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A LINEAR OPERATOR AND ASSOCIATED CLASS OF MULTIVALENT ANALYTIC FUNCTIONS

Abstract. We introduce a certain class $H_k^\alpha(p,\lambda;h)$ of multivalent analytic functions in the open unit disc involving a linear operator L_k^α. The aim of this paper is to extend the similar concept of many earlier papers. We use the techniques of differential subordination and convolution of this class.

1. Introduction

Let $A(p)$ denote the class of functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \qquad (p \in \mathbb{N} = \{1,2,3,\ldots\}),$$

which are analytic in the open unit disc $\Delta = \{z : |z| < 1\}$ on the complex plane $\mathbb{C}$. For $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ the Hadamard product (or convolution) is defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

DEFINITION 1. Let L_0 be a linear operator $L_0 : A(p) \to A(p)$ and $k \in \mathbb{N}$, $\alpha \in \mathbb{C} \setminus \{0\}$. The operator $L_k^\alpha : A(p) \to A(p)$ is defined by

$$L_k^\alpha f = \frac{1}{\alpha}\left[z(L_{k-1}^\alpha f)' + (\alpha - p)L_{k-1}^\alpha f\right], L_0^\alpha = L_0. \tag{1.1}$$

In recent years, many important properties and characteristics of various interesting operators were investigated extensively. The definition (1.1) of the linear operator L_k^α is motivated essentially by work [13] of J. Patel and P. Sahoo and papers B. C. Carlson and D. B. Shaffer [1], H. Saitoh

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[15], H.Saitoh and H. Nunokawa [16], J. Dziok and H. M. Srivastava [2], [3], J.-L. Liu and H. M. Srivastava [7], [8], [9], and others [5], [11], [18]. The above authors consider many problems associated with the operator L_0 of type $L_0 f(z) = \phi(z) * f(z)$, where ϕ is a special function connected with the hypergeometric function, f is analytic multivalent or meromorphic multivalent and $*$ is the Hadamard product. The aim of this paper is to show that many of the results proved earlier (for example connected with the operator of type $L_0 f = \phi * f$) remain true for any linear operator L_0.

It is easily verified from the definition (1.1) that the operator L_k^α is linear and

$$L_0^\alpha f(z) = z^p + \sum_{n=1}^{\infty} A_{p+n} z^{p+n} \Rightarrow L_k^\alpha f(z) = z^p + \sum_{n=1}^{\infty} (n+\alpha)^k A_{p+n} z^{p+n}$$

$$\alpha L_1^\alpha f = z(L_0^\alpha f)' + (\alpha - p) L_0^\alpha f,$$

$$\alpha L_k^\alpha f = z^{p+1}(z^{-p} L_{k-1}^\alpha f)' + \alpha L_{k-1}^\alpha f,$$

$$\frac{L_k^\alpha f}{z^p} = \frac{z}{\alpha}\left[\frac{L_{k-1}^\alpha f}{z^p}\right]' + \frac{L_{k-1}^\alpha f}{z^p}. \tag{1.2}$$

We say that $f \in A(p)$ is subordinate to $g \in A(p)$, written $f \prec g$, if there exists a Schwarz function ω analytic in Δ such that $\omega(0) = 0$, $|\omega(z)| < 1$ and $f(z) = g(\omega(z))$. Making use of the subordination we define the following class of analytic functions.

Definition 2. Let L_0 be a linear operator and let L_k^α be given by (1.1). Let h be analytic and convex univalent in Δ. A function $f \in A(p)$ is said to be in the class $H_k^\alpha(p, \lambda; h)$, $(\lambda \geq 0,\ \alpha \in \mathbb{C} \setminus \{0\},\ k, p \in \mathbb{N})$, if and only if

$$(1-\lambda)\frac{L_k^\alpha f(z)}{z^p} + \lambda \frac{L_{k+1}^\alpha f(z)}{z^p} \prec h(z), \quad (z \in \Delta). \tag{1.3}$$

J. Patel and P. Sahoo [13] considered the class $H_k^\alpha(p, \lambda; h)$ for $k = 0$, $L_0 f(z) = \phi_p(a, c; z) * f(z)$ and $h(z) = \frac{1+Az}{1+Bz}$, $(-1 \leq B < A \leq 1)$, where $\phi_p(a, c; z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{p+n}$, $(c - 1 = \alpha)$, is the hypergeometric function and $(\lambda)_n$ is the Pochhammer symbol

$$(\lambda)_n = \begin{cases} 1 & (n = 0) \\ \lambda(\lambda+1)\cdot \ldots \cdot(\lambda+n-1) & (n \in \mathbb{N}) \end{cases}.$$

By specializing parameters we can obtain many earlier investigated classes of functions (see [13] and the references given there).

2. Main results

The main object of this paper is to present several inclusion and other properties of the Hadamard product of functions in the class $H_k^\alpha(p,\lambda;h)$. We begin by recalling the following lemma which will be required in our present investigation.

LEMMA 1. *Let h be an analytic and convex univalent function in Δ. Let f be analytic in Δ with $h(0)=f(0)=1$. If*

$$f(z)+\frac{zf'(z)}{\gamma}\prec h(z)\quad (z\in\Delta),$$

for $\gamma\neq 0$ and $\mathrm{Re}[\gamma]\geq 0$, then

$$f(z)\prec g(z)=\frac{\gamma}{z^\gamma}\int_0^z t^{\gamma-1}h(t)dt\prec h(z)\quad (z\in\Delta).$$

Moreover, the function $g(z)$ is convex univalent and it is the best dominant in the sense that $f\prec g_1$, then $g\prec g_1$.

The above lemma is due to D. Hallenbeck and S. Rusheweyh [6].

THEOREM 1. *Let $\lambda\geq 0$, $\alpha\in\mathbb{C}\setminus\{0\}$, $\mathrm{Re}[\alpha]>0$, $k,p\in\mathbb{N}$. If f belongs to the class $H_k^\alpha(p,\lambda;h)$, then*

$$\frac{L_k^\alpha f(z)}{z^p}\prec h(z)\quad (z\in\Delta), \tag{2.1}$$

moreover, if $\lambda>0$ then

$$\frac{L_k^\alpha f(z)}{z^p}\prec g(z)=\frac{\alpha}{\lambda}z^{\frac{-\alpha}{\lambda}}\int_0^z t^{\frac{\alpha}{\lambda}-1}h(t)dt\prec h(z)\quad (z\in\Delta), \tag{2.2}$$

and g is convex univalent and it is the best dominant.

Proof. Let f belong to the class $H_k^\alpha(p,\lambda;h)$. Then $h(0)=1$ and by (1.2), (1.3) we have

$$\begin{aligned}(1-\lambda)\frac{L_k^\alpha f(z)}{z^p}&+\lambda\frac{L_{k+1}^\alpha f(z)}{z^p}\\ &=(1-\lambda)\frac{L_k^\alpha f(z)}{z^p}+\lambda\left\{\frac{z}{\alpha}\left[\frac{L_k^\alpha f(z)}{z^p}\right]'+\frac{L_k^\alpha f(z)}{z^p}\right\}\\ &=\frac{L_k^\alpha f(z)}{z^p}+\frac{\lambda z}{\alpha}\left[\frac{L_k^\alpha f(z)}{z^p}\right]'\prec h(z)\quad (z\in\Delta).\end{aligned} \tag{2.3}$$

Now, by using Lemma 1 for (2.3) and $\gamma=\frac{\alpha}{\lambda}$, $\lambda\neq 0$ we obtain (2.2). For $\lambda\neq 0$ the condition (2.2) follows (2.1) while for $\lambda=0$ (1.3) follows (2.1). ■

If we take $L_0 f(z) = f(z) * \sum\limits_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{p+n}$ and $h(z) = \frac{1+Az}{1+Bz}$, $(-1 \leq B < A \leq 1)$, $(c - 1 = \alpha)$, then from the Theorem 1 we obtain the subordination (2.2) with

$$g(z) = \begin{cases} \frac{A}{B} + \left(1 - \frac{A}{B}\right)(1+Bz)^{-1} \ {}_2F_1\left(1, 1; \frac{c-1}{\lambda} + 1; \frac{Bz}{Bz+1}\right), & (B \neq 0) \\ 1 - \frac{c-1}{c-1+\lambda} Az, & (B = 0) \end{cases}$$

given by J. Patel and P. Sahoo in [13], where ${}_2F_1$ is the hypergeometric function

$${}_2F_1(z) = {}_2F_1(\alpha_1, \alpha_2; \beta_1; z) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n}{(\beta_1)_n n!} z^n \quad (z \in \Delta).$$

THEOREM 2. *If* $0 \leq \lambda_1 \leq \lambda_2$, $\mathrm{Re}[\alpha] > 0$, *then* $H_k^\alpha(p, \lambda_2; h) \subset H_k^\alpha(p, \lambda_1; h)$.

Proof. Let $f \in H_k^\alpha(p, \lambda_2; h)$. A simple computation gives

$$(2.4) \quad (1-\lambda_1)\frac{L_k^\alpha f(z)}{z^p} + \lambda_1 \frac{L_{k+1}^\alpha f(z)}{z^p} = \left(1 - \frac{\lambda_1}{\lambda_2}\right) \frac{L_k^\alpha f(z)}{z^p} + \frac{\lambda_1}{\lambda_2}\left\{(1-\lambda_2)\frac{L_k^\alpha f(z)}{z^p} + \lambda_2 \frac{L_{k+1}^\alpha f(z)}{z^p}\right\}.$$

By (1.3) and (2.1) the sum (2.4) is a convex combination of functions subordinated to the convex function h hence (2.4) is subordinated to h and $f \in H_k^\alpha(p, \lambda_1; h)$. ∎

THEOREM 3. *If* $k \in \mathbb{N}$, $\mathrm{Re}[\alpha] > 0$, *then* $H_k^\alpha(p, \lambda; h) \subset H_{k-1}^\alpha(p, \lambda; h)$.

Proof. Let $f \in H_k^\alpha(p, \lambda; h)$. Then by (1.2) we have

$$\left[(1-\lambda)\frac{L_{k-1}^\alpha f(z)}{z^p} + \lambda \frac{L_k^\alpha f(z)}{z^p}\right] + \frac{z}{\alpha}\left[(1-\lambda)\frac{L_{k-1}^\alpha f(z)}{z^p} + \lambda \frac{L_k^\alpha f(z)}{z^p}\right]'$$

$$= (1-\lambda)\left\{\frac{L_{k-1}^\alpha f(z)}{z^p} + \frac{z}{\alpha}\left[\frac{L_{k-1}^\alpha f(z)}{z^p}\right]'\right\} + \lambda\left\{\frac{L_k^\alpha f(z)}{z^p} + \frac{z}{\alpha}\left[\frac{L_k^\alpha f(z)}{z^p}\right]'\right\}$$

$$= \left[(1-\lambda)\frac{L_k^\alpha f(z)}{z^p} + \lambda \frac{L_{k+1}^\alpha f(z)}{z^p}\right] \prec h(z) \quad (z \in \Delta).$$

Using Lemma 1 for $\lambda = \alpha \geq 0$ we deduce that

$$\left[(1-\lambda)\frac{L_{k-1}^\alpha f(z)}{z^p} + \lambda \frac{L_k^\alpha f(z)}{z^p}\right] \prec \frac{\alpha}{z^\alpha}\int_0^z t^{\alpha-1} h(t)\,dt \prec h(z) \quad (z \in \Delta)$$

and the assertion results from definition of the class $H_{k-1}^\alpha(p, \lambda; h)$. ∎

COROLLARY 1. *Let* $\operatorname{Re}[\alpha]>0$. *If* $f\in H_k^\alpha(p,\lambda;h)$, *then for* $s\in\{0,1,2,\dots,k\}$

$$\frac{L_s^\alpha f(z)}{z^p}\prec h(z)\quad(z\in\Delta). \tag{2.5}$$

Proof. The condition (2.5) we obtain from Theorem 3 and (2.2). ∎

In order to get the convolution results of the class H_k^α, it is necessary to put some restrictions on the operator L_k^α:

$$L_k^\alpha(f*g)=(L_k^\alpha f)*g=f*(L_k^\alpha g),\ \ f,g\in H_k^\alpha(p,\lambda;h),k\in\mathbb{N}. \tag{2.6}$$

We begin by recalling the following result which we shall apply in proving our first convolution theorem.

LEMMA 2 ([17]). *Let* q,h *be analytic in* Δ *with* $q(0)=1$ *and* $\operatorname{Re}[q(z)]>\frac{1}{2}$ *in* Δ. *Then*

$$(q*h)(\Delta)\in\operatorname{co}[h(\Delta)],$$

where $\operatorname{co}[h(\Delta)]$ *is the convex hull of* $h(\Delta)$.

THEOREM 4. *Let the operator* L_k^α *satisfy* (2.6). *If* $f\in H_k^\alpha(p,\lambda;h)$ *and* $q\in A(p)$ *with* $\operatorname{Re}\left[\frac{q(z)}{z^p}\right]\ge\frac{1}{2}$ *in* Δ, *then* $f*q\in H_k^\alpha(p,\lambda;h)$.

Proof. Using the properties of convolution and (2.6) we have

$$\begin{aligned}(1-\lambda)&\frac{L_k^\alpha(f*q)(z)}{z^p}+\lambda\frac{L_{k+1}^\alpha(f*q)(z)}{z^p}\\&=\left\{(1-\lambda)\frac{L_k^\alpha f(z)}{z^p}+\lambda\frac{L_{k+1}^\alpha f(z)}{z^p}\right\}*\frac{q(z)}{z^p}.\end{aligned} \tag{2.7}$$

By Lemma 2, the convolution (2.7) takes values in the convex hull of image of Δ under the function h. Moreover, $\operatorname{co}[h(\Delta)]=h(\Delta)$ because h is convex univalent. It is known that the conditions $g_1(0)=g_2(0)$, $g_1(\Delta)\subset g_2(\Delta)$, g_2 is univalent in Δ, follow $g_1\prec g_2$, $(z\in\Delta)$. Then (2.7) is subordinated to h and we get the required result. ∎

LEMMA 3 ([14]). *If* $f\prec F,g\prec G$ *and* F,G *are convex function then* $f*g\prec F*G,z\in\Delta$.

THEOREM 5. *Let the operator* L_k^α *satisfy (2.6). If* $f_1\in H_k^\alpha(p,\lambda;h_1)$ *and* $f_2\in H_k^\alpha(p,\lambda;h_2)$, *then*

$$g(z)=(1-\lambda)L_k^\alpha(f_1*f_2)(z)+\lambda L_{k+1}^\alpha(f_1*f_2)(z)\in H_k^\alpha(p,\lambda;h_1*h_2), \tag{2.8}$$

$$L_k^\alpha(f_1*f_2)(z)\in H_k^\alpha(p,\lambda;h_1*h_2), \tag{2.9}$$

$$\frac{L_k^\alpha\left[L_k^\alpha(f_1*f_2)(z)\right]}{z^p}\prec(h_1*h_2)(z)\quad(z\in\Delta). \tag{2.10}$$

Proof. Since $f_1 \in H_k^\alpha(p,\lambda;h_1)$, $f_2 \in H_k^\alpha(p,\lambda;h_2)$, hence by (1.3)

$$(1-\lambda)\frac{L_k^\alpha f_1(z)}{z^p} + \lambda\frac{L_{k+1}^\alpha f_1(z)}{z^p} \prec h_1(z) \quad (z \in \Delta), \tag{2.11}$$

$$(1-\lambda)\frac{L_k^\alpha f_2(z)}{z^p} + \lambda\frac{L_{k+1}^\alpha f_2(z)}{z^p} \prec h_1(z) \quad (z \in \Delta). \tag{2.12}$$

Furthermore, from (2.11), (2.12), and (2.1) we have

$$\frac{L_k^\alpha f_1(z)}{z^p} \prec h_1(z) \quad (z \in \Delta), \tag{2.13}$$
$$\frac{L_k^\alpha f_2(z)}{z^p} \prec h_2(z) \quad (z \in \Delta).$$

Thus, by making use of (2.6), (2.11), (2.12) and Lemma 3 we get

$$\begin{aligned}(1-\lambda)&\frac{L_k^\alpha\left[(1-\lambda)L_k^\alpha(f_1*f_2)(z)+\lambda L_{k+1}^\alpha(f_1*f_2)(z)\right]}{z^p}\\ &\quad+\lambda\frac{L_{k+1}^\alpha\left[(1-\lambda)L_k^\alpha(f_1*f_2)(z)+\lambda L_{k+1}^\alpha(f_1*f_2)(z)\right]}{z^p}\\ &=(1-\lambda)\frac{L_k^\alpha g(z)}{z^p}+\lambda\frac{L_{k+1}^\alpha g(z)}{z^p}\prec(h_1*h_2)(z)\end{aligned}$$

that is $g \in H_k^\alpha(p,\lambda;h_1*h_2)$. This completes the proof of (2.8). In order to verify (2.9) we use Lemma 3 to (2.11) and (2.13) and we obtain

$$(1-\lambda)\frac{L_k^\alpha\left[L_k^\alpha(f_1*f_2)(z)\right]}{z^p}+\lambda\frac{L_{k+1}^\alpha\left[L_k^\alpha(f_1*f_2)(z)\right]}{z^p}\prec(h_1*h_2)(z). \tag{2.14}$$

which clearly shows (2.9). Finally (2.14) and (2.1) follow (2.10). ∎

For $h_i(z) = \frac{1+A_i z}{1+B_i z}$, $i = 1, 2$ and

$$H_k^\alpha f(z) = f(z) *_q F_s(z)$$

we obtain from Theorem 5 the earlier result of the paper [9], where $\alpha = \alpha_1$, $k = 0$ and ${}_qF_s$ is the generalized hypergeometric function

$${}_qF_s(z) = {}_qF_s(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_s;z) = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_s)_n}\frac{z^n}{n!}$$

for complex parameters $\alpha_1,\ldots,\alpha_q$ and $\beta_1,\ldots,\beta_s$ ($\beta_j \neq 0,-1,-2,\ldots;\ j = 1,\ldots,s$), $q \le s+1$, $q,s \in \mathbb{N}_0 = \mathbb{N}\cup\{0\}$.

3. Concluding remarks

The results presented in this paper can lead to several applications (studied by various authors) by appropriately selecting a linear operator $L_0 : A(p) \to A(p)$, $k \in \mathbb{N}$ and a convex univalent function h.

The examples of the operators:

1. $L_0^\alpha f(z) = \frac{z}{p} f'(z)$, $\alpha = p - 1$.

2. $L_0^\alpha f(z) = \frac{\delta+p}{z^\delta} \int_0^z t^{\delta-1} f(t)dt$, $\alpha = \delta + p$, (the generalized Bernardi-Libera-Livingston operator), [13].

3. $L_0^\alpha f(z) = \Gamma^{-1}(p+1)\Gamma(p+1-\mu) z^\mu D_z^\mu f(z)$, $\alpha = p - \mu$, where $D_z^\mu f$ is the fractional derivative of f of order μ considered by S. Owa, Γ is the Gamma function.

4. $L_0^\alpha f(z) = f(z) * z^p \cdot {}_q F_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z) =$
$f(z) * z^p \sum\limits_{n=0}^{\infty} \frac{(\alpha_1)_n \cdot \ldots \cdot (\alpha_q)_n}{(\beta_1)_n \cdot \ldots \cdot (\beta_s)_n} \frac{z^n}{n!}$, $\alpha = \alpha_i$, $i \in \{1, \ldots, q\}$, [4], [8], [9], [16].

5. $L_0^\alpha f(z) = f(z) * z^p \sum\limits_{n=0}^{\infty} \frac{(\mu)_{n+1}}{(n+1)!} \left[\frac{\lambda}{n+1+\lambda}\right]^\beta$, $\mu > 0, \lambda > 0$, $\beta \geq 0$, [2], [12].

The examples of the convex functions h:

1. If $h(z) = \frac{1+(1-2\alpha)z}{1-z}$, $(0 \leq \alpha < 1)$, then $h(\Delta) = \{z : \mathrm{Re} z > 0\}$.

2. If $h(z) = \left[\frac{1+z}{1-z}\right]^\alpha$, $h(0) = 1$, $(0 < \alpha < 1)$, then $h(\Delta) = \{z : |\mathrm{Arg} z| < \frac{\alpha\pi}{2}\}$.

3. If $h(z) = 1 + \frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2$, $\mathrm{Im}\sqrt{z} > 0$, then $h(\Delta)$ is the interior of the parabola $\{z : (\mathrm{Im} z)^2 = 2\mathrm{Re} z - 1\}$.

4. If $h(z) = \frac{M(1+z)}{M+(1-M)z}$, $M > \frac{1}{2}$, then $h(\Delta) = \{z : |2z - M| < M\}$.

5. If $h(z) = \sqrt{z+1}$, $\mathrm{Re}\sqrt{z+1} \geq 0$, then $h(\Delta)$ is the interior of the right half of the Bernoulli lemniscate.

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