}(V_\beta)) = (p_\beta \circ f)^{-1}(V_\beta) = f_\beta^{-1}(V_\beta)$ is cl-open in X and so f_β is quasi cl-supercontinuous. ∎

6. Interplay between topological properties

Theorem 6.1. *Let $f : X \to Y$ be a quasi cl-supercontinuous function from a weakly cl-normal space X into a space Y. If* (i) *f is an open bijection; or* (ii) *f is a closed surjection, then Y is a weakly θ-normal space.*

Proof. Let A and B be disjoint θ-closed subsets of Y. Since f is quasi cl-supercontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint cl-closed subsets of X. Since X is a weakly cl-normal space, there exist disjoint open sets U and V containing $f^{-1}(A)$ and $f^{-1}(B)$, respectively.

(i) In case f is an open bijection, $f(U)$ and $f(V)$ are disjoint open sets containing A and B respectively.

(ii) In case f is a closed surjection, the sets $W_1 = Y \setminus f(X \setminus U)$ and $W_2 = Y \setminus f(X \setminus V)$ are open in Y. It is easily verified that W_1 and W_2 are disjoint and contain A and B respectively. ∎

Theorem 6.2. *Let $f : X \to Y$ be a quasi cl-supercontinuous injection into a Hausdorff space Y. Then X is an ultra Hausdorff space.*

Corollary 6.3. [5, Theorem 22] *If $f : X \to Y$ is an almost cl-supercontinuous injection and Y is Hausdorff, then X is ultra Hausdorff.*

Proof. Every almost cl-supercontinuous function is quasi cl-supercontinuous. ∎

Theorem 6.4. *Let $f : X \to Y$ be a quasi cl-supercontinuous function from a mildly compact space X onto Y. Then Y is a θ-compact space.*

Theorem 6.5. *Let $f : X \to Y$ be a quasi cl-supercontinuous function from a connected space X onto Y. Then Y is a connected θ-compact space.*

Proof. Since every connected space is mildly compact, in view of Theorem 6.4, Y is a θ-compact. Again, since connectedness is preserved under quasi cl-supercontinuous functions, Y is a connected θ-compact space. ∎

We may recall that a space X is said to be ***θ*-Hausdorff** ([2], [47]) if each pair of distinct points in X are contained in disjoint θ-open sets.

Theorem 6.6. *Let $f, g : X \to Y$ be quasi cl-supercontinuous functions from a space X into a θ-Hausdroff space Y. Then the equalizer $E = \{x \in X : f(x) = g(x)\}$ of the functions f and g is a cl-closed subset of X.*

Theorem 6.7. *Let $f : X \to Y$ be quasi cl-supercontinuous function into a θ-Hausdorff space Y. Then the set $A = \{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is a cl-closed subset of $X \times X$.*

Definition 6.8. [10] A space X is said to be **quasi zero dimensional** if for each $x \in X$ and each θ-open set U containing x there is a clopen set V such that $x \in V \subset U$.

Theorem 6.9. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function defined by $g(x) = (x, f(x))$ for each $x \in X$. If g is quasi cl-supercontinuous, then f is quasi cl-supercontinuous and X is a quasi zero dimensional space.*

Proof. Suppose that g is quasi cl-supercontinuous. Then by Theorem 5.4, the composition $f = p_y \circ g$ is quasi cl-supercontinuous, where p_y denotes the projection map $p_y : X \times Y \to Y$. To show that the space X is quasi zero dimensional, let U be any θ-open set in X and let $x \in U$. Then $U \times Y$ is a θ-open set in $X \times Y$ containing $g(x)$. Since g is quasi cl-supercontinuous, there exists a clopen set V containing x such that $g(V) \subset U \times Y$. Thus $x \in V \subset U$. This shows that the space X is quasi zero dimensional. ■

7. Clopen θ-closed graphs

Definition 7.1. The graph $G(f)$ of a function $f : X \to Y$ is said to be **clopen θ-closed** if for each $(x, y) \notin G(f)$ there exist a clopen set U containing x and a θ-open set V containing y such that $(U \times V) \cap G(f) = \emptyset$.

Theorem 7.2. *Let $f : X \to Y$ be a quasi cl-supercontinuous function into a θ-Hausdorff space Y. Then the graph $G(f)$ of f is clopen θ-closed in $X \times Y$.*

Proof. Suppose $(x, y) \notin G(f)$. Then $f(x) \neq y$. Since Y is θ-Hausdorff, there exist disjoint θ-open sets V and W containing $f(x)$ and y, respectively. Since f is quasi cl-supercontinuous, there exists a clopen set U containing x such that $f(U) \subset V$. Clearly $(U \times W) \cap G(f) = \emptyset$ and so the graph $G(f)$ of f is clopen θ-closed in $X \times Y$. ■

Theorem 7.3. *Let $f : X \to Y$ be an injection such that the graph $G(f)$ is clopen θ-closed in $X \times Y$. Then X is ultra Hausdorff.*

Theorem 7.4. *Let $f : X \to Y$ be an open surjection such that the graph $G(f)$ of f is clopen θ-closed in $X \times Y$. Then Y is Hausdorff. Further, if f maps clopen sets to θ-open sets, then Y is θ-Hausdorff.*

Proof. Let $y, z \in Y, y \neq z$. Since f is a surjection, there exists $x \in X$ such that $f(x) = y$. Then $(x, z) \notin G(f)$. In view of clopen θ-closedness of the graph $G(f)$, there exists a clopen set U containing x and a θ-open set V containing z such that $(U \times V) \cap G(f) = \emptyset$. It follows that $f(U) \cap V = \emptyset$. Since f is an open map, $f(U)$ is an open set containing y. Thus Y is Hausdorff.

Further, if f maps clopen sets to θ-open sets, then $f(U)$ is a θ-open set and so Y is θ-Hausdorff. ■

Definition 7.5. [13](3) A subset A of space X is called ***θ*-set** if every cover of A by θ-open subsets of X has a finite subcover.

Theorem 7.6. *Let $f : X \to Y$ be a function such that the graph $G(f)$ of f is clopen θ-closed in $X \times Y$. Then $f^{-1}(K)$ is cl-closed in X for every θ-set K in Y.*

Proof. Let K be a θ-set in Y and let $x \notin f^{-1}(K)$. Then for each $y \in K$, $(x, y) \notin G(f)$. So in view of clopen θ-closedness of the graph $G(f)$, there exist a clopen set U_y containing x and a θ-open set V_y containing y such that $(U_y \times V_y) \cap G(f) = \emptyset$. The collection $\{V_y : y \in K\}$ is a cover of K by θ-open sets in Y. Since K is a θ-set, there exist a finitely many points $y_1, \ldots, y_n \in K$ such that $K \subset \bigcup\{V_{y_i} : i = 1, \ldots, n\}$. Let $U = \bigcap_{i=1}^{n} U_{y_i}$, $V = \bigcup_{i=1}^{n} V_{y_i}$. Then U is a clopen set containing x and $f(U) \cap K = \emptyset$. So $U \subset X \setminus f^{-1}(K)$ and so $X \setminus f^{-1}(K)$ is cl-open being the union of clopen sets. Thus $f^{-1}(K)$ is cl-closed. ■

Theorem 7.7. *If a function $f : X \to Y$ has a clopen θ-closed graph $G(f)$, then $f(K)$ is $u\theta$-closed in Y for each subset K which is mildly compact relative to X.*

Proof. Let $y \notin f(K)$. Then for each $x \in K, (x, y) \notin G(f)$. Since f has a clopen θ-closed graph, there exists a clopen set U_x in X containing x and a θ-open set V_x of Y containing y such that $f(U_x) \cap V_x = \emptyset$. The collection $\{U_x : x \in K\}$ is a cover of K by clopen sets in X. Since K is mildly compact relative to X, there exist finitely many points $x_1, x_2, \ldots, x_n \in K$ such that $K \subset \bigcup\{U_{x_i} : i = 1, 2, \ldots, n\}$. Let $V = \bigcap_{i=1}^{n} V_{x_i}$. Then V is a θ-open set containing y and $f(K) \cap V = \emptyset$. Thus $f(K)$ is $u\theta$-closed. ■

8. Change of topology

In this section we study the behaviour of a quasi cl-supercontinuous function if its domain and/or range are retopologized in an appropriate way.

8.1 Let (X, τ) be a topological space and let β denote the collection of all clopen subsets of (X, τ). Since the intersection of two clopen sets is a clopen set, the collection β is a base for a topology τ^* on X. Clearly $\tau^* \subset \tau$ and any topological property which is preserved under continuous bijections is transferred from (X, τ) to (X, τ^*). Moreover, the space (X, τ) is zero dimensional if and only if $\tau = \tau^*$. The topology τ^* has been extensively referred to in the mathematical literature (see [5], [45]).

(3) θ-sets' are referred to as θ-compact relative to X in [7]. A θ-set in a topological space need not be θ-compact. For such an example (see [13, Remark 2.2]).

Throughout the section, the symbol τ^* will have the same meaning as in the above paragraph.

8.2 Let (Y,σ) be a topological space, and let σ_θ denote the collection of all θ-open subsets of (Y,σ). Since the finite intersection and arbitrary union of θ-open sets is θ-open (see [51]), the collection σ_θ is a topology for Y considered in [32]. Clearly, $\sigma_\theta \subset \sigma$ and any topological property which is preserved by continuous bijections is transferred from (Y,σ) to (Y,σ_θ). Moreover, the space (Y,σ) is a regular space if and only if $\sigma = \sigma_\theta$.

Throughout the section, the symbol σ_θ will have the same meaning as in the above paragraph.

Theorem 8.3. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following statements are equivalent.*

(a) $f : (X,\tau) \to (Y,\sigma)$ *is quasi cl-supercontinuous.*
(b) $f : (X,\tau) \to (Y,\sigma_\theta)$ *is cl-supercontinuous.*
(c) $f : (X,\tau^*) \to (Y,\sigma)$ *is faintly continuous.*
(d) $f : (X,\tau^*) \to (Y,\sigma_\theta)$ *is continuous.*

9. Function spaces and quasi cl-supercontinuous functions

A topological space X is said to be sum connected [8] if each $x \in X$ is contained in a connected open set, or equivalently each component of X is open in X. The category of sum connected spaces contains all connected spaces as well as all locally connected spaces and represents precisely the coreflective hull of the category of connected spaces. The product of topologist's sine curve with a nondegenerate discrete space is a sum connected space which is neither connected nor locally connected.

Throughout the section we assume that the function space Y^X of all functions from a space X into a space Y is endowed with the topology of pointwise convergence. In general, the set $C(X,Y)$ of all continuous functions from a space X into a space Y is not closed in Y^X. In contrast Naimpally [36] showed that if X is locally connected and Y is Hausdorff, then the set $S(X,Y)$ of all strongly continuous functions from X to Y is closed in Y^X. In [19] Naimpally's result is extended to a larger framework wherein it is shown that if X is sum connected and Y is Hausdorff, then the set $P(X,Y)$ of all perfectly continuous functions as well as the set $L(X,Y)$ of all cl-supercontinuous functions from X to Y is closed in Y^X. This result is further extended in ([20], [28], [46]) to show that the set $P_\Delta(X,Y)$ of all δ-perfectly continuous functions as well as the set $P_\delta(X,Y)$ of all almost perfectly continuous ($\equiv$ regular set connected) functions and the set $P_q(X,Y)$ of all quasi perfectly continuous functions are closed in Y^X under the same hypotheses on X and Y.

In this section we further improve upon these results to show that if X is sum connected and Y is Hausdorff, then the set of $L_q(X,Y)$ of all quasi cl-supercontinuous functions as well as the set $L_\delta(X,Y)$ of almost cl-supercontinuous functions is closed in Y^X in the topology of pointwise convergence. Thus in this case all the eight classes of functions coincide, i.e.

$$\begin{aligned} S(X,Y) &= P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) \\ &= L_\delta(X,Y) = L_q(X,Y). \end{aligned}$$

We may recall that a space X is a $\boldsymbol{\delta T_0}$**-space** [21] if for each pair of distinct points x and y in X there is a regular open set containing one of the points x and y but not the other.

Since an almost perfectly continuous function is almost cl-supercontinuous, the following result generalizes Theorem 4.1 of Singh [46]. Although the proof is similar we include it here for the sake of completeness.

Theorem 9.1. *Let $f : X \to Y$ be an almost cl-supercontinuous function into a δT_0-space Y. If C is a connected set in X, then $f(C)$ is a singleton. In particular, every almost cl-supercontinuous function from a connected space into a δT_0-space is constant and so strongly continuous.*

Proof. Assume contrapositive and let C be a connected subset of X such that $f(C)$ is a not singleton. Let $f(x), f(y) \in f(C), f(x) \neq f(y)$. Since Y is a δT_0-space, there exists a regular open set V containing one of the points $f(x)$ and $f(y)$ but not the other. For definiteness assume that $f(x) \in V$. In view of almost cl-supercontinuity of $f, f^{-1}(V) \cap C$ is a nonempty proper cl-open subset of C, contradicting the fact that C is connected. ∎

Corollary 9.2. *Let $f : X \to Y$ be an almost cl-supercontinuous function into a δT_0-space Y. If X is sum connected, then f is constant on each component of X and hence strongly continuous.*

Proof. Let X be sum connected space. Then every component of X is clopen in X. Indeed X is partitioned into disjoint components which are clopen subsets of X. Hence it follows that any union of components of X and the complement of this union are complementary clopen sets in X. By Theorem 9.1, f is constant on each component of X. So for every subset S of $Y, f^{-1}(S)$ and $X \setminus f^{-1}(S)$ are complementary clopen sets being union of components of X. Hence f is strongly continuous. ∎

Remark 9.3. In view of the fact that every almost perfectly continuous function is almost cl-supercontinuous, Corollary 9.2 generalizes [46, Corollary 4.3] due to Singh.

Remark 9.4. Simple examples can be given to show that the hypothesis of δT_0-space cannot be omitted either in Theorem 9.1 or Corollary 9.2.

Theorem 9.5. *Let $f : X \to Y$ be a quasi cl-supercontinuous function into a Hausdorff space Y. If C is a connected set in X, then $f(C)$ is a singleton. In particular, if X is connected, then f is constant and hence f is strongly continuous.*

Proof. Assume contrapositive and let C be a connected subset of X such that $f(C)$ is not a singleton. Let $f(a)$ and $f(b)$ be any two distinct points in $f(C)$. Since Y is Hausdorff and since every compact set in a Hausdorff space is θ-closed, the set $V = Y \setminus \{f(b)\}$ is a θ-open set containing $f(a)$. In view of quasi cl-supercontinuity of $f, f^{-1}(V) \cap C$ is a nonempty proper cl-open subset of C, contradicting the fact that C is connected. ∎

Corollary 9.6. *Let $f : X \to Y$ be a quasi cl-supercontinuous function from a sum connected space X into a Hausdorff space Y. Then f is a constant on each component of X and hence strongly continuous.*

Proof. Proof is similar to that of Corollary 9.2 and hence omitted. ∎

Theorem 9.7. [46, Theorem 4.5] *Let $f : X \to Y$ be a function from a sum connected space into a δT_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*

Theorem 9.8. *Let $f : X \to Y$ be a function from a sum connected space into a Hausdorff space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*
(f) *f is almost cl-supercontinuous.*
(g) *f is quasi cl-supercontinuous.*

Proof. The implications $(a) \Rightarrow (b) \Rightarrow (d) \Rightarrow (e) \Rightarrow (f) \Rightarrow (g)$ are trivial and the implication $(g) \Rightarrow (a)$ is a consequence of Corollary 9.6. So the result is immediate in view of Theorem 9.7. ∎

Theorem 9.9. *Let X be a sum connected space and let Y be a Hausdorff space. Then all the seven classes of functions $S(X,Y)$, $P(X,Y)$, $L(X,Y)$, $P_\Delta(X,Y)$, $P_\delta(X,Y)$, $L_\delta(X,Y)$, $L_q(X,Y)$ coincide and are closed in Y^X in the topology of pointwise convergence.*

Proof. This is immediate in view of Theorem 9.8 and ([20, Theorem 5.4] or [46, Theorem 4.6]). ∎

In particular, in view of above theorem it follows that if X is a sum connected (e.g. connected or locally connected) space and Y is Hausdorff, then the pointwise limit of a sequence $\{f_n : X \to Y\}$ of almost (quasi) cl-supercontinuous functions is almost (quasi) cl-supercontinuous.

10. Connectedness and existence/non-existence of certain functions

It is shown in [21, Theorem 5.2] that if $f : X \to Y$ is a cl-supercontinuous surjection defined on a connected space X, then Y is an indiscrete space. Thus there exists no cl-supercontinuous surjection from a connected space onto a non-indiscrete space. In contrast for almost cl-supercontinuous functions we have the following.

First we may recall that a space X is said to be **hyperconnected** ([1], [50]) if every nonempty open set in X is dense in X, or equivalently any two nonempty open sets in X intersect.

Theorem 10.1. *Let $f : X \to Y$ be an almost cl-supercontinuous surjection from a connected space X onto Y. Then Y is a hyperconnected space.*

Proof. Suppose Y is not hyperconnected. Then there exists a nonempty proper open subset V of Y which is not dense in Y. So $W = \overline{V}^0$ is a nonempty proper regular open subset of Y. Since f is almost cl-supercontinuous, $f^{-1}(W)$ is a nonempty proper cl-open subset of X contradicting the fact that X is connected. ∎

It follows that there exists no almost cl-supercontinuous surjection from a connected space onto a non-hyperconnected space. Moreover, from Theorem 9.1 it follows that there exists no nonconstant almost cl-supercontinuous function from a connected space into a δT_0-space. Similarly, from Theorem 9.5 it follows that there exists no nonconstant quasi cl-supercontinuous function from a connected space into a Hausdorff space.

Definition 10.2. A space X is said to be **quasi hyperconnected** if there exists no proper θ-closed set in X or equivalently there exists no proper θ-open set in X.

Theorem 10.3. *Let $f : X \to Y$ be a quasi cl-supercontinuous surjection from a connected space X onto Y. Then Y is quasi hyperconnected.*

Proof. Suppose Y is not quasi hyperconnected and let V be a nonempty proper θ-open subset of X. In view of quasi cl-supercontinuity of f, $f^{-1}(V)$

is a nonempty proper cl-open subset of X contradicting the fact that X is connected.

Thus there exists no quasi cl-supercontinuous surjection from a connected space onto a non quasi hyperconnected space. ■

11. M-continuous functions

In this section we give a brief description of notions of minimal structures and M-continuous functions introduced and studied by Popa and Noiri [40]. We show that the notion of 'quasi cl-supercontinuous functions' is related with the notion of 'M-continuous functions' and point out that certain results of preceding sections follow directly from results on M-continuous functions [40].

Definition 11.1. [40] A subfamily m_X of the power $\mathcal{P}(X)$ of a nonempty set X is called a **minimal structure** on X if $\emptyset \in m_X$ and $X \in m_X$.

Definition 11.2. [40] A function $f : (X, m_X) \to (Y, m_Y)$ where (X, m_X) and (Y, m_Y) are nonempty sets X and Y with minimal structures m_X and m_Y, respectively, is said to be **M-continuous** if for each $x \in X$ and $V \in m_Y$ containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$.

Definition 11.3. [34, 40] A minimal structure m_X on a nonempty set X is said to have the **property** ($\mathcal{B}$) if the union of any subfamily of m_X belongs to m_X.

11.4. Let X and Y be topological spaces and let m_X denote the set of all cl-open sets in X and let m_Y denote the set of all θ-open sets in Y. Then it follows that the collections m_X and m_Y are minimal structures with property ($\mathcal{B}$) on X and Y, respectively. Now, it follows that Theorem 4.3 (a), (b), (e), (f) and (g) are particular cases of Theorem 3.1 and Corollary 3.1 of Popa and Noiri [40].

Definition 11.5. [40] A nonempty set X with minimal structure is said to be **m-compact** if every cover of X with members of m_X has a finite subcover. A subset K of a nonempty set X with a minimal structure m_X is said to be **m-compact** if every cover of K by members of m_X has a finite subcover.

Definition 11.6. [40] A function $f : (X, m_X) \to (Y, m_Y)$ is said to have a **strongly m-closed graph** (respectively, **m-closed graph**) if for each $(x, y) \in (X \times Y) \setminus G(f)$ there exists $U \in m_X$ containing x and $V \in m_Y$ containing y such that $f(U) \cap m_Y\text{-}cl(V) = \emptyset$ (respectively $f(U) \cap V = \emptyset$).

11.7. Since θ-open sets in a topological space X constitute a minimal structure with property ($\mathcal{B}$) on X, Theorem 6.4 is a particular case of Theorem 4.2 of [40]. Moreover, Theorem 7.2 is a particular case of Theorem 4.3 of [40].

Acknowledgement. Authors are thankful to the referee for helpful suggestions and pointing out reference [40] on M-continuous functions. In particular, his remarks culminated in the formulation of Section 11.

References

[1] N. Ajmal, J. K. Kohli, *Properties of hyperconnected spaces, their mappings into Hausdorff spaces and embeddings into hyperconnected spaces*, Acta Math. Hungar. 60(1-2) (1992), 41–49.

[2] A. K. Das, *A note on θ-Hausdorff spaces*, Bull. Calutta Math. Soc. 97(1) (2005), 15–20.

[3] J. Dontchev, M. Ganster, I. Reilly, *More on almost s-continuity*, Indian J. Math. 41 (1999), 139–146.

[4] S. Fomin, *Extensions of topological spaces*, Ann. of Math. 44 (1943), 471–480.

[5] E. Ekici, *Generalization of perfectly continuous, regular set-connected and clopen functions*, Acta Math Hungar. 107(3) (2005), 193–206.

[6] N. C. Heldermann, *Developability and some new regularity axioms*, Canadian J. Math. 33(3) (1981), 641–663.

[7] S. Jafari, *Some properties of quasi θ-continuous functions*, Far East J. Math. 6(5) (1998), 689–696.

[8] J. K. Kohli, *A class of spaces containing all connected and all locally connected spaces*, Math. Nachr. 82 (1978), 121–129.

[9] J. K. Kohli, *D-continuous functions, D-regular spaces and D-Hausdorff spaces*, Bull. Calcutta Math. Soc. 84 (1992), 39–46.

[10] J. K. Kohli, *Localization, generalizations and factorizations of zero dimensionality*, (preprint).

[11] J. K. Kohli, A. K. Das, *New normality axioms and factorizations of normality*, Glasnik Mat. 37(57) (2002), 105–114.

[12] J. K. Kohli, A. K. Das, *On functionally θ-normal spaces*, App. Gen. Topol. 6(1) (2005), 1–14.

[13] J. K. Kohli, A. K. Das, *A class of spaces containing all generalized absolutely closed (almost compact) spaces*, App. Gen. Topol. 7(2) (2006), 233–244.

[14] J. K. Kohli, A. K. Das, R. Kumar, *Weakly functionally θ-normal spaces, θ-shrinking of covers and partition of unity*, Note Mat. 19(2) (1999), 293–297.

[15] J. K. Kohli, R. Kumar, *z-supercontinuous functions*, Indian J. Pure Appl. Math. 33(7) (2002), 1097–1108.

[16] J. K. Kohli, D. Singh, *D-supercontinuous functions*, Indian J. Pure Appl. Math. 32(2) (2001), 227–235.

[17] J. K. Kohli, D. Singh, *D_δ-supercontinuous functions*, Indian J. Pure Appl. Math 34(7) (2003), 1089–1100.

[18] J. K. Kohli, D. Singh, *Between weak continuity and set connectedness*, Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau 15 (2005), 55–65.

[19] J. K. Kohli, D. Singh, *Function spaces and strong variants of continuity*, App. Gen. Topol. 9(1) (2008), 33–38.

[20] J. K. Kohli, D. Singh, *δ-perfectly continuous functions*, Demonstratio Math. 42(1) (2009), 221–231.

[21] J. K. Kohli, D. Singh, *Almost cl-supercontinuous functions*, App. Gen. Topol. 10(1) (2009), 1–12.

[22] J. K. Kohli, D. Singh, J. Aggarwal, *R-supercontinuous functions*, Demonstratio Math. 43(3) (2010), 703–723.

[23] J. K. Kohli, D. Singh, J. Aggarwal, *On certain generalizations and factorizations of normality*, (preprint).

[24] J. K. Kohli, D. Singh, C. P. Arya, *Perfectly continuous functions*, Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau 18 (2008), 99–110.

[25] J. K. Kohli, D. Singh, R. Kumar, *Quasi z-supercontinuous functions and pseudo z-supercontinuous functions*, Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau 14 (2004), 43–56.

[26] J. K. Kohli, D. Singh, R. Kumar, *Some properties of strongly θ-continuous functions*, Bull. Calcutta Math. Soc. 100(2) (2008), 185–196.

[27] J. K. Kohli, D. Singh, R. Kumar, *Generalizations of z-supercontinuous functions and D_δ-supercontinuous functions*, App. Gen. Topol. 9(2) (2008), 239–251.

[28] J. K. Kohli, D. Singh, B. K. Tyagi, *Quasi perfectly continuous functions*, Sci. Stud. Res. Ser. Math. Inform. 11(2) (2011), 23–40.

[29] N. Levine, *Strong continuity in topological spaces*, Amer. Math. Monthly 67 (1960), 269.

[30] N. Levine, *A decomposition of continuity in topological space*, Amer. Math. Monthly 68 (1961), 44–46.

[31] P. E. Long, L. Herrington, *Strongly θ-continuous functions*, J. Korean Math. Soc. 8 (1981), 21–28.

[32] P. E. Long, L. Herrington, *T_θ-topology and faintly continuous functions*, Kyungpook Math. J. 22 (1982), 7–14.

[33] J. Mack, *Countable paracompactness and weak normality properties*, Trans. Amer. Math. Soc. 148 (1970), 265–272.

[34] H. Maki, *On generalizing semi-open and preopen sets*, Report for Meeting on Topological Spaces Theory and its Applications, August 1996, Yatsushiro College of Technology, pp. 13–18.

[35] B. M. Munshi, D. S. Bassan, *Supercontinuous mappings*, Indian J. Pure Appl. Math. 13 (1982), 229–236.

[36] S. A. Naimpally, *On strongly continuous functions*, Amer. Math. Monthly 74 (1967), 166–169.

[37] T. Noiri, *On δ-continuous functions*, J. Korean Math. Soc. 16 (1980), 161–166.

[38] T. Noiri, *Supercontinuity and some strong forms of continuity*, Indian J. Pure Appl. Math. 15(3) (1984), 241–250.

[39] T. Noiri, V. Popa, *Weak forms of faint continuity*, Bull. Math. Soc. Sci. Math. Roumanie 34(82) (1990), 263–270.

[40] V. Popa, T. Noiri, *On M-continuous functions*, Anal. Univ. "Dunarea de Jos", Galati, Ser. Mat. Fiz. Mec. Teor. 18(23) (2000), 31–41.

[41] I. L. Reilly, M. K. Vamanamurthy, *On supercontinuous mappings*, Indian J. Pure Appl. Math. 14(6) (1983), 767–772.

[42] M. K. Singal, A. R. Singal, *Almost continuous mappings*, Yokohama Math. J. 16 (1968), 63–73.

[43] M. K. Singal, S. B. Niemse, *z-continuous mappings*, Math. Student 66(1-4) (1997), 193–210.

[44] D. Singh, *D^*-continuous functions*, Bull. Calcutta Math. Soc. 91(5) (1999), 385–390.

[45] D. Singh, *cl-supercontinuous functions*, App. Gen. Topol. 8(2) (2007), 293–300.

[46] D. Singh, *Almost perfectly continuous functions*, Quaestiones Math. 33(2) (2010), 211–221.

[47] S. Sinharoy, S. Bandhopadhyay, *On θ-completely regular spaces and locally θH-closed spaces*, Bull. Calcutta Math. Soc. 87 (1995), 19–28.

[48] A. Sostak, *A class of topological spaces containing all bicompact and all connected spaces*, General Topology and its Relations to Modern Analysis and Algebra IV, Proceedings of the 4th Prague Topological Symposium (1976), Part B, 445–451.

[49] R. Staum, *The algebra of bounded continuous functions into a nonarchimedean field*, Pacific J. Math. 50(1) (1974), 169–185.

[50] L. A. Steen, J. A. Seebach, Jr., *Counter Examples in Topology*, Springer, New York, 1978.

[51] N. V. Veličko, *H-Closed topological spaces*, Amer. Math. Soc. Transl. 78 (1968), 103–118.

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Received February 23, 2010; revised version May 25, 2010.