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REMARKS ON: "OSCILLATION CRITERIA FOR SECOND-ORDER FUNCTIONAL DIFFERENCE EQUATION WITH NEUTRAL TERMS" [DEMONSTRATIO MATH. 41 (2008)]

Abstract. In this paper, we show that the paper mentioned in the title includes some wrong results. We also provide a counter example.

1. Introduction

In [1], the author studies a nonlinear type of second-order difference equations having the following form:

$$\Delta\big[p(n)F\big(\Delta\big[y(n)+a(n)y(n-\tau)\big]\big)\big]+q(n)G\big(\Delta\big[y(n)+a(n)y(n-\tau)\big]\big)$$
$$+H\big(n,y(n),y(n-\sigma_1),\dots,y(n-\sigma_r)\big)=0,$$

where $n \geq n_0$ travels through integers with the following primary assumptions:

(A1) $\{p(n)\}_{n=n_0}^{\infty}, \{q(n)\}_{n=n_0}^{\infty}$ are positive sequences of reals,

(A2) $F(\nu)$ is an increasing continuous function such that $\nu F(\nu) > 0$ holds for all $\nu \neq 0$,

(A3) there exist $M, m > 0$ such that $M \geq G(\nu) \geq m$ holds for all $\nu \in \mathbb{R}$,

(A4) $H : \mathbb{Z} \times \mathbb{R}^{r+1} \to \mathbb{R}$ is a continuous increasing function with respect to its each real component, when the others are fixed; further, $\operatorname{sgn}(H(n,\nu_0,\nu_1,\dots\nu_r)) = \operatorname{sgn}(\nu_0)$ provided that $\operatorname{sgn}(\nu_0) = \operatorname{sgn}(\nu_i)$ for all $i \in \{1,\dots,r\}$,

(A5) τ, σ_i for all $i \in \{1,\dots,r\}$ are positive integers.

[1, Theorem 1] reads as follows:

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Theorem A. *Suppose that $0 \le a(n) < 1$ holds for all sufficiently large n. If*

$$\sum_{i=n_0}^{\infty} F^{-1}\Big(-\frac{\alpha}{p(i)}\Big) = -\infty \text{ for all } \alpha > 0 \tag{1}$$

and

$$\sum_{i=n_0}^{\infty} F^{-1}\Big(\frac{\beta - \sum_{j=n_0}^{i-1} H(j,\gamma,\dots,\gamma)}{p(i)}\Big) = -\infty \tag{2}$$

for every constant β and all $\gamma > 0$,

then every solution of (1) *is oscillatory.*

Now, we state the counter example to Theorem A.

Counter Example. Consider the following difference equation

$$\Delta\Big[\Big(\frac{n+1}{2}\Big)\Delta\Big[y(n) + \frac{n^2-1}{n^2}y(n-1)\Big]\Big] + \Big(\frac{1}{n(n+1)} + 1\Big) + \frac{n-1}{n}y(n-1) = 0 \quad \text{for } n \ge 2, \tag{3}$$

where $p(n) = (n+1)/2$, $\tau = 1$, $a(n) = (n^2-1)/n^2$, $F(\nu) = \nu$, $q(n) = 1/(n+1) + 1$, $G(\nu) = 1$, $\sigma_1 = 1$, $H(n, \nu_0, \nu_1) = (n-1)\nu_1/n$, and $r = 1$. It is not hard to check that (A1)–(A5) hold. On the other hand, we have

$$\sum_{i=2}^{\infty}\Big(-\frac{\alpha}{(i+1)/2}\Big) = -\infty \quad \text{for all } \alpha > 0,$$

and

$$\sum_{i=2}^{\infty}\Big(\frac{\beta - \sum_{j=2}^{i-1}(j-1)\gamma/j}{((i+1)/2)}\Big) = -\infty \quad \text{for every constant } \beta \text{ and all } \gamma > 0,$$

that is (1) and (2) hold for (3). Therefore, by Theorem A every solution of (3) is oscillatory. But unfortunately, one can show by direct substitution that $y(n) = -(n+1)/n$ is a nonoscillatory solution of (3), which tends to -1 from above asymptotically.

Remark 1. One of the mistakes in the proof of [1, Lemma 1, Theorem 1, Theorem 2, Theorem 3] is assuming the existence of the constant $\lambda \in (0,1)$ such that $|y(n)| \ge \lambda|y(n) + a(n)y(n-\tau)|$ holds for all sufficiently large n. Indeed, it is not always possible to find such $\lambda \in (0,1)$, these results are therefore not always true. Moreover, nonsysmetric conditions assumed to hold on G forces us to find an eventually negative solution.

References

[1] Y. Bolat, *Oscillation criteria for second-order functional difference equation with neutral terms*, Demonstratio Math. 41 (2008), 615–625.

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