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NEIGHBOURHOODS OF CERTAIN p-VALENTLY ANALYTIC FUNCTIONS DEFINED BY USING SALAGEAN OPERATOR

Abstract. By making use of the familiar concept of neighbourhood of analytic and p-valent functions, the author prove coefficient bounds and distortion inequalities and associated inclusion relations for the (j, θ)-neighbourhoods of a family of p-valent functions with negative coefficients and defined by using Salagean operator which is defined by means of a certain non-homogenous Cauchy–Euler differential equation.

1. Introduction

Let $T(j,p)$ denote the class of functions of the form :

$$(1.1) \qquad f(z) = z^p - \sum_{k=j+p}^{\infty} a_k z^k \quad (a_k \geq 0;\ p, j \in N = \{1, 2, \ldots.\}),$$

which are analytic and p-valent in the open unit disc $U = \{z : |z| < 1\}$. A function $f(z) \in T(j,p)$, is said to be p-valently starlike of order α if it satisfies the inequality :

$$(1.2) \qquad \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad (z \in U;\ 0 \leq \alpha < p;\ p \in N).$$

We denote by $T_j^*(p,\alpha)$ the class of all p-valently starlike functions of order α. Also a function $f(z) \in T(j,p)$ is said to be p-valently convex of order α if it satisfies the inequality :

$$(1.3) \qquad \operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \quad (z \in U;\ 0 \leq \alpha < p;\ p \in N).$$

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We denote by $C_j(p,\alpha)$ the class of all p-valently convex functions of order α. We note that (see for example Duren [10])

$$f(z) \in C_j(p,\alpha) \Leftrightarrow \frac{zf'(z)}{p} \in T_j^*(p,\alpha) \quad (0 \le \alpha < p;\ p \in N). \tag{1.4}$$

The classes $T_j^*(p,\alpha)$ and $C_j(p,\alpha)$ are studied by Owa [15].

For a function $f(z)$ in $T(j,p)$, we have

$$D_p^0 f(z) = f(z),$$

$$D_p^1 f(z) = Df(z) = \frac{z}{p} f'(z) = z^p - \sum_{k=j+p}^{\infty} \left(\frac{k}{p}\right) a_k z^k,$$

$$D_p^2 f(z) = D(D_p^1 f(z)) = \frac{z}{p}\left(\frac{z}{p} f'(z)\right)' = z^p - \sum_{k=j+p}^{\infty} \left(\frac{k}{p}\right)^2 a_k z^k,$$

and

$$D_p^n f(z) = D(D_p^{n-1} f(z)) \quad (n \in N).$$

It is easy to see that

$$D^n f(z) = z^p - \sum_{k=j+p}^{\infty} \left(\frac{k}{p}\right)^n a_k z^k \quad (n \in N_0 = N \cup \{0\}). \tag{1.5}$$

For $j = p = 1$, the differential operator D^n was introduced by Salagean [19].

Now, making use of the differential operator $D_p^n f(z)$ given by (1.5), we introduce a new class $S_n(j,p,\lambda,b,\beta)$ of the p-valently analytic functions $f(z) \in T(j,p)$ satisfying the following inequality:

$$\left|\frac{1}{b}\left(\frac{zF'_{n,p,\lambda}(z)}{F_{n,p,\lambda}(z)} - p\right)\right| < \beta$$

$$(z \in U;\ p,j \in N;\ n \in N_0; 0 \le \lambda \le 1;\ b \in C\backslash\{0\};\ 0 < \beta \le 1), \tag{1.6}$$

where

$$F_{n,p,\lambda}(z) = (1-\lambda) D_p^n f(z) + \lambda z (D_p^n f(z))'. \tag{1.7}$$

We note that :

(i) $S_0(j,p,\lambda,p-\alpha,1) = T_j(p,\alpha,\lambda)$ $(0 \le \alpha < p)$ (Altintas et al. [3] and [7]);

(ii) $S_0(j,1,\lambda,1-\alpha,1) = P(j,\lambda,\alpha)$ $(j \in N;\ 0 \le \alpha < 1;\ 0 \le \lambda \le 1)$ (Altintas [1]);

(iii) $S_n(j,1,0,1-\alpha,1) = P(j,\alpha,n)$ $(j \in N;\ n \in N_0;\ 0 \le \alpha < 1)$ (Aouf and Srivastava [9]);

(iv) $S_0(j,p,0,p-\alpha,1) = T_j^*(p,\alpha)$ $(p,j \in N;\ 0 \le \alpha < p)$ (Owa [15] and Yamakawa [22]);

(v) $S_0(j,p,1,p-\alpha,1) = C_j(p,\alpha)$ $(p,j \in N;\ 0 \leq \alpha < p)$ (Owa [15] and Yamakawa [22]).

Now, following the earlier investigation by Goodman [11], Ruscheweyh [18], and others including Altintas and Owa [5], Altintas et al. ([6] and [7]), Murgusundaramoorthy and Srivastava [12], Raina and Sirvastava [17], Aouf [8], Prajapat et al. [16] and Srivastava and Orhan [20] (see also [13], [14] and [21]), we define the (j,δ)-neighbourhood of a function $f(z) \in T_p(n)$ by (see, for example, [7, p. 1668])

$$(1.8) \qquad N_{j,\theta}(f;g) = \Big\{ g : g \in T(j,p),\ g(z) = z^p - \sum_{k=j+p}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} k|a_k - b_k| \leq \theta \Big\}.$$

In particular, if

$$(1.9) \qquad h(z) = z^p \quad (p \in N),$$

we immediately have

$$(1.10) \qquad N_{j,\theta}(h;g) = \Big\{ g : g \in T(j,p), g(z) = z^p - \sum_{k=j+1}^{\infty} b_k z^k \text{ and } \sum_{k=j+p}^{\infty} k|b_k| \leq \theta \Big\}.$$

The main object of this paper is to derive several coefficient bounds, distortion inequalities and associated inclusion relations for the (j,θ)-neighbourhood of function in the class $H_n(j,p,\lambda,b,\beta;\delta)$ which consists of functions $f(z) \in T(j,p)$ satisfying the following non-homogenous Cauchy-Euler differential equation :

$$(1.11) \qquad z^2\frac{d^2w}{dz^2} + 2(\delta+1)z\frac{dw}{dz} + \delta(\delta+1)w = (p+\delta)(p+\delta+1)g(z)$$
$$(w = f(z) \in T(j,p); g \in S_n(j,p,\lambda,b,\beta); \delta > -p\ (\delta \in R)).$$

2. Coefficient bounds and distortion inequalities

In our present investigation of the class $S_n(j,p,\lambda,b,\beta)$ we shall require Lemmas 1 and 2 below.

LEMMA 1. *Let the function $f(z) \in T(j,p)$ be defined by* (1.1). *Then $f(z)$ is in the class $S_n(j,p,\lambda,b,\beta)$ if and only if*

$$(2.1) \qquad \sum_{k=j+p}^{\infty} \left(\frac{k}{p}\right)^n (k+\beta|b|-p)[1+\lambda(k-1)]a_k \leq \beta|b|[1+\lambda(p-1)].$$

Proof. Let a function $f(z)$ of the form (1.1) belong to the class $S_n(j,p,\lambda,b,\beta)$. Then, in view of (1.5) and (1.6), we obtain the following inequality:

$$\text{(2.2)} \qquad \operatorname{Re}\left\{\frac{zF'_{n,p,\lambda}(z)}{F_{n,p,\lambda}(z)} - p\right\} > -\beta|b| \quad (z \in U),$$

or, equivalently,

$$\text{(2.3)} \qquad \operatorname{Re}\left\{\frac{-\sum_{k=j+p}^{\infty}(\frac{k}{p})^n(k-p)[1+\lambda(k-1)]a_k z^{k-p}}{[1+\lambda(p-1)] - \sum_{k=j+p}^{\infty}(\frac{k}{p})^n[1+\lambda(k-1)]a_k z^{k-p}}\right\} > -\beta|b|, \quad (z \in U).$$

Setting $z = r$ $(0 \le r < 1)$ in (2.3), we observe that the expression in the denominator of the left-hand side of (2.3) is positive for $r = 0$ and also for $0 < r < 1$. Thus, by letting $r \to 1^-$ through real values, (2.3) leads us to the desired assertion of Lemma 1.

Conversely, by applying the hypothesis (2.1) and letting $|z| = 1$, we find from (1.6) that

$$\text{(2.4)} \qquad \left|\frac{zF'_{n,p,\lambda}(z)}{F_{n,p,\lambda}(z)} - p\right|$$
$$= \left|\frac{\sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n(k-p)[1+\lambda(k-1)]a_k z^{k-p}}{[1+\lambda(p-1)] - \sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n[1+\lambda(k-1)]a_k z^{k-p}}\right|$$
$$\le \frac{\sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n(k-p)[1+\lambda(k-1)]a_k}{[1+\lambda(p-1)] - \sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n[1+\lambda(k-1)]a_k}$$
$$\le \frac{\beta|b|\{[1+\lambda(p-1)] - \sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n[1+\lambda(k-1)]a_k\}}{[1+\lambda(p-1)] - \sum_{k=j+p}^{\infty}\left(\frac{k}{p}\right)^n[1+\lambda(k-1)]a_k} = \beta|b|.$$

Hence, by the maximum modulus theorem, we have $f(z) \in S_n(j,p,\lambda,b,\beta)$, which evidently completes the proof of Lemma 1.

LEMMA 2. *Let the function $f(z)$ given by (1.1) be in the class $S_n(j,p,\lambda,b,\beta)$. Then*

$$\text{(2.4)} \qquad \sum_{k=j+p}^{\infty} a_k \le \frac{\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]}$$

and

$$\text{(2.5)} \qquad \sum_{k=j+p}^{\infty} ka_k \le \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]} \quad (p > |b|).$$

Proof. By using Lemma 1, we find from (2.1) that

$$\left(\frac{j+p}{p}\right)^n (j+\beta|b|)[1+\lambda(j+p-1)] \sum_{k=j+p}^{\infty} a_k$$
$$\leq \sum_{k=j+p}^{\infty} \left(\frac{k}{p}\right)^n [k+\beta|b|-p)[1+\lambda(k-1)]a_k$$
$$\leq \beta|b|[1+\lambda(p-1)],$$

which immediately yields the first assertion (2.4).

For the proof of the second assertion, by appealing to (2.1), we have

$$\left(\frac{j+p}{p}\right)^n [1+\lambda(j+p-1)] \sum_{k=j+p}^{\infty} ka_k$$
$$\leq \beta|b|[1+\lambda(p-1)] + \left(\frac{j+p}{p}\right)^n (p-\beta|b|)[1+\lambda(j+p-1)] \sum_{k=j+p}^{\infty} a_k$$
$$\leq \beta|b|[1+\lambda(p-1)] + (p-\beta|b|)\frac{\beta|b|[1+\lambda(p-1)]}{(j+\beta|b|)}$$
$$= \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(j+\beta|b|)}.$$

Hence

$$\sum_{k=j+p}^{\infty} ka_k \leq \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)]} \qquad (p>|b|),$$

which implies the second assertion (2.5). ∎

Our main distortion inequalities for functions in the class $H_n(j,p,\lambda,b,\beta,\delta)$ are given by Theorem 1 below.

THEOREM 1. *Let a function $f(z)\in T(j,p)$ be in the class $H_n(j,p,\lambda,b,\beta;\delta)$. then for $z\in U$ we have*

$$|f(z)| \leq |z|^p + \frac{\beta|b|[1+\lambda(p-1)](p+\delta)(p+\delta+1)}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)](j+p+\delta)}|z|^{j+p}, \tag{2.6}$$

$$|f(z)| \geq |z|^p - \frac{\beta|b|[1+\lambda(p-1)](p+\delta)(p+\delta+1)}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)](j+p+\delta)}|z|^{j+p}, \tag{2.7}$$

$$|f^{(m)}(z)| \leq \left\{\frac{p!}{(p-m)!} + \right. \tag{2.8}$$
$$\left. + \frac{\beta|b|[1+\lambda(p-1)](p+\delta)(p+\delta+1)(j+p)!}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)](j+p+\delta)(j+p-m)!}|z|^j\right\}|z|^{p-m}$$

and

$$(2.9)\qquad |f^{(m)}(z)| \geq \left\{ \frac{p!}{(p-m)!} - \right.$$
$$\left. - \frac{\beta|b|[1+\lambda(p-1)](p+\delta)(p+\delta+1)(j+p)!}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)](j+p+\delta)(j+p-m)!}|z|^j \right\} |z|^{p-m}.$$

Proof. Suppose that $f(z) \in T(j,p)$ is given by (1.1). Also let the function $g(z) \in S_n(j,p,\lambda,b,\beta)$, occuring in the non-homogenous Cauchy-Euler differential equation (1.11), be given as in the definitions (1.8) and (1.10) with

$$b_k \geq 0 \quad (k = j+p, j+p+1, \ldots\ldots).$$

Then we easily see from (1.11) that

$$(2.10)\qquad a_k = \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)} b_k \quad (k = j+p, j+p+1, \ldots\ldots),$$

so that

$$(2.11)\qquad f(z) = z^p - \sum_{k=j+p}^{\infty} a_k z^k = z^p - \sum_{k=j+p}^{\infty} \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)} b_k z^k,$$

$$(2.12)\qquad |f(z)| \leq |z|^p + |z|^{j+p} \sum_{k=j+p}^{\infty} \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)} b_k$$

and

$$(2.13)\qquad |f(z)| \geq |z|^p - |z|^{j+p} \sum_{k=j+p}^{\infty} \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)} b_k\,.$$

Since $g(z) \in S_n(j,p,\lambda,b,\beta)$, the first assertion (2.4) of Lemma 2 yields the following inequality:

$$(2.14)\qquad b_k \leq \frac{\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]}$$
$$(k = j+p, j+p+1,\ j+p+2, \ldots).$$

This, in conjunction with (2.12) gives

$$(2.15)\qquad |f(z)| \leq |z|^p + \frac{\beta|b|[1+\lambda(p-1)](p+\delta)(p+\delta+1)}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]}|z|^{j+p}$$
$$\times \sum_{k=j+p}^{\infty} \frac{1}{(k+\delta)(k+\delta+1)} \qquad (z \in U).$$

Observe that also the following identity holds:

$$(2.16)\quad \sum_{k=j+p}^{\infty} \frac{1}{(k+\delta)(k+\delta+1)} = \sum_{k=j+p}^{\infty} \left(\frac{1}{(k+\delta)} - \frac{1}{(k+\delta+1)}\right)$$
$$= \frac{1}{j+p+\delta} \qquad (\delta \in R\backslash\{-j-p, -j-p-1, -j-p-2, \dots\}).$$

Now the assertion (2.6) of Theorem 1 follows at once from (2.15) together with (2.16). The second assertion (2.7) of Theorem 1 can be proven by, similarly applying (2.13), (2.14) and (2.16).

REMARK 1. (i) Putting $n = 0$, $\beta = 1$ and $b = p - \alpha$, $0 \le \alpha < p$, in Theorem 1, we obtain the result obtained by Altintas et al. [7, Theorem 1];

(ii) Putting $n = 0$, $\beta = 1$ and $b = p - \alpha$, $0 \le \alpha < p$, in Theorem 1, we obtain the result obtained by Altintas [2, Theorem 1 with $q = 0$];

(iii) Putting $n = 0$, in Theorem 1, we obtain the result obtained by Altintas et al. [4, Theorem 1 with $q = 0$].

3. Neighborhoods for the classes $S_n(j,p,\lambda,b,\beta)$ and $H_n(j,p,\lambda,b,\beta;\delta)$

In this section, we determine inclusion relations for the classes $S_n(j,p,\lambda,b,\beta)$ and $H_n(j,p,\lambda,b,\beta;\delta)$ involving the (j,δ)-neighbourhoods defined by (1.8) and (1.10).

THEOREM 2. *If $f(z) \in T(j,p)$ is in the class $S_n(j,p,\lambda,b,\beta)$, then*

$$(3.1)\qquad S_n(j,p,\lambda,b,\beta) \subset N_{j,\theta}(h;f),$$

where $h(z)$ is given by (1.9) and

$$(3.2)\qquad \theta = \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)]}.$$

Proof. Assertion (3.1) follows easily from the definition of $N_{j,\theta}(h;f)$, which is given by (1.10) with $g(z)$ replaced by $f(z)$, and the second assertion (2.5) of Lemma 2.

THEOREM 3. *Let the function $f(z) \in T(j,p)$ be in the class $H_n(j,p,\lambda,b,\beta;\delta)$. Then*

$$(3.3)\qquad H_n(j,p,\lambda,b,\beta;\delta) \subset N_{j,\theta}(g;f),$$

where $g(z)$ is given by (1.11) and

$$(3.4)\qquad \theta = \frac{(j+p)\beta|b|[1+\lambda(p-1)][j+(p+\delta)(p+\delta+2)]}{(\frac{j+p}{p})^n (j+\beta|b|)[1+\lambda(j+p-1)](j+p+\delta)} \qquad (p > |b|).$$

Proof. Suppose that $f(z) \in H_n(j,p,\lambda,b,\beta;\delta)$. Then we obtain

$$\sum_{k=j+p}^{\infty} k|b_k - a_k| \leq \sum_{k=j+p}^{\infty} kb_k + \sum_{k=j+p}^{\infty} \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)} kb_k. \tag{3.6}$$

Next, since $g(z) \in S_n(j,p,\lambda,b,\beta)$, the second assertion (2.5) of Lemma 2 yields

$$kb_k \leq \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]} \quad (k = j+p; j+p+1; j+p+2; \dots). \tag{3.7}$$

Finally, by making use of (2.5) as well as (3.7) on the right - hand side of (3.6), we find that

$$\sum_{k=j+p}^{\infty} k|b_k - a_k| \leq \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]} \times \left(1 + \sum_{k=j+p}^{\infty} \frac{(p+\delta)(p+\delta+1)}{(k+\delta)(k+\delta+1)}\right), \tag{3.8}$$

which, by virture of the identity (2.16), immediately yields that

$$\sum_{k=j+p}^{\infty} k|b_k - a_k| \leq \frac{(j+p)\beta|b|[1+\lambda(p-1)]}{(\frac{j+p}{p})^n(j+\beta|b|)[1+\lambda(j+p-1)]} \times \left(\frac{j+(p+\delta)(p+\delta+2)}{j+p+\delta}\right) = \theta \quad (p > |b|). \tag{3.9}$$

Thus, by definition (1.8) with $g(z)$ interchanged by $f(z)$, $f(z) \in N_{j,\theta}(g;f)$. This evidently completes the proof of Theorem 3.

REMARK 2. (i) Putting $n = 0$, $\beta = 1$ and $b = p - \alpha$, $0 \leq \alpha < p$, in Theorem 3, we obtain the result obtained by Altintas et al. [7, Theorem 3];

(ii) Putting $n = 0$, $\beta = 1$ and $b = p - \alpha$, $0 \leq \alpha < p$, in Theorem 3, we obtain the result obtained by Altintas [2, Theorem 3 with $q = 0$];

(iii) Putting $n = 0$, in Theorem 3, we obtain the result obtained by Altintas et al. [4, Theorem 3 with $q = 0$].

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