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CERTAIN SUFFICIENCY CONDITIONS ON FOX–WRIGHT FUNCTIONS

Abstract. The main object of this paper is to find certain conditions for the function $z\{{}_p\psi_q(z)\}$ to be a member of certain subclasses of analytic functions. Our results provides generalization of some recent results due to Swaminathan [19] and Chaurasia and Srivastava [20].

1. Introduction

As usual, let A denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

analytic in the open unit disk $\Delta = \{z : |z| < 1\}$ and S denote the subclass of A that are univalent in Δ. We begin with the following.

DEFINITION 1.1 ([2]). Let $f \in A$, $0 \le k < \infty$, and $0 \le \alpha < 1$. Then $f \in k - UCV(\alpha)$ if and only if

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \ge k\left|\frac{zf''(z)}{f'(z)}\right| + \alpha. \tag{1.2}$$

This class generalizes various other classes which are worthy of mention. The class $k - UCV(0)$ called the k-uniformly convex is due to Kanas and Wiśniowska [8] and has its geometric characterization given in the following way: let $0 \le k < \infty$. The function $f \in A$ is said to be k-uniformly convex in Δ, if f is convex in Δ, and the image of every circular arc γ contained in Δ, with center ζ, where $|\zeta| \le k$, is convex.

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The class $0-UCV(\alpha)=\kappa(\alpha)$ is the well known class of convex functions of order α that satisfy the analytic conditions

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha.$$

In particular, for $\alpha=0$, f maps the unit disk onto a convex domain (for details, see [6]). We have $1-UCV(0)=UCV$ [7]. Denoting

$$p(z)=\left\{1+\frac{zf''(z)}{f'(z)}\right\}\qquad(z\in\Delta)$$

and assuming that $f\in UCV(\alpha)$ we have that p is in conic region

$$\Omega=\{\omega\in C:(\mathrm{Im}\omega)^2<2\,\mathrm{Re}\,\omega-1\}.$$

The classes $UCV(\alpha)$ and $ST(\alpha)$ are unified and studied using certain fractional calculus operator methods in [13]. We refer to [8, 9, 10] and references therein for basic results related to this paper.

For $\tau\in C\setminus\{0\}$, Swaminathan [19] introduce the class $P_\gamma^\tau(\beta)$, with $0\le\gamma<1$ and $\beta<1$, as

(1.3) $$P_\gamma^\tau(\beta) := \left\{f\in A:\left|\frac{(1-\gamma)\frac{f(z)}{z}+\gamma f'(z)-1}{2\tau(1-\beta)+(1-\gamma)\frac{f(z)}{z}+\gamma f'(z)-1}\right|<1, z\in\Delta\right\}.$$

We list a few particular cases of this class discussed in the literature.

(i) The class $P_\gamma^\tau(\beta)$, with $0\le\gamma<1$ and $\beta<1$ was studied by Dixit and Pal in [3]. Properties of that class related to the operator $I_{a,b;c}(f)(z)=zF(a,b;c;z)*f(z)$ were considered in [5].

(ii) The class $P_\gamma^\tau(\beta)$, with $0\le\gamma<1$ and $\beta<1$ for $\tau=e^{i\eta}\cos\eta$ where $-\pi/2<\eta<\pi/2$ was examined in [12] and discussed by many authors with the reference to the Carlson-Schaffer operator $G_{b,c}(f)(z)=zF(1,b;c;z)*f(z)$ using duality techniques (for example, see [1, 4, 11, 12, 14, 15]).

We denote by T a subclass S with negative coefficients, e.g.

(1.4) $$f(z)=z-\sum_{n=2}^{\infty}a_nz^n,\, a_n\ge 0.$$

This class is due to H. Silverman [17] and has many interesting results (see [17] and [18]).

In the link of $k-UCV(\alpha)$ the following class was defined in [2].

Definition 1.2 ([2]). Let $k-UCT(\alpha)$ be the class of functions $f(z)$ of the form (1.4) that satisfies the condition (1.2). Using the analytic condition (1.2) and an Alexander type theorem, the following classes are defined in [2].

DEFINITION 1.3 ([2]). Let $0 \leq k < \infty$, and $0 \leq \alpha < 1$. Then

(i) $f \in k - ST(\alpha)$ if and only if f has the form (1.1) and satisfies the condition

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} \geq k\left|\frac{zf'(z)}{f(z)} - 1\right| + \alpha. \tag{1.5}$$

(ii) $f \in k - STT(\alpha)$ if and only if f has the form (1.4) and satisfies the inequality given by the expression (1.5).

For $k = 0$, we obtain the well known class of starlike functions of order α, which has the analytic characterization $\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha (z \in \Delta)$. In particular, for $\alpha = 0$, f maps the unit disk onto starlike domain (for details, see [4]). We further note that, $1 - ST(\alpha)$ is the well known class discussed in [16]. We also need the following sufficient condition on the coefficients for the functions in the class $k - UCV(\alpha)$.

LEMMA 1.1 ([2]). *If f of the form* (1.1) *satisfies a condition*

$$\sum_{n=2}^{\infty} n[n(k+1) - (k+\alpha)]\,|a_n| \leq 1 - \alpha \tag{1.6}$$

for some $0 \leq k < \infty$ and $\alpha \in [0, 1)$, then $f \in k - UCV(\alpha)$. Moreover, the above condition is necessary and sufficient for f to be in $k - UCT(\alpha)$, further the condition

$$\sum_{n=2}^{\infty} [n(k+1) - (k+\alpha)]\,|a_n| \leq 1 - \alpha \tag{1.7}$$

is sufficient for f to be in $k - ST(\alpha)$ and it is both necessary and sufficient for f to be in $k - STT(\alpha)$.

Another sufficient condition is also given for the class k- UCV (α) in [8] which is given by the following

LEMMA 1.2 ([8]). *If $f \in S$ and be of the form* (1.1) *satisfies a condition*

$$\sum_{n=2}^{\infty} n(n-1)\,|a_n| \leq \frac{1}{k+2}, \tag{1.8}$$

for some k, $0 \leq k < \infty$ then $f \in k - UCV(\alpha)$. The number $1/k + 2$ cannot be increased.

LEMMA 1.3 ([19]). *If f of the form* (1.1) *satisfies a sufficient condition*

$$\sum_{n=2}^{\infty}[1+\gamma(n-1)]\,|a_n| \le |\tau|\,(1-\beta), \tag{1.9}$$

then $f \in P_\gamma^\tau(\beta)$. This condition is also necessary if f is of the form (1.4) *and $\tau = 1$.*

The Fox–Wright function [21, p. 50, equation 1.5] appearing in the present paper is defined by

$$ {}_p\psi_q(z) = {}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p}; \\ (b_j,\beta_j)_{1,q};\end{matrix}\; z\right] = \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)}\cdot\frac{z^n}{n!}, \tag{1.10}$$

where $\alpha_j (j = 1,\ldots,p)$ and $\beta_j (j = 1,\ldots,q)$ are real and positive and $1+\sum_{j=1}^{q}\beta_j > \sum_{j=1}^{p}\alpha_j$.

2. Main results

THEOREM 2.1. *If $\sum_{j=1}^{q}|b_j| > \sum_{j=1}^{p}|a_j| + 2, a_j > 0$ and $1+\sum_{j=1}^{q}\beta_j > \sum_{j=1}^{p}\alpha_j$, then a sufficient condition for the function $z\{{}_p\psi_q(z)\}$ to be in the class $k-UCV(\alpha)$, and both necessary and sufficient conditions for $z\{{}_p\psi_q(z)\}$ to be in $k-UCT(\alpha), (0 \le k < \infty, 0 \le \alpha < 1)$ is*

$$\left(\frac{1+k}{1-\alpha}\right){}_p\psi_q\left[\begin{matrix}(a_j+2\alpha_j,\alpha_j)_{1,p}; \\ (b_j+2\beta_j,\beta_j)_{1,q};\end{matrix}\;1\right] \tag{2.1}$$

$$+\left(\frac{3+2k-\alpha}{1-\alpha}\right){}_p\psi_q\left[\begin{matrix}(a_j+\alpha_j,\alpha_j)_{1,p}; \\ (b_j+\beta_j,\beta_j)_{1,q};\end{matrix}\;1\right]+{}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p}; \\ (b_j,\beta_j)_{1,q};\end{matrix}\;1\right] \le 1+\frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j}.$$

Proof. By virtue of Lemma 1.1, equation (1.6), it suffices to prove that

$$\sum_{n=2}^{\infty} n[n(1+k)-(k+\alpha)]\left[\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n-1)](n-1)!}\right] \le 1-\alpha. \tag{2.2}$$

The above inequality may be expressed as

$$(1+k)\sum_{n=1}^{\infty}(n+1)^2\left[\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)n!}\right] - (k+\alpha)\sum_{n=1}^{\infty}(n+1)\left[\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)n!}\right] \leq 1-\alpha.$$

The left hand side of the above inequality is equal to

$$(1+k)\sum_{n=2}^{\infty}\left[\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)(n-2)!}\right]$$

$$+(3+2k-\alpha)\sum_{n=0}^{\infty}\left[\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n+1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n+1)]n!}\right]+(1-\alpha)\sum_{n=1}^{\infty}\left[\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)n!}\right]$$

$$=(1+k)_p\psi_q\left[\begin{matrix}(a_j+2\alpha_j,\alpha_j)_{1,p};\\(b_j+2\beta_j,\beta_j)_{1,q};\end{matrix}\ 1\right]+(3+2k-\alpha)_p\psi_q\left[\begin{matrix}(a_j+\alpha_j,\alpha_j)_{1,p};\\(b_j+\beta_j,\beta_j)_{1,q};\end{matrix}\ 1\right]$$

$$+(1-\alpha)\left\{{}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p};\\(b_j,\beta_j)_{1,q};\end{matrix}\ 1\right]-\frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j}\right\},$$

and is bounded by $(1-\alpha)$ if and only if (2.1) holds. It ends the Theorem.

THEOREM 2.2. *If* $\sum_{j=1}^{q}|b_j| > \sum_{j=1}^{p}|a_j|+1, a_j>0$ *and* $1+\sum_{j=1}^{q}\beta_j > \sum_{j=1}^{p}\alpha_j$, *then a sufficient condition for the function* $z\{{}_p\psi_q(z)\}$ *to be in the class* $k-ST(\alpha)$ *and it is both necessary and sufficient conditions for* $z\{{}_p\psi_q(z)\}$ *to be in* $k-STT(\alpha)$ $(0\leq k<\infty,\ 0\leq\alpha<1)$ *is*

$$\left(\frac{1+k}{1-\alpha}\right){}_p\psi_q\left[\begin{matrix}(a_j+\alpha_j,\alpha_j)_{1,p};\\(b_j+\beta_j,\beta_j)_{1,q};\end{matrix}\ 1\right]+{}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p};\\(b_j,\beta_j)_{1,q};\end{matrix}\ 1\right]\leq 1+\frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j}. \tag{2.3}$$

Proof. Since

$$z\{{}_p\psi_q(z)\} = \frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j} z + \sum_{n=2}^{\infty} \frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n-1)]z^n}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n-1)](n-1)!}$$

then, by virtue of Lemma 1.1 (equation (1.7)), we only need to show that

$$\sum_{n=2}^{\infty}[n(1+k)-(k+\alpha)]\left[\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n-1)](n-1)!}\right] \le 1-\alpha. \tag{2.4}$$

Now, we have

$$\begin{aligned}
&\sum_{n=2}^{\infty}[n(1+k)-(k+\alpha)]\left[\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n-1)](n-1)!}\right]\\
&\qquad= \sum_{n=0}^{\infty}[(n+2)(1+k)-(k+\alpha)]\left[\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n+1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n+1)](n+1)!}\right]\\
&\qquad= (1+k)\sum_{n=0}^{\infty}\left[\frac{\prod_{j=1}^{p}\Gamma[(a_j+\alpha_j)+n\alpha_j]}{\prod_{j=1}^{q}\Gamma[(b_j+\beta_j)+n\beta_j]n!}\right]\\
&\qquad\quad + (1-\alpha)\left[\sum_{n=0}^{\infty}\left[\frac{\prod_{j=1}^{p}\Gamma[(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma[(b_j+\beta_j n)n!}\right] - \frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j}\right]\\
&\qquad= (1+k)\,{}_p\psi_q\left[\begin{matrix}(a_j+\alpha_j,\alpha_j)_{1,p};\\(b_j+\beta_j,\beta_j)_{1,q};\end{matrix}\ 1\right] + (1-\alpha)\,{}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p};\\(b_j,\beta_j)_{1,q};\end{matrix}\ 1\right]\\
&\qquad\quad - (1-\alpha)\frac{\prod_{j=1}^{p}\Gamma a_j}{\prod_{j=1}^{q}\Gamma b_j} \le 1-\alpha
\end{aligned}$$

by the assertion (2.3). Hence $z\{{}_p\psi_q(z)\} \in k-ST(\alpha)$.

REMARK 1. In the special case, when $k = 2 - \alpha$, Theorem 2.2 corresponds to a result given earlier by Chaurasia and Srivastava [20, p. 2, Theorem 2.1].

THEOREM 2.3. *If* $\sum_{j=1}^{q} |b_j| > \sum_{j=1}^{p} |a_j| + 1, a_j > 0$ *and* $1 + \sum_{j=1}^{q} \beta_j > \sum_{j=1}^{p} \alpha_j$, *then a sufficient condition for the function* $z\{{}_p\psi_q(z)\}$ *to be in the class* $k-UCV(\alpha)$, $(0 \le k < \infty)$ *is*

$$(k+2)_p\psi_q\left[\begin{matrix}(|a_j+2\alpha_j|,\alpha_j)_{1,p};\\(|b_j+2\beta_j|,\beta_j)_{1,q};\end{matrix}\,1\right] + 2(k+2)_p\psi_q\left[\begin{matrix}(|a_j+\alpha_j|,\alpha_j)_{1,p};\\(|b_j+\beta_j|,\beta_j)_{1,q};\end{matrix}\,1\right] \le 1. \tag{2.5}$$

The number $1/k+2$ *cannot be increased.*

Proof. By virtue of Lemma 1.2, equation (1.8), it suffices to prove that

$$\sum_{n=2}^{\infty} n(n-1)\left|\frac{\prod_{j=1}^{p}\Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q}\Gamma[b_j+\beta_j(n-1)](n-1)!}\right| \le \frac{1}{k+2}, \tag{2.6}$$

we note, that

$$\sum_{n=1}^{\infty}(n+1)^2\left|\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)n!}\right| - \sum_{n=1}^{\infty}(n+1)\left|\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)n!}\right|$$
$$= \sum_{n=2}^{\infty}\left|\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)(n-2)!}\right| + 2\sum_{n=1}^{\infty}\left|\frac{\prod_{j=1}^{p}\Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)(n-1)!}\right|$$
$$= {}_p\psi_q\left[\begin{matrix}(|a_j+2\alpha_j|,\alpha_j)_{1,p};\\(|b_j+2\beta_j|,\beta_j)_{1,q};\end{matrix}\,1\right] + 2{}_p\psi_q\left[\begin{matrix}(|a_j+\alpha_j|,\alpha_j)_{1,p};\\(|b_j+\beta_j|,\beta_j)_{1,q};\end{matrix}\,1\right]$$

which is bounded by $1/k+2$ if (2.5) holds. Hence the theorem is proved.

REMARK 2. In the special case, when

$$p = 2, q = 1, \alpha_1 = \alpha_2 = \beta_1 = 1, a_1 = a, a_2 = b, b_1 = c, \alpha = 0 \tag{2.7}$$

and after some manipulation, Theorem 2.3 provides a similar results obtained earlier by Swaminathan [19, p. 6, Theorem 2.10, Corollary 2.11, Corollary 2.12].

THEOREM 2.4. *If* $\sum_{j=1}^{q} |b_j| > \sum_{j=1}^{p} |a_j| + 1, a_j > 0$ *and* $1 + \sum_{j=1}^{q} \beta_j > \sum_{j=1}^{p} \alpha_j$, *then a sufficient condition for the function* $z\{{}_p\psi_q(z)\}$ *to be in the class* $P_\gamma^\tau(\beta)$ *is*

$$(2.8) \quad \gamma\, {}_p\psi_q \left[\begin{matrix}(|a_j+\alpha_j|,\alpha_j)_{1,p};\\ (|b_j+\beta_j|,\beta_j)_{1,q};\end{matrix}\ 1\right] + {}_p\psi_q \left[\begin{matrix}(|a_j|,\alpha_j)_{1,p};\\ (|b_j|,\beta_j)_{1,q};\end{matrix}\ 1\right] \le |\tau|\,(1-\beta) + \frac{\prod_{j=1}^{p} \Gamma a_j}{\prod_{j=1}^{q} \Gamma b_j}.$$

Proof. By virtue of Lemma 1.3 (equation (1.9)) it suffices to prove that

$$(2.9) \quad \sum_{n=2}^{\infty} [1+\gamma(n-1)] \left| \frac{\prod_{j=1}^{p} \Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q} \Gamma[b_j+\beta_j(n-1)](n-1)!} \right| \le |\tau|\,(1-\beta).$$

Now, we have

$$\sum_{n=2}^{\infty} [\gamma n + (1-\gamma)] \left| \frac{\prod_{j=1}^{p} \Gamma[a_j+\alpha_j(n-1)]}{\prod_{j=1}^{q} \Gamma[b_j+\beta_j(n-1)](n-1)!} \right|$$
$$= \sum_{n=0}^{\infty} \gamma(n+1) \left| \frac{\prod_{j=1}^{p} \Gamma[a_j+\alpha_j(n+1)]}{\prod_{j=1}^{q} \Gamma[b_j+\beta_j(n+1)](n+1)!} \right|$$
$$+ \sum_{n=0}^{\infty} \left| \frac{\prod_{j=1}^{p} \Gamma(a_j+\alpha_j n)}{\prod_{j=1}^{q} \Gamma(b_j+\beta_j n)} \right| \frac{1}{n!} - \frac{\prod_{j=1}^{p} \Gamma a_j}{\prod_{j=1}^{q} \Gamma b_j}$$

which is bounded by $|\tau|\,(1-\beta)$ if (2.8) holds. It ends the proof.

REMARK 3. Applying the parametric substitutions listed in (2.7) and after some manipulation, Theorem 2.4 would yield the similar known results due to Swaminathan [19, p. 5, Theorem 2.5, Corollary 2.6, Theorem 2.7, Theorem 2.8, Corollary 2.9].

3. Particular cases

By specifying the parameters suitably, the results of this paper readily yield some results due to Swaminathan [19] and Chaurasia and Srivastava [20].

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