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GENERIC SUBMANIFOLDS OF QUASI-SASAKIAN MANIFOLDS

1. Introduction

A. Bejancu [1], [2] defined and studied CR-submanifolds of Kaehlerian manifolds. CR-submanifolds of Sasakian manifolds were studied by M. Kobayshi [11] and M. Hasan Shahid [16]. B.-Y. Chen [8] introduced the notion of a generic submanifold of a Kaehler manifold. Generic submanifolds of Sasakian manifolds were studied by P. Verheyen [17] and M.Hasan Shahid [14]. Generic submanifolds of trans-Sasakian manifolds were studied by M. Hasan Shahid and I. Mihai [15].

D. E. Blair [3] initiated the study of normal almost contact metric manifolds with closed fundamental 2-form Φ and such manifolds were called quasi-Sasakian manifolds. Z. Olszak [12], [13] extensively studied 3-dimensional quasi-Sasakian manifolds. C. Calin [6] studied contact CR-submanifolds of a quasi-Sasakian manifold.

The purpose of the present paper is to study generic submanifolds of a quasi-Sasakian manifold.

2. Preliminaries

Let $\overline{M}$ be a $(2n+1)$-dimensional almost contact metric manifold with (ϕ, ξ, η, g) as its almost contact metric structure, where ϕ is a tensor field of type (1,1), ξ is a vector field, η is a 1-form and g is a Riemannian metric on $\overline{M}$ satisfying

$$(1) \qquad \phi^2 = -I + \eta \otimes \xi, \quad \phi\xi = 0, \quad \eta \circ \phi = 0, \quad \eta(\xi) = 1,$$

$$(2) \qquad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \text{ for } X,\ Y \in T(\overline{M}).$$

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The fundamental 2-form of $\overline{M}$ is defined by $\Phi(X,Y) = g(X,\phi Y)$, $X, Y \in T(\overline{M})$. The Nijenhuis tensor of ϕ is the tensor field N_ϕ given by

$$N_\phi(X,Y) = [\phi X,\phi Y] + \phi^2[X,Y] - \phi[X,\phi Y] - \phi[\phi X,Y], \quad X,\ Y \in T(\overline{M}).$$

The almost contact structure (ϕ,ξ,η,g) is said to be normal if [4], [5] $N_\phi + 2d\eta \otimes \xi = 0$.

The manifold $\overline{M}$ is said to be a quasi-Sasakian manifold if its almost contact structure (ϕ,ξ,η) is normal and the fundamental 2-form Φ is closed. A quasi-Sasakian manifold has been characterised by S. Kanemaki [10] as follows: a differentiable manifold $\overline{M}$ is quasi-Sasakian if and only if it is endowed with an almost contact metric structure (ϕ,ξ,η,g) and a tensor field F of type (1,1) such that

$$(\overline{\nabla}_X\phi)Y = \eta(Y)FX - g(FX,Y)\xi, \tag{3}$$

$$\phi FX = F\phi X, \quad g(FX,Y) = g(X,FY), \tag{4}$$

for X, Y $\in T(\overline{M})$, where $\overline{\nabla}$ is the Levi-Civita connection with respect to the metric g. From (3), we obtain

$$\overline{\nabla}_X\xi = \phi FX, \quad X \in T(\overline{M}). \tag{5}$$

Using (1) and (4) we get

$$F\xi = \eta(F\xi)\xi. \tag{6}$$

Hence from (3) it follows that $\overline{\nabla}_\xi\xi = 0$. A quasi-Sasakian manifold $\overline{M}$ is Sasakian if $F = -I$.

Let M be an m-dimensional submanifold isometrically immersed in a quasi-Sasakian maifold $\overline{M}$ such that the structure vector field ξ of $\overline{M}$ is tangent to the submanifold M. We denote by $\{\xi\}$ the 1-dimensional distribution spannned by ξ on M and by $\{\xi\}^\perp$ the complementary orthogonal distribution to $\{\xi\}$ in $T(M)$. For any $X \in T(M)$, we have $\phi X \in \{\xi\}^\perp$. For $X \in T(M)$, we put

$$\phi X = bX + cX, \tag{7}$$

where $bX \in \{\xi\}^\perp$ and $cX \in T^\perp(M)$. Thus $X \mapsto bX$ is an endomorphism of the tangent bundle $T(M)$ and $X \mapsto cX$ is a normal bundle valued 1-form on M.

If the maximal invariant subspaces under ϕ, $D_p = T_p(M) \cap \phi T_p(M)$, define a differentiable distribution D on M, then M is called a generic submanifold of $\overline{M}$. Thus for a generic submanifold M, we have the orthogonal decomposition $T(M) = D \oplus D^\perp \oplus \{\xi\}$, where $D^\perp$ is the subbundle complementary orthogonal to $D \oplus \{\xi\}$ in $T(M)$. Then it follows immediately that $bD = 0$, $bD^\perp \subset D^\perp$.

Let $T^{\perp}M$ be the normal bundle of M. If ν_p is the maximal invariant vector subspace of $T_p^{\perp}(M)$, i. e., $\nu_p = T_p^{\perp}(M) \cap \phi T_p^{\perp}(M)$, then for $p \in$M, ν_p defines a differentiable subbundle of $T_p^{\perp}(M)$ and $\nu = (cD^{\perp})^{\perp}$, $T^{\perp}(M) = cD^{\perp} \oplus \nu$. For $V \in T^{\perp}(M)$, suppose

$$\phi V = tV + fV, \tag{8}$$

where tV and fV denote the tangential and normal component of ϕV respectively. We have $t\left(T^{\perp}(M)\right) \subset D^{\perp}$.

A generic submanifold with $\phi D^{\perp} \subset T^{\perp}(M)$ will be called a CR-submanifold of the quasi-Sasakian manifold $\overline{M}$.

We denote by g the metric tensor field of $\overline{M}$ as well as the induced metric on M. Let $\overline{\nabla}$ (resp., ∇) be the Levi-Civita connection on $\overline{M}$ (resp., M). The Gauss and Weingarten formulas for M are respectively given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \quad \overline{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{9}$$

for $X, Y \in T(M)$ and $V \in T^{\perp}(M)$, where h (resp., A) is the second fundamental form (resp., tensor) of M in $\overline{M}$ and $\nabla^{\perp}$ denotes the covariant differentiation with respect to the normal connection. From (9), it follows

$$g\left(h(X,Y), V\right) = g(A_V X, Y), \tag{10}$$

for $X, Y \in T(M)$ and $V \in T^{\perp}(M)$.

3. Basic results

For $X \in T(\overline{M})$, we put $FX = \alpha X + \beta X$, where αX and βX are the tangential and normal component of FX respectively.
For $X, Y \in T(M)$, we put

$$(\nabla_X b)Y = \nabla_X bY - b(\nabla_X Y),$$
$$(\nabla_X c)Y = \nabla^{\perp}_X cY - c(\nabla_X Y).$$

LEMMA 1. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. Then for $X, Y \in T(M)$,*

$$(\nabla_X b)Y = A_{cY}X + t\,h(X,Y) + \eta(Y)\alpha X - g(FX,Y)\xi, \tag{11}$$

$$(\nabla_X c)Y = fh(X,Y) - h(X,bY) + \eta(Y)\beta X. \tag{12}$$

Proof. From (3), (7), (8) and (9) we have

$$\nabla_X bY + h(X,bY) - A_{cY}X + \nabla^{\perp}_X cY - b\nabla_X Y - c\nabla_X Y - t\,h(X,Y) - f\,h(X,Y)$$
$$= \eta(Y)\alpha X + \eta(Y)\beta X - g(FX,Y)\xi,$$

for $X, Y \in T(M)$. Hence, by equating the tangent and normal parts, we obtain (11) and (12).

LEMMA 2. *Let M be a CR-submanifold of a quasi-Sasakian manifold $\overline{M}$. Then for $Z, W \in D^{\perp}$,*

$$A_{cZ}W = A_{cW}Z.$$

Proof. For $Z, W \in D^{\perp}$ and $X \in T(M)$, by using (3), (9) and (10),we have

$$g(A_{cZ}W, X) = g(h(X, W), \phi Z) = g\left(\overline{\nabla}_X W, \phi Z\right) = -g\left(\phi\overline{\nabla}_X W, Z\right)$$
$$= -g\left(\overline{\nabla}_X \phi W, Z\right) = g(A_{cW}X, Z) = g(A_{cW}Z, X),$$

which proves the lemma.

PROPOSITION 1. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. If $T(M)$ is invariant under F, then both the distributions D and $D^{\perp}$ are invariant under F.*

Proof. As T(M) is invariant under F, we have $X \in D \Longrightarrow FX \in T(M)$. Also $X \in D \Longrightarrow \phi X \in T(M)$. Now by (4), $\phi FX = F\phi X \in T(M)$, since $\phi X \in T(M)$ and $T(M)$ is invariant under F. But $FX \in T(M)$ and $\phi FX \in T(M) \Longrightarrow FX \in D$. Thus D is invariant under F.

For $X \in D^{\perp}$ and $Y \in D$, by using the invariance of D under F and (4), we obtain $g(FX, Y) = g(X, FY) = 0$. On the other hand, by using (4) and (6), we have $g(FX, \xi) = g(X, F\xi) = g(X, \eta(F\xi)\xi) = \eta(f\xi)g(X, \xi) = 0$, for $X \in D^{\perp}$. Hence $D^{\perp}$ is invariant under F.

REMARK. In Corollary 1 of [6] to prove the invariance of D, Calin used a method which is applicable only for CR-submanifolds. The method used above can be applied for any type of submanifolds having invariant distribution D.

Now we give an example of a generic submanifold, which is not a CR-submanifold, of a quasi-Sasakian manifold.

EXAMPLE. Let the quasi-Sasakian structure (ϕ, ξ, η, g) of rank 5 of $\mathbf{R}^7$ [7], be given by $\phi = \phi_i^h dx^i \otimes \frac{\partial}{\partial x^h}$; $g = g_{ij} dx^i \otimes dx^j$,

$$\left[\phi_i^j\right] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2y^1 & 2y^2 & 0 & 0 \end{pmatrix},$$

and

$$[g_{ij}] = \begin{pmatrix} 1+4(y^1)^2 & 4y^1y^2 & 0 & 0 & 0 & 0 & -2y^1 \\ 4y^1y^2 & 1+4(y^2)^2 & 0 & 0 & 0 & 0 & -2y^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -2y^1 & -2y^2 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

with $\xi = (0,0,0,0,0,0,1)^t$; $\eta = dz - 2y^1dx^1 - 2y^2dx^2$, where $(x^1, x^2, x^3, x^4, x^5, x^6, x^7) = (x^1, x^2, x^3, y^1, y^2, y^3, z)$ are the Cartesian coordinate on R^7. With respect to the Cartesian coordinate, let $\left\{e_i = \frac{\partial}{\partial x^i}\right\}$ be the global field of frames of R^7.

The quasi-Sasakian structure defined in the above is in fact the product of 5-dimensional Sasakian structure and the flat 2-dimensional Kählerian structure.

Now let M be a submanifold of R^7, $dimM = 5$, defined by

$$\begin{aligned} (13) \qquad & x^1 = u^3 \sinh^2 u^2;\ x^2 = \frac{u^2}{2} + \frac{1}{4}\sinh 2u^2;\ x^3 = u^1;\ x^4 = u^2; \\ & x^5 = u^3;\ x^6 = u^4;\ x^7 = u^5. \end{aligned}$$

The global field of frames for M is $\{\frac{\partial}{\partial u^\alpha}, 1 \leq \alpha \leq 5\}$, where

$$\begin{aligned} &\frac{\partial}{\partial u^1} = \frac{\partial}{\partial x^3};\quad \frac{\partial}{\partial u^4} = \frac{\partial}{\partial x^6};\quad \frac{\partial}{\partial u^5} = \frac{\partial}{\partial x^7} = \xi; \\ &\frac{\partial}{\partial u^2} = u^3 \sinh 2u^2 \frac{\partial}{\partial x^1} + \cosh^2 u^2 \frac{\partial}{\partial x^2} + \frac{\partial}{\partial x^4}; \\ &\frac{\partial}{\partial u^3} = \sinh^2 u^2 \frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^5}. \end{aligned}$$

It is easy to see that the normal vector bundle $T^\perp(M)$ is generated by the normal vector fields N_1, N_2 of M defined by

$$\begin{aligned} N_1 &= \left(1, 0, 0, -u^3 \sin 2u^2, -\sinh^2 u^2, 0, 2u^2\right); \\ N_2 &= \left(0, 1, 0, -\cosh^2 u^2, 0, 0, 2u^3\right). \end{aligned}$$

By straightforward calculation we have $\phi(D) = D$; $g\left(b(\frac{\partial}{\partial u^2}), \frac{\partial}{\partial u^3}\right) \neq 0$, $g\left(c(\frac{\partial}{\partial u^2}), N_1\right) \neq 0$ that is, $bD^\perp \subseteq D^\perp$ and $cD^\perp \subseteq T^\perp(M)$ with $bD^\perp \neq \{0\}$ and $cD^\perp \neq \{0\}$. Thus the submanifold M, defined by (13) is a proper generic submanifold of the quasi-Sasakian manifold R^7 with $D = span\left\{\frac{\partial}{\partial u^1}, \frac{\partial}{\partial u^4}\right\}$; $D^{\perp} = span\left\{\frac{\partial}{\partial u^2}, \frac{\partial}{\partial u^3}\right\}$ and $T^{\perp}(M) = span\{N_1, N_2\}$.

4. Integrability of distributions

THEOREM 1. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. If $T(M)$ is invariant under F and F is a non-zero tensor field, then the distribution D is not integrable.*

Proof. For $X, Y \in D$, by using (4), (5) and (9), we obtain

$$
\begin{aligned}
g([X,Y],\xi) &= g(\nabla_X Y,\xi) - g(\nabla_Y X,\xi) \\
&= g\left(\overline{\nabla}_X Y,\xi\right) - g\left(\overline{\nabla}_Y X,\xi\right) \\
&= -g\left(Y,\overline{\nabla}_X \xi\right) + g\left(X,\overline{\nabla}_Y \xi\right) \\
&= -g(Y,\phi F X) + g(X,\phi F Y) \\
&= g(FX,\phi Y) + g(FX,\phi Y) = 2g(FX,\phi Y).
\end{aligned} \tag{14}
$$

As $T(M)$ is invariant under F, by Proposition 1, D is invariant under F. Then, by taking $Y = \phi F X$ in (14) the theorem follows.

COROLLARY 1. [14] *Let M be a generic submanifold of a Sasakian manifold $\overline{M}$. Then the distribution D is not integrable.*

Proof. Since $F = -I$, $T(M)$ is invariant under F and therefore by Theorem 1, the corollary follows.

THEOREM 2. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. Then the distribution D is integrable if and only if $FD \perp D$ and*

$$h(X,\phi Y) = h(\phi X, Y), \quad \text{for } X, Y \in D.$$

Proof. For X, $Y \in D$, by using (12), we have

$$c\nabla_X Y = h(X,\phi Y) - f\,h(X,Y) - \eta(Y)\beta X,$$

and therefore

$$c[X,Y] = h(X,\phi Y) - h(Y,\phi X) \tag{15}$$

since $\eta(X) = \eta(Y) = 0$. Hence the theorem follows from (14) and (15).

In the example, given earlier, it is easy to verify that the distributions $D^{\perp}$ and $D^{\perp} \oplus \{\xi\}$ of the generic submanifold M of the quasi-Sasakian manifold R^7 are not integrable.

Necessary and sufficient conditions for the integrability of the distributions $D^{\perp}$ and $D^{\perp} \oplus \{\xi\}$ are obtained by the following:

THEOREM 3. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. Then the distribution $D^{\perp}$ is integrable if and only if*

$$A_{cX}Y - A_{cY}X + \nabla_X bY - \nabla_Y bX \in D^{\perp} \oplus \{\xi\}$$

for $X, Y \in D^{\perp}$ and $\alpha D^{\perp} \subset D$ and $\beta D^{\perp} \subset \nu$.

Proof. For $X, Y \in D^{\perp}$ and $Z \in D$ by using (3), (7) and (9) we have

$$\begin{aligned}(16)\quad g([X,Y],\phi Z) &= g(\nabla_X Y,\phi Z) - g(\nabla_Y X,\phi Z)\\ &= g\left(\overline{\nabla}_X Y,\phi Z\right) - g\left(\overline{\nabla}_Y X,\phi Z\right)\\ &= g\left((\overline{\nabla}_X\phi)Y - \overline{\nabla}_X\phi Y, Z\right) - g\left((\overline{\nabla}_Y\phi)X - \overline{\nabla}_Y\phi X, Z\right)\\ &= -g(\nabla_X bY - A_{cY}X, Z) + g(\nabla_Y bX - A_{cX}Y, Z)\\ &= -g(A_{cX}Y - A_{cY}X + \nabla_X bY - \nabla_Y bX, Z).\end{aligned}$$

Further, for $X, Y \in D^{\perp}$, using (5), we obtain

$$\begin{aligned}(17)\qquad g([X,Y],\xi) &= g(\nabla_X Y,\xi) - g(\nabla_Y X,\xi)\\ &= f\left(\overline{\nabla}_X Y,\xi\right) - g\left(\overline{\nabla}_Y X,\xi\right)\\ &= -g\left(Y,\overline{\nabla}_X\xi\right) + g\left(X,\overline{\nabla}_Y\xi\right)\\ &= -g(Y,\phi FX) + g(X,\phi FY)\\ &= 2g(FX,\phi Y)\\ &= 2g(\alpha X, bY) + 2g(\beta X, cY).\end{aligned}$$

Hence the theorem follows from (16) and (17).

COROLLARY 2. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. Then the distribution $D^{\perp}$ is integrable if M is a CR-submanifold of $\overline{M}$ and $\beta D^{\perp} \subset \nu$.*

Proof. With the help of Lemma 2, the result follows from (16) and (17).

COROLLARY 3. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. If $T(M)$ is invariant under F and $D^{\perp}$ is integrable, then M is an invariant submanifold of $\overline{M}$.*

Proof. As $D^{\perp}$ is integrable, by Theorem 3 we have $\alpha X \in D$, for $X \in D^{\perp}$. Also as $T(M)$ is invariant under F, by Proposition 1, we have $FX = \alpha X \in D^{\perp}$, for $X \in D^{\perp}$. Thus $FX = \alpha X = 0$, for $X \in D^{\perp}$ and therefore $D^{\perp} \subset kerF$. For $X \in D^{\perp}$, as $FX = 0$, using (4) we obtain $F\phi X = \phi FX = 0$, which gives $\phi X \in KerF \subseteq T(M)$ and therefore we have $D^{\perp} \subset D$. Thus $D^{\perp} = \{0\}$.

Hence the corollary follows.

THEOREM 4. *Let M be a generic submanifold of quasi-Sasakian manifold $\overline{M}$. Then the distribution $D^\perp \oplus \{\xi\}$ is integrable if and only if*

$$A_{cX}Y - A_{cY}X + \nabla_X bY - \nabla_Y bX \in D^\perp \text{ for } X, Y \in D^\perp,$$

and

$$\alpha X + \nabla_\xi \phi X \in D \text{ for } X \in D. \tag{18}$$

Proof. For $Z \in D^\perp$ and $X \in D$, by using (2), (3), (4) and (5) we have

$$\begin{aligned}
(19)\quad g([Z,\xi],X) &= g(\nabla_Z\xi, X) - g(\nabla_\xi Z, X)\\
&= g\left(\overline{\nabla}_Z\xi, X\right) - g\left(\overline{\nabla}_\xi Z, X\right)\\
&= g(\phi FZ, X) + g\left(Z, \overline{\nabla}_\xi X\right)\\
&= g(\phi FZ, X) + g\left(\phi Z, \phi\overline{\nabla_\xi} X\right)\\
&= g(\phi Z, FX) + g\left(\phi Z, \overline{\nabla}_\xi \phi X\right)\\
&= g(bZ, FX) + g(cZ, FX) + g(bZ, \nabla_\xi\phi X) + g(cZ, h(\xi, \phi X))\\
&= g(bZ, FX) + g(cZ, FX) + g(bZ, \nabla_\xi\phi X) + g\left(cZ, \overline{\nabla}_{\phi X}\xi\right)\\
&= g(bZ, FX) + g(cZ, FX) + g(bZ, \nabla_\xi\phi X) + g(cZ, \phi F\phi X)\\
&= g(bZ, FX) + g(cZ, FX) + g(bZ, \nabla_\xi\phi X) + g(cZ, -FX + \eta(FX)\xi)\\
&= g(bZ, FX) + g(cZ, FX) + g(bZ, \nabla_\xi\phi X) - g(cZ, FX)\\
&= g(bZ, \alpha X) + g(bZ, \nabla_\xi\phi X)\\
&= g(bZ, \alpha X + \nabla_\xi\phi X)\,.
\end{aligned}$$

By Theorem 3 and formula (19) the theorem follows.

COROLLARY 4. *Let M be a generic submanifold of a quasi-Sasakian manifold $\overline{M}$. If $T(M)$ is invariant under F, then $D^\perp \oplus \{\xi\}$ is integrable if and only if $\nabla_\xi\phi X \in D$ for $X \in D$.*

Proof. Since $T(M)$ is invariant under F, by Proposition 1, D is invariant under F and therefore the corollary follows from (18) of Theorem 4.

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References

[1] A. Bejancu, *CR-submanifolds of Kaehler manifold I*, Proc. Amer. Math. Soc. 69, (1978), 135–142.

[2] A. Bejancu, *CR-submanifolds of Kaehler manifold II*, Trans. Amer. Math. Soc. 250, (1979), 333–345.

[3] D. E. Blair, *The theory of quasi-Sasakian structures*, J. Diff. Geometry, 1 (1967), 331–345.
[4] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, 509 Springer-Verlag, Berlin, 1976.
[5] D. E. Blair, *Riemannian geometry of contact and symplectic manifold,* Progress Math. Vol. 203, Birkhäuser, Boston-Basel-Berlin, 2003.
[6] C. Calin, *Contact CR-submanifolds of quasi-Sasakian manifolds*, Bull. Math. Soc. Sci. Math. Roumanie, 36(84) (1992), nr 3-4, 217–226.
[7] C. Calin, *On some properties of a quasi-Sasakian manifold*, Algebras, Groups and Geometries, 16 (1999), 39–46.
[8] B. -Y. Chen, *Differential geometry of real submanifold*, Monatsh Math. 91 (1981), 257–274.
[9] U. C. De and A. K. Sengupta, *Generic submanifolds of a Lorentzian Para-Sasakian manifold*, Soochow J. Math., 27, No 1. (2001), 29–36.
[10] S. Kanemaki, *On Quasi-Sasakian Manifolds*, Banach Center Publications, 12 (1984), 95–125.
[11] M. Kobayashi, *CR-submanifolds of a Sasakian manifold*, Tensor N. S. 35 (1981), 297–307.
[12] Z. Olszak, *Curvature properties of quasi-Sasakian manifolds*, Tensor N. S. 38 (1982), 19–28.
[13] Z. Olszak, *On three-dimensional conformally flat quasi-Sasakian manifolds*, Period. Math. Hungar., Vol. 33(2) (1996), 105–113.
[14] M. H. Shahid and S. Ali, *Generic submanifold of a Sasakian manifold*, Math. Student, 52, No. 1 (1989), 197–202.
[15] M. H. Shahid and I. Mihai, *Generic submanifolds of trans-Sasakian manifolds*, Publ. Math. Debrecen, 47 (1995), 209–218.
[16] M. H. Shahid, A. Sharfuddin and S. I. Hussain, *CR-submanifolds of a Sasakian manifold*, Review Research, Fac. Sc. Yugoslavia, 15 No. 1 (1985), 263–278.
[17] P. Verheyen, *Generic submanifolds of Sasakian Manifold*, Med. Wisk. Inst. K. U. Leuven, 157 (1982), 1–21.

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