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V. B. L. Chaurasia, Mukesh Agnihotri

THE PRODUCT OF GENERALIZED POLYNOMIALS SETS PERTAINING TO A CLASS OF CONVOLUTION INTEGRAL EQUATIONS

Abstract. The purpose of this paper is to obtain a certain class of convolution integral equation of Fredholm type with the product of two generalized polynomials sets. Using of the Mellin transform technique; we have established solution of the integral equation.

1. Introduction

The polynomial set $R_n^{a,b}[x]$ is introduced by Agrawal and Chaubey [2].

$$\begin{aligned}(1.1)\quad R_n^{a,b}[x,\alpha,\beta,\gamma,\delta;c,d,\mu,\nu;w(x)] &= R_n^{a,b}[x]\\ &= \frac{(\alpha x^c+\beta)^{-a}(\gamma x^d+\delta)^{-b}}{K_n w(x)} T_{\xi;\varsigma}^{n}\left[(\alpha x^c+\beta)^{a+\mu n}(\gamma x^d+\delta)^{b+\nu n} w(x)\right],\end{aligned}$$

$n = 0,1,2,\ldots$, where

$$(1.2)\qquad T_{\xi;\varsigma} = x^{\xi}(\varsigma + xD_x), \qquad D_x = \frac{d}{dx},$$

$\alpha,\beta,\gamma,\delta,a,b,c,d,\mu,\nu$ are constants, $\{K_n\}_{n=0}^{\infty}$ is a sequence of constants, and $w(x)$ is any general function of x, differentiable an arbitrary number of times.

The polynomial set $R_n^{a,b}[x]$ is general in nature and gets a number of known polynomials as its special cases.

If, we take $c = d = 1$, $K_n = n!$, $\varsigma = 0$, $n = -1$, the polynomial set $R_n^{a,b}[x]$ reduces to $S_n^{a,b}[x,\alpha,\beta,\gamma,\delta;,\mu,\nu;w(x)]$, it is defined by Srivastava and Panda [6].

In this paper, we shall investigate the inversion of the integral

$$(1.3)\qquad \prod_{i=1,2} g_i(x) = \prod_{i=1,2}\int_0^{\infty} h_i\left(\frac{x}{y}\right) f_i(y)\left(\frac{dy}{y}\right), \qquad (x>0),$$

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where g_i is given function, f_i is an unknown function to be determined , and the kernel h_i is given by

$$(1.4)\quad \prod_{i=1,2} h_i(x)$$
$$= \prod_{i=1,2}\left[\frac{(\alpha_i x^{c_i}+\beta_i)^{a_i}(\gamma_i x^{d_i}+\delta_i)^{b_i}}{e^{t_i x^{\rho_i}}} \cdot K_{n_i} R_{n_i}^{a_i,b_i}[x,\alpha_i,\beta_i,\gamma_i,\delta_i;c_i,d_i,\mu_i,\nu_i;e^{-t_i x^{\rho_i}}]\right]$$
$$= \prod_{i=1,2}[\{x^{\xi_i}(\varsigma_i+xD_x)\}^{n_i}\{(\alpha_i x^{c_i}+\beta_i)^{a_i+\mu_i n_i}(\gamma_i x^{d_i}+\delta_i)^{b_i+v_i n_i}e^{-t_i x^{\rho_i}}\}].$$

2. Mellin Transform of $\prod_{i=1,2} h_i(x)$

We start with the following Lemma:

Lemma. *If* $\prod_{i=1,2} H_i(s) = \prod_{i=1,2} M\{h_i(x);s\}$, *where* $\prod_{i=1,2} h_i(x)$ *is defined by* (1.4), *then*

$$(2.1)\quad \prod_{i=1,2} H_i(s) = \prod_{i=1,2}\left[\sum_{k_i=0}^{n_i}\sum_{p_i=0}^{a_i+\mu_i n_i}\sum_{q_i=0}^{b_i+v_i n_i}(-1)^{k_i+p_i+q_i} \cdot \frac{(-n_i)_{k_i}}{k_i!}\frac{(-a_i-\mu_i n_i)_{p_i}}{p_i!}\frac{(-b_i-v_i n_i)_{q_i}}{q_i!} \cdot \varsigma_i^{n_i-k_i}\beta_i^{a_i+\mu_i n_i-p_i}\alpha_i^{p_i}\delta_i^{b_i+v_i n_i-q_i}\gamma_i^{q_i}\xi_i^{k_i}\left(-\frac{s+\xi_i n_i}{\xi_i}\right)_{k_i} \cdot \frac{1}{|\rho_i|}\Gamma\left(\frac{s+\xi_i n_i+c_i p_i+d_i q_i}{\rho_i}\right) \cdot (t_i)^{-\frac{s+\xi_i n_i+c_i p_i+d_i q_i}{\rho_i}}\right],$$

provided that

$0 < \mathrm{Re}(s+\xi_i n_i+c_i p_i+d_i q_i) < \rho_i$, *when* $\rho_i > 0$;
$\rho_i < \mathrm{Re}(s+\xi_i n_i+c_i p_i+d_i q_i) < 0$, *when* $\rho_i < 0$; $\xi_i \neq 0$ *and*
$n_i, (a_i+\mu_i n_i), (b_i+v_i n_i) \in N_0$, *where* $i = 1,2$.

Proof. Making use of the binomial expansions for $(\alpha_i x^{c_i}+\beta_i)^{a_i+\mu_i n_i}$, $(\gamma_i x^{d_i}+\delta_i)^{b_i+v_i n_i}$ and $[x^{\xi_i}(\varsigma_i+xD_x)]^{n_i}$, we find

$$\begin{aligned}(2.2)\quad \prod_{i=1,2} h_i(x) = \prod_{i=1,2}\Bigg[&\sum_{k_i=0}^{n_i}\sum_{p_i=0}^{a_i+\mu_i n_i}\sum_{q_i=0}^{b_i+v_i n_i}(-1)^{k_i+p_i+q_i}\\ &\cdot\frac{(-n_i)_{k_i}}{k_i!}\frac{(-a_i-\mu_i n_i)_{p_i}}{p_i!}\frac{(-b_i-v_i n_i)_{q_i}}{q_i!}\varsigma_i^{n_i-k_i}\beta_i^{a_i+\mu_i n_i-p_i}\\ &\cdot\alpha_i^{p_i}\delta_i^{b_i+v_i n_i-q_i}\gamma_i^{q_i}x^{\xi_i(n_i-k_i)}(x^{\xi_i+1}D_x)^{k_i}\{x^{c_i p_i+d_i q_i}e^{-t_i x^{\rho_i}}\}\Bigg].\end{aligned}$$

Taking Mellin transform of both sides of equation (2.2) and applying the following known formulas [7, p. 1.4, eq. (2.2)]; [1, p. 307, eq. (7)].

$$(2.3)\qquad M\{(x^{k+1}D_x)^n f(x);s\} = k^n\left(-\frac{s+kn}{k}\right)_n F(s+kn),$$

and

$$(2.4)\qquad M\{x^{\mu}f(x);s\} = F(s+\mu),$$

we have

$$\begin{aligned}(2.5)\quad \prod_{i=1,2} H_i(s) = \prod_{i=1,2}\Bigg[&\sum_{k_i=0}^{n_i}\sum_{p_i=0}^{a_i+\mu_i n_i}\sum_{q_i=0}^{b_i+v_i n_i}(-1)^{k_i+p_i+q_i}\\ &\cdot\frac{(-n_i)_{k_i}}{k_i!}\frac{(-a_i-\mu_i n_i)_{p_i}}{p_i!}\frac{(-b_i-v_i n_i)_{q_i}}{q_i!}\\ &\cdot\varsigma_i^{n_i-k_i}\beta_i^{a_i+\mu_i n_i-p_i}\alpha_i^{p_i}\delta_i^{b_i+v_i n_i-q_i}\gamma_i^{q_i}\xi_i^{k_i}\left(-\frac{s+\xi_i n_i}{\xi_i}\right)_{k_i}\\ &\cdot M\{x^{c_i p_i+d_i q_i}e^{-t_i x^{\rho_i}}, s+\xi_i n_i\}\Bigg],\end{aligned}$$

again, applying [1, p. 307, eq. (7)] with the following known result [1, p. 313]

$$(2.6)\qquad M\{(e^{-ax^h});s\} = \frac{1}{|h|}a^{-s/h}\Gamma(\frac{s}{h}),$$

we obtain the desired result (2.1).

3. Solution of the Integral Equation (1.3)

Theorem. *Let the Mellin transforms $F_i(s)$, $G_i(s)$ and $H_i(s)\neq 0$ of the functions $f_i(x)$, $g_i(x)$ and $h_i(x)$ defined by (1.4) exist and be analytic in some infinite strip $\eta_i < \mathrm{Re}(s) < \lambda_i$ of the complex s-plane. Also let for a find $r_i \in (\eta_i, \lambda_i)$, $h_i^*(x)$ is defined by*

$$(3.1)\ \prod_{i=1,2} h_i^*(x) = \prod_{i=1,2}\left[M^{-1}\{H_i^*(s);x\}\right] = \prod_{i=1,2}\frac{1}{2\pi\varpi}\int_{r_i-\varpi\infty}^{r_i+\varpi\infty} x^{-s}H_i^*(s)ds,$$

where, $\varpi = \sqrt{-1}$,

$$(3.2)\quad \prod_{i=1,2} H_i^*(s)$$

$$= \prod_{i=1,2}\Bigg[U_i^{W_i}\frac{\Gamma(-\frac{s}{U_i})}{\Gamma(-W_i-\frac{s}{U_i})}\sum_{k_i=0}^{n_i}\sum_{p_i=0}^{a_i+\mu_i n_i}\sum_{q_i=0}^{b_i+v_i n_i}(-1)^{k_i+p_i+q_i}\frac{(-n_i)_{k_i}}{k_i!}$$

$$\cdot\frac{(-a_i-\mu_i n_i)_{p_i}}{p_i!}\frac{(-b_i-v_i n_i)_{q_i}}{q_i!}\varsigma_i^{n_i-k_i}\beta_i^{a_i+\mu_i n_i-p_i}\alpha_i^{p_i}\delta_i^{b_i+v_i n_i-q_i}\gamma_i^{q_i}\frac{\xi_i^{k_i}}{|\rho_i|}$$

$$\cdot\frac{\Gamma(1+\frac{s+\xi_i n_i+U_iW_i+V_i}{\xi_i})\Gamma(\frac{s+U_iW_i+V_i+\xi_i n_i+c_ip_i+d_iq_i}{\rho_i})}{\Gamma(\frac{s+\xi_i n_i+U_iW_i+V_i}{\xi_i}-k_i+1)\,(t_i)^{\frac{s+U_iW_i+V_i+\xi_i n_i+c_ip_i+d_iq_i}{\rho_i}}}\Bigg]^{-1},$$

provided that

$0 < \mathrm{Re}(s+U_iW_i+V_i+\xi_i n_i+c_ip_i+d_iq_i) < \rho_i$, *when* $\rho_i > 0$;
$\rho_i < \mathrm{Re}(s+U_iW_i+V_i+\xi_i n_i+c_ip_i+d_iq_i) < 0$, *when* $\rho_i < 0$; $\rho_i, U_i, \xi_i \neq 0$
n_i, $(a_i+\mu_i n_i), (b_i+v_i n_i) \in N_0$, *where* $i = 1, 2$.

Then the integral equation (1.3) *has its solution given by*

$$(3.3)\quad \prod_{i=1,2} f_i(x) = \prod_{i=1,2}\left[x^{-U_iW_i-V_i}\int_0^\infty y^{-1}h_i^*\left(\frac{x}{y}\right)(y^{U_i+1}D_y)^{W_i}\{y^{V_i}g_i(y)\}dy\right],$$

provided that the integral exists.

Proof. By convolution Theorem for Mellin transforms [1, p. 308, eq. (14)], equation (1.3) converts into

$$(3.4)\qquad \prod_{i=1,2} H_i(s)F_i(s) = \prod_{i=1,2} G_i(s),$$

where $H_i(s), F_i(s)$ and $G_i(s)$ are Mellin transforms of $h_i(x), f_i(x)$ and $g_i(x)$ respectively. Replacing s in (3.4) by $(s+U_iW_i+V_i)$, we get

$$(3.5)\quad \prod_{i=1,2} F_i(s+U_iW_i+V_i)$$

$$= \prod_{i=1,2}\left[H_i^*(s)\left\{U_i^{W_i}\left(-\frac{s+U_iW_i}{U_i}\right)_{W_i}\right\}G_i(s+U_iW_i+V_i)\right],$$

where $H_i^*(s)$ is given by (3.2).

Now, applying eq. (2.3) and (2.4), we find

$$(3.6)\quad \prod_{i=1,2} F_i(s+U_iW_i+V_i) = \prod_{i=1,2} H_i^*(s)[M\{(y^{U_i+1}D_y)^{W_i}y^{V_i}g_i(y)\}dy; s],$$

and by the application of the known results [1, p. 307, eq. (7)] and [1, p. 308, eq. (14)], in (3.6) , leads to

$$(3.7)\quad \prod_{i=1,2} M[x^{U_iW_i+V_i} f_i(x); s]$$

$$= \prod_{i=1,2} \left[M\left\{ \int_0^\infty y^{-1} h_i^*\left(\frac{x}{y}\right)(y^{U_i+1}D_y)^{W_i}(y^{V_i}g_i(y))dy; s \right\}\right].$$

Inverting both sides of (3.7) by taking the Mellin inversion Theorem [1, p. 307, eq. (1)], we find the required solution (3.3).

4. Applications

Since the polynomial set $R_n(x)$ incorporates in itself several classical as well as other polynomials, solutions of a large number of convolution integral equations for the above mentioned polynomials may be obtained by assigning different values to the parameters in $R_n^{a,b}(x)$.

For example, on taking $c_i = d_i = 1$, $K_n = n_i!$, $\varsigma_i = 0$, $\xi_i = -1$, we have the following corollary

Corollary 1. *The convolution integral equation*

$$(4.1)\qquad \prod_{i=1,2} g_i(x) = \prod_{i=1,2} \int_0^\infty y^{-1} h_i'\left(\frac{x}{y}\right), f_i(y)dy, \qquad (x > 0),$$

where, the kernel

$$(4.2)\quad \prod_{i=1,2} h_i'(x)$$

$$= \prod_{i=1,2} \left[\frac{(\alpha_i x + \beta_i)^{a_i}(\gamma_i x + \delta_i)^{b_i}}{e^{t_i x^{\rho_i}}} n_i! S_{n_i}^{a_i b_i} \left[x, \alpha_i, \beta_i, \gamma_i, \delta_i; \mu_i, \nu_i; e^{-t_i x^{\rho_i}}\right]\right]$$

$$= \prod_{i=1,2} \left[D_x^{n_i}(\alpha_i x + \beta_i)^{a_i+\mu_i n_i}(\gamma_i x + \delta_i)^{b_i+v_i n_i} e^{-t_i x^{\rho_i}} \right],$$

has the solution

$$(4.3)\quad \prod_{i=1,2} f_i(x)$$

$$= \prod_{i=1,2} \left[x^{-U_iW_i-V_i} \int_0^\infty y^{-1} h_i'^*\left(\frac{x}{y}\right)\left(y^{U_i+1}D_y\right)^{W_i} \{y^{V_i}g_i(y)\}\, dy \right],$$

provided that the integral exists, and $\prod_{i=1,2} h_i'^*(x)$ *is the Mellin inverse transform of*

$$(4.4)\quad \prod_{i=1,2} H_i'^*(s) = \prod_{i=1,2}\Bigg[U_i^{W_i}\frac{\Gamma(-\frac{s}{U_i})}{\Gamma(-W_i-\frac{s}{U_i})}\sum_{p_i=0}^{a_i+\mu_i n_i}\sum_{q_i=0}^{b_i+v_i n_i}\frac{(-1)^{k_i+p_i+q_i}}{|\rho_i|}$$

$$\cdot\frac{(-a_i-\mu_i n_i)_{p_i}}{p_i!}\frac{(-b_i-v_i n_i)_{q_i}}{q_i!}\varsigma_i^{n_i-k_i}\beta_i^{a_i+\mu_i n_i-p_i}\alpha_i^{p_i}\delta_i^{b_i+v_i n_i-q_i}\gamma_i^{q_i}$$

$$\cdot\frac{\Gamma(s+U_iW_i+V_i)\,\Gamma(\frac{s+U_iW_i+V_i-n_i+p_i+q_i}{\rho_i})}{\Gamma(s-n_i+U_iW_i+V_i)\,(t_i)^{\frac{s+U_iW_i+V_i+n_i+p_i+q_i}{\rho_i}}}\Bigg]^{-1},$$

provided that

$\mathrm{Re}(s+U_iW_i+V_i-n_i+p_i+q_i)>0$, *when* $\rho_i>0$;

$\mathrm{Re}(s+U_iW_i+V_i-n_i+p_i+q_i)<0$, *when* $\rho_i<0$; $\rho_i, U_i\neq 0$,

$n_i, (a_i+\mu_i n_i), (b_i+v_i n_i)$ *and* $W_i\in N_0$, *where* $i=1,2$.

Also, letting $\alpha_i=1$, $\beta_i=0$, $b_i=0$, $v_i=0$, $a_i=\alpha_i'-\sigma_i$, $\mu_i=\sigma_i$, $c_i=1$, $\rho_i=-1$, $\varsigma_i=0$, $\xi_i=1-\sigma_i$, we find

$$(4.5)\quad R_{n_i}^{\alpha_i'-\sigma_i}\left[x_i,\,1,\,0,\,\gamma_i,\,\delta_i;\,1,\,0,\,\sigma_i;\,e^{-t_ix^{\rho}}\right]=t_i^{n_i}M_{n_i}^{\sigma_i}\left[x,\,\alpha_i',\,t_i\right],$$

where $M_{n_i}^{\sigma_i}[x,\alpha_i',t_i]$ is known as generalized Bessel polynomial [8]. Thus, we have the following result contained in the following corollary

Corollary 2. *Corresponding to the hypothesis of Theorem, the integral equation*

$$(4.6)\qquad \prod_{i=1,2} g_i(x)=\prod_{i=1,2}\int_0^{\infty} y^{-1}h_i''\left(\frac{x}{y}\right)f_i(y)dy,\qquad (x>0),$$

where the kernel

$$(4.7)\qquad \prod_{i=1,2} h_i''(x)=\prod_{i=1,2}[x^{\alpha_i'-\sigma_i}\,e^{-\frac{t_i}{x}}\,t_i^{n_i}M_{n_i}^{\sigma_i}[x,\alpha_i',t_i]]$$

$$=\prod_{i=1,2}[(x^{2-\sigma_i}D_x)\{x^{\alpha_i'-\sigma_i+\sigma_in_i}e^{\frac{-t_i}{x}}\}],$$

possesses the solution

$$(4.8)\quad \prod_{i=1,2} f_i(x)$$

$$=\prod_{i=1,2}\left[x^{-U_iW_i-V_i}\int_0^{\infty} y^{-1}h_i''^*\left(\frac{x}{y}\right)(y^{U_i+1}D_y)^{W_i}\{y^{V_i}g_i(y)\}dy\right],$$

if the integral exits, and $\prod_{i=1,2} h_i''^*(x)$ *is the Mellin inverse transform of*

$$(4.9)\quad \prod_{i=1,2} H_i^{''*}(s) = \prod_{i=1,2}\left[\frac{(-1)^{n_i}\, t_i^{-s-n_i-\alpha_i'+\sigma_i-U_iW_i-V_i}}{\sigma_i^{n_i}U_i^{W_i}\Gamma(-\frac{s}{U_i})\Gamma(\frac{s+U_iW_i+V_i+n_i-\sigma_in_i}{\sigma_i}+1)} \cdot \frac{\Gamma(-W_i-\frac{s}{U_i})\Gamma(\frac{s+U_iW_i+V_i+n_i-2\sigma_in_i}{\sigma_i}+1)}{\Gamma(-s-n_i-U_iW_i-V_i-\alpha_i'+\sigma_i)}\right],$$

provided that $\mathrm{Re}(s+U_iW_i+V_i+\alpha_i') < \sigma_i - n_i,\ \mathrm{Re}(t_i) > 0,$ *when* $U_i \neq 0,$ n_i *and* $W_i \in N_0,$ *where* $i = 1, 2.$

Now, by the application of the Mellin inversion formula (3.1) and replacing s by $-s$ given as

$$(4.10)\quad \prod_{i=1,2} h_i^{''*}(x)$$

$$= \prod_{i=1,2}\left[\frac{(-1)^{n_i}\, t_i^{-ni+\alpha_i'-\sigma_i+U_iW_i+V_i}}{\sigma_i^{n_i}U_i^{W_i}}\frac{1}{2\pi\varpi}\int_{r-\varpi\infty}^{r+\varpi\infty} x^s t_i^s \frac{\Gamma(-W_i+\frac{s}{U_i})}{\Gamma(\frac{s}{U_i})} \cdot \frac{\Gamma(1+\frac{U_iW_i+V_i+n_i-2\sigma_in_i}{\sigma_i}-\frac{s}{\sigma_i})}{\Gamma(1+\frac{U_iW_i+V_i+n_i-\sigma_in_i}{\sigma_i}-\frac{s}{\sigma_i})\Gamma(-U_iW_i-V_i-\alpha_i'-n_i-\sigma_i+s)}ds\right].$$

The contour integral in (4.10) can be represented in terms of Fox's H-function [4]. Thus the solution of the integral equation (4.8) can be expressed as

$$(4.11)\quad \prod_{i=1,2} f_i(x)$$

$$= \prod_{i=1,2}\left\{\frac{(-1)^{n_i} t_i^{-n_i+\alpha_i'-\sigma_i+U_iW_i+V_i}}{\sigma_i^{n_i}U_i^{W_i}} x^{-U_iW_i-V_i}\int_0^\infty y^{-1}(y^{U_i+1}D_y)^{W_i} \{y^{V_i}g_i(y)\} H_{2,3}^{1,1}\left[\frac{xt_i}{y}\left|\begin{matrix}(1+W_i,\frac{1}{U_i}),(\frac{\sigma_i+U_iW_i+V_i+n_i-\sigma_in_i}{\sigma_i},\frac{1}{\sigma_i}), \\ (\frac{\sigma_i+U_iW_i+V_i+n_i-\sigma_in_i}{\sigma_i},\frac{1}{\sigma_i}),(1,\frac{1}{U_i}),\end{matrix}\right.\begin{matrix}(....) \\ (1+U_iW_i+V_i+\alpha_i'+n_i-\sigma_i,1)\end{matrix}\right]dy\right\}.$$

I. On taking $i = 1$, $\alpha_i = 1$, $\beta_i = 0$, $c_i = 1$, $b_i = 0$, $\nu_i = 0$, $\varsigma_i = 0$ and a_i by $(\alpha_i + \xi_i n_i)$ and μ_i by $-\xi_i$ in (1.4), the main Theorem reduces to a known result given by Srivastava [7] under less stringent conditions.

II. Also, letting $i = 1$, $\alpha_i = 1$, $\beta_i = 0$, $c_i = 1$, $V_i = 0$, $\nu_i = 0$, $\varsigma_i = 0 = \mu_i$ and $\xi_i = U_i = -1$ in (2.1), Theorem seems to correspond to result given by Sriastava [7] .

III. The result recently obtained by Chaurasia and Patni in [3] contained in our result given in the form of the main Theorem, for $i = 1$.

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V. B. L Chaurasia
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF RAJASTHAN
JAIPUR - 302004, INDIA

M. Agnihotri
DEPARTMENT OF MATHEMATICS
SHANKARA INSTITUTE OF TECHNOLOGY
Delhi Road
KUKAS, JAIPUR-302001, RAJASTHAN, INDIA
E-mail: agnivasu@yahoo.co.in

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