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δ-PERFECTLY CONTINUOUS FUNCTIONS

Abstract. A new class of functions called 'δ-perfectly continuous functions' is introduced and their basic properties are studied. Their place in the hierarchy of other variants of continuity that already exist in the literature is elaborated. Further, it is shown that if X is sum connected (e.g. connected or locally connected) and Y is Hausdorff, then the function space $P_{\Delta}(X,Y)$ of all δ-perfectly continuous functions from X into Y is closed in Y^X in the topology of pointwise convergence.

1. Introduction

Several weak, strong and other variants of continuity occur in the lore of mathematical literature and arise in diverse situations in mathematics and applications of mathematics. The main purpose of this paper is to introduce a new class of functions called 'δ-perfectly continuous function' and to elaborate on their basic properties and discuss their interplay and interrelations with other variants of continuity that already exist in the mathematical literature. It turns out that in general the notion of δ-perfect continuity is independent of continuity but coincides with perfect continuity [15], a significantly strong form of continuity if Y is a semiregular space. The class of δ-perfectly continuous functions properly includes the class of perfectly continuous functions defined by Noiri [15] and studied by Kohli, Singh and Arya [8] which in turn strictly contains the class of strongly continuous functions introduced by Levine [10]. Moreover, the class of δ-perfectly continuous functions is properly contained in the class of almost perfectly

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continuous (≡regular set connected) functions due to Dontchev, Reilly and Vamanamurthy [2] which in turn is strictly contained in the class of almost cl-supercontinuous functions (≡almost clopen maps) defined and studied by Ekici [3] and further investigated by Kohli and Singh [7]. The other variants of continuity with which we shall be dealing in this paper include almost z-supercontinuous functions [9], almost D_δ-supercontinuous functions [9], almost strongly θ-continuous functions defined by Noiri and Kang [16], δ-continuous functions due to Noiri [14] and almost continuous functions introduced by Singal and Singal [18].

Section 2 is devoted to preliminaries and basic definitions. In Section 3 of this paper we introduce the notion of 'δ-perfectly continuous function', wherein we also elaborate on the interrelations that exist between δ-perfect continuity and other variants of continuity that already exist in the literature. Basic properties of δ-perfectly continuous functions are studied in Section 4. The function space $P_\Delta(X,Y)$ of all δ-perfectly continuous functions from X to Y with the topology of pointwise convergence is considered in Section 5 and the sufficient conditions on X and Y are outlined for it to be closed in Y^X.

2. Preliminaries and basic definitions

DEFINITIONS 2.1. A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) strongly continuous [10] if $f(\overline{A}) \subset f(A)$ for each subset A of X.
(b) perfectly continuous ([15]) if $f^{-1}(V)$ is clopen in X for every open set $V \subset Y$.
(c) almost perfectly continuous (≡ regular set connected [2]) if $f^{-1}(V)$ is clopen for every regular open set V in Y.
(d) cl-supercontinuous [20] (≡ clopen continuous [17]) if for each $x \in X$ and each open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.
(e) almost cl-supercontinuous [7] (≡ almost clopen [3]) if for each $x \in X$ and for each regular open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.
(f) (almost) strongly θ-continuous ([16]) [14] if for each $x \in X$ and for each (regular) open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.
(g) supercontinuous [12] if for each $x \in X$ and for each open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subset V$.
(h) almost z-supercontinuous [9] if for each $x \in X$ and each regular open set V containing $f(x)$, there exists a cozero set U containing x such that $f(U) \subset V$.

(i) almost D_δ-supercontinuous [9] if for each $x \in X$ and each regular open set V containing $f(x)$, there exists a regular F_σ-set U containing x such that $f(U) \subset V$.
(j) δ-continuous [14] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subset V$.
(k) almost continuous [18] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists an open set U containing x such that $f(U) \subset V$.

Definition 2.2. A set G is said to be δ-open [24] if for each $x \in G$, there exists a regular open set H such that $x \in H \subset G$, or equivalently, G is expressible as an arbitrary union of regular open sets. The complement of a δ-open set will be referred to as a δ-closed set.

Definition 2.3. Let X be a topological space and let $A \subset X$. A point $x \in X$ is called a δ-adherent point [24] (cl-adherent point [20]) of A if every regular open set (clopen set) containing x has non-empty intersection with A. Let A_δ ($[A]_{cl}$) denote the set of all δ-adherent points (cl-adherent points) of A. The set A is δ-closed (cl-closed) if and only if $A = A_\delta$ ($[A]_{cl} = A$).

Definition 2.4. A filterbase $\mathcal{F}$ is said to δ-converge[24] (cl-converge [20]) to a point x, written as $\mathcal{F} \xrightarrow{\delta} x$ ($\mathcal{F} \xrightarrow{cl} x$) if every regular open (clopen) set containing x contains a member of $\mathcal{F}$.

Definition 2.5. A net (x_λ) in X is said to δ-converge (cl-converge) to a point x, written as $x_\lambda \xrightarrow{\delta} x$ ($x_\lambda \xrightarrow{cl} x$) if it is eventually in every regular open (clopen) set containing x.

Definitions 2.6. A space X is said to be endowed with a

(1) partition topology [23] if every open set in X is closed; and
(2) δ-partition topology if every δ-open set in X is closed or equivalently every δ-closed set in X is open.

Clearly a space endowed with a partition topology is equipped with a δ-partition topology. Conversely, a semiregular space with a δ-partition topology has a partition topology. Moreover, an infinite set with cofinite topology has a δ-partition topology which is not a partition topology.

3. δ-perfectly continuous functions

A function $f : X \to Y$ from a topological space X into a topological space Y is said to be δ-perfectly continuous if for each δ-open set V in Y, $f^{-1}(V)$ is a clopen set in X.

The following diagram well illustrates the interrelations that exist among δ-perfectly continuity and other variants of continuity that already exist in the literature and have some relevance to the contents of this paper.

strongly continuous				
↓				
perfectly continuous	→	δ-perfectly continuous	→	almost perfectly continuous (≡ regular set connected)
	↘			↓
		clopen map (≡ cl-supercontinuous)	→	almost clopen map (≡ almost cl-supercontinuous)
		↓		↓
		z-supercontinuous	→	almost z-supercontinuous
		↓		↓
		D_δ-supercontinuous	→	almost D_δ-supercontinuous
		↓		↓
		strongly θ-continuous	→	almost strongly θ-continuous
		↓		↓
		supercontinuous	→	δ-continuous
		↓		↓
		continuous	→	almost continuous

It is either well known or simple examples can be given to show that none of the above implications is reversible. Nevertheless we narrate the following observations/examples.

3.1. Let X be endowed with a non-discrete partition topology. For example, let X denote the set of positive integers with odd even topology [23, p. 43]. Then the identity function defined on X is perfectly continuous but not strongly continuous.

3.2. Let X be endowed with a δ-partition topology which is not a partition topology. For example, any infinite set with cofinite topology is such a space. Then the identity function defined on X is δ-perfectly continuous but not perfectly continuous.

3.3. A δ-perfectly continuous function need not even be continuous. For let X denote the real line with indiscrete topology and Y be the real line with cofinite topology. If f denotes the identity mapping of X onto Y, then f is δ-perfectly continuous but not continuous.

3.4. Let $A = K \cup \{a_+, a_-\}$ be the regular space (hence a semi regular space) but not a completely regular space due to Hewitt [4]. Then the identity function defined on A is continuous but not δ-perfectly continuous.

3.5. If X is endowed with a partition topology, then every almost continuous function $f : X \to Y$ is δ-perfectly continuous.

3.6. If Y is semiregular, then every δ-perfectly continuous function $f : X \to Y$ is perfectly continuous.

3.7. Let X be endowed with a δ-partition topology. If $f : X \to Y$ is δ-continuous, then f is δ-perfectly continuous. Further, if f is supercontinuous, then f is perfectly continuous.

In view of 3.3 and 3.4 it is clear that δ-perfect continuity and continuity are independent of each other.

4. Basic properties of δ-perfectly continuous functions

THEOREM 4.1. *For a function $f : X \to Y$ from a topological space X into a topological space Y, the following statements are equivalent:*

(a) *f is δ-perfectly continuous.*
(b) *$f^{-1}(F)$ is clopen in X for every δ-closed set F in Y.*
(c) *$f(\mathcal{F}) \xrightarrow{\delta} f(x)$ for every filter $\mathcal{F} \xrightarrow{cl} x$.*
(d) *$f(x_\lambda) \xrightarrow{\delta} f(x)$ for every net $x_\lambda \xrightarrow{cl} x$.*
(e) *$f(A_{cl}) \subset [f(A)]_\delta$ for every subset $A \subset X$.*

Proof. Easy. ∎

DEFINITION 4.2. Let Y be a subspace of a space Z. Then Y is said to be δ-embedded in Z if every δ-open set in Y is the restriction of a δ-open set in Z with Y; or equivalently every δ-closed set in Y is the restriction of a δ-closed set in Z with Y.

THEOREM 4.3. *Let $f : X \to Y$ be a δ-perfectly continuous function. If $f(X)$ is δ-embedded in Y, then the surjection $f : X \to f(X)$ is δ-perfectly continuous.*

Proof. Let V be a δ-open set in $f(X)$. Then there exists a δ-open set W in Y such that $V = W \cap f(X)$. It follows that $f^{-1}(V) = f^{-1}(W) \cap X = f^{-1}(W)$. ∎

THEOREM 4.4. *If $f : X \to Y$ is δ-perfectly continuous function and $g : Y \to Z$ is a δ-continuous function, then $g \circ f$is δ-perfectly continuous. In particular, the composition of two δ-perfectly continuous functions is δ-perfectly continuous.*

Proof. Let W be a δ-open set in Z. Since g is δ-continuous, $g^{-1}(W)$ is δ-open in Y (see [14]). In view of δ-perfect continuity of f, $f^{-1}(g^{-1}(W))$ is clopen in X. Since $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$, $g \circ f$ is δ-perfectly continuous. ∎

It is routine to verify that δ-perfect continuity is invariant under the restriction of domain

THEOREM 4.5. *Let $f : X \to Y$ be a function and let $Q = \{X_\alpha : \alpha \in \Lambda\}$ be a locally finite clopen cover of X. For each $\alpha \in \Lambda$, let $f_\alpha = f|X_\alpha$ denote the restriction map. Then f is δ-perfectly continuous if and only if each f_α is δ-perfectly continuous.*

Proof. Necessity is immediate since δ-perfect continuity is invariant under restriction of domain. To prove sufficiency, let V be a δ-open set in Y. Then $f^{-1}(V) = \bigcup_{\alpha\in\Lambda} (f|X_\alpha)^{-1}(V) = \bigcup_{\alpha\in\Lambda} (f^{-1}(V) \cap X_\alpha)$. Since each $f^{-1}(V) \cap X_\alpha$ is clopen in X_α and hence in X. Thus $f^{-1}(V)$ is open being the union of clopen sets. Moreover, since the collection Q is locally finite, the collection $\{f^{-1}(V) \cap X_\alpha : \alpha \in \Lambda\}$ is a locally finite collection of clopen sets. Since the union of a locally finite collection of closed sets is closed, $f^{-1}(V)$ is also closed and hence clopen. ∎

THEOREM 4.6. *If $f : X \to Y$ is a δ-perfectly continuous surjection which maps clopen sets to closed sets (open sets). Then Y is endowed with a δ-partition topology. Further, if in addition f is a bijection which maps δ-open (δ-closed) sets to δ-open (δ-closed) sets, then X is also equipped with a δ-partition topology.*

Proof. Suppose f maps clopen sets to closed (open) sets. Let V be a δ-open (δ-closed) set in Y. Since f is δ-perfectly continuous, $f^{-1}(V)$ is a clopen set in X. Again, since f is a surjection which maps clopen sets to closed (open) sets, the set $f(f^{-1}(V)) = V$ is closed (open) in Y and hence clopen. Thus Y is endowed with a δ-partition topology.

To prove the last part of the theorem assume that f is a bijection which maps δ-open (δ-closed) sets to δ-open (δ-closed) sets. To show that X possesses a δ-partition topology, let A be a δ-open (δ-closed) set in X. Then $f(A)$ is a δ-open (δ-closed) set in Y. Since f is a δ-perfectly continuous bijection, $f^{-1}(f(A)) = A$ is a clopen set in X. This proves that X is endowed with a δ-partition topology. ∎

THEOREM 4.7. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. If g is δ-perfectly continuous, then so is f and the space X possesses a δ-partition topology. Further, if f is δ-perfectly continuous and X is endowed with a δ-partition topology, then g is δ-perfectly continuous.*

Proof. Suppose that the graph function $g : X \to X \times Y$ is δ-perfectly continuous. Now, it is easily verified that the projection map $p_y : X \times Y \to Y$ is δ-continuous so in view of Theorem 4.4 the function $f = p_y \circ g$ is δ-perfectly continuous. To prove that X is endowed with a δ-partition topology, let U be a δ-open set in X. Then $U \times Y$ is a δ-open in $X \times Y$. Since g is δ-perfectly continuous, $g^{-1}(U \times Y) = U$ is clopen in X and so the topology of X is a δ-partition topology.

Conversely, suppose that f is δ-perfectly continuous and let X be endowed with a δ-partition topology. To show that g is δ-perfectly continuous let W be δ-open set in $X \times Y$. Suppose $W = \{U_\alpha \times V_\alpha : U_\alpha$ is regular open in X and V_α is regular open in $Y\}$. Then $p_y(W) = \bigcup_{\alpha \in \Lambda} V_\alpha = V$ is δ-open in Y. Since f is δ-perfectly continuous, $f^{-1}(V)$ is clopen in X. Now, since $g^{-1}(W) = f^{-1}(p_y(W)) = f^{-1}(V)$, g is δ-perfectly continuous. ∎

The following example shows that the hypothesis that 'X is endowed with a δ-partition topology' in Theorem 4.7 cannot be omitted.

Example 4.8. Let $X = Y = \{a, b, c, d\}$. Let the topology on X be given by $\tau = \{\phi, X, \{a, b\}, \{d\}, \{a, b, d\}\}$ and let Y be equipped with indiscrete topology. Let $f : X \to Y$ be the constant function which takes the value b. Then f is δ-perfectly continuous but the graph function $g : X \to X \times Y$ is not δ-perfectly continuous.

Definition 4.9. ([1]) A topological space X is called an Alexandroff space if any intersection of open sets in X is itself an open set.

In [11] Lorrain calls Alexandroff spaces as saturated spaces.

Theorem 4.10. *For each $\alpha \in \Lambda$, let $f_\alpha : X \to X_\alpha$ be a function and let $f : X \to \prod_{\alpha \in \Lambda} X_\alpha$ be defined by $f(x) = (f_\alpha(x))$ for each $x \in X$. If f is δ-perfectly continuous, then each f_α is δ-perfectly continuous. Further, if X is a saturated space and each f_α is δ-perfectly continuous, then f is δ-perfectly continuous.*

Proof. Suppose that f is δ-perfectly continuous. Then for each α, $f_\alpha = \Pi_\alpha \circ f$, where Π_α denotes the projection map $\Pi_\alpha : \prod X_\alpha \to X_\alpha$. Since each projection map Π_α is δ-continuous so in view of Theorem 4.4, each f_α is δ-perfectly continuous.

Conversely, suppose that X is a saturated space and each f_α is δ-perfectly continuous. To show that the function f is δ-perfectly continuous, it is sufficient to show that $f^{-1}(U)$ is clopen for each δ-open set U in the product space $\prod_{\alpha \in \Lambda} X_\alpha$. Since X is a saturated space, it suffices to prove that $f^{-1}(S)$ is clopen for every subbasic δ-open set S in the product space $\prod_{\alpha \in \Lambda} X_\alpha$. Let $U_\beta \times \prod_{\alpha \in \Lambda} X_\alpha$ be a subbasic δ-open set in $\prod_{\alpha \in \Lambda} X_\alpha$ where U_β is a δ-open set in X_β. Then $f^{-1}(U_\beta \times \prod_{\alpha \neq \beta} X_\alpha) = f^{-1}(\Pi_\beta^{-1}(U_\beta)) = f_\beta^{-1}(U_\beta)$ is clopen in X. Hence f is δ-perfectly continuous. ∎

We may recall that a space X is mildly compact [22] ($\equiv$ clustered space [21]) if every clopen cover of X has a finite subcover and that a space X is nearly compact [19] if every regular open cover of X has a finite subcover, or equivalently every δ-open cover of X has a finite subcover.

THEOREM 4.11. *Let $f : X \to Y$ be δ- perfectly continuous and let $A \subset X$ be mildly compact. Then $f(A)$ is nearly compact. In particular, δ-perfectly continuous image of a mildly compact space is nearly compact.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(A)$ by δ-open sets in Y. Since f is δ-perfectly continuous, $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is a clopen cover of A. Since A is mildly compact, there exist finitely many $\alpha_1, \dots, \alpha_n$ in Λ such that $A \subset \bigcup_{i=1}^{n} f^{-1}(V_{\alpha_i})$. Clearly $f(A) \subset \bigcup_{i=1}^{n} V_{\alpha_i}$ and so $f(A)$ is nearly compact. ∎

DEFINITIONS 4.12. A topological space X is said to be

(1) δT_0-space [7] if for each pair of distinct points x and y in X there exists a regular open set containing one of the points x and y but not the other.
(2) ultra Hausdorff [22] if for each pair of distinct points x and y in X there exist disjoint clopen sets U and V containing x and y, respectively.

An infinite space with co-finite topology is a T_1-space which is not a δT_0-space. Similarly an uncountable space with cocountable topology is a KC-space ($\equiv$ compact sets are closed) and hence T_1 but not a δT_0-space.

THEOREM 4.13. *Let $f : X \to Y$ be a δ-perfectly continuous injection. If Y is a δT_0-space, then X is an ultra-Hausdorff space.*

Proof. Let x_1 and x_2 be any two distinct points in X. Then $f(x_1) \neq f(x_2)$. Since Y is a δT_0-space, there exists a regular open set V containing one of the points $f(x_1)$ or $f(x_2)$ but not the other. To be precise, assume that $f(x_1) \in V$. Since f is δ-perfectly continuous, $f^{-1}(V)$ is a clopen set containing x_1 such that $x_2 \notin f^{-1}(V)$. Then $f^{-1}(V)$ and $X \setminus f^{-1}(V)$ are disjoint clopen sets containing x_1 and x_2 respectively and so X is ultra-Hausdorff. ∎

THEOREM 4.14. *Let $f : X \to Y$ be a δ-perfectly continuous function into a δT_0-space Y. If C is a connected set in X, then $f(C)$ is a singleton. In particular, every δ-perfectly continuous function from a connected space X into a δT_0-space is constant and hence strongly continuous.*

Proof. Assume contrapositive and let C be connected subset of X such that $f(C)$ is not a singleton. Let $f(x)$, $f(y)$ be two distinct points of $f(C)$. Since Y is a δT_0-space, there exists a regular open set V containing one of the points $f(x)$ and $f(y)$ but not other. To be precise, let $f(x) \in V$. Since f is δ-perfectly continuous, $f^{-1}(V) \cap C$ is a nonempty proper clopen subset of C, contradicting connectedness of C. ∎

COROLLARY 4.15. *Let $f : X \to Y$ be a δ-perfectly continuous function into a δT_0-space Y. If X is sum connected, then f is constant on each component of X.*

5. Function spaces and δ-perfectly continuous functions

A space X is said to be *sum connected* [5] if each $x \in X$ has a connected neighbourhood, or equivalently each component of X is open in X. The category of sum connected spaces represents the coreflective hull of the category of connected spaces and includes all locally connected spaces as well. The disjoint topological sum of two copies of topologist's sine curve [23] is a sum connected space which is neither connected nor locally connected.

In general the set of all continuous functions from a space X into a space Y is not closed in Y^X in the topology of pointwise convergence. In contrast, Naimpally [13] showed that if X is locally connected and Y is Hausdorff, then the set $S(X,Y)$ of all strongly continuous functions from X to Y is closed in Y^X in the topology of pointwise convergence. In [6] we generalized Naimpally's result to show that if X is sum connected and Y is Hausdorff, then $S(X,Y) = P(X,Y)$ i.e. the set of all strongly continuous functions as well as the set of all perfectly continuous functions from X into Y is closed in Y^X in the topology of pointwise convergence. In this section we improve upon this result to show that if X is sum connected and Y is a δT_0-space, then all the three classes of functions coincide, i.e. $S(X,Y) = P(X,Y) = P_\Delta(X,Y)$. So in view of results of [6] we conclude that if X is sum connected and Y is Hausdorff, then $P_\Delta(X,Y)$ the set of all δ-perfectly continuous functions from X into Y is closed in the topology of pointwise convergence.

THEOREM 5.1. *Let $f : X \to Y$ be a δ-perfectly continuous function from a sum connected space X into a δT_0-space Y. Then f is strongly continuous.*

Proof. Let X be a sum connected space. Then every component of X is clopen in X. Hence it follows that any union of components of X and the complement of this union are complementary clopen sets in X. By Corollary 4.15 f is constant on each component of X and hence for every subset A of Y, $f^{-1}(A)$ and its complement $X \setminus f^{-1}(A)$ are complementary clopen sets in X being the unions of components of X. Thus f is strongly continuous. ∎

Next we quote the following result from [6].

THEOREM 5.2. ([6, Theorem 3.4]) *Let $f : X \to Y$ be a function from a sum connected space X into a T_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*

THEOREM 5.3. *Let $f : X \to Y$ be a function from a sum connected space X into a δT_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ- perfectly continuous.*

Proof. The implications (a)$\Rightarrow$(b)$\Rightarrow$(d) are trivial and (d)$\Rightarrow$(a) is a consequence of Theorem 5.1. So the result is immediate in view of Theorem 5.2.

Let $L = L(X,Y)$, $P = P(X,Y)$, $S = S(X,Y)$ and $P_\Delta = P_\Delta(X,Y)$ denote the function spaces of all cl-supercontinuous, perfectly continuous, strongly continuous and δ-perfectly continuous functions from X into Y, respectively with the topology of pointwise convergence. ■

THEOREM 5.4. *Let X be a sum connected space and let Y be a Hausdorff space. Then $L = P = S = P_\Delta$ is closed in Y^X in the topology of pointwise convergence.*

Proof. This is immediate in view of Theorem 5.3 and [6, Theorem 3.7]. ■

In view of Theorem 5.4 it follows that if X is sum connected (e.g. connected or locally connected) and Y is Hausdorff, then the pointwise limit of a sequence $\{f_n : X \to Y\}$ of δ-perfectly continuous functions is δ-perfectly continuous.

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