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THE NATURAL AFFINORS ON THE r-JET PROLONGATIONS OF A VECTOR BUNDLE

Abstract. It is known that for integers $m \geq 2$, $n \geq 1$ and $r \geq 3$ there are only three r-jet prolongations of a vector bundle E with m-dimensional bases and n-dimensional fibers. The first one is the usual r-jet prolongation J^rE, the second one is the vertical r-jet prolongation J^r_vE and the third one is the $[r]$-jet prolongation $J^{[r]}E$. In this paper for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classify all natural affinors on F^rE, where F^rE denotes J^rE or J^r_vE or $J^{[r]}E$. As corollaries we obtain similar results for F^rE^*, $(F^rE)^*$ and $(F^rE^*)^*$ instead of F^rE.

Introduction

One can prove (a paper in preparation) that for integers $r \geq 3$ and $m \geq 2$ there are only three r-jet prolongations of a vector bundle E with m-dimensional basis. Namely, we have the usual r-jet prolongation J^rE of E, the vertical r-jet prolongation J^r_vE of E and the $[r]$-jet prolongation $J^{[r]}E$ of E.

In [15] for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classified all natural linear operators A lifting a linear vector field X from a vector bundle E with m-dimensional basis and n-dimensional fibers into a vector field $A(X)$ on F^rE, where F^rE denotes J^rE or J^r_vE or $J^{[r]}E$. In the case $F^rE = J^rE$ we proved that $A(X)$ is a constant multiple of the flow operator $\mathcal{J}^rX$. In the case $F^rE = J^r_vE$ we proved that $A(X)$ is a linear combination of the flow operator $\mathcal{J}^r_vX$ and some explicitely constructed linear natural operator $V^{<1>}(X)$. In the case $F^rE = J^{[r]}E$ we proved that $A(X)$ is a linear combination of the flow operator $\mathcal{J}^{[r]}X$ and some explicitely constructed linear natural operator $U^{(1)}(X)$.

An affinor B on a manifold M is a tensor field of type $(1,1)$ on M.

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A natural affinor B on F^rE is a system of invariant (with respect to vector bundle isomorphisms onto open vector subbundles) affinors $B : TF^rE \to TF^rE$ on F^rE for any vector bundle E with m-dimensdional basis and n-dimensional fibers.

In the present paper for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classify all natural affinors B on F^rE, where F^rE denotes J^rE or J^r_vE or $J^{[r]}E$. In the case $F^rE = J^rE$ we prove that B is a constant multiple of the identity affinor Id on J^rE. In the case $F^rE = J^r_vE$ we proved that B is a linear combination of the identity affinor Id on J^r_vE and some explicitely constructed natural affinor U on J^r_vE. In the case $F^rE = J^{[r]}E$ we prove that B is a linear combination of the identity affinor Id on $J^{[r]}E$ and some explicitely constructed natural affinor V an $J^{[r]}E$. As corollaries we obtain similar results for F^rE^*, $(F^rE)^*$ and $(F^rE^*)^*$ instead of F^rE.

Natural affinors can be used to study torsions of connections, see [5]. That is why they have been classified in many papers, [1]–[4], [6]–[14], [17]. e.t.c.

The category of vector bundles with m-dimensional bases and vector bundle maps with local diffeomorphisms as base maps will be denoted by $\mathcal{VB}_m$.

The category of vector bundles with m-dimensional bases and n-dimensional fibers and vector bundle isomorphisms onto open vector subbundles will be denoted by $\mathcal{VB}_{m,n}$.

The trivial vector bundle $\mathbf{R}^m \times \mathbf{R}^n$ over $\mathbf{R}^m$ with standard fiber $\mathbf{R}^n$ will be denoted by $\mathbf{R}^{m,n}$.

The coordinates on $\mathbf{R}^m$ will be denoted by $x^1, ..., x^m$. The fiber coordinates on $\mathbf{R}^{m,n}$ will be denoted by $y^1, ..., y^n$.

All manifolds are assumed to be finite dimensional and smooth. Maps are assumed to be smooth, i.e. of class $\mathcal{C}^\infty$.

1. The r-jet prolongations of a vector bundle

The r-jet prolongation functor

Given a $\mathcal{VB}_m$-object $p : E \to M$ the r-jet prolongation J^rE of E is a vector bundle

$$J^rE = \{j^r_x\sigma \mid \sigma \text{ is a local section of } E \ , \ x \in M\}$$

over M. Every $\mathcal{VB}_m$-map $f : E_1 \to E_2$ covering $\underline{f} : M_1 \to M_2$ induces a vector bundle map $J^rf : J^rE_1 \to J^rE_2$ covering $\underline{f}$ such that

$$J^rf(j^r_x\sigma) = j^r_{\underline{f}(x)}(f \circ \sigma \circ \underline{f}^{-1}), \quad j^r_x\sigma \in J^rE_1 \ .$$

The functor $J^r : \mathcal{VB}_m \to \mathcal{VB}_m$ is a fiber product preserving vector gauge bundle functor.

The vertical r-jet prolongation functor

Given a $\mathcal{VB}_m$-object $p : E \to M$ the vertical r-jet prolongation $J^r_v E$ of E is a vector bundle

$$J^r_v E = \{j^r_x \sigma \mid \sigma : M \to E_x \ , \ x \in M\}$$

over M. Every $\mathcal{VB}_m$-map $f : E_1 \to E_2$ covering $\underline{f} : M_1 \to M_2$ induces a vector bundle map $J^r_v f : J^r_v E_1 \to J^r_v E_2$ covering $\underline{f}$ such that

$$J^r_v f(j^r_x \sigma) = j^r_{\underline{f}(x)}(f \circ \sigma \circ \underline{f}^{-1}), \quad j^r_x \sigma \in J^r_v E_1 \ .$$

The functor $J^r_v : \mathcal{VB}_m \to \mathcal{VB}_m$ is a fiber product preserving vector gauge bundle functor.

The [r]-jet prolongation functor

Let $p : E \to M$ be a $\mathcal{VB}_m$-object. For any $x \in M$ we have an unital associative algebra homomorphism $t^{[r]}_x : J^r_x(M, \mathbf{R}) \to gl(J^r_x(M, \mathbf{R}))$ given by

$$t^{[r]}_x(j^r_x \gamma)(j^r_x \eta) = j^r_x(\gamma\eta) - j^r_x(\eta(x)\gamma) + j^r_x(\eta(x)\gamma(x)) \ ,$$

$j^r_x\eta, j^r_x\gamma \in J^r_x(M, \mathbf{R})$, $\eta(x), \gamma(x) : M \to \mathbf{R}$ are constant maps. We have a vector bundle

$$J^{[r]}E = \bigcup_{x \in M} Hom_{t^{[r]}_x}(J^r \mathcal{C}^{\infty, f.l}_x(E), J^r_x(M, \mathbf{R}))$$

over M. Here $Hom_{t^{[r]}_x}(J^r \mathcal{C}^{\infty, f.l}_x(E), J^r_x(M, \mathbf{R}))$ is the vector space of all module homomorphisms over $t^{[r]}_x : J^r_x(M, \mathbf{R}) \to gl(J^r_x(M, \mathbf{R}))$ from the (free) $J^r_x(M, \mathbf{R})$-module $J^r(\mathcal{C}^{\infty, f, l}_x(E))$ of r-jets at x of germs at x of fiber linear maps $E \to \mathbf{R}$ into the $gl(J^r_x(M, \mathbf{R}))$-module $J^r_x(M, \mathbf{R})$. We call $J^{[r]}E$ *the* $[r]$-*jet prolongation of* E. Every $\mathcal{VB}_m$-map $f : E_1 \to E_2$ covering $\underline{f} : M_1 \to M_2$ induces a vector bundle map $J^{[r]}f : J^{[r]}E_1 \to J^{[r]}E_2$ covering $\underline{f}$ such that

$$J^{[r]}f(\Phi)(j^r_{\underline{f}(x)}\xi) = J^r(\underline{f}, id_{\mathbf{R}}) \circ \Phi(j^r_x(\xi \circ f))$$

for any $\Phi \in Hom_{t^{[r]}_x}(J^r \mathcal{C}^{\infty, f.l}_x(E), J^r_x(M, \mathbf{R}))$, $x \in M_1$ and any fiber linear map $\xi : E_2 \to \mathbf{R}$. The correspondence $J^{[r]} : \mathcal{VB}_m \to \mathcal{VB}_m$ is a fiber product preserving gauge bundle functor of order r, [16].

REMARK 1. One can show that $J^r E$ and $J^r_v E$ can be constructed similarly as $J^{[r]}E$ using some other algebra homomorphisms $t_x : J^r_x(M, \mathbf{R}) \to gl(J^r_x(M, \mathbf{R}))$ instead of $t^{[r]}_x$. This justifies the name $[r]$-jet prolongation. If $r \geq 3$ and $m \geq 2$ then only $J^r E$, $J^r_v E$ and $J^{[r]}E$ admit such reconstruction (a paper in preparation).

From now on $F^r E$ denotes $J^r E$ or $J^r_v E$ or $J^{[r]}E$.

2. The natural linear operators lifting linear vector fields to F^rE

In this section we will cite some results of [15].

Let $p : E \to M$ be a $\mathcal{VB}_{m,n}$-object. A projectable vector field X on E is called linear if $X : E \to TE$ is a vector bundle map from $p : E \to M$ into $Tp : TE \to TM$. Equivalently, the flow Fl^X_t of X is formed by $\mathcal{VB}_{m,n}$-maps. The space of linear vector fields on E will be denoted by $\mathcal{X}_{lin}(E)$.

A natural linear operator $A : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TF^r$ is an $\mathcal{VB}_{m,n}$-invariant family of $\mathbf{R}$-linear operators $A : \mathcal{X}_{lin}(E) \to \mathcal{X}(F^rE)$ for any $\mathcal{VB}_{m,n}$-object E. The $\mathcal{VB}_{m,n}$-invariance means that for any $\mathcal{VB}_{m,n}$-map $f : E_1 \to E_2$ and any f-conjugate linear vector fields X and Y on E_1 and E_2 the vector fields $A(X)$ and $A(Y)$ are F^rf-conjugate.

EXAMPLE 1. (*The flow operator*) Let X be a linear vector field on a $\mathcal{VB}_{m,n}$-object $p : E \to M$. The flow Fl^X_t of X is formed by $\mathcal{VB}_{m,n}$-maps on E. Applying functor F^r we obtain a flow $F^r(Fl^X_t)$ on F^rE. The vector field $\mathcal{F}^rX$ on F^rE corresponding to the flow $F^r(Fl^X_t)$ is called the flow prolongation of X. The correspondence $\mathcal{F}^r : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TF^r$, $X \to \mathcal{F}^rX$, is a natural linear operator.

EXAMPLE 2. Given a linear vector field X on a $\mathcal{VB}_{m,n}$-object E covering a vector field $\underline{X}$ on M we define a vertical vector field $V^{<1>}(X)$ on J^r_vE as follows. Let $y = j^r_x\sigma \in J^r_vE$, $\sigma : M \to E_x$, $x \in M$. We put

$$V^{<1>}(X)(y) = (y, j^r_x(\underline{X}\sigma(x))) \in \{y\} \times (J^r_v)_xE = V_yJ^r_vE \subset T_yJ^r_vE \ ,$$

where $\underline{X}\sigma(x) : M \to E_x$ is the constant map. The correspondence $V^{<1>} : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TJ^r_v$ is a natural operator.

EXAMPLE 3. Given a linear vector field X on a $\mathcal{VB}_{m,n}$-object E covering a vector field $\underline{X}$ on M and a module homomorphism $\Phi : J^r\mathcal{C}^{\infty,f.l}_x(E) \to J^r_x(M,\mathbf{R})$ over $t^{[r]}_x : J^r_x(M,\mathbf{R}) \to gl(J^r_x(M,\mathbf{R}))$ (i.e. $\Phi \in J^{[r]}_xE$, $x \in M$) we have a linear map $\Phi_X : J^r\mathcal{C}^{\infty,f.l}_x(E) \to J^r_x(M,\mathbf{R})$ given by

$$\Phi_X(\sigma) = j^r_x(\underline{X}\gamma(x)) \ ,$$

$\sigma \in J^r\mathcal{C}^{\infty,f.l}_x(E)$, $\gamma : M \to \mathbf{R}$, $j^r_x\gamma = \Phi(\sigma)$, $\underline{X}\gamma(x) : M \to \mathbf{R}$ is the constant map. The linear map $\Phi_X : J^r\mathcal{C}^{\infty,f.l}_x(E) \to J^r_x(M,\mathbf{R})$ is module homomorphisms over $t^{[r]}_x : J^r_x(M,\mathbf{R}) \to gl(J^r_x(M,\mathbf{R}))$ as easily to verify. Consequently, we have vertical vector field $U^{(1)}(X)$ on $J^{[r]}E$ by

$$U^{(1)}(X)_\Phi = (\Phi, \Phi_X) \in \{\Phi\} \times J^{[r]}_xE = V_\Phi J^{[r]}E \ ,$$

$\Phi \in J^{[r]}_xE$, $x \in M$. The correspondence $U^{(1)} : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TJ^{[r]}$ is a natural linear operator.

In [15] we proved the following classification theorem.

THEOREM 1. ([15]) *Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.*

(a) *Any natural linear operator $A : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TJ^r$ is a constant multiple of the flow operator $\mathcal{J}^r$.*

(b) *Any natural linear operator $A : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TJ^r_v$ is a linear combination with real coefficients of the flow operator $\mathcal{J}^r_v$ and $V^{<1>}$.*

(c) *Any natural linear operator $A : T_{lin|\mathcal{VB}_{m,n}} \rightsquigarrow TJ^{[r]}$ is a linear combination with real coefficients of the flow operator $\mathcal{J}^{[r]}$ and the operator $U^{(1)}$.*

3. Examples of natural affinors on F^rE

A $\mathcal{VB}_{m,n}$-natural affinor B on F^rE is a family of $\mathcal{VB}_{m,n}$-invariant affinors $B : TF^rE \to TF^rE$ for any $\mathcal{VB}_{m,n}$-object E. The invariance means that $B \circ TF^rf = TF^rf \circ B$ for any $\mathcal{VB}_{m,n}$-map f.

EXAMPLE 4. (*The identity affinor*) For any $\mathcal{VB}_{m,n}$-object E we have the identity map $Id : TF^rE \to TF^rE$. The family Id is a $\mathcal{VB}_{m,n}$-natural affinor on F^rE.

EXAMPLE 5. Let $p : E \to M$ be a $\mathcal{VB}_{m,n}$-object. Define $U : TJ^r_vE \to VJ^r_vE$ by

$$U(v) = (y, j^r_x(Tp(v)\sigma)) \in \{y\} \times (J^r_v)_xE \tilde{=} V_yJ^r_vE \ ,$$

$v \in T_yJ^r_vE$, $y = j^r_x\sigma \in (J^r_v)_xE$, $x \in M$. Here $Tp(v)\sigma : M \to E_x$ is the constant map, the differential of $\sigma : M \to E_x$ at $Tp(v)$. The family U is a $\mathcal{VB}_{m,n}$-natural affinor on J^r_vE.

EXAMPLE 6. Let $p : E \to M$ be a $\mathcal{VB}_{m,n}$-object. Define $V : TJ^{[r]}E \to VJ^{[r]}E$ by

$$V(v) = (\Phi, \Phi_{Tp(v)}) \in \{\Phi\} \times J^{[r]}_xE \tilde{=} V_\Phi J^{[r]}E \ ,$$

$v \in T_\Phi J^{[r]}E$, $\Phi \in J^{[r]}_xE$, $x \in M$. More precisely, $\Phi : J^r\mathcal{C}^{\infty, f.l}_c(E) \to J^r_x(M, \mathbf{R})$ is a module homomorphism over $t^{[r]}_x : J^r_x(M, \mathbf{R}) \to gl(J^r_x(M, \mathbf{R}))$. $\Phi_{Tp(v)} : J^r\mathcal{C}^{\infty, f.l}_c(E) \to J^r_x(M, \mathbf{R})$, $\Phi_{Tp(v)}(j^r_x\xi) = j^r_x(Tp(v)\gamma)$, $j^r_x\gamma = \Phi(j^r_x\xi)$, $j^r_x\xi \in J^r\mathcal{C}^{\infty, f.l}_x(E)$, is also a module homomorphism over $t^{[r]}_x$, i.e. $\Phi_{Tp(v)} \in J^{[r]}_xE$, see Example 3. The family V is a $\mathcal{VB}_{m,n}$-natural affinor on $J^{[r]}E$.

4. The main result

The main result of the present paper is the following classification theorem.

THEOREM 2. *Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.*

(a) *Any $\mathcal{VB}_{m,n}$-natural affinor B on J^rE is a constant multiple of the identity affinor Id.*

(b) *Any $\mathcal{VB}_{m,n}$-natural affinor B on J^r_vE is a linear combination with real coefficients of Id and U.*

(c) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $J^{[r]}E$ is a linear combination with real coefficients of Id and V.*

To prove Theorem 2 we need the following lemma.

LEMMA 1. *Let r, m, n be as in Theorem 1. Let B be a $\mathcal{VB}_{m,n}$-natural affinor on F^rE such that $B \circ \mathcal{F}^rX = 0$ for any linear vector field on E. Then $B = 0$.*

Proof of Lemma 1. It is sufficient to show that $B = 0$ over $0 \in \mathbf{R}^m$. We fix a basis in the vector space $F_0^r\mathbf{R}^{m,n}$.

Step 1. B is of vertical type. Consider

$$T\pi \circ B : (TF^r\mathbf{R}^{m,n})_0 \tilde{=} \mathbf{R}^m \times F_0^r\mathbf{R}^{m,n} \times F_0^r\mathbf{R}^{m,n} \to T_0\mathbf{R}^m .$$

Using the invariance of B with respect to the fiber homotheties we deduce that $T\pi \circ B(a, u, v) = T\pi \circ B(a, tu, tv)$ for any $u, v \in F_0^r\mathbf{R}^{m,n}$, $a \in \mathbf{R}^m$, $t \neq 0$. Then $T\pi \circ B(a, u, v) = T\pi \circ B(a, u, 0)$ for u, v, a as above. But $(a, u, 0) = \mathcal{F}^r(a^i \frac{\partial}{\partial x^i})_u$. Then $T\pi \circ B(a, u, 0) = 0$ because of the assumption of the lemma. Then B is of vertical type.

Step 2. B=0. Consider

$$pr_2 \circ B : (TF^r\mathbf{R}^{m,n})_0 \tilde{=} \mathbf{R}^m \times F_0^r\mathbf{R}^{m,n} \times F_0^r\mathbf{R}^{m,n} \to F_0^r\mathbf{R}^{m,n} ,$$

where $pr_2 : (VF^r\mathbf{R}^{m,n})_0 \tilde{=} F_0^r\mathbf{R}^{m,n} \times F_0^r\mathbf{R}^{m,n} \to F_0^r\mathbf{R}^{m,n}$ is the projection onto the second factor. Using the invariance of B with respect to the fiber homotheties we deduce that $pr_2 \circ B(a, tu, tv) = tpr_2 \circ B(a, u, v)$ for a, u, v as in Step 1. Then $pr_2 \circ B(a, u, v)$ is a linear combination of the coefficients of u and v (with respect to the obvious basis in the vector space $F_0^r\mathbf{R}^{m,n}$) with coefficient being smooth maps in a because of the homogeneous function theorem. On the other hand, since B is an affinor $B(a, u, v)$ is a linear combination of the coefficients of a and v with coefficient being smooth functions in u. We see that $pr_2 \circ B(a, u, 0) = 0$ by the same reason as in Step 1. We also see that $(0, v, v) = \mathcal{F}^rL_v$, where L is the Liouville vector field on $\mathbf{R}^{m,n}$, and consequently $B(0, v, v) = 0$ because of the assumption of the lemma. Hence $B(a, u, v) = 0$ for all a, u, v as above. □

Proof of Theorem 2. Lemma 1 says that a $\mathcal{VB}_{m,n}$-natural affinor B on F^rE is uniquely determined by the vector fields $B \circ \mathcal{F}^rX$ for linear vector fields X on E. On the other hand $B \circ \mathcal{F}^rX$ is a $\mathcal{VB}_{m,n}$-natural linear operator lifting linear vector fields on E into F^rE. Using Theorem 1 (a) we know that $B \circ \mathcal{J}^rX = a\mathcal{J}^rX$. Hence $B = aId$. This complete the proof of Theorem 2 for $F^rE = J^rE$. Using Theorem 1 (b) we complete the proof of Theorem 2 for $F^rE = J_v^rE$. Using Theorem 1 (c) we complete the proof of Theorem 2 for $F^rE = J^{[r]}E$. □

5. Some versions on the main result

We say that an affinor $B : TE \to TE$ on a vector bundle E is linear if $B(X)$ is for any linear vector field X on E.

PROPOSITION 1. *Let B be a $\mathcal{VB}_{m,n}$-natural affinor on F^rE (resp. F^rE^*, $(F^rE)^*$, $(F^rE^*)^*$). Then B is linear.*

P r o o f. Observe that a vector field X on a vector bundle E is linear iff $(b_t)_*X = tX$ for $t \neq 0$, where b_t is the fiber homothety on E.

Observe also that F^rb_t is the fiber homothety on $F^r\mathbf{R}^{m,n}$ if b_t is the fiber homothety on $\mathbf{R}^{m,n}$.

Let X be a linear vector field on $F^r\mathbf{R}^{m,n}$. Then $(F^rb_t)_*X = tX$ for any $t \neq 0$. Then $(F^rb_t)_*(B(X)) = tB(X)$ because of the invariance of B with respect to b_t. Then $B(X)$ is a linear vector field on $F^r(\mathbf{R}^{m,n})$.

Similar method we use for F^rE^*, $(F^rE)^*$ and $(F^rE^*)^*$ □

There is a natural involution (dualization) $()^* : \mathcal{VB}_{m,n} \to \mathcal{VB}_{m,n}$, $E \to E^*$, $f \to (f^{-1})^*$. Given a linear vector field on a vector bundle E we have the dual linear vector field X^* on E^* such that if f_t is the flow of X then $(f_t^{-1})^*$ is the flow of X^*.

LEMMA 2. *Let $B : TE \to TE$ be a linear affinor on a vector bundle E. Then there is one and only one linear affinor $B^* : TE^* \to TE^*$ on the dual vector bundle E^* such that $B^*(X^*) = (B(X))^*$ for any linear vector field X on E.*

P r o o f. We use local vector bundle cordinate argument. If

$$B = a_i^j(x)dx^i \otimes \frac{\partial}{\partial x^j} + b_{ik}^s(x)y^kdx^i \otimes \frac{\partial}{\partial y^s} + c_k^s(x)dy^k \otimes \frac{\partial}{\partial y^s}$$

then

$$B^* = a_i^j(x)dx^i \otimes \frac{\partial}{\partial x^j} + b_{ik}^s(x)v^kdx^i \otimes \frac{\partial}{\partial v^s} + c_k^s(x)dv^k \otimes \frac{\partial}{\partial v^s} ,$$

where (x^i, y^k) are vector bundle coordinates on E and (x^i, v^k) are the dual vector bundle coordinates on E^*. □

Using Proposition 1 and Lemma 2 one can easily deduce from Theorem 2 the following versions of Theorem 2.

THEOREM 3. *Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.*

(a) *Any $\mathcal{VB}_{m,n}$-natural affinor B on J^rE^* is a constant multiple of the identity affinor Id.*

(b) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $J_v^rE^*$ is a linear combination with real coefficients of Id and U.*

(c) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $J^{[r]}E^*$ is a linear combination with real coefficients of Id and V.*

THEOREM 4. *Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.*

(a) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^rE)^*$ is a constant multiple of the identity affinor Id.*

(b) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^r_vE)^*$ is a linear combination with real coefficients of Id and U^*.*

(c) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^{[r]}E)^*$ is a linear combination with real coefficients of Id and V^*.*

THEOREM 5. *Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.*

(a) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^rE^*)^*$ is a constant multiple of the identity affinor Id.*

(b) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^r_vE^*)^*$ is a linear combination with real coefficients of Id and U^*.*

(c) *Any $\mathcal{VB}_{m,n}$-natural affinor B on $(J^{[r]}E^*)^*$ is a linear combination with real coefficients of Id and V^*.*

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