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B.Y. CHEN INEQUALITIES FOR BI-SLANT SUBMANIFOLDS IN SASAKIAN SPACE FORMS

Abstract. In the present paper, we establish Chen inequalities for bi-slant submanifolds in Sasakian space forms, by using subspaces orthogonal to the Reeb vector field ξ.

1. Preliminaries: Riemannian invariants

The Riemannian invariants of a Riemannian manifold are the intrinsic characteristics of the Riemannian manifold. In this section, we recall a string of Riemannian invariants on a Riemannian manifold introduced in [6].

Let M be a Riemannian manifold. Denote by $K(\pi)$ the sectional curvature of M associated with a plane section $\pi \subset T_pM$, $p \in M$.

For any orthonormal basis $\{e_1, e_2, \dots, e_n\}$ of the tangent space T_pM, the scalar curvature τ at p is defined by

$$\tau(p) = \sum_{i<j} K(e_i \wedge e_j). \tag{1.1}$$

One denotes by

$$(\inf K)(p) = \inf\{K(\pi) \mid \pi \subset T_pM,\ \dim \pi = 2\}, \tag{1.2}$$

and one introduces the Chen invariant

$$\delta_M(p) = \tau(p) - (\inf K)(p). \tag{1.3}$$

We recall the following lemma of Chen [5].

LEMMA 1.1. *Let $a_1, \dots, a_n, c$ be $n+1$, $n \geq 2$, real numbers such that*

$$\Big(\sum_{i=1}^{n} a_i\Big)^2 = (n-1)\Big(\sum_{i=1}^{n} a_i^2 + c\Big).$$

Then $2a_1a_2 \geq c$ and the equality holds if and only if $a_1 + a_2 = a_3 = \dots = a_n$.

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Let M be an n-dimensional submanifold of a Riemannian manifold $\tilde{M}$. We denote by ∇ the Riemannian connection of M. Also, let h be the second fundamental form and R the Riemann curvature tensor on M. Then the equation of Gauss is given by

$$\tilde{R}(X,Y,Z,W) = R(X,Y,Z,W) + \\ + g(h(X,W),h(Y,Z)) - g(h(X,Z),h(Y,W)),$$

for any vectors X, Y, Z, W tangent to M.

Let $p \in M$ and $\{e_1, \dots, e_n\}$ be an orthonormal basis of the tangent space T_pM. We denote by H the mean curvature vector, that is

$$H(p) = \frac{1}{n}\sum_{i=1}^{n} h(e_i, e_i). \tag{1.4}$$

Also, we set

$$h_{ij}^r = g(h(e_i, e_j), e_r)$$

and

$$\|h\|^2 = \sum_{i,j=1}^{n} g(h(e_i,e_j), h(e_i,e_j)).$$

2. Bi-slant submanifolds in Sasakian manifolds

A $(2m+1)$-dimensional Riemannian manifold $(\tilde{M}, g)$ is said to be a *Sasakian manifold* if it admits an endomorphism ϕ of its tangent bundle $T\tilde{M}$, a vector field ξ and a 1-form η satisfying:

$$\begin{cases} \phi^2 = -Id + \eta\otimes\xi, \ \eta(\xi) = 1, \ \phi\xi = 0, \ \eta\circ\phi = 0, \\ g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \ \eta(X) = g(X,\xi), \\ (\tilde{\nabla}_X\phi)Y = -g(X,Y)\xi + \eta(Y)X, \ \tilde{\nabla}_X\xi = \phi X, \end{cases}$$

for any vector fields X, Y on $T\tilde{M}$, where $\tilde{\nabla}$ denotes the Riemannian connection with respect to g.

A plane section π in $T_p\tilde{M}$ is called a *ϕ-section* if it is spanned by X and ϕX, where X is a unit tangent vector orthogonal to ξ. The sectional curvature of a ϕ-section is called a *ϕ-sectional curvature.* A Sasakian manifold with constant ϕ-sectional curvature c is said to be a *Sasakian space form* and is denoted by $\tilde{M}(c)$.

The curvature tensor of $\tilde{M}(c)$ of a Sasakian space form $\tilde{M}(c)$ is given by [1]

$$\tilde{R}(X,Y)Z = \frac{c+3}{4}\{g(Y,Z)X - g(X,Z)Y\} + \\ + \frac{c-1}{4}\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi + \\ + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\},$$

for any tangent vector fields X, Y, Z on $\tilde{M}(c)$.

As examples of Sasakian space forms we mention $\mathbb{R}^{2m+1}$ and S^{2m+1}, with standard Sasakian structures (see [1]).

DEFINITION. We call a differentiable distribution $\mathcal{D}$ on M a *slant distribution* if for each $x \in M$ and each nonzero vector $X \in \mathcal{D}_x$, the angle $\theta_{\mathcal{D}}(X)$ between ϕX and the vector subspace $\mathcal{D}_x$ is constant, which is independent of the choice of $x \in M$ and $X \in \mathcal{D}_x$. In this case, the constant angle $\theta_{\mathcal{D}}$ is called the *slant angle* of the distribution $\mathcal{D}$.

DEFINITION. We say that a submanifold M tangent to ξ is a *bi-slant* submanifold of $\tilde{M}$ if there exist two orthogonal distributions $\mathcal{D}_1$ and $\mathcal{D}_2$ on M such that:

i) TM admits the orthogonal direct decomposition $TM = \mathcal{D}_1 \oplus \mathcal{D}_2 \oplus \{\xi\}$.

ii) For any $i = 1, 2$, $\mathcal{D}_i$ is slant distribution with slant angle θ_i.

Let $2d_1 = \dim \mathcal{D}_1$ and $2d_2 = \dim \mathcal{D}_2$.

REMARK. If either d_1 or d_2 vanishes, the bi-slant submanifold ia a slant submanifold. Thus, slant submanifolds (and, therefore, invariant and anti-invariant submanifolds) are special cases of bi-slant submanifolds.

Invariant and *anti-invariant immersions* are slant immersions with slant angle $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively. A slant immersion which is neither invariant nor anti-invariant is called a *proper slant immersion.*

For the properties and examples of bi-slant submanifolds in Sasakian manifolds, we refer to [2].

For any tangent vector field X to M, we put $\phi X = PX + FX$, where PX and FX are the tangential and normal components of ϕX, respectively. We denote by

$$\|P\|^2 = \sum_{i,j=1}^{n} g^2(Pe_i, e_j). \tag{2.1}$$

3. B.Y. Chen inequality

We prove the Chen inequality for bi-slant submanifolds in a Sasakian space form.

We consider plane sections π orthogonal to ξ.

THEOREM 3.1. *Let M be an $(n = 2d_1 + 2d_2 + 1)$ dimensional bi-slant submanifold in a $(2m+1)$-dimensional Sasakian space form $\tilde{M}(c)$. Then we have:*

$$\inf K \geq \tau - \frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\right\} - \frac{(c-1)}{4}[3(d_1-1)\cos^2\theta_1 + 3d_2\cos^2\theta_2 - (n-1)], \tag{3.1}$$

on $\mathcal{D}_1$ *and*

$$(3.1') \quad \inf K \geq \tau - \frac{n-2}{2}\Big\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\Big\} - \frac{(c-1)}{4}[3d_1\cos^2\theta_1 + 3(d_2-1)\cos^2\theta_2 - (n-1)].$$

on $\mathcal{D}_2$.

The equality case of the inequality (3.1) *and* (3.1$'$) *holds at a point* $p \in M$ *if and only if there exists an orthonormal basis* $\{e_1, \dots, e_n = \xi\}$ *of* T_pM *and an orthonormal basis* $\{e_{n+1}, \dots, e_{2m}, e_{2m+1}\}$ *of* $T_p^{\perp}M$ *such that the shape operators of* M *in* $\tilde{M}(c)$ *at* p *have the following forms:*

$$(3.2) \qquad A_{n+1} = \begin{pmatrix} a & 0 & 0 & . & . & & . & 0 \\ 0 & b & 0 & . & . & & . & 0 \\ 0 & 0 & & & \mu I_{n-2} & & & \end{pmatrix}, \qquad a + b = \mu,$$

$$(3.3) \quad A_r = \begin{pmatrix} h_{11}^r & h_{12}^r & 0 & . & . & & . & 0 \\ h_{12}^r & -h_{11}^r & 0 & . & . & & . & 0 \\ 0 & 0 & & & 0_{n-2} & & & \end{pmatrix}, \qquad r \in \{n+2, \dots, 2m+1\}.$$

P r o o f. We recall the Gauss equation for the submanifold M

$\tilde{R}(X,Y,Z,W) = R(X,Y,Z,W) + g(h(X,W),h(Y,Z)) - g(h(X,Z),h(Y,W))$,

for all $X, Y, Z, W \in \Gamma(TM)$.

Since $\tilde{M}(c)$ is a Sasakian space form, then we have

$$(3.4) \qquad \tilde{R}(X,Y,Z,W) = \frac{c+3}{4}\{-g(Y,Z)g(X,W) + g(X,Z)g(Y,W)\} +$$
$$+\frac{c-1}{4}\{-\eta(X)\eta(Z)g(Y,W) + \eta(Y)\eta(Z)g(X,W) - g(X,Z)\eta(Y)g(\xi,W) +$$
$$+g(Y,Z)\eta(X)g(\xi,W) - g(\phi Y,Z)g(\phi X,W) + g(\phi X,Z)g(\phi Y,W) +$$
$$+2g(\phi X,Y)g(\phi Z,W)\}, \qquad \forall X,Y,Z,W \in \Gamma(TM).$$

Let $p \in M$ and $\{e_1, \dots, e_n = \xi\}$ an orthonormal basis of T_pM and $\{e_{n+1}, \dots, e_{2m}, e_{2m+1}\}$ an orthonormal basis of $T_p^{\perp}M$. For $X = Z = e_i, Y = W = e_j$, $\forall i,j \in \{1, \dots, n\}$, from the equation (3.4), it follows that

$$(3.5) \qquad \tilde{R}(e_i,e_j,e_i,e_j) = \frac{c+3}{4}(-n+n^2) + \frac{c-1}{4}\Big\{-2(n-1) + 3\sum_{i,j=1}^{n} g^2(\phi e_i, e_j)\Big\}.$$

Let $M \subset \tilde{M}(c)$ be a bi-slant submanifold, $\dim M = n = 2d_1 + 2d_2 + 1$.

We consider an adapted bi-slant orthonormal frame

$$e_1, e_2 = \frac{1}{\cos\theta_1} P e_1, \dots, e_{2d_1-1},$$
$$e_{2d_1} = \frac{1}{\cos\theta_1} P e_{2d_1-1}, e_{2d_1+1},$$
$$e_{2d_1+2} = \frac{1}{\cos\theta_2} P e_{2d_1+1}, \dots, e_{2d_1+2d_2-1},$$
$$e_{2d_1+2d_2} = \frac{1}{\cos\theta_2} P e_{2d_1+2d_2-1}, e_{2d_1+2d_2+1} = \xi.$$

We have

$$g(\phi e_1, e_2) = g\left(\phi e_1, \frac{1}{\cos\theta_1} P e_1\right) = \frac{1}{\cos\theta_1} g(\phi e_1, P e_1)$$
$$= \frac{1}{\cos\theta_1} g(P e_1, P e_1) = \cos\theta_1$$

and, in the same way,

$$g^2(\phi e_i, e_{i+1}) = \begin{cases} \cos^2\theta_1, & \text{for } i \in \{1, \dots, 2d_1 - 1\}, \\ \cos^2\theta_2, & \text{for } i \in \{2d_1 + 1, \dots, 2d_1 + 2d_2 - 1\}. \end{cases}$$

Then

$$\sum_{i,j=1}^{n} g^2(\phi e_i, e_j) = 2(d_1 \cos^2\theta_1 + d_2 \cos^2\theta_2).$$

The relation (3.5) implies that

$$\text{(3.6)} \quad \tilde{R}(e_i, e_j, e_i, e_j) = \frac{c+3}{4}(n^2 - n) + \frac{c-1}{4}[6(d_1 \cos^2\theta_1 + d_2 \cos^2\theta_2) - 2(n-1)].$$

Denoting by

$$\text{(3.7)} \qquad \|h\|^2 = \sum_{i,j=1}^{n} g(h(e_i, e_j), h(e_i, e_j)),$$

from the relation (3.6), one has

$$\text{(3.8)} \qquad \frac{c+3}{4} n(n-1) + \frac{c-1}{4}[6(d_1 \cos^2\theta_1 + d_2 \cos^2\theta_2) - 2n + 2] = 2\tau - n^2\|H\|^2 + \|h\|^2,$$

or equivalently,

$$\text{(3.9)} \quad 2\tau = n^2\|H\|^2 - \|h\|^2 + \frac{c+3}{4} n(n-1) + \frac{c-1}{4}[6(d_1 \cos^2\theta_1 + d_2 \cos^2\theta_2) - 2n + 2].$$

If we put

$$\varepsilon = 2\tau - \frac{n^2}{n-1}(n-2)\|H\|^2 - \frac{c+3}{4}n(n-1) - \frac{c-1}{4}[6(d_1\cos^2\theta_1 + d_2\cos^2\theta_2) - 2n + 2], \tag{3.10}$$

we obtain

$$n^2\|H\|^2 = (n-1)(\varepsilon + \|h\|^2). \tag{3.11}$$

Let $p \in M$, $\pi \subset T_pM$, $\dim\pi = 2$, π orthogonal to ξ.

We consider two cases:

i) π is tangent to $\mathcal{D}_1$. We may assume $\pi = sp\{e_1, e_2\}$.

We put $e_{n+1} = \frac{H}{\|H\|}$. The relation (3.11) becomes

$$\Big(\sum_{i=1}^{n} h_{ii}^{n+1}\Big)^2 = (n-1)\Big\{\sum_{i,j=1}^{n}\sum_{r=n+1}^{2m+1}(h_{ij}^r)^2 + \varepsilon\Big\},$$

or equivalently,

$$\Big(\sum_{i=1}^{n} h_{ii}^{n+1}\Big)^2 = (n-1)\Big\{\sum_{i=1}^{n}[(h_{ii}^{n+1})^2 + \sum_{i\neq j}(h_{ij}^{n+1})^2 + \sum_{r=n+2}^{2m+1}\sum_{i,j=1}^{n}(h_{ij}^r)^2 + \varepsilon\Big\}. \tag{3.12}$$

By using Lemma 1.1, we derive from (3.12):

$$2h_{11}^{n+1}h_{22}^{n+1} \geq \sum_{i\neq j}(h_{ij}^{n+1})^2 + \sum_{i,j=1}^{n}\sum_{r=n+2}^{2m+1}(h_{ij}^r)^2 + \varepsilon. \tag{3.13}$$

From the Gauss equation for $X = Z = e_1, Y = W = e_2$, we obtain

$$\begin{aligned}
K(\pi) &= \frac{c+3}{4} + 3\cos^2\theta_1 \cdot \frac{c-1}{4} + \sum_{r=n+1}^{2m+1}[h_{11}^r h_{22}^r - (h_{12}^r)^2] \geq \\
&\geq \frac{c+3}{4} + 3\cos^2\theta_1 \cdot \frac{c-1}{4} + \frac{1}{2}\sum_{i\neq j}(h_{ij}^{n+1})^2 + \frac{1}{2}\sum_{i,j=1}^{n}\sum_{r=n+2}^{2m+1}(h_{ij}^r)^2 + \frac{\varepsilon}{2} + \\
&\quad + \sum_{r=n+2}^{2m+1} h_{11}^r h_{22}^r - \sum_{r=n+1}^{2m+1}(h_{12}^r)^2 = \\
&= \frac{c+3}{4} + 3\cos^2\theta_1 \cdot \frac{c-1}{4} + \frac{1}{2}\sum_{i\neq j}(h_{ij}^{n+1})^2 + \frac{1}{2}\sum_{r=n+2}^{2m+1}\sum_{i,j>2}(h_{ij}^r)^2 +
\end{aligned}$$

$$+\frac{1}{2}\sum_{r=n+2}^{2m+1}(h_{11}^r+h_{22}^r)^2+\sum_{j>2}[(h_{1j}^{n+1})^2+(h_{2j}^{n+1})^2]+\frac{\varepsilon}{2}\geq$$

$$\geq\frac{c+3}{4}+3\cos^2\theta_1\cdot\frac{c-1}{4}+\frac{\varepsilon}{2},$$

or equivalently,

$$K(\pi)\geq\frac{c+3}{4}+3\cos^2\theta_1\cdot\frac{c-1}{4}+\frac{\varepsilon}{2}. \tag{3.14}$$

Then

$$\inf K-\tau\geq\frac{c+3}{4}+3\cos^2\theta_1\cdot\frac{c-1}{4}- \tag{3.15}$$

$$-\left\{\frac{c+3}{8}(n^2-n)+\frac{c-1}{8}[6(d_1\cos^2\theta_1+d_2\cos^2\theta_2)-2n+2]\right\}-\frac{n^2(n-2)}{2(n-1)}\|H\|^2.$$

The last relation implies that

$$\inf K\geq\tau-\frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2+\frac{(c+3)(n+1)}{4}\right\}-$$

$$-\frac{(c-1)}{4}[3(d_1\cos^2\theta_1+d_2\cos^2\theta_2)-(n-1)]+3\cos^2\theta_1\cdot\frac{c-1}{4}.$$

ii) Similary, if π is tangent to $\mathcal{D}_2$, we obtain

$$\inf K\geq\tau-\frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2+\frac{(c+3)(n+1)}{4}\right\}-$$

$$-\frac{(c-1)}{4}[3(d_1\cos^2\theta_1+d_2\cos^2\theta_2)-(n-1)]+3\cos^2\theta_2\frac{c-1}{4}.$$

These relations represent the inequalities to prove.

The case of equality at a point $p\in M$ holds if and only if it achieves the equalities in the previous inequalities and we have the equality in the Lemma 1.1

$$\begin{cases} h_{ij}^{n+1}=0, & \forall i\neq j, i,j>2,\\ h_{ij}^r=0, & \forall i\neq j, i,j>2, r=n+1,\dots,2m+1,\\ h_{11}^r+h_{22}^r=0, & \forall r=n+2,\dots,2m+1,\\ h_{1j}^{n+1}=h_{2j}^{n+1}=0, & \forall j>2,\\ h_{11}^{n+1}+h_{22}^{n+1}=h_{33}^{n+1}=\dots=h_{nn}^{n+1}. \end{cases}$$

We may choose $\{e_1,e_2\}$ such that $h_{12}^{n+1}=0$ and we denote by $a=h_{11}^r, b=h_{22}^r, \mu=h_{33}^{n+1}=\dots=h_{nn}^{n+1}$.

It follows that the shape operators take the desired forms.

COROLLARY 3.2. *Let M be an n-dimensional contact CR submanifold ($\theta_1=0,\theta_2=\frac{\pi}{2}$) in a $(2m+1)$-dimensional Sasakian space form $\tilde{M}(c)$.*

Then we have:

$$\inf K \geq \tau - \frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\right\} - \frac{(c-1)}{4}[3d_1 - (n+2)],$$

on $\mathcal{D}_1$ and

$$\inf K \geq \tau - \frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\right\} - \frac{(c-1)}{4}[3d_1 - (n-1)],$$

on $\mathcal{D}_2$.

In particular if $\theta_1 = \theta_2 = \theta$, for slant submanifolds, one derives

THEOREM 3.3.[7] *Let M be an $(n = 2k+1)$-dimensional θ-slant submanifold in a $(2m+1)$-dimensional Sasakian space form $\tilde{M}(c)$. Then we have:*

$$\delta_M \leq \frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\right\} + \frac{(c-1)}{8}[3(n-3)\cos^2\theta - 2(n-1)].$$

The equality case of the inequality (3.1) holds at a point $p \in M$ if and only if there exists an orthonormal basis $\{e_1, \ldots, e_n = \xi\}$ of T_pM and an orthonormal basis $\{e_{n+1}, \ldots, e_{2m}, e_{2m+1}\}$ of $T_p^{\perp}M$ such that the shape operators of M in $\tilde{M}(c)$ at p have the following forms:

$$A_{n+1} = \begin{pmatrix} a & 0 & 0 & . & . & & . & 0 \\ 0 & b & 0 & . & . & & . & 0 \\ 0 & 0 & & & \mu I_{n-2} & & & \end{pmatrix}, \quad a + b = \mu,$$

$$A_r = \begin{pmatrix} h_{11}^r & h_{12}^r & 0 & . & . & & . & 0 \\ h_{12}^r & -h_{11}^r & 0 & . & . & & . & 0 \\ 0 & 0 & & & 0_{n-2} & & & \end{pmatrix}, \quad r \in \{n+2, \ldots, 2m+1\}.$$

COROLLARY 3.4. *Let M be an $(n = 2k+1)$-dimensional invariant submanifold in a $(2m+1)$-dimensional Sasakian space form $\tilde{M}(c)$.*

Then we have:

$$\delta_M \leq \frac{(c+3)(n-2)(n+1)}{8} + \frac{(c-1)(n-7)}{8}.$$

COROLLARY 3.5. *Let M be an n-dimensional anti-invariant submanifold in a $(2m+1)$-dimensional Sasakian space form $\tilde{M}(c)$.*

Then we have:

$$\delta_M \leq \frac{n-2}{2}\left\{\frac{n^2}{n-1}\|H\|^2 + \frac{(c+3)(n+1)}{4}\right\} - \frac{(c-1)(n-1)}{4}.$$

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