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ON (α,β)-DERIVATIONS OF SEMIPRIME RINGS, II

Abstract. Let α,β be centralizing automorphisms of a semiprime ring R. Then we show that: (i) If R is 2-torsion free and 3-torsion free and d is an (α,β)-derivation of R such that the mapping $x \to [d(x),x]$ is centralizing on R, then d is commuting and $d(u)[x,y] = 0$ for all $x,y,u \in R$; in particular, d is central. (ii) Let R be 2-torsion free and d,g be (α,β)-derivations of R such that d commutes with both α and β and the mapping $x \to d^2(x) + g(x)$ is centralizing on R, then d and g are both commuting and $d(u)[x,y] = 0 = g(u)[x,y]$ for all $x,y,u \in R$; in particular d and g are central. (iii) If R admits an (α,β)-derivation d which is strong commutativity-preserving on R, then R is commutative. (iv) An additive mapping d on R is an (α,β)-reverse derivation if and only if it is a central (α,β)-derivation. We also show that if α,β are automorphisms and d an (α,β)-reverse derivation on R which is strong commutativity-preserving, then R is commutative.

1. Introduction

Throughout, R denotes a ring with center $Z(R)$. We write $[x,y]$ for $xy-yx$. Then $[xy,z] = x[y,z]+[x,z]y$ and $[x,yz] = y[x,z]+[x,y]z$ hold in R. R is *prime* if $aRb = (0)$ implies either $a = 0$ or $b = 0$; it is *semiprime* if $aRa = (0)$ implies $a = 0$. A prime ring is obviously semiprime. An additive mapping d from R into itself is called a *derivation* (reverse derivation) if $d(xy) = xd(y)+d(x)y(d(xy) = d(y)x+yd(x))$ for all $x,y \in R$. A mapping f from R into itself is *commuting* if $[f(x),x] = 0$; and *centralizing* if $[f(x),x] \in Z(R)$ for all $x \in R$. If f is commuting then it is trivially centralizing but the converse is not true, in general. However, if f is a centralizing automorphism or a centralizing derivation of a semiprime ring, then it is commuting [1, Lemmas 2 and 4]. We call a mapping $f : R \to R$ *central* if $f(x) \in Z(R)$ for all $x \in R$. Every central mapping is obviously commuting but not conversely,

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in general. Recall that if f is an additive commuting mapping from R into itself, then a linearization of $[f(x), x] = 0$ yields $[f(x), y] = [x, f(y)]$ for all $x, y \in R$. A mapping $f : R \to R$ is called strong commutativity preserving (SCP) on a set $S \subseteq R$ if $[f(x), f(y)] = [x, y]$ for all $x, y \in S$. For more information on SCP, we refer to [2, 6] and references therein.

Derivations are generalized as α-derivations and (α, β)-derivations, and have been extensively studied in pure algebra. They have played an important role in the solution of some functional equations (see, e.g., Brešar [4] and references therein). Recently, α-derivations have been used in [7] in connection with the noncommutative Singer-Wermer conjecture for derivations on Banach algebras. Let α, β be automorphisms of R. An additive mapping d of R into itself is called an α-derivation (α-reverse derivation) if $d(xy) = \alpha(x)d(y) + d(x)y (d(xy) = d(y)\alpha(x) + yd(x))$ for all $x, y \in R$. It is called an (α, β)-derivation ((α, β)-reverse derivation) if $d(xy) = \alpha(x)d(y) + d(x)\beta(y) (d(xy) = d(y)\alpha(x) + \beta(y)d(x))$ for all $x, y \in R$. Of course a 1-derivation (1-reverse derivation) or a $(1, 1)$-derivation ($(1, 1)$-reverse derivation) is a derivation (reverse derivations), where 1 is the identity mapping of R. Simple calculations show that $\alpha - 1$ is an α-derivation and $\alpha - \beta$ is an (α, β)-derivation. For more information on α-derivations and (α, β)-derivations, we refer to [4, 5, 7–11, 13, 14]. For information on reverse derivations, we refer to [3].

This research is inspired by the work of Vukman [15, 16]. Vukman [16] has proved the following results:

THEOREM A. *Let R be a 2-torsion free and 3-torsion free semiprime ring and $d : R \to R$ a derivation. Suppose that the mapping $x \to [d(x), x]$ is centralizing on R. In this case d is commuting on R.*

THEOREM B. *Let R be a 2-torsion free semiprime ring and $d : R \to R, g : R \to R$ be derivations. Suppose that the mapping $x \to d^2(x) + g(x)$ is centralizing on R. In this case D and G are both commuting on R.*

The purpose of this paper is to establish analogues of Theorems A and B, as well as to prove some other results, for (α, β)-derivations of semiprime rings. We prove the following:

(i) Let α, β be centralizing automorphisms and d an (α, β)-derivation of a 2-torsion free and 3-torsion free semiprime ring R such that the mapping $x \to [d(x), x]$ is centralizing on R. Then d is commuting and $d(u)[x, y] = 0$ for all $x, y, u \in R$; in particular d is central.

(ii) Let α, β be centralizing automorphisms and d, g be (α, β)-derivations of a 2-torsion free semiprime ring R such that the mapping $x \to d^2(x) + g(x)$ is centralizing on R. If d commutes with both α and β, then d, g are commuting and $d(u)[x,y] = 0 = g(u)[x,y]$ for all $x, y, u \in R$; in particular d and g are central.

(iii) Let α, β be centralizing automorphisms and d an (α,β)-derivation of a semiprime ring R, respectively. If d is strong commutativity-preserving on R, then R is commutative.

(iv) An additive mapping d on a semiprime ring R is an (α,β)-reverse derivation if and only if it is a central (α,β)-derivation.

We also show that if α, β are automorphisms of a semiprime ring R and R admits an (α,β)-reverse derivation d which is also strong commutativity-preserving, then R is commutative.

We shall need the following results of Chaudhry and Thaheem in the sequel.

THEOREM C [10, Theorem 2.3]. *Let α, β be centralizing automorphisms and d an (α,β)-derivation of a 2-torsion free semiprime ring R, respectively, such that $[[d(x),x],x] = 0$ for all $x \in R$, then d is commuting and $d(u)[x,y] = 0$ for all $x, y, u \in R$; in particular d is central.*

THEOREM D [9, Proposition 2.3]. *Let β be a centralizing automorphism and d a commuting (α,β)-derivation of a semiprime ring R. Then $d(u)[x,y] = 0$ for all $x, y, u \in R$; in particular d maps R into its center.*

2. The results

We now prove our main results.

THEOREM 2.1. *Let α, β be centralizing automorphisms and d an (α,β)-derivation of a 2-torsion free and 3-torsion free semiprime ring R, such that the mapping $x \to [d(x),x]$ is centralizing on R. Then d is commuting and $d(u)[x,y] = 0$ for all $x, y, u \in R$; in particular d is central.*

Proof. According to the hypothesis we have

$$(1) \qquad [[d(x),x],x] \in Z(R) \text{ for all } x \in R.$$

Linearizing (1) (and using (1) again), we get

$$(2) \qquad [[d(x),x],y] + [[d(x),y],x] + [[d(x),y],y] + [[d(y),x],x] + [[d(y),x],y] + [[d(y),y],x] \in Z(R) \text{ for all } x, y \in R.$$

Replacing x by $-x$ in (2), we get

$$(3) \qquad [[d(x),x],y] + [[d(x),y],x] - [[d(x),y],y] + [[d(y),x],x] - [[d(y),x],y] - [[d(y),y],x] \in Z(R) \text{ for all } x, y \in R.$$

Adding (2) and (3) and using the hypothesis that R is 2-torsion free, we get

$$(4) \qquad [[d(x),x],y] + [[d(x),y],x] + [[d(y),x],x] \in Z(R) \text{ for all } x, y \in R.$$

Replacing y by x^2 in (4), we get $[[d(x),x],x^2]+[[d(x),x^2],x]+[[d(x^2),x],x] = x[[d(x),x],x] + [[d(x),x],x]x + [x[d(x),x] + [d(x),x]x,x] + [[\alpha(x)d(x) + d(x)\beta(x),x],x] \in Z(R)$, for all $x \in R$. That is,

$$(5)\quad x[[d(x),x],x]+[[d(x),x],x]x+x[[d(x),x],x]+[[d(x),x],x]x+[(\alpha(x)[d(x),x] + [\alpha(x),x]d(x)+[d(x),x]\beta(x)+d(x)[\beta(x),x]),x] \in Z(R) \text{ for all } x \in R.$$

Since α and β are centralizing, therefore commuting by [1, Lemma 2]. Thus $\alpha - 1$ and $\beta - 1$ are commuting α-derivation and β-derivation, respectively. Hence by [14, Proposition 2.3],

$$(6)\quad \begin{cases} \alpha(u)[x,y] = u[x,y]; [x,y]\alpha(u) = [x,y]u \text{ for all } x,y,u \in R, \\ \beta(u)[x,y] = u[x,y]; [x,y]\beta(u) = [x,y]u \text{ for all } x,y,u \in R, \quad \text{and} \\ \alpha(u) - u \in Z(R), \beta(u) - u \in Z(R) \text{ for all } u \in R. \end{cases}$$

Thus, $0 = [\alpha(u) - u, x] = [\alpha(u), x] - [u, x]$ for all $u, x \in R$. That is,

$$(7)\quad [\alpha(u), x] = [u, x] \text{ for all } x, u \in R.$$

Similarly,

$$(8)\quad [\beta(u), x] = [u, x] \text{ for all } x, u \in R.$$

Using (6) - (8) and the hypothesis $[[d(x),x],x] \in Z(R)$, from (5) we get $4x[[d(x),x],x] + [x[d(x),x] + [d(x),x]x, x] = 4x[[d(x),x],x] + x[[d(x),x],x] + [[d(x),x],x]x = 6x[[d(x),x],x] \in Z(R)$. Thus we have

$$(9)\quad 6x[[d(x),x],x] = 6[[d(x),x],x]x \in Z(R) \text{ for all } x \in R.$$

Since R is 2-torsion free and 3-torsion free, therefore $[[d(x),x],x]x \in Z(R)$, which gives $[[d(x),x],x]x,y] = 0$. That is,

$$(10)\quad [d(x),x],x][x,y] = 0 \text{ for all } x,y \in R.$$

Replacing y by $y[d(x),x]$ in (10) (and using (10) again), we get $0 = [[d(x),x],x]y[x,[d(x),x]] = -[[d(x),x],x]y[[d(x),x],x]$. Since R is semiprime, we get

$$(11)\quad [[d(x),x],x] = 0 \text{ for all } x \in R,$$

which by Theorem C implies that

$$(12)\quad d(u) \in Z(R) \text{ and } d(u)[x,y] = 0 \text{ for all } x,y,u \in R. \blacksquare$$

REMARK 2.2. (i) It will be interesting to prove the above theorem without one or both assumptions: R is 2-torsion free and 3-torsion free.

(ii) Since derivations are $(1,1)$-derivations, therefore Theorem A becomes a special case of Theorem 2.1.

THEOREM 2.3. *Let α, β be centralizing automorphisms and d and g be (α,β)-derivations of a 2-torsion free semiprime ring R such that the mapping $x \to$*

$d^2(x) + g(x)$ is centralizing on R. If d commutes with both α and β, then d and g are both commuting and $d(u)[x, y] = 0 = g(u)[x, y]$ for all $x, y, u \in R$; in particular d and g are central.

Proof. Since R is 2-torsion free and the mapping $x \to d^2(x) + g(x)$ is centralizing, therefore by Brešar [5, Proposition 3.1], it is commuting. Thus we have

$$[d^2(x) + g(x), x] = 0 \text{ for all } x \in R. \tag{13}$$

Linearizing (13) (and using (13) again), we get

$$[d^2(x) + g(x), y] + [d^2(y) + g(y), x] = 0 \text{ for all } x, y \in R. \tag{14}$$

Replacing y by yx in (14), we get

$$[d^2(x) + g(x), yx] + [d^2(yx) + g(yx), x] = 0 \text{ for all } x, y \in R. \tag{15}$$

Now,

$$\begin{aligned}[d^2(yx) + g(yx), x] &= [d(d(yx)) + g(yx), x] \\ &= [d(\alpha(y)d(x) + d(y)\beta(x)) + \alpha(y)g(x) + g(y)\beta(x), x] \\ &= [\alpha^2(y)d^2(x) + d(\alpha(y))\beta(d(x)) + \alpha(d(y))d(\beta(x) \\ &\quad + d^2(y)\beta^2(x) + \alpha(y)g(x) + g(y)\beta(x), x] \\ &= [\alpha^2(y)d^2(x) + \alpha(d(y))\beta(d(x)) + \alpha(d(y))\beta(d(x)) \\ &\quad + d^2(y)\beta^2(x) + \alpha(y)g(x) + g(y)\beta(x), x].\end{aligned}$$

That is,

$$\begin{aligned}[d^2(yx) + g(yx), x] &= [\alpha^2(y)d^2(x) + \alpha(y)g(x), x] + [d^2(y)\beta^2(x) \\ &\quad + g(y)\beta(x), x] + 2[\alpha(d(y))\beta(d(x)), x] \text{ for all } x, y \in R.\end{aligned} \tag{16}$$

Since α is commuting, therefore $[\alpha(x), x] = 0$, which after linearization gives

$$[\alpha(x), y] + [\alpha(y), x] = 0 \text{ for all } x, y \in R. \tag{17}$$

Replacing y by $\alpha(x)$ in (17) we get $[\alpha^2(x), x] = 0$ for all $x \in R$. Hence α^2 is a commuting automorphism. Similarly, one can show that β^2 is a commuting automorphism. Thus $\alpha^2 - 1$ and $\beta^2 - 1$ are commuting α^2-derivation and β^2-derivation, respectively. Using (6)–(8) for α, β, α^2 and β^2, from (16) we get

$$\begin{aligned}[d^2(yx) + g(yx), x] &= \alpha^2(y)[d^2(x), x] + [\alpha^2(y), x]d^2(x) + \alpha(y)[g(x), x] \\ &\quad + [\alpha(y), x]g(x) + [d^2(y), x]\beta^2(x) + d^2(y)[\beta^2(x), x] + g(y)[\beta(x), x] \\ &\quad + [g(y), x]\beta(x) + 2\alpha(d(y))[\beta(d(x)), x] + 2[\alpha(d(y)), x]\beta(d(x)) \\ &= y[d^2(x), x] + [y, x]d^2(x) + y[g(x), x] + [y, x]g(x) + [d^2(y), x]x \\ &\quad + [g(y), x]x + 2d(y)[d(x), x] + 2[d(y), x]d(x) \\ &= y[d^2(x) + g(x), x] + [y, x](d^2(x) + g(x)) + [d^2(y) + g(y), x]x \\ &\quad + 2d(y)[d(x), x] + 2[d(y), x]d(x) \text{ for all } x, y \in R.\end{aligned}$$

Replacing this in (15), we get

(18) $[d^2(x)+g(x),y]x+y[d^2(x)+g(x),x]+[y,x](d^2(x)+g(x))+[d^2(y)$
$+g(y),x]x+2d(y)[d(x),x]+2[d(y),x]d(x)=0$ for all $x,y\in R$.

Using (13) and (14), from (18) we get

(19) $[y,x](d^2(x)+g(x))+2[d(y),x]d(x)+2d(y)[d(x),x]=0$
for all $x,y\in R$.

Replacing y by xy in (19), we get

$$\begin{aligned}0&=[xy,x](d^2(x)+g(x))+2[d(xy),x]d(x)+2d(xy)[d(x),x]\\&=x[y,x](d^2(x)+g(x))+2[\alpha(x)d(y)+d(x)\beta(y),x]d(x)\\&\quad+2(\alpha(x)d(y)+d(x)\beta(y))[d(x),x]\\&=x[y,x](d^2(x)+g(x))+2\alpha(x)[d(y),x]d(x)+2[\alpha(x),x]d(x)\\&\quad+2d(x)[\beta(y),x]d(x)+2[d(x),x]\beta(y)d(x)+2((\alpha(x)-x)dy+xd(y)\\&\quad+d(x)\beta(y))[d(x),x]\\&=x[y,x](d^2(x)+g(x))+2x[d(y),x]d(x)+2d(x)[y,x]d(x)+2[d(x),x]yd(x)\\&\quad+2(d(y)(\alpha(x)-x)+xd(y)+d(x)\beta(y)[d(x),x]\\&=x[y,x](d^2(x)+g(x))+2x[d(y),x]d(x)+2d(x)[y,x]d(x)+2[d(x),x]yd(x)\\&\quad+2xd(y)[d(x)x]+2d(x)y[d(x)x].\end{aligned}$$

That is,

(20) $x[y,x](d^2(x)+g(x))+2x[d(y),x]d(x)+2d(x)[y,x]d(x)$
$+2[d(x),x]yd(x)+2xd(y)[d(x),x]+2d(x)y[d(x),x]=0$ for all $x,y\in R$.

Using (19), from (20) we get $0=2d(x)[y,x]d(x)+2[d(x),x]yd(x)+2d(x)y[d(x),x]$, which gives

(21) $[d(x),x]yd(x)+d(x)[y,x]d(x)+d(x)y[d(x),x]=0$ for all $x,y\in R$.

Replacing y by $yd(x)z$ in (21), we get $0=[d(x),x]yd(x)zd(x)+d(x)[yd(x)z,x]d(x)+d(x)yd(x)z[d(x),x]$. That is,

(22) $[d(x),x]yd(x)zd(x)+d(x)[y,x]d(x)zd(x)+d(x)y[d(x),z]zd(x)$
$+d(x)yd(x)[z,x]d(x)+d(x)yd(x)z[d(x),x]=0$ for all $x,y,z\in R$.

Using (21), from (22) we get $d(x)yd(x)[z,x]d(x)+d(x)yd(x)z[d(x),x]=0$, which (using (21) again) reduces to

(23) $d(x)y[d(x),x]zd(x)=0$ for all $x,y,z\in R$.

Multiplying (23) by $y[d(x),x]$ on right, we get

$0 = d(x)y[d(x),x]zd(x)y[d(x),x]$, which, by semiprimeness of R, gives

(24) $$d(x)y[d(x),x] = 0 \text{ for all } x, y \in R.$$

Replacing y by xy in (24), we get

(25) $$d(x)xy[d(x),x] = 0 \text{ for all } x, y \in R.$$

Left multiplication of (24) by x gives

(26) $$xd(x)y[d(x),x] = 0 \text{ for all } x, y \in R.$$

Subtracting (26) from (25), we get $[d(x),x]y[d(x),x] = 0$, which, by semiprimeness of R, implies

(27) $$[d(x),x] = 0 \text{ for all } x \in R.$$

Linearizing (27), we get $[d(x),y]+[d(y),x] = 0$ for all $x, y \in R$. In particular for $y = d(x)$, we get

(28) $$[d^2(x),x] = 0 \text{ for all } x \in R.$$

Using (28), from (13) we get $[g(x),x] = 0$ for all $x \in R$. Thus d and g are both commuting (α,β)-derivations. Hence, using Theorem C, we get $d(u), g(u) \in Z(R)$ and $d(u)[x,y] = 0 = g(u)[x,y]$ for all $x, y, u \in R$. ∎

REMARK 2.4. Taking $\alpha = \beta = 1$ in Theorem 2.3, we note that Theorem B is a special case of Theorem 2.3.

PROPOSITION 2.5. *Let α,β be centralizing automorphisms of a semiprime ring R. If R admits an (α,β)-derivation d which is strong commutativity-preserving on R, then R is commutative.*

Proof. Brešar (in the proof of Theorem 1 [6, page 458]) has proved that every additive strong commutativity-preserving mapping on a semiprime ring is commuting. Thus d is a commuting (α,β)-derivation. By Theorem D we conclude that d is central. Thus, $0 = [d(x),d(y)] = [x,y]$ for all $x, y \in R$. Hence R is commutative. ∎

REMARK 2.6. Taking $\alpha = \beta = 1$ in Proposition 2.5, we get the result of Bell and Daif [2, Corollary 1] which states: If R is a semiprime ring admitting a derivation which is strong commutativity-preserving on R, then R is commutative.

We now establish a characterization of (α,β)-reverse derivations of semiprime rings.

THEOREM 2.7. *Let α,β be automorphisms of a semiprime ring R. An additive mapping $d : R \to R$ is an (α,β)-reverse derivation if and only if it is a central (α,β)-derivation.*

Proof. Let d be an (α,β)-reverse derivation. Then

$$d(xy^2) = d(y^2)\alpha(x)+\beta(y^2)d(x) = (d(y)\alpha(y)+\beta(y)d(y))\alpha(x)+\beta(y)\beta(y)d(x)$$
$$= d(y)\alpha(y)\alpha(x)+\beta(y)d(y)\alpha(x)+\beta(y)\beta(y)d(x).$$

That is,

(29) $d(xy^2) = d(y)\alpha(y)\alpha(x)+\beta(y)d(y)\alpha(x)+\beta(y)\beta(y)d(x)$ for all $x, y \in R$.

Also,

$$\begin{aligned} d((xy)y) &= d(y)\alpha(xy) + \beta(y)d(xy) \\ &= d(y)\alpha(x)\alpha(y) + \beta(y)[d(y)\alpha(x) + \beta(y)d(x)] \\ &= d(y)\alpha(x)\alpha(y) + \beta(y)d(y)\alpha(x) + \beta(y)\beta(y)d(x). \end{aligned}$$

That is,

(30) $d((xy)y) = d(y)\alpha(x)\alpha(y)+\beta(y)d(y)\alpha(x)+\beta(y)\beta(y)d(x)$ for all $x, y \in R$.

From (29) and (30), we get $d(y)\alpha(x)\alpha(y) = d(y)\alpha(y)\alpha(x)$. Thus,

(31) $$d(y)[\alpha(x), \alpha(y)] = 0 \text{ for all } x, y \in R.$$

Since α is onto, therefore (31) gives $0 = d(y)[x, \alpha(y)] = d(\alpha^{-1}(y))[x, y]$. Thus,

(32) $$d(\alpha^{-1}(y))[x, y] = 0 \text{ for all } x, y \in R.$$

Replacing x by zx in (32) (and using (32) again), we get

$$\begin{aligned} 0 &= d(\alpha^{-1}(y))[zx, y] = d(\alpha^{-1}(y))z[x, y] + d(\alpha^{-1}(y))[z, y]x \\ &= d(\alpha^{-1}(y))z[x, y]. \end{aligned}$$

That is,

(33) $$d(\alpha^{-1}(y))z[x, y] = 0 \text{ for all } x, y, z \in R.$$

Linearizing (32) in y (and using (32) again), we get

$$\begin{aligned} 0 &= d(\alpha^{-1}(y+u))[x, y+u] = (d\alpha^{-1}(y) + d\alpha^{-1}(u))([x, y] + [x, u]) \\ &= d(\alpha^{-1}(y))[x, y] + d(\alpha^{-1}(y))[x, u] + d(\alpha^{-1}(u))[x, y] + d(\alpha^{-1}(u))[x, u] \\ &= d(\alpha^{-1}(y))[x, u] + d(\alpha^{-1}(u))[x, y]. \end{aligned}$$

That is,

(34) $$d(\alpha^{-1}(y))[x, u] = -d(\alpha^{-1}(u))[x, y] \text{ for all } u, x, y \in R.$$

Replacing z by $[x, u]zd(\alpha^{-1}(u))$ in (33) and then using (34), we get $0 = d(\alpha^{-1}(y))[x, u]\ zd(\alpha^{-1}(u))[x, y] = -d(\alpha^{-1}(u))[x, y]zd(\alpha^{-1}(u))[x, y]$, which, by semiprimeness of R, implies

(35) $$d(\alpha^{-1}(u))[x, y] = 0 \text{ for all } x, y, u \in R.$$

Using Herstein [12, Lemma 1.1.8], from (35) we get that $d(\alpha^{-1}(u)) \in Z(R)$ for all $u \in R$. Since α^{-1} is onto, therefore $d(u) \in Z(R)$ for all $u \in R$. Thus

d is central. Now, $d(xy) = d(y)\alpha(x) + \beta(y)d(x) = \alpha(x)d(y) + d(x)\beta(y)$ for all $x, y \in R$. This implies that d is an (α, β)-derivation.

Conversely, let d be a central (α, β)-derivation. Then, $d(xy) = \alpha(x)d(y) + d(x)\beta(y) = d(y)\alpha(x) + \beta(y)d(x)$. Thus d is an (α, β)-reverse derivation. ∎

COROLLARY 2.8. *Let α, β be automorphisms of a semiprime ring R. If R admits an (α, β)-reverse derivation d which is strong commutativity-preserving on R, then R is commutative.*

Proof. By Theorem 2.7, d is central. Since d is strong commutativity-preserving, therefore $[x, y] = [d(x), d(y)] = 0$ for all $x, y \in R$. Hence R is commutative. ∎

Taking $\alpha = \beta = 1$ in Theorem 2.7, the following corollary is immediate.

COROLLARY 2.9. *Let R be a semiprime ring. An additive mapping $d: R \to R$ is a reverse derivation on R if and only if it is a central derivation.*

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