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EXISTENCE AND GLOBAL EXPONENTIAL STABILITY OF PERIODIC SOLUTION OF HIGH-ORDER COHEN-GROSSBERG NEURAL NETWORK WITH IMPULSES

Abstract. Sufficient conditions are obtained for the existence and global exponential stability of periodic solution of high-order Cohen-Grossberg neural network with impulses by using Mawhin's continuation theorem of coincidence degree and by means of a method based differential inequality.

1. Introduction

The study of the existence of periodic solutions and almost periodic solutions of the nonautonomous neutral networks has received much attention, see, for instance, Refs [1-5] and references cited therein. Most widely studied and used neural networks can be classified as either continuous or discrete. Recently, there has been a somewhat new category of neural networks which is neither purely continuous-time nor purely discrete-time ones; these are called impulsive neural networks. This third category of neural networks display a combination of characteristics of both the continuous-time and discrete-time systems [6-9]. In this paper, we will study the existence and exponential stability of periodic solution of high-order Cohen-Grossberg neural network with variable delays and impulses

$$(1.1)\quad \begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(x_i(t))\Big[b_i(t)x_i(t) + \displaystyle\sum_{j=1}^{n} c_{ij}(t)f_j(x_j(t-\tau_j(t))) \\ \qquad\qquad + \displaystyle\sum_{j=1}^{n}\sum_{s=1}^{n} d_{ijs}(t)f_j(x_j(t-\tau_j(t)))f_s(x_s(t-\tau_s(t))) + I_i(t)\Big] \\ \triangle x_i(t_k) = J_i(x_i(t_k)) = -\gamma_{ik}x_i(t_k), \quad i = 1,2,\cdots,n, \quad k = 1,2,\cdots, \end{cases}$$

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where $\triangle x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ are the impulses at moments t_k and $0 < t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{t\to\infty} t_k = +\infty$; $x_i(t)$ is the state of neuron $i = 1, 2, \cdots, n$, and n is the number of neurons; $C(t) = (c_{ij}(t))_{n\times n}$ and $D(t) = (d_{i1j}(t) + d_{i2j}(t) + \cdots + d_{inj}(t))_{n\times n}$ are connection matrix functions, $I(t) = (I_1(t), I_2(t), \cdots, I_n(t))^T : R^+ \longmapsto R^n$ is continuous periodic functions with period $\omega > 0, f(x) = (f_1(x_1), f_2(x_2), \cdots, f_n(x_n))$ is the activation function of the neurons. The delays $0 \le \tau_i(t) \le \tau$ $(i = 1, 2, \cdots, n)$ are bounded function.

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \longmapsto x_i(t)$ we assume that $x_i(t_k) \equiv x_i(t_k^-)$. It is clear that the derivatives $x_i'(t_k)$, $i = 1, 2, \cdots, n$, $k = 1, 2, \cdots$ do not exist. On the other hand, according to the first equality of (1.1) there exists the limits $x_i(t_k^{\mp})$. According to the above convention, we assume $x_i'(t_k) \equiv x_i'(t_k^-)$, $i = 1, 2, \cdots, n$, $k = 1, 2, \cdots$.

The initial conditions of system (1.1) are of the form

$$x_i(s) = \phi_i(s) \neq 0, \quad s \in [-\tau, 0], \quad i = 1, 2, \cdots, n,$$

where $\phi_i \in C([-\tau, 0], R)$, $i = 1, 2, \cdots, n$.

Throughout this paper, we assume that

(H_1) The delays $0 \le \tau_i(t) \le \tau$ $(i = 1, 2, \cdots, n)$ are bounded continuous $\omega-$periodic functions.

(H_2) $a_i(u)$, $i = 1, 2, \cdots, n$ are positive and bounded continuous $\omega-$periodic functions and $0 < \underline{a_i} \le a_i(u) \le \overline{a_i}$, $\forall u \in R$, $i = 1, 2, \cdots, n$.

(H_3) $f_i \in C(R, R)$, $j = 1, 2, \cdots, n$ are Lipschitzian with Lipschitz constants $L_j > 0$,

$$|f_j(x) - f_j(y)| \le L_j|x - y| \quad \text{for} \quad j = 1, 2, \cdots, n, \quad x, y \in R.$$

(H_4) There exists positive constants $M_j > 0$ such that $|f_j(x)| \le M_j$ for $j = 1, 2, \cdots, n$, $x \in R$.

(H_5) $b_i(t), c_{ij}(t)$ and $d_{ijs}(t)$, $i, j, s = 1, 2, \cdots, n$ are bounded continuous $\omega-$periodic functions.

(H_6) There exists a positive integer q such that $t_{k+q} = t_k + \omega, \gamma_{i(k+q)} = \gamma_{ik}$, for $k = 1, 2, 3, \cdots$, $i = 1, 2, \cdots, n$.

(H_7) $\prod_{0\le t_k<t} (1 - \gamma_{ik})$, $i = 1, 2, \cdots, n$ are $\omega-$periodic functions.

For convenience, for $i, j, s = 1, 2, \cdots, n$, we introduce the following notations:

$$\underline{b}_i = \inf\{|b_i(t)|,\ t \in [0, \omega]\}, \quad \overline{b}_i = \sup\{|b_i(t)|,\ t \in [0, \omega]\},$$
$$\overline{c}_{ij} = \sup\{|c_{ij}(t)|,\ t \in [0, \omega]\}, \quad \overline{d}_{ijs} = \sup\{|d_{ijs}(t)|,\ t \in [0, \omega]\},$$

$$\overline{I}_i = \sup\{|I_i(t)|,\ t \in [0,\omega]\}, \quad N_i = \left(\int_0^\omega \prod_{0\le t_k<t} (1-\gamma_{ik})^{-2}\right)^{\frac{1}{2}}.$$

The organization of this paper is as follows. In Section 2, we introduce some notations and definitions, and state some preliminary results needed in later sections. In Section 3, we study the existence of periodic solutions of system (1.1) by using the continuation theorem of coincidence degree proposed by Gains and Mawhin [15]. In Section 4, we shall derive sufficient conditions to ensure that the periodic solution of (1.1) is globally exponentially stable. In Section 5, an illustrate example is given to demonstrate the effectiveness of the obtained results.

2. Preliminaries

In this section, we shall introduce some notations and definitions, and state some preliminary results. Consider the impulsive system

$$(2.1)\quad \begin{cases} x'(t) = f(t, x(t), x(t-\tau_1(t)), \dots, x(t-\tau_n(t))),\ t \neq t_k,\ k = 1, 2, \cdots \\ \triangle x(t)|_{t=t_k} = J_k(x(t_k^-)) \end{cases}$$

where $x \in R^n$, $f : R\times R^n \longrightarrow R^n$ is continuous; $f(t+\omega, x(t-\tau_1(t)), \dots, x(t-\tau_n(t))) = f(t, x(t-\tau_1(t)), \dots, x(t-\tau_n(t)))$ and $J_k : R \longrightarrow R$, $k = 1, 2, \dots$ are continuous; $\tau_i \in C(R, [0, \tau])$, $i = 1, 2, \dots, n$ is $\omega-$ periodic functions and $t - \tau_i(t) \longrightarrow \infty$ as $t \longrightarrow \infty$, $i = 1, 2, \dots, n$, and there exists a positive integer q such that $t_{k+q} = t_k + \omega$, $J_{k+q}(x) = J_k(x)$ with $t_k \in R$, $t_{k+1} > t_k$, $\lim\limits_{k\longrightarrow\infty} t_k = \infty$, $\triangle(t)|_{t=t_k} = x_{t_k^+} - x_{t_k^-}$. For $t_k \neq 0$ $(k = 1, 2, \dots)$, $[0, \omega] \cap \{t_k\} = \{t_1, t_2, \dots, t_q\}$. As we know, $\{t_k, k = 1, 2, \cdots\}$ are called points of jump.

Definition 2.1. A function $x \in ([0, \infty), R)$ is said to be a solution of system (2.1) on $[0, \infty)$ satisfying the initial value condition

$$x(s) = \phi(s) \neq 0, \quad s \in [-\tau, 0],$$

where $\phi \in C([-\tau, 0], R^n)$, if the following conditions are satisfied

(i) $x(t)$ is absolutely continuous on each interval $(t_k, t_{k+1}) \subset [0, \infty)$;
(ii) for any $t_k \in [0, \infty)$, $k = 1, 2, \dots, x(t_k^-)$ and $x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$;
(iii) $x(t)$ satisfied (2.1) for almost everywhere in $[0, \infty)$ and at impulsive points t_k situated in $[0, \infty)$ may have discontinuity of the first kind.

Definition 2.2. The periodic solution x^* of system (1.1) is said to be globally exponentially stable (GES), if there exists constants $\alpha > 0$ and $\beta > 0$ such that

$$|x_i(t) - x_i^*| \le \beta \parallel \phi - x^* \parallel e^{-\alpha t}$$

for all $t \geq 0$, where

$$\| \phi - x^* \| = \sup_{s \in [-\tau, 0]} \left(\sum_{i=1}^{n} |\phi_i(s) - x_i^*| \right).$$

Consider the nonimpulsive delay differential system

$$(2.2) \quad \frac{dy_i(t)}{dt} = - \prod_{0 \leq t_k < t} (1 - \gamma_{ik})^{-1} a_i \left(\prod_{0 \leq t_k < t} (1 - \gamma_{ik}) y_i(t) \right)$$

$$\cdot \left\{ b_i(t) \prod_{0 \leq t_k < t} (1 - \gamma_{ik}) y_i(t) + \sum_{j=1}^{n} c_{ij}(t) f_j \left(\prod_{0 \leq t_k < t - \tau_j(t)} (1 - \gamma_{jk}) y_j(t - \tau_j(t)) \right) \right.$$

$$+ \sum_{j=1}^{n} \sum_{s=1}^{n} d_{ijs}(t) f_j \left(\prod_{0 \leq t_k < t - \tau_j(t)} (1 - \gamma_{jk}) y_j(t - \tau_j(t)) \right)$$

$$\left. \cdot f_s \left(\prod_{0 \leq t_k < t - \tau_s(t)} (1 - \gamma_{sk}) y_s(t - \tau_s(t)) \right) + I_i(t) \right\}, \quad t \geq 0, \quad i = 1, 2, \ldots, n.$$

with initial conditions $y_i(s) = \phi_i(s) \neq 0, \quad s \in [-\tau, 0], \quad i = 1, 2, \ldots, n.$

LEMMA 2.1. [6] *Assume* (H_7) *holds, then*

(i) *if* $y = (y_1, \ldots, y_n)$ *is a solution of* (2.2), *then*

$$x = \left(\prod_{0 \leq t_k < t} (1 - \gamma_{1k}) y_1, \ldots, \prod_{0 \leq t_k < t} (1 - \gamma_{nk}) y_n \right)$$

is a solution of (1.1);

(ii) *if* $x = (x_1, \ldots, x_n)$ *is a solution of* (1.1), *then*

$$y = \left(\prod_{0 \leq t_k < t} (1 - \gamma_{1k}) x_1, \ldots, \prod_{0 \leq t_k < t} (1 - \gamma_{nk}) x_n \right)$$

is a solution of (2.2).

Let $\mathbb{X}, \mathbb{Y}$ be real Banach space, $L : \mathtt{Dom\,L} \subset \mathbb{X} \longrightarrow \dim \mathbb{Y}$ be a linear mapping, and $N : \mathbb{X} \longrightarrow \mathbb{Y}$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim $\mathtt{Ker}\ L = \mathtt{codim}\ \mathtt{Im}\ L < +\infty$ and $\mathtt{Im}\ L$ is closed in $\mathbb{Y}$. If L is a Fredholm mapping of index zero and there exists continuous projectors $P : \mathbb{X} \longrightarrow \mathbb{X}$ and $Q : \mathbb{Y} \longrightarrow \mathbb{Y}$ such that $\mathtt{Im}\ P = \mathtt{Ker}\ L$, $\mathtt{Ker}\ Q = \mathtt{Im}\ (I - Q)$, it follows that mapping $L|_{\mathtt{Dom}L \cap \mathtt{Ker}P : (I-P)} \mathbb{X} \longrightarrow \mathtt{Im}\ L$ is invertible. We denote the inverse of the mapping by K_p. If Ω is an open bounded subset of $\mathbb{X}$, then the mapping N will be called $L-$compact on $\overline{\Omega}$. If $QN(\overline{\Omega})$ is bounded, then $K_p(I - Q)N$:

$\overline{\Omega} \longrightarrow \mathbb{X}$ is compact. Since $\mathtt{Im}\ Q$ is isomorphic to $\mathtt{Ker}\ L$, there exists an isomorphism $J : \mathtt{Im\ Q} \longrightarrow \mathtt{Ker}\ L$.

Now, we introduce Mawhin's continuation theorem [11] as follows.

LEMMA 2.2. [10] *Let $\Omega \subset \mathbb{X}$ be an open bounded set and let $N : \mathbb{X} \longrightarrow \mathbb{Y}$ be a continuous operator which is $L-$compact on $\overline{\Omega}$. Assume*

(a) *for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \mathit{Dom}\ L$, $Lx \neq \lambda Nx$,*
(b) *for each $x \in \partial\Omega \cap \mathit{Ker}\ L$, $QNx \neq 0$, and* $\deg(JQN, \Omega \cap \mathit{Ker}\ L, 0) \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathit{Dom}\ L$.

DEFINITION 2.3. Let the $n \times n$ matrix $A = (a_{ij})_{n\times n}$ have nonpositive off-diagonal elements and all principal minors of A are positive, then A is said to be an $M-$matrix.

LEMMA 2.3. [11] *Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be a solution of the differential inequality*

$$x'(t) \leq Ax(t) + B\overline{x}(t), \quad t \geq t_0,$$

where

$$\overline{x}(t) = \left(\sup_{t-\tau\leq s\leq t} \{x_1(s)\}, \sup_{t-\tau\leq s\leq t} \{x_2(s)\}, \ldots, \sup_{t-\tau\leq s\leq t} \{x_n(s)\} \right)^T,$$
$$A = (a_{ij})_{n\times n}, \quad B = (b_{ij})_{n\times n}.$$

If

(A_1) *$a_{ij} \geq 0$ $(i \neq j)$, $b_{ij} \geq 0$, $i, j = 1, 2, \ldots, n$; $\sum_{j=1}^{n} \overline{x}_j(t_0) > 0$;*
(A_2) *The matrix $-(A+B)$ is an $M-$matrix.*

Then there always exists constants $\lambda > 0$, $r_i > 0$ $(i = 1, 2, \ldots, n)$ such that

$$x_i(t) \leq r_i \sum_{j=1}^{n} \overline{x}_j(t_0) e^{\lambda(t-t_0)}.$$

3. Existence of periodic solutions

In this section, based on the Mawhin's continuation theorem, we shall study the existence of periodic solution of (1.1). For convenience, we introduce the following notations:

$$G_i(t) = -\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})y_i(t)\Big)\Big\{b_i(t)\prod_{0\le t_k<t}(1-\gamma_{ik})y_i(t)$$
$$+\sum_{j=1}^{n}c_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})y_j(t-\tau_j(t))\Big)$$
$$+\sum_{j=1}^{n}\sum_{s=1}^{n}d_{ijs}(t)f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})y_j(t-\tau_j(t))\Big)$$
$$\cdot f_s\Big(\prod_{0\le t_k<t-\tau_s(t)}(1-\gamma_{sk})y_s(t-\tau_s(t))\Big)+I_i(t)\Big\},\quad t\ge 0,$$

where $y=(y_1,y_2,\dots,y_n)^T$ is $\omega-$periodic function, $i=1,2,\dots,n$. Our main result of this section is as follows.

Theorem 3.1. *Assume that* $(H_1)-(H_7)$ *hold, then the system* (1.1) *has at least one* $\omega-$*periodic solution.*

Proof. According to the discussion in Section 2, we need only to prove that non-impulsive delay differential system (2.2) has an $\omega-$periodic solution. In order to use the continuous theorem of coincidence degree theory to establish the existence of solution of (2.2), we take

$$\mathbb{X}=\mathbb{Z}=\{x(t)\in C(R,R^n): x(t+\omega)=x(t),\ t\in R,\ x=(x_1,x_2,\dots,x_n)^T\}$$

with the norm

$$\| x \| = \sum_{k=1}^{n}|x_k|_0,\quad |x_k|_0=\sup_{t\in[0,\omega]}|x_k(t)|,\quad k=1,2,\dots,n,$$

then $\mathbb{X}$ and $\mathbb{Z}$ are Banach spaces.

Set

$$Lx=x' \quad\text{and}\quad Px=\frac{1}{\omega}\int_0^{\omega}x(t)dt,\ x\in\mathbb{X};\quad Qx=\frac{1}{\omega}\int_0^{\omega}z(t)dt,\ z\in\mathbb{Z}$$

and

$$N_y=(G_1(t),G_2(t),\dots,G_n(t))^T,\quad y\in\mathbb{X}.$$

Obviously $\mathtt{Ker}\,L=\{y|y\in\mathbb{X},\ y=h,\ h\in R^n\}$, $\mathtt{Im}\,L=\{x|x\in\mathbb{X},\ \int_0^{\omega}x(s)ds=0\}$ and

$$\dim\ \mathtt{Ker}\,L=n=\mathtt{comdim}\ \mathtt{Im}\,L.$$

So $\mathtt{Im}\,L$ is closed in $\mathbb{Z}$, L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projectors satisfying

$$\mathtt{Im}\,P=\mathtt{Ker}\,L,\quad \mathtt{Im}\,L=\mathtt{Ker}\,Q=\mathtt{Im}\,(I-Q).$$

Furthermore, through an easy computation, we can find that the inverse $K_p : \mathrm{Im}\, L \longrightarrow \mathrm{Ker}\, P \cap \mathrm{dom}\, L$ of L_p has the form

$$K_p(z) = \int_0^t z(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t z(s)dsdt.$$

Thus

$$QN_y = \left(\frac{1}{\omega}\int_0^\omega G_1(t)dt, \cdots, \int_0^\omega G_n(t)dt\right)^T, \quad y \in \mathbb{X}$$

and

$$K_p(I-Q)N_y$$
$$= \begin{pmatrix} \int_0^t G_1(s)ds \\ \vdots \\ \int_0^t G_j(s)ds \\ \vdots \\ \int_0^t G_n(s)ds \end{pmatrix} - \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \int_0^t G_1(s)dsdt \\ \vdots \\ \frac{1}{\omega}\int_0^\omega \int_0^t G_j(s)dsdt \\ \vdots \\ \frac{1}{\omega}\int_0^\omega \int_0^t G_n(s)dsdt \end{pmatrix} - \begin{pmatrix} (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega G_1(s)ds \\ \vdots \\ (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega G_j(s)ds \\ \vdots \\ (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega G_n(s)ds \end{pmatrix}.$$

Clearly, QN and $K_p(I-Q)N$ are continuous. Using the Arzela-Ascoli theorem, it is not difficult to show that $QN(\overline{\Omega})$, $K_p(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset \mathbb{X}$.

Now we reach the position to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $L_x = \lambda Nx, \lambda \in (0,1)$, we have

$$\begin{aligned} (3.1)\quad x_i'(t) = \lambda\Bigg\{ &- \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1} a_i\Bigg(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Bigg) \\ &\cdot\Bigg[b_i(t)\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t) \\ &+\sum_{j=1}^n c_{ij}(t)f_j\Bigg(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Bigg) \\ &+\sum_{j=1}^n\sum_{s=1}^n d_{ijs}(t)f_j\Bigg(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Bigg) \\ &\cdot f_s\Bigg(\prod_{0\le t_k<t-\tau_s(t)}(1-\gamma_{sk})x_s(t-\tau_s(t))\Bigg) + I_i(t)\Bigg]\Bigg\}, \quad x\in X, \\ &\qquad\qquad i = 1,2,\dots,n. \end{aligned}$$

Suppose that $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{X}$ is a solution of system (3.1) for some $\lambda \in (0,1)$. Integrating $x_i(t)x_i'(t)$ over the interval $[0,\omega]$, we obtain

$$
\begin{aligned}
0 &= \frac{1}{2}x_i^2(t)|_0^\omega = \int_0^\omega x_i(t)x_i'(t)dt \\
&= \lambda\int_0^\omega\Big\{-\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Big) \\
&\quad\cdot\Big[b_i(t)\prod_{0\le t_k<t}(1-\gamma_{ik})x_i^2(t) \\
&\quad+\sum_{j=1}^n c_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Big)x_i(t) \\
&\quad+\sum_{j=1}^n\sum_{s=1}^n d_{ijs}(t)f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Big) \\
&\quad\cdot f_s\Big(\prod_{0\le t_k<t-\tau_s(t)}(1-\gamma_{sk})x_s(t-\tau_s(t))\Big)x_i(t)+I_i(t)x_i(t)\Big]\Big\}dt, \\
&\quad i=1,2,\ldots,n.
\end{aligned}
$$

That is

$$
\begin{aligned}
&\int_0^\omega a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Big)b_i(t)x_i^2(t)dt \\
&= \textstyle\int_0^\omega\Big[-\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Big) \\
&\cdot\sum_{j=1}^n c_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Big)x_i(t)\Big]dt \\
&+\int_0^\omega\Big[-\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Big)\sum_{j=1}^n\sum_{s=1}^n d_{ijs}(t) \\
&\cdot f_j\Big(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\Big) \\
&\cdot f_s\Big(\prod_{0\le t_k<t-\tau_s(t)}(1-\gamma_{sk})x_s(t-\tau_s(t))\Big)x_i(t)\Big]dt \\
&+\int_0^\omega\Big[-\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}a_i\Big(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\Big)I_i(t)x_i(t)dt\Big], \\
&i=1,2,\ldots,n.
\end{aligned}
$$

From conditions (H_2), (H_4) and (H_5), it follows that

$$\underline{a}_i\underline{b}_i\int_0^\omega |x_i(t)|^2dt \le \int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}\left|a_i\left(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\right)\right|$$

$$\cdot\sum_{j=1}^n |c_{ij}(t)|\left|f_j\left(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\right)\right||x_i(t)|dt$$

$$+\int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}\left|a_i\left(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\right)\right|\sum_{j=1}^n\sum_{s=1}^n |d_{ijs}(t)|$$

$$\cdot\left|f_j\left(\prod_{0\le t_k<t-\tau_j(t)}(1-\gamma_{jk})x_j(t-\tau_j(t))\right)\right.$$

$$\left.\cdot f_s\left(\prod_{0\le t_k<t-\tau_s(t)}(1-\gamma_{sk})x_s(t-\tau_s(t))\right)\right||x_i(t)|dt$$

$$+\int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}\left|a_i\left(\prod_{0\le t_k<t}(1-\gamma_{ik})x_i(t)\right)\right||I_i(t)||x_i(t)|dt$$

$$\le \int_0^\omega \overline{a}_i\sum_{j=1}^n \overline{c}_{ij}M_j|x_i(t)|\prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}dt$$

$$+\int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}\overline{a}_i\sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s|x_i(t)|dt$$

$$+\int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-1}\overline{a}_i\overline{I}_i|x_i(t)|dt$$

$$\le \overline{a}_i\left(\sum_{j=1}^n \overline{c}_{ij}M_j+\sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s+\overline{I}_i\right)\left(\int_0^\omega \prod_{0\le t_k<t}(1-\gamma_{ik})^{-2}dt\right)^{\frac{1}{2}}$$

$$\cdot\left(\int_0^\omega |x_i(t)|^2dt\right)^{\frac{1}{2}}$$

$$= N_i\overline{a}_i\left(\sum_{j=1}^n \overline{c}_{ij}M_j+\sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s+\overline{I}_i\right)\left(\int_0^\omega |x_i(t)|^2dt\right)^{\frac{1}{2}}, i=1,2,\ldots,n.$$

Hence,

$$\left(\int_0^\omega |x_i(t)|^2dt\right)^{\frac{1}{2}} \le \frac{\overline{a}_iN_i}{\underline{a}_i\underline{b}_i}\left(\sum_{j=1}^n \overline{c}_{ij}M_j+\sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s+\overline{I}_i\right) := S_i, \quad i=1,2,\ldots,n. \tag{3.2}$$

Let $\underline{t}_i \in [0,\omega] \ne t_k$, $k=1,2,\ldots,m$, such that $|x_i(\underline{t}_i)| = \inf_{t\in[0,\omega]}|x_i(t)|$,

$i = 1, 2, \ldots, n$. Then, by (3.2), we have

$$\big|x_i(\underline{t}_i)\big|\sqrt{\omega} = \big|x_i(\underline{t}_i)\big|\Big(\int_0^\omega dt\Big)^{\frac{1}{2}} \leq \Big(\int_0^\omega |x_i(t)|^2 dt\Big)^{\frac{1}{2}} \leq S_i.$$

Thus,

$$|x_i(\underline{t}_i)| \leq \frac{S_i}{\sqrt{\omega}}. \tag{3.3}$$

From (3.3), and since $x_i(t) = x_i(\underline{t}_i) + \int_{\underline{t}_i}^t x_i'(s)ds$, it follows that

$$|x_i(t)| \leq |x_i(\underline{t}_i)| + \int_0^\omega |x_i'(t)|dt = \frac{S_i}{\sqrt{\omega}} + \int_0^\omega |x_i'(t)|dt. \tag{3.4}$$

On the other hand, from

$$\begin{aligned}
&\int_0^\omega |x_i'(t)|dt < \overline{a}_i\overline{b}_i \int_0^\omega |x_i(t)|dt \\
&\quad +\overline{a}_i\Big(\sum_{j=1}^n \overline{c}_{ij}M_j + \sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s + \overline{I}_i\Big)\int_0^\omega \prod_{0\leq t_k<t}(1-\gamma_{ik})^{-1}dt \\
&\leq \overline{a}_i\overline{b}_i\sqrt{\omega}\Big(\int_0^\omega |x_i(t)|^2dt\Big)^{\frac{1}{2}} \\
&\quad +\overline{a}_i\Big(\sum_{j=1}^n \overline{c}_{ij}M_j + \sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s + \overline{I}_i\Big)\sqrt{\omega}\Big(\int_0^\omega \prod_{0\leq t_k<t}(1-\gamma_{ik})^{-2}dt\Big)^{\frac{1}{2}} \\
&= \overline{a}_i\overline{b}_i\sqrt{\omega}\Big(\int_0^\omega |x_i(t)|^2dt\Big)^{\frac{1}{2}} + \overline{a}_iN_i\sqrt{\omega}\Big(\sum_{j=1}^n \overline{c}_{ij}M_j + \sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s + \overline{I}_i\Big).
\end{aligned}$$

Together with (3.2), we get

$$\begin{aligned}
\int_0^\omega |x_i'(t)|dt &< \overline{a}_i\overline{b}_i\sqrt{\omega}S_i \\
&\quad +\overline{a}_iN_i\sqrt{\omega}\Big(\sum_{j=1}^n \overline{c}_{ij}M_j + \sum_{j=1}^n\sum_{s=1}^n \overline{d}_{ijs}M_jM_s + \overline{I}_i\Big) := D_i.
\end{aligned} \tag{3.5}$$

In view of (3.3), (3.4) and (3.5), we obtain

$$|x_i(t)| \leq \frac{S_i}{\sqrt{\omega}} + D_i := R_i, \quad i = 1, 2, \ldots, n.$$

Denote $A = \sum_{i=1}^m R_i + K$, where K is a sufficiently large positive constant, clearly, A is independent of λ. Now, take $\Omega = \{x \in \mathbb{X} : \| x(t) \| < A\}$. It is obvious that Ω satisfies the requirement (a) in Lemma 2.2.

When $x \in \partial\Omega \cap \mathtt{Ker}\, L$, $x = (x_1, x_2, \ldots, x_n)^T$ is a constant vector in R^n with $\| x \| = A$. There

$$QNx = \left(\frac{1}{\omega}\int_0^{\omega} G_1 dt, \ldots, \frac{1}{\omega}\int_0^{\omega} G_n dt\right)^T, \quad x \in \mathbb{X}$$

where

$$\begin{aligned} G_i = &- \prod_{0 \le t_k < t} (1-\gamma_{ik})^{-1} a_i\Big(\prod_{0 \le t_k < t} (1-\gamma_{ik}) y_i\Big)\Big[b_i(t) \prod_{0 \le t_k < t} (1-\gamma_{ik}) y_i \\ &+ \sum_{j=1}^{n} c_{ij}(t) f_j\Big(\prod_{0 \le t_k < t-\tau_j} (1-\gamma_{jk}) y_j\Big) \\ &+ \sum_{j=1}^{n}\sum_{s=1}^{n} d_{ijs}(t) f_j\Big(\prod_{0 \le t_k < t-\tau_j} (1-\gamma_{jk}) y_j\Big) f_s\Big(\prod_{0 \le t_k < t-\tau_s} (1-\gamma_{sk}) y_s\Big) + I_i(t)\Big], \end{aligned}$$

$i = 1, 2, \ldots, n.$

Take $J : \mathtt{Im}\, Q \longrightarrow \mathtt{Ker}\, L$, $R \longrightarrow R$. Then, if necessary, we can let K be greater such that $x^T JQNx < 0$. So, for any $x \in \partial\Omega \cap \mathtt{Ker}\, L$, $QNx \neq 0$. Furthermore, let $\Phi(\gamma; x) = -\gamma x + (1-\gamma) JQNx$, then for any $x \in \partial\Omega \cap \mathtt{Ker}\, L$, $x^T \Phi(\gamma; x) < 0$, we get

$$\deg\{JQN, \Omega \cap \mathtt{Ker}\, L, 0\} = \deg\{-x, \Omega \cap \mathtt{Ker}\, L, 0\} \neq 0.$$

So, condition (b) of Lemma 2.2 is also satisfied. We now know that ω satisfies all the requirements in Lemma 2.2. Therefore, (2.2) has at least one $\omega-$periodic solution. As a sequence system (1.1) has at least one $\omega-$periodic solution. The proof is complete. □

4. Global exponential stability of the periodic solution

Suppose that $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is a periodic solution of system (1.1). In this section, we will use a technique of differential inequality to study the exponential stability of this periodic solution.

THEOREM 4.1. *Assume $(H_1) - (H_7)$ hold. Moveover, suppose that matrix $A - \alpha\beta\overline{A}(C + m(D + \overline{D}))L$ is an $M-$matrix, where $A = diag(\underline{a}_1\underline{b}_1, \underline{a}_2\underline{b}_2, \ldots, \underline{a}_n\underline{b}_n)$, $\overline{A} = diag(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$, $C = (\overline{c}_{ij})_{n\times n}$, $D = diag(\overline{d}_{i1j} + \overline{d}_{i2j} + \cdots + \overline{d}_{inj})_{n\times n}$, $\overline{D} = diag(\overline{d}_{ij1} + \overline{d}_{ij2} + \cdots + \overline{d}_{ijn})_{n\times n}$, $L = diag(L_1, L_2, \ldots, L_n)$, $\alpha = \max_{1\le i\le n}\{\sup_{t\in[0,\omega]} \prod_{0\le t_k<t} (1-\gamma_{ik})\}$, $\beta = \max_{1\le i\le n}\{\sup_{t\in[0,\omega]} \prod_{0\le t_k<t} (1-\gamma_{ik})^{-1}\}$, $m = \max\{M_j, M_s\}$. Then the $\omega-$periodic solution of system (1.1) is globally exponentially stable.*

Proof. According to Theorem 3.1, we know that (1.1) has an ω−periodic solution $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$. Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is an arbitrary solution of (1.1).

Let $y(t) = x(t) - x^*$, then (1.1) can be written as

$$(4.1)\quad \begin{cases} \dfrac{dy_i(t)}{dt} = -\alpha_i(y_i(t))\Big[b_i(t)y_i(t) + \displaystyle\sum_{j=1}^{n} c_{ij}(t)g_j(y_j(t-\tau_j(t))) \\ \quad + \displaystyle\sum_{j=1}^{n}\sum_{s=1}^{n} d_{ijs}(t) \\ \quad \cdot\Big[\beta_j(y_j(t-\tau_j(t)))g_s(y_s(t-\tau_s(t))) + f_s(x_s^*)g_j(y_j(t-\tau_j(t)))\Big]\Big] \\ \triangle y_i(t_k) = -\gamma_{ik}y_i(t_k), \quad t \geq 0, \quad i = 1, 2, \ldots, n, k = 1, 2, \ldots, \end{cases}$$

where

$\alpha_i(y_i(t)) = a_i(y_i(t) + x_i^*), \quad g_j(y_j(t-\tau_j(t))) = f_j(y_j(t-\tau_j(t)) + x_j^*) - f_j(x_j^*)$

$f_j(y_j(t-\tau_j(t)) + x_j^*) = \beta_j(y_j(t-\tau_j(t))), \quad j = 1, 2, \ldots, n.$

Due to the assumption of Lemma 2.1, we consider the following nonimpulsive delay differential system

$$(4.2)\quad \begin{aligned} \frac{du_i(t)}{dt} = &-\prod_{0\leq t_k<t}(1-\gamma_{ik})^{-1}\alpha_i\Big(\prod_{0\leq t_k<t}(1-\gamma_{ik})u_i(t)\Big) \\ &\cdot\Big\{b_i(t)\prod_{0\leq t_k<t}(1-\gamma_{ik})u_i(t) \\ &+\sum_{j=1}^{n} c_{ij}(t)g_j\Big(\prod_{0\leq t_k<t-\tau_j(t)}(1-\gamma_{jk})u_j(t-\tau_j(t))\Big) \\ &+\sum_{j=1}^{n}\sum_{s=1}^{n} d_{ijs}(t)\Big[\beta_j\Big(\prod_{0\leq t_k<t-\tau_j(t)}(1-\gamma_{jk})u_j(t-\tau_j(t))\Big) \\ &\cdot g_s\Big(\prod_{0\leq t_k<t-\tau_s(t)}(1-\gamma_{sk})u_s(t-\tau_s(t))\Big) \\ &+f_s(x_s^*)g_j(\prod_{0\leq t_k<t-\tau_j(t)}(1-\gamma_{jk})u_j(t-\tau_j(t))\Big]\Big\}, \quad i = 1, 2, \ldots, n. \end{aligned}$$

with initial condition $u(s) = \psi(s) = \phi(s) - x^*, s \in [-\tau, 0]$.

Let $z_i(t) = |u_i(t)|$, then the upper right derivative $D^+z_i(t)$ along the solutions of system (4.2) is as follows:

$$(4.3)\quad D^+z_i(t) = D^+|u_i(t)| = u_i'(t)sgn(u_i(t))$$
$$\leq -\underline{a}_i\underline{b}_i|u_i(t)| + \prod_{0\leq t_k<t}(1-\gamma_{ik})^{-1}\overline{a}_i\Big\{\sum_{j=1}^{n}\overline{c}_{ij}\prod_{0\leq t_k<t}(1-\gamma_{ik})L_j|u_j(t-\tau_j(t))|$$
$$+\sum_{j=1}^{n}\sum_{s=1}^{n}\overline{d}_{ijs}\Big[M_j\prod_{0\leq t_k<t-\tau_s(t)}(1-\gamma_{sk})L_s|u_s(t-\tau_s(t))|$$
$$+M_s\prod_{0\leq t_k<t-\tau_j(t)}(1-\gamma_{jk})L_j|u_j(t-\tau_j(t))|\Big]\Big\}$$
$$= -\underline{a}_i\underline{b}_i z_i(t) + \alpha\beta\overline{a}_i\sum_{j=1}^{n}\overline{c}_{ij}L_j|\overline{u}_j(t)| + \alpha\beta\overline{a}_i m\sum_{j=1}^{n}\sum_{s=1}^{n}\overline{d}_{ijs}(L_s|\overline{u}_s(t)| + L_j|\overline{u}_j(t)|)$$
$$= -\underline{a}_i\underline{b}_i z_i(t) + \alpha\beta\overline{a}_i\sum_{j=1}^{n}\overline{c}_{ij}L_j\overline{z}_j(t) + \alpha\beta\overline{a}_i m\sum_{j=1}^{n}\sum_{s=1}^{n}\overline{d}_{ijs}(L_s\overline{z}_s(t) + L_j\overline{z}_j(t)).$$

That is

$$D^+z(t) \leq -Az(t) + \alpha\beta\overline{A}CL\overline{z}(t) + \alpha\beta m\overline{A}(DL + \overline{D}L)\overline{z}(t)$$
$$= -Az(t) + \alpha\beta\overline{A}\big(C + m(D + \overline{D})\big)L\overline{z}(t),$$

where

$$A = diag(\underline{a}_1\underline{b}_1, \underline{a}_2\underline{b}_2, \ldots, \underline{a}_n\underline{b}_n),$$
$$\overline{A} = diag(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n),$$
$$C = (c_{ij})_{n\times n},$$
$$D = (\overline{d}_{i1j} + \overline{d}_{i2j} + \cdots + \overline{d}_{inj})_{n\times n},$$
$$\overline{D} = diag(\overline{d}_{ij1} + \overline{d}_{ij2} + \cdots + \overline{d}_{ijn})_{n\times n}.$$

By initial conditions $x_i(s) = \phi_i(s) \neq 0$, $s \in [-\tau, 0]$, $i = 1, 2, \ldots, n$, we know that $\overline{z}_i(0) > 0$, according to Lemma 2.3, if the matrix $A - \alpha\beta\overline{A}(C + m(D + \overline{D}))L$ is an $M-$matrix, then there must exist constants $\mu > 0, r_i > 0$ $(i = 1, 2, \ldots, n)$ such that

$$z_i(t) = |u_i(t)| \leq r_i\sum_{j=1}^{n}\overline{z}_i(0)e^{-\mu t} = r_i\sum_{j=1}^{n}|\overline{u}_i(0)|e^{-\mu t}, \quad i = 1, 2, \ldots, n.$$

By initial conditions, we have $\overline{u}(0) = \overline{\psi}(0) = \overline{\phi}(0) - x^*$, then the solution of (4.1) satisfies

$$|y_i(t)| = \prod_{0\le t_k<t}(1-\gamma_{jk})|u_i(t)| \le \prod_{0\le t_k<t}(1-\gamma_{jk})r_i\sum_{j=1}^{n}|\overline{u}_i(0)|e^{-\mu t}$$
$$= \prod_{0\le t_k<t}(1-\gamma_{jk})r_i\sum_{j=1}^{n}|\overline{\phi}_i(0)-x^*|e^{-\mu t} \le \alpha r_i\sum_{j=1}^{n}|\overline{\phi}_i(0)-x^*|e^{-\mu t},$$
$$i=1,2,\ldots,n.$$

That is

$$|x_i(t)-x_i^*| \le \alpha r_i\sum_{j=1}^{n}|\overline{\phi}_i(0)-x_i^*|e^{-\mu t}$$
$$= \alpha r_i\left[\sup_{s\in[-\tau,0]}\left(\sum_{j=1}^{n}|\overline{\phi}_i(0)-x_i^*|\right)\right]e^{-\mu t}$$
$$= \alpha r_i \parallel \phi - x^* \parallel e^{-\mu t}, \quad i=1,2,\ldots,n.$$

From Definition 2.2, the $\omega-$periodic solution $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T$ of system (1.1) is globally exponentially stable. ∎

5. An illustrative example

In this section, we give an example to illustrate the effectiveness of our results. Consider the following Conhen-Grossberg type neural network model with delays and impulses

$$(5.1)\quad \begin{cases} \dfrac{dx_t(t)}{dt} = -a_i(x_i(t))\Big[b_i(t)x_i(t)+\sum_{j=1}^{2}c_{ij}(t)f_j(x_j(t-\tau_j(t))) \\ \qquad +\sum_{j=1}^{2}\sum_{s=1}^{2}d_{ijs}(t)f_j(x_j(t-\tau_j(t)))f_s(x_s(t-\tau_s(t)))+I_i(t)\Big] \\ \triangle x_i(t_k) = J_i(x_i(t_k)) = -\gamma_{ik}x_i(t_k),\ i=1,2, k=1,2,\ldots, \end{cases}$$

where $a_i(u)=2+\left(\dfrac{1}{3\pi}\right)\arctan u$, $b_i(t)=4(\sin t+6)$, $c_{ij}(t)=\dfrac{1}{9}\cos(t+i+j)+\dfrac{1}{12}$, $d_{ijs}(t)=d_{isj}(t)=\dfrac{1}{9}\sin(t+i+j)+\dfrac{1}{12}$, $I_i(t)=2+2\cos t$ $(i=1,2)$ are continuous $2\pi-$periodic functions. $f_i(u)=\dfrac{1}{2}(|u+1|-|u-1|)$ $(i=1,2)$, $q=2$, $[0,2\pi]\cap\{t_k\}=\{t_1,t_2\}$, $r_{11}=0.3$, $r_{12}=0.2$, $r_{21}=0.2$ and $r_{22}=0.3$, $\tau_1(t)=\tau_2(t)=2\pi$.

The system (5.1) is supplement with initial values given by

$$x_i(s)=\varphi_i(s)=\sin s, \quad s\in[-\frac{\pi}{2},0], \quad i=1,2.$$

Obviously, $f_1(0) = f_2(0) = 0$, $f_1(u)$ and $f_2(u)$ satisfy the Lipschitz condition (H_3) with constants $L_1 = L_2 = 1$, and $|f_i(u)| \le 1$ satisfies (H_4); $b_i(t)$, $c_{ij}(t)$, $d_{ijs}(t)$ satisfy the condition (H_1) with $\overline{b}_i = 28$, $\underline{b}_i = 20$, $\overline{c_{ij}} = \frac{7}{36}$ and $\overline{d}_{ijs} = \frac{7}{36}$, respectively; $a_1(u)$ and $a_2(u)$ satisfy the conditions (H_2), (H_7), (H_8) with $a_{11} = a_{12} = \frac{11}{6}, a_{21} = a_{22} = \frac{13}{6}$. It is easily verified that

$$A = \begin{pmatrix} \frac{220}{6} & 0 \\ 0 & \frac{220}{6} \end{pmatrix}, \quad \overline{A} = \begin{pmatrix} \frac{13}{6} & 0 \\ 0 & \frac{13}{6} \end{pmatrix}, \quad C = \begin{pmatrix} \frac{7}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{7}{36} \end{pmatrix},$$

$$D = \begin{pmatrix} \frac{7}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{7}{36} \end{pmatrix}, \quad \overline{D} = \begin{pmatrix} \frac{7}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{7}{36} \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

So

$$A - \alpha\beta\overline{A}\Big(C + m(D + \overline{D})\Big)L = \begin{pmatrix} \frac{2549}{72} & -\frac{91}{72} \\ -\frac{91}{72} & \frac{2549}{72} \end{pmatrix}$$

is an $M-$matrix. It then follows from Theorem 2 that system (5.1) has a unique $2\pi-$periodic solution which is globally exponentially stable.

References

[1] J. Cao, *New results concerning exponential stability and periodic solutions of delayed cellular neural networks*, Phys. Lett. A 307 (2003), 136–147.

[2] J. Cao, J. Liang, *Boundedness and stability for Cohen-Grossberg neural networks with time-varying delays*, J. Math. Anal. Appl. 296 (2004), 665–685.

[3] S. Guo, L. Huang, *Periodic oscillation for a class of neural networks with variable coefficients*, Nonlinear Anal. 6 (2005), 545–561.

[4] Z. Liu, L. Liao, *Existence and global exponential stability of periodic solution of cellular neural networks with time-varying delays*, J. Math. Anal. Appl. 290 (2004), 247–262.

[5] B. Liu, L. Huang, *Existence and exponential stability of periodic solution of cellular neural networks with time-varying delays*, J. Math. Anal. Appl. 290 (2004), 247–262.

[6] Y. K. Li, Z. Y. Xing, L. H. Lu, *Existence and global exponential stability of periodic solution of a class of neural networks with impulses*, Chaos, Solitons and Fractals 27 (2006), 437–445.

[7] Y. K. Li, Z. Y. Xing, *Existence and global exponential stability of periodic solution of CNNs with impulses*, Chaos, Solitons and Fractals 33 (2007), 1686–1693.

[8] H. Akca, R. Alassar, Z. Covacheva, E. Al-Zahrani, *Continuous-time additive Hopfield neural networks with impulses*, J. Math. Anal. Appl. 290 (2004), 436–451.

[9] Y. K. Li, L. Lu, *Global exponential stability and existence of periodic solution of Hopfield-type neural networks with impulses*, Phys. Lett. A. 333 (2004), 62–71.
[10] R. E. Gains, J. L. Mawhin, *Coincidence Degree and Nonlinear Differential Equations*, Springer Verlag, Berlin 1977.
[11] H. Tokumarn, N. Adachi, T. Amemiya, *Macroscopic stability of interconnected systems*, In: 6th IFAC Congress, paper ID44, 4 (1975), 1–7.

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