DEMONSTRATIO MATHEMATICA
Vol. XXXVI No 1 2003

Ivan Chajda

TERNARY t-DEDUCTIVE SYSTEMS

Abstract. The concept of deductive systems was introduced by A. Diego [6] in Hilbert algebras. For universal algebras, it was generalized in [3], and the ternary version was occured in [1] where a basic connection between ternary deductive systems and congruence classes was established. An approach using Galois connections for binary deductive systems was developed in [4]. In the present paper, an approach similar to that of [4] is applied for a modified version of ternary deductive systems.

The concept of deductive system was introduced by A. Diego [6] in the so called Hilbert algebras which form an algebraic counterpart of the implication reduct of an arbitrary intuitionistic logic. It is relatively easy to show that every such a deductive system of a Hilbert algebra $\mathcal{H}$ is a congruence kernel (i.e. 1-class) of some $\Theta \in Con\,\mathcal{H}$ and vice versa. It was pointed by the author in [3] that this property of deductive systems is true also on every algebra of a weakly reguar variety. Of course, one can generalize the concept of deductive system for universal algebras what is the proper reason of the papers [3], [4]. A certain ternary version of this concept is given in [1] where connections with congruence classes are studied in general and in particular with respect to the computational complexity, see also [2]. A bit more different approach was developed in [4] where connections between the binary term function inducing deductive systems and the properties of their lattices are studied by means of Galois connections. Our aim is to set up a definition of deductive system induced by a ternary term function t and study the correspondence between t and congruence classes.

For an algebra $\mathcal{A} = (A, F)$, we denote by $\mathbf{T}_3(\mathcal{A})$ the set of all ternary term functions on $\mathcal{A}$ and by $\mathcal{P}(\mathcal{A})$ the set of all subsets of A, i.e. the so called power set of A. Let $\mathcal{V}$ be a variety of algebras and t be an n-ary term

Key words and phrases: ternary t-deductive system, congruence class, regular algebra, Csákány's term.

1991 *Mathematics Subject Classification*: 08A30, 08B05.

of $\mathcal{V}$. If $\mathcal{A} \in \mathcal{V}$, the n-ary term function t^A induced on $\mathcal{A}$ by the term t will be denoted by the same symbol t for the sake of brevity since there is no danger of misunderstanding. We are ready to introduce the basic concepts:

DEFINITION 1. Let $\mathcal{A} = (A, F)$ be an algebra and $t \in \mathbf{T}_3(\mathcal{A})$. By a *$t$-translation* is meant a unary function $\tau : A \to A$ such that either it is a translation, i.e.

$$\tau(x) = f(a_1, \dots, a_{i-1}, x, a_{i+1}, \dots, a_n)$$

for some n-ary $f \in F$, $i \in \{1, \dots, n\}$ and $a_1, \dots, a_n \in A$ or

$$\tau(x) = t(a, x, b) \quad \text{or} \quad \tau(x) = t(x, a, b)$$

for some $a, b \in A$.

DEFINITION 2. Let $\mathcal{A} = (A, F)$ be an algebra, $z \in A$ and $t \in \mathbf{T}_3(\mathcal{A})$. A subset $D \subseteq A$ is called a *t-deductive system of $\mathcal{A}$ relative to z* whenever

(i) $z \in D$

(ii) $a \in D$ and $t(a, b, z) \in D$, $t(b, a, z) \in D$ imply $b \in D$

(iii) $a \in D$ implies $t(z, a, z) \in D$ and $t(a, z, z) \in D$

(iv) $t(a, b, z) \in D$ and $t(b, a, z) \in D$ imply $t(\tau(a), \tau(b), z) \in D$ for any t-translation τ.

DEFINITION 3. Let $\mathcal{A} = (A, F)$ be an algebra, $z \in A$, $t \in \mathbf{T}_3(\mathcal{A})$ and $D \subseteq A$. A binary relation Θ_D on A defined by the setting

$$\langle a, b \rangle \in \Theta_D \text{ if and only if } t(a, b, z) \in D \text{ and } t(b, a, z) \in D \qquad (*)$$

will be caled *t-induced by D and z.*

At first, we are interested in the relationship between t-deductive systems and congruence classes:

THEOREM 1. *Let $\mathcal{A} = (A, F)$ be an algebra, $z \in A, D \subseteq A$ and $t \in \mathbf{T}_3(\mathcal{A})$. Consider the relation Θ_D that is t-induced by D and z. If Θ_D is a congruence on $\mathcal{A}$ and $D = [z]_{\Theta_D}$ then D is a t-deductive system of $\mathcal{A}$ relative to z.*

Proof. Suppose $\Theta_D \in Con\,\mathcal{A}$ and $D = [z]_{\Theta_D}$. Then D satisfies (i) of Definition 2 trivially. Prove (ii): let $a \in D$ and $t(a, b, z) \in D$, $t(b, a, z) \in D$. Since $D = [z]_{\Theta_D}$, we have $\langle a, z \rangle \in \Theta_D$ and, by $(*)$, also $\langle b, a \rangle \in \Theta_D$. Due to transitivity of Θ_D, we conclude $\langle b, z \rangle \in \Theta_D$ proving $b \in [z]_{\Theta_D} = D$.

Prove (iii): let $a \in D$. Then $\langle a, z \rangle \in \Theta_D$ and (iii) follows directly by $(*)$. Prove (iv): let $t(a, b, z) \in D$ and $t(b, a, z) \in D$. By $(*)$, we have $\langle a, b \rangle \in \Theta_D$ and, due to reflexivity and the substitution property of Θ_D, also $\langle \tau(a), \tau(b) \rangle \in \Theta_D$ for every t-translation τ if $\mathcal{A}$. Applying $(*)$ once more, we get $t(\tau(a), \tau(b), z) \in D$. ∎

The converse of Theorem 1 is also valid under a certain natural assumption on the term function t:

THEOREM 2. *Let $\mathcal{A} = (A, F)$ be an algebra, $z \in A$, $z \in D \subseteq A$ and $t \in \mathbf{T}_3(\mathcal{A})$ satisfy the identity $t(x,x,z) = z$. If D is a t-deductive system of $\mathcal{A}$ relative to z then the relation Θ_D t-induced by D and z is a congruence on $\mathcal{A}$ and $D = [z]_{\Theta_D}$.*

Proof. By using of the identity $t(x,x,z) = z$ we get by $(*)$ that the relation Θ_D is reflexive. Symmetry of Θ_D is evident. Prove transitivity: let $\langle a,b\rangle \in \Theta_D$ and $\langle b,c\rangle \in \Theta_D$. By $(*)$ we have

$$t(a,b,z) \in D,\ t(b,a,z) \in D \text{ and } t(b,c,z) \in D,\ t(c,b,z) \in D.$$

Consider the t-translation $\tau(x) = t(c,x,z)$. Due to (iv) of Definition 2, we have also

$$t(\tau(b),\tau(a),z) \in D \quad\text{and}\quad t(\tau(a),\tau(b),z) \in D,$$

i.e.

$$t(t(c,b,z),t(c,a,z),z) \in D \text{ and } t(t(c,a,z),t(c,b,z),z) \in D.$$

Since $t(b,c,z) \in D$ and $t(c,b,z) \in D$, the foregoing relationship infer by (ii) of Definition 2 also

$$t(c,a,z) \in D.$$

Applying the t-translation $\tau'(x) = t(x,c,z)$, we obtain analogously also

$$t(a,c,z) \in D.$$

Hence, by means of $(*)$, we have $\langle a,c\rangle \in \Theta_D$.

Applying (iv) of Definition 2 for a translation $\tau(x)$ (induced by basic operations of F), we derive easily the substitution property of Θ_D with respect to each $f \in F$. Thus $\Theta_D \in Con\,\mathcal{A}$.

It remains to show that $D = [z]_{\Theta_D}$. If $a \in D$ then, by (iii) of Definition 2, also $t(z,a,z) \in D$ and $t(a,z,z) \in D$ thus $\langle a,z\rangle \in \Theta_D$, i.e. $a \in [z]_{\Theta_D}$. Conversely, if $a \in [z]_\Theta$ then $\langle a,z\rangle \in \Theta_D$ and, by $(*)$, $t(a,z,z) \in D$ and $t(z,a,z) \in D$. By (i), $z \in D$ and, due to (ii), also $a \in D$. ∎

REMARK 1. Under the assumption of Theorem 2, Θ_D is the greatest congruence on $\mathcal{A}$ having the class D. Indeed, if $\Phi \in Con\,\mathcal{A}$ and $[z]_\Phi = D$ then for $\langle a,b\rangle \in \Phi$ we have

$$\langle t(a,b,z),z\rangle = \langle t(a,b,z),t(a,a,z)\rangle \in \Phi \text{ and}$$
$$\langle t(b,a,z),z\rangle = \langle t(b,a,z),t(a,a,z)\rangle \in \Phi$$

whence $t(a,b,z) \in D$ and $t(b,a,z) \in D$ giving $\langle a,b\rangle \in \Theta_D$, i.e. $\Phi \subseteq \Theta_D$.

Recall that an algebra $\mathcal{A} = (A,F)$ is *regular* if every congruence on $\mathcal{A}$ is determined by every of its class, i.e. if $\Theta, \Phi \in Con\,\mathcal{A}$ and $[a]_\Theta = [a]_\Phi$

for some $a \in A$ then $\Theta = \Phi$. A variety $\mathcal{V}$ is *regular* if each $\mathcal{A} \in \mathcal{V}$ has this property. Denote by ω_A the identity relation on a set A; of course, ω_A is the least congruence on $\mathcal{A}$. Hence, if $\mathcal{A}$ is regular and $\Theta \in Con\,\mathcal{A}$ has a class which is a singleton then $\Theta = \omega_A$.

The following characterization of regular varieties was involved by B. Csákány in [5]:

PROPOSITION. *A variety $\mathcal{V}$ is regular if and only if there exist $n \geq 1$ and ternary terms $t_1, \ldots, t_n$ such that*

$$t_1(x,y,z) = \ldots = t_n(x,y,z) = z \text{ if and only if } x = y.$$

For groups, rings and Boolean algebras , one can take $n = 1$ and

$$t_1(x,y,z) = x \cdot y^{-1} \cdot z \quad \text{or}$$
$$t_1(x,y,z) = x - y + z \quad \text{or}$$
$$t_1(x,y,z) = x \oplus y \oplus z,$$

respectively (where for Boolean algebras the symbol $\oplus$ denotes the so called *symmetrical difference*). All of those terms have one property in common which is a particular case of a bit more general one:

$$(C) \qquad t(x,y,z) = t(y,x,z) = z \text{ if and only if } x = y.$$

To remember the original result included in the Proposition, a ternary term $t(x,y,z)$ satisfying (C) will be called a *Csákány's term*. Hence, every variety of groups, rings, quasigroups as well as the variety of Boolean algebras has a Csákány's term. Conversely, if a variety $\mathcal{V}$ has a Csákány's term then, by the Proposition, $\mathcal{V}$ is regular.

The following lemma is easy:

LEMMA. *Let $t(x,y,z)$ be a Csákány's term of a variety $\mathcal{V}$, let $\mathcal{A} \in \mathcal{V}$ and $\Theta \in Con\,\mathcal{A}$. Then*

$$\langle a,b\rangle \in \Theta \text{ if and only if } t(a,b,z) \in [z]_\Theta \text{ and } t(b,a,z) \in [z]_\Theta.$$

Proof. Let $\mathcal{A} \in \mathcal{V}$, $\Theta \in Con\,\mathcal{A}$ and suppose $\langle a,b\rangle \in \Theta$. Then

$$\langle t(a,b,z), z\rangle = \langle t(a,b,z), t(a,a,z)\rangle \in \Theta \text{ and}$$
$$\langle t(b,a,z), z\rangle = \langle t(b,a,z), t(a,a,z)\rangle \in \Theta$$

proving $t(a,b,z) \in [z]_\Theta$ and $t(b,a,z) \in [z]_\Theta$.

Conversely, let $t(a,b,z) \in [z]_\Theta$ and $t(b,a,z) \in [z]_\Theta$ for some $\Theta \in Con\,\mathcal{A}$. In the quotient algebra $\mathcal{A}/\Theta \in \mathcal{V}$ we have

$$t([b]_\Theta, [a]_\Theta, [z]_\Theta) = [t(b,a,z)]_\Theta = [z]_\Theta \text{ and}$$
$$t([a]_\Theta, [b]_\Theta, [z]_\Theta) = [t(a,b,z)]_\Theta = [z]_\Theta.$$

Since t is a Csákány's term of $\mathcal{V}$, we conclude $[a]_\Theta = [b]_\Theta$ whence $\langle a, b\rangle \in \Theta$. ∎

For the Csákány's term, we can get together our previous results to state the following

THEOREM 3. *Let t be a Csákány's term of a variety $\mathcal{V}$. Let $\mathcal{A} = (A, F) \in \mathcal{V}$ and $D \subseteq A$. Then D is a congruence class containing z, i.e. $D = [z]_\Theta$ for some $\Theta \in Con\,\mathcal{A}$, if and only if D is a t-deductive system of $\mathcal{A}$ relative to z.*

Proof. Let t be a Csákány's term of $\mathcal{V}$ and $\mathcal{A} = (A, F) \in \mathcal{V}$. Suppose $D = [z]_\Theta$ for some $\Theta \in Con\,\mathcal{A}$. Clearly $z \in D$. Prove (ii) of Definition 2: if $a \in D$ and $t(a, b, z) \in D$, $t(b, a, z) \in D$ then $\langle a, z\rangle \in \Theta$ and $\langle b, a\rangle \in \Theta$ thus also $\langle b, z\rangle \in \Theta$, i.e. $b \in [z]_\Theta = D$.

Prove (iii): let $a \in D$. Then $\langle a, z\rangle \in \Theta$ and, due to the Lemma, we conclude (iii) immediately.

For (iv), let $t(a, b, z) \in D$ and $t(b, a, z) \in D$, let τ be a t-translation of $\mathcal{A}$. Then $\langle a, b\rangle \in \Theta$ and hence also $\langle \tau(a), \tau(b)\rangle \in \Theta$. Applying the Lemma, we are done. Thus D is a t-deductive system of $\mathcal{A}$ relative to z.

The converse follows directly by Theorem 2. ∎

Let $\mathcal{A} = (A, F)$ be an algebra and $z \in A$. Define a binary relation $R_z \subseteq \mathbf{T}_3(\mathcal{A}) \times \mathcal{P}(A)$ as follows:

$$\langle t, D\rangle \in R_z \text{ iff } D \text{ is a t-deductive system of } \mathcal{A} \text{ relative to } z.$$

This relation defines a pair of mappings (ρ, σ) which clearly forms a Galois connection:

for $\mathcal{S} \subseteq \mathbf{T}_3(\mathcal{A})$, $\rho(\mathcal{S}) = \{D \subseteq A;\ D$ is a t-deductive system of $\mathcal{A}$ relative to z for each $t \in \mathcal{S}\}$,

for $\mathcal{L} \subseteq \mathcal{P}(\mathcal{A})$, $\sigma(\mathcal{L}) = \{t \in \mathbf{T}_3(\mathcal{A})$; each $D \in \mathcal{L}$ is a t-deductive system of $\mathcal{A}$ relative to $z\}$.

For the sake of brevity, we will write $\rho(t)$ instead of $\rho(\{t\})$ whenever $\mathcal{S} = \{t\}$ is a singleton. The proof of the following assertion is almost evident and hence omitted:

THEOREM 4. *Let $\mathcal{A} = (A, F)$ be an algebra, $z \in A$, $\mathcal{L} \subseteq \mathcal{P}(A)$ and $t \in \mathbf{T}_3(\mathcal{A})$. Then*

(a) *$\rho(t)$ is the complete lattice of all t-deductive systems of $\mathcal{A}$ relative to z (with respect to set inclusion); denote it by $Ded_{\mathcal{A}}(t, z)$.*

(b) *$\rho(\sigma(\mathcal{L})) = \bigcap\{Ded_{\mathcal{A}}(t, z); t \in \sigma(\mathcal{L})\}$.*

Theorems 3 and 4 yield the following

COROLLARY. *Let t be a Csákány's term of a variety $\mathcal{V}$ and $\mathcal{A} = (A, F) \in \mathcal{V}$, $z \in A$. Then $\rho(t) = Ded_{\mathcal{A}}(t, z) \simeq Con\,\mathcal{A}$.*

Proof. If t is a Csákány's term of $\mathcal{V}$, $\mathcal{V}$ is regular and hence the mapping $[z]_\Theta \to \Theta$ is an isomorphism of the lattice of all congruence classes containing z onto $Con\,\mathcal{A}$. By Theorem 3, the lattice of congruence classes containing z coincides with $Ded_{\mathcal{A}}(t,z)$. Together with Theorem 4, we obtain the assertion. ∎

Of course, the operation meet in the lattice $Ded_{\mathcal{A}}(t,z)$ coincides with set intersection and A is the greatest element of it. On the other hand, one can ask about the least element of $Ded_{\mathcal{A}}(t,z)$. In general, it need not be a singleton $\{z\}$. We can prove the following:

THEOREM 5. *Let t be a ternary term of a variety $\mathcal{V}$ satisfying $t(x,x,z)=z$. The following conditions are equivalent:*

(a) *for each $\mathcal{A}=(A,F)\in\mathcal{V}$ and each $z\in A$, $\{z\}$ is a t-deductive system of $\mathcal{A}$ relative to z;*

(b) *for each $\mathcal{A}=(A,F)\in\mathcal{V}$, each $z\in A$ and $\Theta\in Con\,\mathcal{A}$, $[z]_\Theta$ is a t-deductive system of $\mathcal{A}$ relative to z;*

(c) *for each $\mathcal{A}=(A,F)\in\mathcal{V}$ and each $z\in A$, the relation $\Theta_{\{z\}}$ t-induced by $\{z\}$ and z is a congruence on $\mathcal{A}$ having the class $\{z\}$.*

Proof. (b)⇒(a) is trivial and (a)⇒(c) by Theorem 2. It remains to prove (c)⇒(b). Suppose $\mathcal{A}=(A,F)\in\mathcal{V}$ and $z\in A$. Since $\{z\}$ is a class of $\Theta_{\{z\}}\in Con\,\mathcal{A}$, we have

$$(**)\qquad t(z,c,z)=z=t(c,z,z)\ \text{iff}\ \langle c,z\rangle\in\Theta_{\{z\}}$$

immediately by $(*)$ of Definition 3.

Suppose now $\Theta\in Con\,\mathcal{A}$ and $D=[z]_\Theta$. Then $z\in D$ and we need to verify (ii), (iii) and (iv) of Definition 2. For (ii), let $a\in D$ and $t(a,b,z)\in D$, $t(b,a,z)\in D$. Then $[a]_\Theta=[z]_\Theta$ and hence

$$t([z]_\Theta,[b]_\Theta,[z]_\Theta)=t([a]_\Theta,[b]_\Theta,[z]_\Theta)=[t(a,b,z)]_\Theta=D=[z]_\Theta,$$

analogously

$$t([b]_\Theta,[z]_\Theta,[z]_\Theta)=[z]_\Theta.$$

Applying $(**)$ on the quotient algebra $\mathcal{A}/\Theta\in\mathcal{V}$, we get $\langle[b]_\Theta,[z]_\Theta\rangle\in\Theta_{\{[z]_\Theta\}}$ whence $[b]_\Theta=[z]_\Theta$, i.e. $b\in[z]_\Theta=D$.

Prove (iii): if $a\in D=[z]_\Theta$ then $[a]_\Theta=[z]_\Theta$ and, by means of the identity $t(x,x,z)=z$, we have

$$t([a]_\Theta,[z]_\Theta,[z]_\Theta)=t([z]_\Theta,[z]_\Theta,[z]_\Theta)=[z]_\Theta$$
$$t([z]_\Theta,[a]_\Theta,[z]_\Theta)=t([z]_\Theta,[z]_\Theta,[z]_\Theta)=[z]_\Theta$$

thus $t(a,z,z)\in D$ and $t(z,a,z)\in D$.

For (iv), let $t(a,b,z) \in D$ and $t(b,a,z) \in D$. By the assumption, $\Theta_{\{[z]_\Theta\}} \in Con\,\mathcal{A}/\Theta$ and hence

$$t([a]_\Theta, [b]_\Theta, [z]_\Theta) = [t(a,b,z)]_\Theta = D = [z]_\Theta \text{ and}$$
$$t([b]_\Theta, [a]_\Theta, [z]_\Theta) = [t(a,b,z)]_\Theta = D = [z]_\Theta$$

thus $\langle [a]_\Theta, [b]_\Theta \rangle \in \Theta_{\{[z]_\Theta\}}$. Hence also

$$\langle \tau([a]_\Theta), \tau([b]_\Theta) \rangle \in \Theta_{\{[z]_\Theta\}}$$

for every t-translation τ on $\mathcal{A}/\Theta$. One can derive easily

$$[t(\tau(a), \tau(b), z)]_\Theta = t([\tau(a)]_\Theta, [\tau(b)]_\Theta, [z]_\Theta) = [z]_\Theta$$

whence $t(\tau(a), \tau(b), z) \in [z]_\Theta = D$. ∎

For regular varieties, we can state stronger result concerning the term t:

THEOREM 6. *Let $\mathcal{V}$ be a regular variety and t be a ternary term of $\mathcal{V}$ satisfying $t(x,x,z) = z$. If for each $\mathcal{A} = (A,F) \in \mathcal{V}$ and every $z \in A$ the singleton $\{z\}$ is a t-deductive system of $\mathcal{A}$ relative to z then t is a Csákány's term of $\mathcal{V}$.*

Proof. Suppose $t(x,x,z) = z$ holds in $\mathcal{V}$, $\mathcal{A} = (A,F) \in \mathcal{V}$ and $z \in A$. Let $\{z\}$ be a t-deductive system of $\mathcal{A}$ relative to z. Let $a,b \in A$ and $t(a,b,z) = t(b,a,z) = z$. Then $t(a,b,z) \in \{z\}$ and $t(b,a,z) \in \{z\}$ which yield $\langle a,b \rangle \in \Theta_{\{z\}}$. By Theorem 2, $\Theta_{\{z\}} \in Con\,\mathcal{A}$ and $[z]_{\Theta_{\{z\}}} = \{z\}$. Since $\mathcal{V}$ (and hence also $\mathcal{A}$) is regular, we have $\Theta_{\{z\}} = \omega_A$. Thus $a = b$. Altogether,

$$t(x,y,z) = t(y,x,z) = z \text{ if and only if } x = y$$

holds in $\mathcal{V}$, i.e. t is a Csákány's term of $\mathcal{V}$. ∎

THEOREM 7. *Let t be a Csákány's term of a variety $\mathcal{V}$. Let t_0 be a ternary term of $\mathcal{V}$ such that $t_0(x,x,z) = z$ holds in $\mathcal{V}$. Then $t_0 \in \sigma(\rho(t))$ for each $\mathcal{A} \in \mathcal{V}$ if and only if t_0 is also a Csákány's term of $\mathcal{V}$.*

Proof. Let t be a Csákány's term of $\mathcal{V}$. Then $\mathcal{V}$ is regular. Consider $\mathcal{A} = (A,F) \in \mathcal{V}$ and $z \in A$. By Theorem 3, $\{z\} \in Ded_{\mathcal{A}}(t,z)$ since it is a class of ω_A. Thus $\{z\} \in \rho(t)$. Since $t_0 \in \sigma(\rho(t))$, $\{z\}$ is also a t_0-deductive system of $\mathcal{A}$ relative to z.

Suppose $t_0(a,b,z) = z = t_0(b,a,z)$ for $a,b \in A$. Then $t_0(a,b,z) \in \{z\}$ and $t_0(a,b,z) \in \{z\}$ and, due to Theorem 2, $\langle a,b \rangle \in \Theta_{\{z\}}$. Since $\mathcal{V}$ is regular, we have $\Theta_{\{z\}} = \omega_A$ whence $a = b$. Thus t_0 is a Csákány's term of $\mathcal{V}$. ∎

References

[1] R. Bělohlávek and I. Chajda, *Relative deductive systems and congruence classes*, Multiple Val. Logic 5 (2000), 259–266.

[2] R. Bělohlávek and I. Chajda, *A polynomial characterization of congruence classes*, Algebra Universalis 37 (1997), 235–242.
[3] I. Chajda, *Congruence kernels in weakly regular varieties*, Southeast Asian Bull. Math. 24 (2001), 15–18.
[4] I. Chajda and R. Halaš, *Deductive systems and Galois connections*, Algebra and Discrete Math., to appear.
[5] B. Csákány, *Characterization of regular varieties*, Acta Sci.Math (Szeged) 31 (1971), 187–189.
[6] A. Diego, *Sur les algébres de Hilbert*, Collection de Logique Math., Ser. A (Ed. Hermann, Paris), 21 (1967), 177-189.

DEPARTMENT OF ALGEBRA AND GEOMETRY
PALACKÝ UNIVERSITY OLOMOUC
Tomkova 40
779 00 OLOMOUC
CZECH REPUBLIC
E-mail: chajda@risc.upol.cz

Received January 28, 2002.