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REMARKS ON δ-SEMI-OPEN SETS AND δ-PREOPEN SETS

Abstract. It is shown that a subset A of a topological space (X, τ) is δ-semi-open (resp. δ-preopen) in (X, τ) if and only if it is semi-open (resp. preopen) in (X, τ_s), where τ_s denotes the semi-regularization of τ. By using this fact, we can obtain several new characterizations of s-closed spaces, semi-connected spaces, and some separation axioms.

1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and δ-open sets play an important role in the study of generalizations of continuity in topological spaces. By using these sets, many authors introduced and studied various types of modifications of continuity. In 1963, Levine [18] introduced the notions of semi-open sets and semi-continuity in topological spaces. It is shown in [30] that semi-continuity is equivalent to quasicontinuity due to Marcus [20]. Park et al. [34] defined and studied the notion of δ-semi-open sets in topological spaces. Recently, in [17], they obtained the further properties of δ-semi-open sets and related sets. On the other hand, Mashhour et al. [24] introduced the notions of preopen sets and precontinuous functions. As generalizations of these notions, Raychaudhuri and Mukherjee [38] introduced δ-preopen sets and δ-almost continuous functions.

A topological property R is said to be *semi-regular* [6] provided that a topological space (X, τ) has property R if and only if (X, τ_s) has property R, where τ_s denotes the semi-regularization of τ. In [28], Mršević et al. discussed semi-regular properties for separation axioms, connectedness and covering property.

In this paper, for several topological spaces (X, τ) defined by using semi-open sets or preopen sets, we investigate the relationship between the prop-

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erties of (X,τ) and those of (X,τ_s). As a result, it turned out that many properties concerning δ-semi-open sets and δ-preopen sets easily follow from known results concerning semi-open sets and preopen sets, respectively. In Section 3, we show that a subset A of a topological space (X,τ) is δ-semi-open (resp. δ-preopen) in (X,τ) if and only if it is semi-open (resp. preopen) in (X,τ_s). In Section 4, we introduce the notion of δ-semi-continuous functions. It turns out that semi-continuity is implied by both δ-semi-continuity and continuity which are independent of each other. Section 5 deals with δ_p-closed spaces and some related functions. In the last section, we obtain new characterizations of s-closed spaces, semi-connected spaces and certain separation axioms in term of δ-semi-open sets. Furthermore, we show that s-closedness, semi-connectedness and semi-Hausdorffness have semi-regular property.

2. Preliminaries

Let (X,τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be *regular closed* (resp. *regular open*) if $\mathrm{Cl}(\mathrm{Int}(A)) = A$ (resp. $\mathrm{Int}(\mathrm{Cl}(A)) = A$). A subset A is said to be *δ-open* [43] if for each $x \in A$ there exists a regular open set G such that $x \in G \subset A$. A point $x \in X$ is called a *δ-cluster* point of A if $\mathrm{Int}(\mathrm{Cl}(V)) \cap A \neq \emptyset$ for every open set V containing x. The set of all δ-cluster points of A is called the *δ-closure* of A and is denoted by $\mathrm{Cl}_\delta(A)$. The set $\{x \in X : x \in U \subset A$ for some regular open set U of $X\}$ is called the *δ-interior* of A and is denoted by $\mathrm{Int}_\delta(A)$.

DEFINITION 2.1. Let (X,τ) be a topological space. A subset A of X is said to be

(1) *semi-open* [18] (resp. *preopen* [24], *α-open* [31], *β-open* [1] or *semi-preopen* [3]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A))))$,

(2) *δ-preopen* [38] (resp. *δ-semi-open* [34]) if $A \subset \mathrm{Int}(\mathrm{Cl}_\delta(A))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}_\delta(A)))$.

The family of all semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) sets in X is denoted by $\mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\delta\mathrm{PO}(X)$, $\delta\mathrm{SO}(X)$).

DEFINITION 2.2. The complement of a semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) set is said to be *semi-closed* [7] (resp. *preclosed* [24], *α-closed* [27], *β-closed* [1] or *semi-preclosed* [3], *δ-preclosed* [38], *δ-semi-closed* [34]).

DEFINITION 2.3. The intersection of all semi-closed (resp. preclosed, α-closed, β-closed, δ-preclosed, δ-semi-closed) sets of X containing A is called the *semi-closure* [7] (resp. *preclosure* [15], *α-closure* [27], *β-closure* [2] or *semi-preclosure* [3], *δ-preclosure* [38], *δ-semi-closure* [34]) of A and is denoted by $\mathrm{sCl}(A)$ (resp. $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$ or $\mathrm{spCl}(A)$, $\mathrm{pCl}_{\delta}(A)$, $\mathrm{sCl}_{\delta}(A)$).

DEFINITION 2.4. The union of all semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) sets of X contained in A is called the *semi-interior* (resp. *preinterior*, *α-interior*, *β-interior* or *semi-preinterior*, *δ-preinterior*, *δ-semi-interior*) of A and is denoted by $\mathrm{sInt}(A)$ (resp. $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_{\beta}\mathrm{Int}(A)$ or $\mathrm{spInt}(A)$, $\mathrm{pInt}_{\delta}(A)$, $\mathrm{sInt}_{\delta}(A)$).

3. δ-semi-open sets and δ-preopen sets

First we recall the relationship among some generalizations of open sets. If a subset A of a topological space (X,τ) is semi-open and semi-closed, then it is said to be *semi-regular* [10]. The set of all semi-regular sets of (X,τ) is denoted by $\mathrm{SR}(X)$.

LEMMA 3.1. *For a subset A of a topological space (X,τ), the following properties hold:*

(1) *If A is a semi-regular set, then it is δ-semi-open,*

(2) *If A is a δ-semi-open set, then it is semi-open,*

(3) *If A is a semi-open set, then $\mathrm{sCl}(A)$ is semi-regular.*

Proof. (1) Let A be a semi-regular set. Then since A is semi-open and semi-closed, we have $\mathrm{Int}(\mathrm{Cl}(A)) \subset A \subset \mathrm{Cl}(\mathrm{Int}(A))$. Since $\mathrm{Int}(\mathrm{Cl}(A))$ is regular open, we obtain $\mathrm{Int}(\mathrm{Cl}(A)) \subset \mathrm{Int}_{\delta}(A)$ and hence

$$A \subset \mathrm{Cl}(\mathrm{Int}(A)) \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A))) \subset \mathrm{Cl}(\mathrm{Int}_{\delta}(A)).$$

This shows that A is δ-semi-open.

(2) Since $\mathrm{Int}_{\delta}(A) \subset \mathrm{Int}(A)$, $A \subset \mathrm{Cl}(\mathrm{Int}_{\delta}(A))$ implies $A \subset \mathrm{Cl}(\mathrm{Int}(A))$. This shows that A is semi-open.

(3) This is shown in Proposition 2.2 of [10].

By Lemma 3.1, we have the following diagram in which the converses of implications need not be true as shown by the three examples stated below.

DIAGRAM I

regular open	→	δ-open	→	open	→	preopen
↓		↓		↓		↓
semi-regular	→	δ-semi-open	→	semi-open		δ-preopen

EXAMPLE 3.1. Let $X = \{a,b,c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Then $\{a,b\}$ is a δ-open set of (X,τ) which is not semi-regular. A subset $\{a,c\}$ is semi-regular but not open.

EXAMPLE 3.2. (Park et al. [34]) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Then $\{a, c, d\}$ is an open set of (X, τ) which is not δ-semi-open. A subset $\{c, d\}$ is δ-semi-open but not open.

EXAMPLE 3.3. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$. Then $\{a, b, c\}$ is a pre-open set of (X, τ) which is not semi-open. A subset $\{b, c\}$ is semi-open but not δ-preopen.

LEMMA 3.2. *Let (X, τ) be a topological space and A be a subset of X.*

(1) *If A is open, then* $\mathrm{Cl}_\delta(A) = \mathrm{Cl}(A)$,

(2) *If A is closed, then* $\mathrm{Int}_\delta(A) = \mathrm{Int}(A)$.

Proof. (1) is known in [43] and (2) follows obviously from (1).

For a topological space (X, τ), the family of all δ-open sets of (X, τ) forms a topology for X, which is weaker than τ. This topology has a base consisting of all regular open sets in (X, τ). We shall denote it by τ_δ although it is usually denoted by τ_s. Now, we have the following interesting theorem.

THEOREM 3.1. *Let (X, τ) be a topological space and A be a subset of X.*

(1) *A is δ-semi-open in (X, τ) if and only if A is semi-open in (X, τ_δ),*

(2) *A is δ-preopen in (X, τ) if and only if A is preopen in (X, τ_δ).*

Proof. This follows from Lemma 3.2 and the next facts:

(1) $\mathrm{Cl}(\mathrm{Int}_\delta(A)) = \mathrm{Cl}_\delta(\mathrm{Int}_\delta(A)) = \tau_\delta\text{-}\mathrm{Cl}(\tau_\delta\text{-}\mathrm{Int}(A))$,

(2) $\mathrm{Int}(\mathrm{Cl}_\delta(A)) = \mathrm{Int}_\delta(\mathrm{Cl}_\delta(A)) = \tau_\delta\text{-}\mathrm{Int}(\tau_\delta\text{-}\mathrm{Cl}(A))$.

Let A be a subset of a topological space (X, τ). A point x of X is called a *semi-θ-cluster point* of A if $\mathrm{sCl}(U) \cap A \neq \emptyset$ for every $U \in \mathrm{SO}(X)$ containing x. The set of all semi-θ-cluster points of A is called the *semi-θ-closure* [10] of A and is denoted by $\mathrm{sCl}_\theta(A)$. A subset A is said to be *semi-θ-closed* if $A = \mathrm{sCl}_\theta(A)$. The complement of a semi-θ-closed set is said to be *semi-θ-open.* The family of all semi-θ-open sets of (X, τ) is denoted by $\theta\mathrm{SO}(X)$.

THEOREM 3.2. *Let (X, τ) be a topological space. Then every semi-regular set is semi-θ-open and every semi-θ-open set is δ-semi-open.*

Proof. Every semi-regular set is semi-θ-open by Propostion 2.3 of [10]. Let A be a semi-θ-open set. For each $x \in A$, there exists $U_x \in \mathrm{SO}(X)$ such that $x \in U_x \subset \mathrm{sCl}(U_x) \subset A$. By Lemma 3.1, $\mathrm{sCl}(U_x)$ is semi-regular and δ-semi-open. Therefore, $A = \bigcup_{x \in A} \mathrm{sCl}(U_x)$ is δ-semi-open.

REMARK 3.1. For families of subsets of a topological space (X, τ), we have the relations: $\mathrm{SR}(X) \subset \theta\mathrm{SO}(X) \subset \delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$.

THEOREM 3.3. *Let (X, τ) be a topological space. Then* $\mathrm{sCl}_\theta(V) = \mathrm{sCl}_\delta(V) = \mathrm{sCl}(V)$ *for any* $V \in \mathrm{SO}(X)$.

Proof. In general, we have $\mathrm{sCl}_\theta(S) \supset \mathrm{sCl}_\delta(S) \supset \mathrm{sCl}(S)$ for any subset S of X. Suppose that $V \in \mathrm{SO}(X)$ and $x \notin \mathrm{sCl}(V)$. Then there exists $U \in \mathrm{SO}(X)$ containing x such that $U \cap V = \emptyset$ and hence $\mathrm{sCl}(U) \cap V = \emptyset$. This shows that $x \notin \mathrm{sCl}_\theta(V)$. Hence, we have $\mathrm{sCl}_\theta(V) \subset \mathrm{sCl}(V)$. Therefore, we obtain that $\mathrm{sCl}_\theta(V) = \mathrm{sCl}_\delta(V) = \mathrm{sCl}(V)$.

4. δ-semi-continuous functions

DEFINITION 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *super-continuous* [29] if for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \tau$ containing x such that $f(\mathrm{Int}(\mathrm{Cl}(U))) \subset V$,

(2) *semi-continuous* [18] if for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \mathrm{SO}(X)$ containing x such that $f(U) \subset V$.

DEFINITION 4.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *δ-semi-continuous* if for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $U \in \delta\mathrm{SO}(X)$ containing x such that $f(U) \subset V$.

LEMMA 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is δ-semi-continuous (resp. semi-continuous, super-continuous) if and only if $f^{-1}(V)$ is δ-semi-open (resp. semi-open, δ-open) in (X, τ) for each $V \in \sigma$.*

THEOREM 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is δ-semi-continuous if and only if $f : (X, \tau_\delta) \to (Y, \sigma)$ is semi-continuous.*

Proof. This is an immediate consequence of Theorem 3.1.

REMARK 4.1. By DIAGRAM I we have the following diagram in which the converses of each implications need not be true as shown by the examples stated below. Moreover, the examples show that δ-semi-continuity and continuity are independent of each other.

DIAGRAM II

super-continuous ⇒ continuous

⇓ ⇓

δ-semi-continuous ⇒ semi-continuous

EXAMPLE 4.1. Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then, the identity function $f : (X, \tau) \to (Y, \sigma)$ is a δ-semi-continuous function which is not continuous.

EXAMPLE 4.2. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Then the identity function $f : (X, \tau) \to (X, \tau)$ is continuous but it is not δ-semi-continuous.

It is shown in [30] that semi-continuity and quasicontinuity are equivalent of each other. The properties of these functions are investigated by many authors. Therefore, by using Theorem 4.1 we can obtain properties of δ-semi-continuous functions. For examples, characterizations of δ-semi-continuous functions are obtained by Theorem 4 of [5]. For the product function of δ-semi-continuous functions, we can use Theorem 5 of [32].

DEFINITION 4.3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *irresolute* [8] if $f^{-1}(V) \in \mathrm{SO}(X)$ for every $V \in \mathrm{SO}(Y)$,

(2) *quasi-irresolute* [11] if for each $x \in X$ and each $V \in \mathrm{SO}(Y)$ containing $f(x)$, there exists $U \in \mathrm{SO}(X)$ containing x such that $f(U) \subset \mathrm{sCl}(V)$.

DEFINITION 4.4. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *δ-irresolute* if $f^{-1}(V) \in \delta\mathrm{SO}(X)$ for every $V \in \delta\mathrm{SO}(Y)$,

(2) *quasi-δ-irresolute* if for each $x \in X$ and each $V \in \delta\mathrm{SO}(Y)$ containing $f(x)$, there exists $U \in \delta\mathrm{SO}(X)$ containing x such that $f(U) \subset \mathrm{sCl}_\delta(V)$.

THEOREM 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties hold:*

(1) *f is δ-irresolute if and only if $f : (X, \tau_\delta) \to (Y, \sigma_\delta)$ is irresolute,*

(2) *Every δ-irresolute function is quasi-δ-irresolute.*

THEOREM 4.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *$f : (X, \tau) \to (Y, \sigma)$ is quasi-irresolute;*

(2) *For each $x \in X$ and each $V \in \mathrm{SO}(Y)$ containing $f(x)$, there exists $U \in \mathrm{SO}(X)$ containing x such that $f(\mathrm{sCl}(U)) \subset \mathrm{sCl}(V)$;*

(3) *For each $x \in X$ and each $V \in \delta\mathrm{SO}(Y)$ containing $f(x)$, there exists $U \in \delta\mathrm{SO}(X)$ containing x such that $f(\mathrm{sCl}_\delta(U)) \subset \mathrm{sCl}_\delta(V)$;*

(4) *$f : (X, \tau) \to (Y, \sigma)$ is quasi-δ-irresolute.*

Proof. (1) $\Leftrightarrow$ (2): This is shown in Proposition 3.3 of [11].

(2) $\Rightarrow$ (3): Let $x \in X$ and V be any δ-semi-open set containing $f(x)$. Since $\delta\mathrm{SO}(Y) \subset \mathrm{SO}(Y)$, by (2) there exists $U \in \mathrm{SO}(X)$ containing x such that $f(\mathrm{sCl}(U)) \subset \mathrm{sCl}(V)$. It follows from Theorem 3.3 $\mathrm{sCl}(V) = \mathrm{sCl}_\delta(V)$. Moreover, $\mathrm{sCl}(U) \in \mathrm{SR}(X) \subset \delta\mathrm{SO}(X)$ and $\mathrm{sCl}_\delta(\mathrm{sCl}(U)) = \mathrm{sCl}(U)$. Therefore, there exists a δ-semi-open set $\mathrm{sCl}(U)$ containing x such that $f(\mathrm{sCl}_\delta(\mathrm{sCl}(U))) = f(\mathrm{sCl}(U)) \subset \mathrm{sCl}_\delta(V)$.

(3) $\Rightarrow$ (4): This is obvious.

(4) $\Rightarrow$ (1): Let $x \in X$ and $V \in \mathrm{SO}(Y)$ containing $f(x)$. Since $\mathrm{sCl}(V)$ is δ-semi-open, there exists $U \in \delta\mathrm{SO}(X)$ containing x such that $f(U) \subset \mathrm{sCl}_\delta(\mathrm{sCl}(V)) = \mathrm{sCl}(V)$. Since every δ-semi-open set is semi-open, f is quasi-irresolute.

COROLLARY 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is quasi-irresolute if and only if $f : (X, \tau_\delta) \to (Y, \sigma_\delta)$ is quasi-irresolute.*

Proof. The proof follows from Theorems 3.1 and 4.3.

5. δ_p-closed spaces and some functions

DEFINITION 5.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *δ-almost continuous* [38] if $f^{-1}(V) \in \delta\mathrm{PO}(X)$ for each $V \in \sigma$,

(2) *δ*-almost continuous* [37] if $f^{-1}(V) \in \delta\mathrm{PO}(X)$ for each $V \in \delta\mathrm{PO}(Y)$,

(3) *p-continuous* [40] if for each $x \in X$ and each $V \in \delta\mathrm{PO}(Y)$ containing $f(x)$, there exists, an open set U containing x such that $f(U) \subset \mathrm{pCl}_\delta(V)$.

DEFINITION 5.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *almost continuous* [16] or *precontinuous* [24] if $f^{-1}(V) \in \mathrm{PO}(X)$ for each $V \in \sigma$,

(2) *preirresolute* [41] if $f^{-1}(V) \in \mathrm{PO}(X)$ for each $V \in \mathrm{PO}(Y)$,

(3) *p(θ)-continuous* [9] if for each $x \in X$ and each $V \in \mathrm{PO}(Y)$ containing $f(x)$, there exists, an open set U containing x such that $f(U) \subset \mathrm{pCl}(V)$.

THEOREM 5.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties hold:*

(1) *f is δ-almost continuous if and only if $f : (X, \tau_\delta) \to (Y, \sigma)$ is precontinuous,*

(2) *f is δ*-almost continuous if and only if $f : (X, \tau_\delta) \to (Y, \sigma_\delta)$ is preirresolute,*

(3) *f is p-continuous if and only if $f : (X, \tau) \to (Y, \sigma_\delta)$ is p(θ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.1.

REMARK 5.1. Since precontinuous functions and preirresolute functions are well-known, by using Theorem 5.1 we can obtain many properties of δ-almost continuiuty and δ*-almost continuity. For instance, we can mention as follows:

(1) The characterizations of δ-almost continuous functions obtained in Theorem 5 of [38] follow from Theorem 5.1 and Theorem 1 of [36] or Theorem 1 of [24],

(2) Concerning the product function of δ-almost continuous functions obtained in Theorem 10 of [38], the generalized form follows from Theorem 5.1 and Theorem 5 of [36] or Theorem 2.6 of [26].

DEFINITION 5.3. A subset A of a topological space (X, τ) is said to be

(1) *δ_p-compact relative to (X, τ)* if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by δ-preopen sets of X, there exists a finite subset Δ_0 of Δ such that $A \subset \bigcup_{\alpha \in \Delta_0} V_\alpha$,

(2) *δ_p-closed relative to (X, τ)* [40] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by δ-preopen sets of X, there exists a finite subset Δ_0 of Δ such that $A \subset \bigcup_{\alpha \in \Delta_0} \mathrm{pCl}_\delta(V_\alpha)$.

If $A = X$, then (X, τ) is said to be (1) *δ_p-compact* (resp. (2) *δ_p-closed*).

Definition 5.4. A subset A of a topological space (X, τ) is said to be

(1) *strongly compact relative to* (X, τ) [25] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by preopen sets of X, there exists a finite subset Δ_0 of Δ such that $A \subset \bigcup_{\alpha\in\Delta_0} V_\alpha$,

(2) *p-closed relative to* (X, τ) [12] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by preopen sets of X, there exists a finite subset Δ_0 of Δ such that $A \subset \bigcup_{\alpha\in\Delta_0} \mathrm{pCl}(V_\alpha)$.

If $A = X$, then (X, τ) is said to be (1) *strongly-compact* (resp. (2) *p-closed*).

Theorem 5.2. *Let (X, τ) be a topological space and A be a subset of X. Then the following properties hold:*

(1) *A is δ_p-compact relative to (X, τ) if and only if it is strongly compact relative to (X, τ_δ),*

(2) *A is δ_p-closed relative to (X, τ) if and only if it is p-closed relative to (X, τ_δ),*

(3) *(X, τ) is δ_p-compact (resp. δ_p-closed) if and only if (X, τ_δ) is strongly compact (resp. p-closed).*

Proof. This is an immediate consequence of Theorem 3.1.

6. New characterizations of some topological spaces

Definition 6.1. A topological space (X, τ) is said to be *s-closed* [10] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of X by semi-open sets of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}(V_\alpha)$.

Theorem 6.1. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is s-closed;*

(2) *For every δ-semi-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}(V_\alpha)$;*

(3) *For every δ-semi-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}_\delta(V_\alpha)$.*

Proof. (1) $\Rightarrow$ (2): Let $\{V_\alpha : \alpha \in \Delta\}$ be a δ-semi-open cover of X. By Lemma 3.1, $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$ and there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}(V_\alpha)$.

(2) $\Rightarrow$ (3): Let $\{V_\alpha : \alpha \in \Delta\}$ be a δ-semi-open cover of X. By Lemma 3.1, $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$ and it follows from Theorem 3.3 that $\mathrm{sCl}_\delta(V_\alpha) = \mathrm{sCl}(V_\alpha)$ for each $\alpha \in \Delta$.

(3) $\Rightarrow$ (1): Let $\{V_\alpha : \alpha \in \Delta\}$ be a semi-open cover of X. Then we have $X = \bigcup_{\alpha\in\Delta} \mathrm{sCl}(V_\alpha)$. By Lemma 3.1, we have $\mathrm{sCl}(V_\alpha) \in \mathrm{SR}(X) \subset \delta\mathrm{SO}(X)$ and there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}_\delta(\mathrm{sCl}(V_\alpha))$.

By Theorem 3.3, $\mathrm{sCl}_\delta(\mathrm{sCl}(V_\alpha)) = \mathrm{sCl}(\mathrm{sCl}(V_\alpha)) = \mathrm{sCl}(V_\alpha)$. Therefore, we obtain $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}(V_\alpha)$. This shows that X is s-closed.

COROLLARY 6.1. *A topological space (X,τ) is s-closed if and only if (X,τ_δ) is s-closed.*

Proof. This is an immediate consequence of Theorems 3.1 and 6.1.

THEOREM 6.2. *A topological space (X,τ) is s-closed if and only if for every semi-θ-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} V_\alpha$.*

Proof. *Necessity.* Let $\{V_\alpha : \alpha \in \Delta\}$ be a semi-θ-open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in V_{\alpha(x)}$. Since $V_{\alpha(x)}$ is semi-θ-open, there exists $G_{\alpha(x)} \in \mathrm{SO}(X)$ such that $x \in G_{\alpha(x)} \subset \mathrm{sCl}(G_{\alpha(x)}) \subset V_{\alpha(x)}$. Since $\{G_{\alpha(x)} : x \in X\}$ is a semi-open cover of X, there exist finite points, say, $x_1, x_2, ..., x_n$ such that $X = \bigcup_{i=1}^n \mathrm{sCl}(G_{\alpha(x_i)})$. Hence $X = \bigcup_{i=1}^n V_{\alpha(x_i)}$.

Sufficiency. Let $\{V_\alpha : \alpha \in \Delta\}$ be a semi-open cover of X. By Lemma 3.1, $\{\mathrm{sCl}(V_\alpha) : \alpha \in \Delta\}$ is a semi-regular cover of X and hence a semi-θ-open cover of X. Therefore, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\alpha\in\Delta_0} \mathrm{sCl}(V_\alpha)$. This shows that (X,τ) is s-closed.

DEFINITION 6.2. A topological space (X,τ) is said to be *semi-connected* [35] if X cannot be expressed by the disjoint union of two nonempty semi-open sets.

Levine [19] called a topological space X a *D-space* if $\mathrm{Cl}(V) = X$ for every nonempty open set V of X. In [42], D-spaces are called *hyperconnected.* We obtain several characterizations of semi-connected spaces by using δ-semi-open sets. Here we should note that open sets and δ-semi-open sets are independent of each other.

THEOREM 6.3. *For a topological space (X,τ), the following properties are equivalent:*

(1) $\mathrm{Cl}(V) = X$ *for every nonempty open set V of X;*

(2) (X,τ) *is semi-connected;*

(3) *X cannot be expressed by the disjoint union of two nonempty δ-semi-open sets;*

(4) $\mathrm{sCl}_\delta(V) = X$ *for every nonempty $V \in \delta\mathrm{SO}(X)$.*

Proof. (1) ⇔ (2): This is shown in Theorem 4.3 of [35].

(2) ⇒ (3): Suppose that there exist two nonempty δ-semi-open sets V_1, V_2 such that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = X$. Since $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$, this shows that (X,τ) is not semi-connected.

(3) ⇒ (4): $\mathrm{sCl}_\delta(V) \neq X$ for some nonempty $V \in \delta\mathrm{SO}(X)$. Then, $X - \mathrm{sCl}_\delta(V) \neq \emptyset$, $\mathrm{sCl}_\delta(V) \neq \emptyset$ and $X = (X - \mathrm{sCl}_\delta(V)) \cup \mathrm{sCl}_\delta(V)$. Since $\delta\mathrm{SO}(X) \subset$

SO(X), by Theorem 3.3 and Lemma 3.1 $\mathrm{sCl}_\delta(V) = \mathrm{sCl}(V) \in \mathrm{SR}(X)$. Moreover, since $\mathrm{SR}(X) \subset \delta\mathrm{SO}(X)$, $(X - \mathrm{sCl}_\delta(V))$ and $\mathrm{sCl}_\delta(V)$ are δ-semi-open.

(4) $\Rightarrow$ (1): Let V be any nonempty open set of (X, τ). Then $\mathrm{Cl}(V)$ is regular closed and hence semi-regular. Therefore, $\mathrm{Cl}(V)$ is δ-semi-open and $X = \mathrm{sCl}_\delta(\mathrm{Cl}(V)) = \mathrm{sCl}(\mathrm{Cl}(V)) = \mathrm{Cl}(V)$.

THEOREM 6.4. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is semi-connected;*
(2) *$\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \beta(X)$;*
(3) *$\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \mathrm{SO}(X)$;*
(4) *$\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \mathrm{PO}(X)$;*
(5) *$\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \alpha(X)$;*
(6) *$\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \tau$.*

Proof. (1) $\Rightarrow$ (2): Let V be any nonempty β-open set and U any nonempty δ-semi-open set. Then $\mathrm{Int}(\mathrm{Cl}(V)) \neq \emptyset$ and $\mathrm{Int}(U) \neq \emptyset$ by Lemma 4 of [32]. By Theorem 6.3, we have $\emptyset \neq \mathrm{Int}(U) \cap \mathrm{Int}(\mathrm{Cl}(V)) \subset U \cap \mathrm{Int}(\mathrm{Cl}(V)) \subset U \cap (V \cup \mathrm{Int}(\mathrm{Cl}(V))) = U \cap \mathrm{sCl}(V) \subset U \cap \mathrm{sCl}_\delta(V)$. Since $U \in \delta\mathrm{SO}(X)$, $U \cap V \neq \emptyset$. This shows that $\mathrm{sCl}_\delta(V) = X$.

(6) $\Rightarrow$ (1): Let U, V be any nonempty δ-semi-open sets. Since $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$ and $\mathrm{Int}(V) \neq \emptyset$, we have $\emptyset \neq U \cap \mathrm{Int}(V) \subset U \cap V$. This shows that $\mathrm{sCl}_\delta(V) = X$ for every nonempty $V \in \delta\mathrm{SO}(X)$. Therefore, by Theorem 6.3 (X, τ) is semi-connected.

Other implications are obvious since $\tau \subset \alpha(X) \subset \mathrm{SO}(X) \cap \mathrm{PO}(X)$ and $\mathrm{SO}(X) \cup \mathrm{PO}(X) \subset \beta(X)$.

COROLLARY 6.2. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is semi-connected;*
(2) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \beta(X)$ and $V \in \delta\mathrm{SO}(X)$;*
(3) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \mathrm{PO}(X)$ and $V \in \delta\mathrm{SO}(X)$;*
(4) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \mathrm{SO}(X)$ and $V \in \delta\mathrm{SO}(X)$;*
(5) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \alpha(X)$ and $V \in \delta\mathrm{SO}(X)$;*
(6) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \tau$ and $V \in \delta\mathrm{SO}(X)$;*
(7) *$U \cap V \neq \emptyset$ for any nonempty sets $U \in \delta\mathrm{SO}(X)$ and $V \in \delta\mathrm{SO}(X)$.*

Proof. This is an immediate consequence of Theorems 6.3 and 6.4.

COROLLARY 6.3. *A topological space (X, τ) is semi-connected if and only if (X, τ_δ) is semi-connected.*

Proof. It is shown in Theorem 3.1 of [33] that (X, τ) is hyperconnected if and only if $\mathrm{sCl}(U) = X$ for every nonempty $U \in \mathrm{SO}(X)$. The proof follows from this fact and Theorems 3.1 and 6.3.

DEFINITION 6.3. A topological space (X, τ) is said to be *semi-T_2* [22] if for each pair of distinct points x, y, there exist $U, V \in \mathrm{SO}(X)$ such that $x \in U, y \in V$ and $U \cap V = \emptyset$.

THEOREM 6.5. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is semi-T_2;*

(2) *For each pair of distinct points x, y, there exist $U, V \in \mathrm{SR}(X)$ such that $x \in U, y \in V$ and $U \cap V = \emptyset$;*

(3) *For each pair of distinct points x, y, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $x \in U, y \in V$ and $\mathrm{sCl}_\delta(U) \cap \mathrm{sCl}_\delta(V) = \emptyset$;*

(4) *For each pair of distinct points x, y, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $x \in U, y \in V$ and $\mathrm{sCl}(U) \cap \mathrm{sCl}(V) = \emptyset$;*

(5) *For each pair of distinct points x, y, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $x \in U, y \in V$ and $U \cap V = \emptyset$.*

Proof. (1) $\Rightarrow$ (2): Let (X, τ) is semi-T_2. Then for each pair of distinct points x, y, there exist $G, H \in \mathrm{SO}(X)$ such that $x \in G, y \in H$ and $G \cap H = \emptyset$. We have $\mathrm{sCl}(G) \cap H = \emptyset$. By Lemma 3.1, $\mathrm{sCl}(G) \in \mathrm{SR}(X)$ and $\mathrm{sCl}(G) \cap \mathrm{sCl}(H) = \emptyset$. Now set $U = \mathrm{sCl}(G)$ and $V = \mathrm{sCl}(H)$. Then (2) is obtained.

(2) $\Rightarrow$ (3): This follows from the facts that $\mathrm{SR}(X) \subset \delta\mathrm{SO}(X)$ and $\mathrm{sCl}_\delta(U) = \mathrm{sCl}(U) = U$ for every $U \in \mathrm{SR}(X)$.

(3) $\Rightarrow$ (4): This follows from the fact that $\mathrm{sCl}(U) = \mathrm{sCl}_\delta(U)$ for every $U \in \delta\mathrm{SO}(X)$.

(4) $\Rightarrow$ (5): This is obvious.

(5) $\Rightarrow$ (1): This is obvious since $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$.

COROLLARY 6.4. *A topological space (X, τ) is semi-T_2 if and only if (X, τ_δ) is semi-T_2.*

Proof. This is an immediate consequence of Theorems 3.1 and 6.5.

DEFINITION 6.4. A topological space (X, τ) is said to be *s-Urysohn* [4] if for each pair of distinct points x, y, there exist $U, V \in \mathrm{SO}(X)$ such that $x \in U, y \in V$ and $\mathrm{Cl}(U) \cap \mathrm{Cl}(V) = \emptyset$.

THEOREM 6.6. *A topological space (X, τ) is s-Urysohn if and only if for each pair of distinct points x, y of X, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $x \in U, y \in V$ and $\mathrm{Cl}(U) \cap \mathrm{Cl}(V) = \emptyset$.*

Proof. *Necessity.* Suppose that (X, τ) is s-Urysohn. Then for each pair of distinct points x, y, there exist $U, V \in \mathrm{SO}(X)$ such that $x \in U, y \in V$ and $\mathrm{Cl}(U) \cap \mathrm{Cl}(V) = \emptyset$. Since $U \in \mathrm{SO}(X)$, by Lemma 2 of [32] $\mathrm{Cl}(U) = \mathrm{Cl}(\mathrm{Int}(U))$ and $\mathrm{Cl}(U)$ is regular closed. Therefore, we obtain $\mathrm{Cl}(U), \mathrm{Cl}(V) \in \mathrm{SR}(X) \subset \delta\mathrm{SO}(X)$. It is obvious that $x \in \mathrm{Cl}(U), y \in \mathrm{Cl}(V)$ and $\mathrm{Cl}(\mathrm{Cl}(U)) \cap \mathrm{Cl}(\mathrm{Cl}(V)) = \mathrm{Cl}(U) \cap \mathrm{Cl}(V) = \emptyset$.

Sufficiency. The proof is obvious since $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$.

DEFINITION 6.5. A topological space (X, τ) is said to be *s-regular* [21] (resp. *semi-regular* [13]) if for each closed (resp. semi-closed) set F of X and each point $x \notin F$, there exist $U, V \in \mathrm{SO}(X)$ such that $x \in U, F \subset V$ and $U \cap V = \emptyset$.

THEOREM 6.7. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is s-regular (resp. semi-regular);*

(2) *For each closed (resp. semi-closed) set F and each point $x \notin F$, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $x \in U, F \subset V$ and $U \cap V = \emptyset$;*

(3) *For each point $x \in X$ and each open (resp. semi-open) set V containing x, there exists $U \in \delta\mathrm{SO}(X)$ such that $x \in U \subset \mathrm{sCl}_\delta(U) \subset V$.*

Proof. (1) $\Rightarrow$ (2): Let F be a closed (resp. semi-closed) set and $x \notin F$. there exist $G, H \in \mathrm{SO}(X)$ such that $x \in G, F \subset H$ and $G \cap H = \emptyset$. By Lemma 3.1, $\mathrm{sCl}(G)$ is semi-regular and $\mathrm{sCl}(G) \cap H = \emptyset$. Therefore, we obtain $\mathrm{sCl}(G) \cap \mathrm{sCl}(H) = \emptyset$. Now, we set $U = \mathrm{sCl}(G)$ and $V = \mathrm{sCl}(H)$, then we obtain (2).

(2) $\Rightarrow$ (3): Let $x \in X$ and V be any open (resp. semi-open) set containing x. Since $x \notin X - V$, there exist $U, G \in \delta\mathrm{SO}(X)$ such that $x \in U, X - V \subset G$ and $U \cap G = \emptyset$. Since $X - G$ is δ-semi-closed and $U \subset X - G$, we obtain that $x \in U \subset \mathrm{sCl}_\delta(U) \subset X - G \subset V$.

(3) $\Rightarrow$ (1): Let F be a closed (resp. semi-closed) set X and $x \notin F$. Then $X - F$ is an open (resp. semi-open) set containing x. By (3), there exists $U \in \delta\mathrm{SO}(X)$ such that $x \in U \subset \mathrm{sCl}_\delta(U) \subset X - F$. Therefore, we obtain $x \in U, F \subset X - \mathrm{sCl}_\delta(U)$ and $U \cap (X - \mathrm{sCl}_\delta(U)) = \emptyset$. Since $\delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$, (X, τ) is s-regular (resp. semi-regular).

DEFINITION 6.6. A topological space (X, τ) is said to be *s-normal* [23] (resp. *semi-normal* [14]) if for each disjoint closed (resp. semi-closed) sets F, K of X, there exist $U, V \in \mathrm{SO}(X)$ such that $F \subset U, K \subset V$ and $U \cap V = \emptyset$.

THEOREM 6.8. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is s-normal (resp. semi-normal);*

(2) *For each disjoint closed (resp. semi-closed) sets F, K of X, there exist $U, V \in \delta\mathrm{SO}(X)$ such that $F \subset U, K \subset V$ and $U \cap V = \emptyset$;*

(3) *For each closed (resp. semi-closed) set F and each open (resp. semi-open) set V containing F, there exists $U \in \delta\mathrm{SO}(X)$ such that $F \subset U \subset \mathrm{sCl}_\delta(U) \subset V$.*

Proof. The proof is analogous to that of Theorem 6.7 and is omitted.

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