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# EXCEPTIONAL SETS FOR FUNCTIONS FROM THE WEIGHTED BERGMAN SPACES IN THE BALL

**Abstract.** In this note we study the behaviour of holomorphic functions in the unit ball $\mathbb{B}_N$ in $\mathbb{C}^N$ on one-dimensional complex subspaces of $\mathbb{C}^N$. The behaviour of functions is described in terms of $L^2$-integrability with weights on the sets $L \cap \mathbb{B}_N$, where $L$ runs over different families $E$ of one-dimensional complex subspaces of $\mathbb{C}^N$.

## 1. Introduction

Denote by $\mathbb{B}_N$ the unit ball in $\mathbb{C}^N$. Given an open subset $D$ of $\mathbb{C}^N$, denote by $\mathcal{O}(D)$ the space of all functions holomorphic in $D$, and let $L^2H(D)$ be the space of those holomorphic functions in $D$ for which $\int_D |f(z)|^2 dm(z) < +\infty$ (where $m$ denotes the Lebesgue measure in $\mathbb{C}^N$).

We have proved in [1] that there exists a function $f$ holomorphic in $\mathbb{B}_N$ such that for every complex subspace $L$ of $\mathbb{C}^N$, $f|_{L\cap B_N} \notin L^2(L \cap B_N)$. Note that such function $f$ cannot be in $L^2H(\mathbb{B}_N)$, by Fubini's theorem. On the other hand, in [2] the following result concerning the functions from the space $L^2H(\mathbb{B}_N)$ was proved:

THEOREM 1 ([2], Theorem 1). *Let $E$ be a subset of one-dimensional complex subspaces of $\mathbb{C}^N$ such that the set $\bigcup\{\Pi \cap \mathbb{B}_N | \Pi \in E\}$ is closed in $\mathbb{B}_N$ and of Lebesgue measure zero. Then there exists a function $f \in L^2H(\mathbb{B}_N)$ such that for every one-dimensional complex subspace $\Pi$ of $\mathbb{C}^N$,*

$$\int_{\Pi\cap\mathbb{B}_N} |f(w)|^2 dm(w) = +\infty \tag{1}$$

*iff $\Pi \in E$.*

(Here $m$ is the Lebesgue measure in $\Pi \cap \mathbb{B}_N$).

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It is also known that the following result holds: Given $s > -1$, denote by $A^{2,s}(\mathbb{B}_N)$ the space of all functions holomorphic in $\mathbb{B}_N$ such that

$$\int_{\mathbb{B}_N} |f(z)|^2(1-|z|^2)^s dm(z) < +\infty.$$

Let $\Pi$ be an arbitrary one-dimensional complex subspace of $\mathbb{C}^N$. Then for every $f \in A^{2,s}(\mathbb{B}_N)$, $f|_{\Pi\cap\mathbb{B}_N} \in A^{2,N-1+s}(\Pi\cap\mathbb{B}_N)$, i.e.

$$\int_{\Pi\cap\mathbb{B}_N} |f(w)|^2(1-|w|^2)^{N-1+s} dm(w) < +\infty. \tag{2}$$

(This can be proved e.g. by the use of the orthogonality of the monomials $z_1^{\alpha_1}\dots z_N^{\alpha_N}$ in $\mathbb{B}_N$ and the integration in polar coordinates in $\mathbb{C}^N$).

Note that $L^2H(\mathbb{B}_N) = A^{2,0}(\mathbb{B}_N)$. Hence for every function $f \in L^2H(\mathbb{B}_N)$ and every one-dimensional complex subspace $\Pi$ of $\mathbb{C}^N$, we have, by (2),

$$\int_{\Pi\cap\mathbb{B}_N} |f(w)|^2(1-|w|^2)^{N-1} dm(w) < +\infty. \tag{3}$$

Because of (1) and (3) the question arises whether for every $E$ as in the assumption of Theorem 1 one can construct the function $f \in L^2H(\mathbb{B}_N)$ such that $f$ satisfies not only (1), but also such that for every $\Pi \in E$, and for every $0 \le \eta < N-1$,

$$f|_{\Pi\cap\mathbb{B}_N} \notin A^{2,N-1-\eta}(\Pi\cap\mathbb{B}_N). \tag{4}$$

In this note we shall show that this is impossible in general :

THEOREM 2. *For every $N \ge 3$ there exists a subset $E$ of one-dimensional complex subspaces of $\mathbb{C}^N$ such that the set $\bigcup\{\Pi\cap\mathbb{B}_N | \Pi \in E\}$ is closed in $\mathbb{B}_N$ and of Lebesgue measure zero, and such that for every function $f \in L^2H(\mathbb{B}_N)$ there exists $\Pi \in E$ and $\eta$ with $0 < \eta < N-1$, such that*

$$f|_{\Pi\cap\mathbb{B}_N} \in A^{2,N-1-\eta}(\Pi\cap\mathbb{B}_N).$$

As was already said above, if $f \in L^2H(\mathbb{B}_N) = A^{2,0}(\mathbb{B}_N)$ and $\Pi$ is any one-dimensional complex subspace of $\mathbb{C}_N$, then the condition (3) holds. One can ask whether the converse is true, i.e. if a function $f$ holomorphic in $\mathbb{B}_N$ satisfies (3), then $f \in L^2H(\mathbb{B}_N)$. We show that this also is not true in general:

THEOREM 3. *For every $N \ge 2$ there exists a function $f \in \mathcal{O}(\mathbb{B}_N)$ such that for every one-dimensional complex subspace $\Pi$ of $\mathbb{C}^N$,*

$$f|_{\Pi\cap\mathbb{B}_N} \in A^{2,N-1}(\Pi\cap\mathbb{B}_N), \tag{5}$$

*but $f \notin L^2H(\mathbb{B}_N)$.*

We also prove the following result, concerning the functions satisfying (1) and (2):

THEOREM 4. *There exists a function $f$ holomorphic in $\mathbb{B}_N$ such that for every one-dimensional complex subspace $\Pi$ of $\mathbb{C}_N$,*

$$f|_{\Pi\cap\mathbb{B}_N} \in A^{2,N-1}(\Pi\cap\mathbb{B}_N), \tag{6}$$

*but for every $0 < \varepsilon < N$:*

$$f|_{\Pi\cap\mathbb{B}_N} \notin A^{2,N-1-\varepsilon}(\Pi\cap\mathbb{B}_N). \tag{7}$$

## 2. The exceptional sets

In this part we give the proofs of Theorems 2, 3 and 4. We begin with the proof of Theorem 4.

Proof of Theorem 4. The following characterization of the class $A^{2,s}(U)$, where $U$ is the unit disc in the complex plane, and $s > -1$, is well-known, and can be obtained by integration in polar coordinates in $\mathbb{C}$:

Let $h(w) \in \mathcal{O}(U)$, $h(w) = \sum_{n=0}^{\infty} b_n w^n$. Then $h \in A^{2,s}(U)$, i.e.

$$\int_U |h(w)|^2(1-|w|^2)^s dm(w) < +\infty$$

iff $\sum_{n=1}^{\infty} \frac{1}{n^{s+1}}|b_n|^2 < +\infty$.

In the sequel we will use the sequence of homogeneous polynomials in $\mathbb{C}^N$, constructed by Wojtaszczyk in [3]:

THEOREM 5 ([3], Thm 1). *There exists an integer $K = K(N)$ and a sequence $\{p_n\}$ of homogeneous polynomials in $\mathbb{C}^N$ of degree $n$ such that*

(8) $|p_n(z)| \le 2$ for all $z \in \partial\mathbb{B}_N$ and for $n$ large enough, say $n \ge N_0$;

(9) for each $s$ large enough, say $s \ge s_0$, $\displaystyle\sum_{n=Ks}^{K(s+1)-1} |p_n(z)| \ge 0,5$ for all $z \in \partial\mathbb{B}_N$.

The function $f$ from Theorem 2, satisfying (6) and (7) will have the form

$$f(z) = \sum_{n=1}^{\infty} c_n p_n(z), \tag{10}$$

where $\{p_n(z)\}_{n=0}^{\infty}$ are as in Theorem 5, and the coefficients $c_n$, $n = 1, 2, \ldots$ will be chosen later. Let $f$ be any function of the form (10), and let $z$ be an arbitrary point of the boundary $\partial\mathbb{B}_N$. Consider the function $f_z$, defined in the unit disc $U$ in $\mathbb{B}$ by the formula

$$f_z : U \ni \longrightarrow f(wz).$$

Then, for a given $s$, using the properties of polynomials $p_n$ and polar coordinates we obtain

$$\begin{aligned}\int_U |f_z(w)|^2(1-|w|^2)^s dm(w) &= \int_U |f(wz)|^2(1-|w|^2)^s dm(w)\\ &= \sum_{n=0}^{\infty} |c_n|^2 \int_U |p_n(wz)|^2(1-|w|^2)^s dm(w)\\ &= \sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \int_U |w|^{2n}(1-|w|^2)^s dm(w)\\ &= 2\pi \sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \int_0^1 r^{2n+1}(1-r^2)^s dr\\ &= \pi \sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \int_0^1 t^n(1-t)^s dt.\end{aligned}$$

For $n \geq 0$ and $s > -1$ we have

$$\int_0^1 t^n(1-t)^s dt = \frac{n!}{(s+1)(s+2)\dots(s+n+1)}.$$

Therefore

$$\begin{aligned}\pi \sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \int_0^1 t^n(1-t)^s dt\\ = \pi \sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \frac{n!}{(s+1)(s+2)\dots(s+n+1)}.\end{aligned}$$

In particular, according to (6), (7), and the above, if $f$ has the form (10), then: $f$ satisfies (6) (resp. (7)) iff for every $z \in \partial\mathbb{B}_N$

$$\sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \frac{n!}{N(N+1)\dots(N+n)} < +\infty$$

(resp. for every $z \in \partial\mathbb{B}_N$ and every $\varepsilon$ with $0 < \varepsilon < N$:

$$\sum_{n=0}^{\infty} |c_n|^2 |p_n(z)|^2 \frac{n!}{(N-\varepsilon)(N+1-\varepsilon)\dots(N+n-\varepsilon)} < +\infty).$$

We have

$$\frac{n!}{N(N+1)\dots(N+n)} =$$

$$= \frac{(N-1)!N(N+1)\dots n}{N(N+1)\dots(N+(n-N))(N+(n-N)+1)\dots(N+n)}$$
$$= \frac{(N-1)!}{(n+1)\dots(n+N)},$$

and, for $n$ sufficiently large,

$$\frac{(N-1)!}{n^N} \geq \frac{(N-1)!}{(n+1)\dots(n+N)} \geq \frac{(N-1)!}{(n+N)^N}$$
$$= \frac{(N-1)!}{n^N}\frac{n^N}{(n+N)^N} = \frac{(N-1)!}{n^N}\frac{1}{(1+\frac{N}{n})^N} \geq \frac{1}{2}\frac{(N-1)!}{n^N}.$$

Therefore

$$\sum_{n=0}^{\infty}|c_n|^2|p_n(z)|^2\frac{n!}{N(N+1)\dots(N+n)} < +\infty$$

iff

$$\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^N} < +\infty.$$

Similarly we can prove that

$$\sum_{n=0}^{\infty}|c_n|^2|p_n(z)|^2\frac{n!}{(N-\varepsilon)(N+1-\varepsilon)\dots(N+n-\varepsilon)} < +\infty$$

iff

$$\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^{N-\varepsilon}} < +\infty.$$

Hence, to construct the function $f$ of the form (10) satisfying (6) and (7), it is sufficient to find the coefficients $\{c_n\}_{n=0}^{\infty}$ in such a way that:

For every $z \in \partial\mathbb{B}_N$

$$\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^N} < +\infty, \tag{11}$$

and for every $z \in \partial\mathbb{B}_N$ and every $\varepsilon$ with $0 < \varepsilon < N$:

$$\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^{N-\varepsilon}} = +\infty. \tag{12}$$

Set

$$c_n = \frac{n^{\frac{N-1}{2}}}{\log(n+1)}, \quad n = 1, 2, \dots, \tag{13}$$

and let, according to (10),

$$f(z)=\sum_{n=1}^{\infty} c_n p_n(z) \tag{14}$$

with the coefficients $c_n$ defined by (13).

Since the polynomials $p_n$ are homogeneous and for $n \geq N_0$ we have $|p_n| \leq 2$ on $\mathbb{B}_N$ by (8), and $|c_n|$ are given by the formula (13), it is not difficult to prove that the series in (14) is locally uniformly convergent in $\mathbb{B}_N$ and defines here a holomorphic function; hence $f \in \mathcal{O}(\mathbb{B}_N)$.

Consider the condition (11). For every $z \in \partial\mathbb{B}_N$, by (8) and (13), we have for some constant $d$ (depending on $c_l$ and $p_l(z)$ with $1 \leq l \leq N_0-1$, where $N_0$ is the constant from the condition (8), i.e. for that (finite) set of polynomials $p_l(z)$ for which (8) does not hold),

$$\begin{aligned}\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^N} &= \sum_{n=1}^{\infty}\frac{n^{N-1}}{\log^2(n+1)}|p_n(z)|^2\frac{1}{n^N}\\ &\leq d+4\sum_{n=N_0}^{\infty}\frac{n^{N-1}}{n^N\log^2(n+1)}\\ &= d+4\sum_{n=N_0}^{\infty}\frac{1}{n\log^2(n+1)}<+\infty,\end{aligned}$$

which gives (11).

We pass now to (12). Let $\varepsilon$ with $0<\varepsilon<N$ be given. Let $s_0$ be the constant from the condition (9). Then for every $z \in \partial\mathbb{B}_N$ we have, by (9) and (13),

$$\begin{aligned}&\sum_{n=1}^{\infty}|c_n|^2|p_n(z)|^2\frac{1}{n^{N-\varepsilon}}\\ &\quad=\sum_{n=1}^{\infty}\frac{n^{N-1}}{\log^2(n+1)}|p_n(z)|^2\frac{1}{n^{N-\varepsilon}}\\ &\quad=\sum_{n=1}^{\infty}\frac{1}{n^{1-\varepsilon}\log^2(n+1)}|p_n(z)|^2\\ &\quad\geq\sum_{s\geq s_0}\sum_{p=0}^{K-1}\frac{1}{(Ks+p)^{1-\varepsilon}\log^2(Ks+p+1)}|p_{Ks+p}(z)|^2\\ &\quad\geq\sum_{s\geq s_0}\sum_{p=0}^{K-1}\frac{1}{(Ks+K-1)^{1-\varepsilon}\log^2(Ks+K)}|p_{Ks+p}(z)|^2\end{aligned}$$

$$= \sum_{s \geq s_0} \frac{1}{(Ks + K - 1)^{1-\varepsilon} \log^2(Ks + K)} \sum_{p=0}^{K-1} |p_{Ks+p}(z)|^2$$

$$\geq c \sum_{s \geq s_0} \frac{1}{(Ks + K - 1)^{1-\varepsilon} \log^2(Ks + K)} = +\infty,$$

where $c > 0$ is some constant independent of $s$. This proves (12). Hence the function $f$ given by (14) has the desired properties, which ends the proof of Theorem 4. ∎

Proof of Theorem 2. Let $N \geq 3$. Set

(15) $$E = \{L = \{\lambda(0, w_2, \ldots, w_N) | \lambda \in \mathbb{C}\} | \; |(w_2, \ldots, w_N) \in \mathbb{C}^{N-1}, \sqrt{|w_2|^2 + \ldots + |w_N|^2} = 1\};$$

then $E$ is a set of complex one-dimensional subspaces of $\mathbb{C}^N$ generated by the vectors $w = (w_1, \ldots, w_N)$ with $w_1 = 0$. We have $m(\bigcup E \cap \mathbb{B}_N) = 0$; moreover, the set

(16) $$L = \bigcup E = \{w \in \mathbb{C}^N | w_1 = 0\}$$

is a subspace of $\mathbb{C}^N$ of complex dimension $N - 1$.

We need the additional characterization of behavior of functions from the space $A^{2,s}(\mathbb{B}_N)$ on lower-dimensional slices:

If $f \in A^{2,s}(\mathbb{B}_N)$, $s > -1$, and $\Lambda$ is a complex $k$-dimensional subspace of $\mathbb{C}^N$, $k < N$, then

(17) $$f|_{\Lambda \cap \mathbb{B}_N} \in A^{2,N-k+s}(\Lambda \cap \mathbb{B}_N),$$

i.e.

(18) $$\int_{\Lambda \cap \mathbb{B}_N} |f(w)|^2 (1 - |w|^2)^{N-k+s} dm(w) < +\infty$$

(as before, this characterization can be obtained by a computation on the coefficients of the Taylor series of the function $f$, and the integration in polar coordinates in $\mathbb{B}_N$).

Now suppose that $f$ is a function from Theorem 1, constructed with respect to the set $E$ defined in (15), i.e. $f \in L^2H(\mathbb{B}_N) = A^{2,0}(\mathbb{B}_N)$, and for every complex one-dimensional subspace $\Pi$ of $\mathbb{C}^N$,

$$f|_{\Pi \cap \mathbb{B}_N} \notin L^2(\Pi \cap \mathbb{B}_N) \text{ iff } \Pi \in E,$$

with $E$ defined by (15). Suppose moreover that for every $\Pi \in E$, and every $\eta$ with $0 < \eta < N - 1$,

(19) $$f|_{\Pi \cap \mathbb{B}_N} \notin A^{2,N-1-\eta}(\Pi \cap \mathbb{B}_N).$$

Let $L$ be as in (16). Then, by (17), with $\Lambda = L$, $k = N-1$ and $s = 0$,

(20) $$f|_{L\cap \mathbb{B}_N} \in A^{2,1}(L \cap \mathbb{B}_N).$$

Hence, by Fubini's theorem, for almost all slices of the form $\Pi \cap \mathbb{B}_N$, where $\Pi \in E$, we have

(21) $$f|_{\Pi\cap \mathbb{B}_N} \in A^{2,1}(\Pi \cap \mathbb{B}_N).$$

Then (19), (21) and the assumption that $N \geq 3$ give the contradiction, say for $\eta = 1/2$. ■

Proof of Theorem 3. Fix $N \geq 2$. For every $n = 1, 2, \ldots$, let

(22) $$c_n = \left( \int_{\mathbb{B}_N} |(z_1 z_2)^n|^2 dm(z) \right)^{-1/2},$$

where $z = (z_1, z_2, \ldots, z_N)$, and $m$ is, as before, the Lebesgue measure in $\mathbb{C}^N$. Set

(23) $$f(z) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} c_n (z_1 z_2)^n.$$

We will use the following equality:

If $\alpha = (\alpha_1, \ldots, \alpha_N)$ is a multiindex with $\alpha_i$ being positive integers, $i = 1, \ldots, N$, then

$$\int_{\mathbb{B}_N} z^\alpha \overline{z}^\alpha dz = \frac{\pi^N \alpha!}{(|\alpha| + N)!},$$

where, as usually, $\alpha! = \alpha_1! \ldots \alpha_N!$, $|\alpha| = \alpha_1 + \ldots + \alpha_N$. Hence

(24) $$c_n = \frac{\sqrt{(2n+N)!}}{\sqrt{\pi^N} n!}.$$

We have, by Stirling's formula,

$$\left( \frac{\sqrt{(2n+N)!}}{\sqrt{\pi^N} n!} \right)^{1/n} \approx \left( \frac{((2n+N)^{2n+N} e^{-(2n+N)} \sqrt{2\pi(2n+N)})^{1/2}}{n^n e^{-n} \sqrt{2\pi n}} \right)^{1/n}$$
$$= \frac{(2n+N)^{1+N/2n} e^{-(1+N/2n)} (2\pi(2n+N))^{1/4n}}{n e^{-1} (2\pi n)^{1/n}}.$$

This expression tends to 2 as $n \to \infty$. Since for $z = (z_1, z_2, \ldots, z_N) \in \mathbb{B}_N$ we have

$$|z_1 z_2| < \frac{1}{2},$$

the series in (23) is convergent uniformly on compact subsets of $\mathbb{B}_N$ and defines a holomorphic function there.

By the orthogonality of the monomials $(z_1z_2)^n$ on $\mathbb{B}_N$ for different values of $n$ we have, by (22),

$$\int_{\mathbb{B}_N} |f(z)|^2 dm(z) = \sum_{n=1}^{\infty} \frac{1}{n} c_n^2 \int_{\mathbb{B}_N} |(z_1z_2)^n|^2 dm(z) = \sum_{n=1}^{\infty} \frac{1}{n} = +\infty.$$

Now let $w = (w_1, w_2, \ldots, w_N)$ be an arbitrary point of $\partial\mathbb{B}_N$. Let $\Pi = \{tw | t \in \mathbb{C}\}$. Then we have, by the orthogonality of the monomials $(z_1z_2)^n$ on $\Pi \cap \mathbb{B}_N$,

$$\text{(25)} \qquad \int_{\Pi\cap\mathbb{B}_N} |f(z)|^2(1-|z|^2)^{N-1} dm(z) = \int_U |f(tw)|^2(1-|tw|^2)^{N-1} dm(t)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} c_n^2 |w_1w_2|^{2n} \int_U |t|^{4n}(1-|t|^2)^{N-1} dm(t),$$

where $U$ denotes the unit disc in the complex plane. Because of (24) and the fact that for $w \in \partial\mathbb{B}_N$ we have $|w_1w_2| \le \frac{1}{2}$, the right-hand side in (25) is not greater than

$$\text{(26)} \qquad \sum_{n=1}^{\infty} \frac{(2n+N)!}{n\pi^N(n!)^2 4^n} \int_U |t|^{4n}(1-|t|^2)^{N-1} dm(t)$$

$$= \sum_{n=1}^{\infty} \frac{(2n+N)!}{n\pi^N(n!)^2 4^n} \int_0^1 r \int_0^{2\pi} r^{4n}(1-r^2)^{N-1} d\varphi dr,$$

$$\sum_{n=1}^{\infty} \frac{(2n+N)!2\pi}{n\pi^N(n!)^2 4^n} \int_0^1 r^{4n+1}(1-r^2)^{N-1} dr$$

$$= \sum_{n=1}^{\infty} \frac{(2n+N)!}{n\pi^{N-1}(n!)^2 4^n} \int_0^1 s^{2n}(1-s)^{N-1} ds$$

(we have used the substitution $r^2 = s$). We will use another equality:

$$\int_0^1 s^k(1-s)^p ds = \frac{k!}{(p+1)(p+2)\ldots(p+k+1)}, \quad p, k = 0, 1, 2, \ldots.$$

Hence

$$\int_0^1 s^{2n}(1-s)^{N-1} ds = \frac{(2n)!}{N(N+1)\ldots(N+2n)}, \quad n = 1, 2, \ldots.$$

Therefore the last sum in (26) is equal to

$$\sum_{n=1}^{\infty} \frac{(2n+N)!(2n)!}{n\pi^{N-1}(n!)^2 4^n N(N+1)\ldots(N+2n)} = \sum_{n=1}^{\infty} \frac{(N-1)!(2n)!}{n\pi^{N-1}4^n(n!)^2}.$$

By Stirling's formula we have

$$\frac{(2n)!}{(n!)^2} \approx \frac{(2n)^{2n}e^{-2n}\sqrt{4\pi n}}{(n^n e^{-n}\sqrt{2\pi n})^2} = \frac{2^{2n}\sqrt{4\pi n}}{2\pi n} = \frac{4^n}{\sqrt{\pi n}}.$$

Therefore, for some $d > 0$,

$$\sum_{n=1}^{\infty} \frac{(N-1)!(2n)!}{n\pi^{N-1}4^n(n!)^2} \leq \sum_{n=1}^{\infty} \frac{d(N-1)!}{\pi^{N-1}\sqrt{\pi n}\sqrt{n}} < +\infty.$$

This shows that

$$\int\limits_{\Pi\cap\mathbb{B}_N} |f(z)|^2(1-|z|^2)^{N-1}dm(z) < +\infty,$$

which ends the proof of Theorem 3. ∎

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## References

[1] P. Jakóbczak, *Highly nonintegrable functions in the unit ball*, Israel J. Math. 97 (1997), 175–181.

[2] P. Jakóbczak, *Exceptional sets of slices for functions from the Bergman space in the ball*, Canad. Math. Bull. 44, (2001), 150–159.

[3] P. Wojtaszczyk, *On highly nonintegrable functions and homogeneous polynomials*, Ann. Polon. Math. 65 (1997), 245–251.

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