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ON OPTIMAL STOPPING OF A DISCRETE TIME RISK PROCESS

Abstract. Optimal stopping time problem for a discrete time risk process $U_n = u + cn - (X_1 + \ldots + X_n)$ is analyzed. At a random moment θ, which is unobserved, there is a change in common distribution of subsequent claim sizes $X_1, X_2, \ldots$. In the case when the mean of a new distribution of claim sizes is greater than the premium c there is a need to stop the process to recalculate the premium. The existence of optimal stopping rule is proved and the way to find it efficiently is described.

1. Introduction

The risk process in actuarial mathematically oriented literature has been investigated heavily. The main task of research, apart from modelling the insurer's surplus (reserves) is devoted to ruin probability problems. So far optimal stopping time problems have not gained much of interest. In this paper we deal with a discrete time risk process for which an optimal stopping time problem arises quite naturally, and finding an efficient form of the optimal stopping rule seems to be a crucial goal from the insurer's point of view. The paper of Jensen (1997) in [5] has been our inspiration. Jensen considers a continuous time risk process $\{U(t), t \geq 0\}$ of the form $U(t) = u + G(t) - S(t)$, where $u > 0$ is an initial capital of an insurer, the process of accumulated premiums $\{G(t), t \geq 0\}$ is a Brownian motion with mean ct, $G(t)$ is the premium collected till time t. The process of aggregated claims $\{S(t), t \geq 0\}$ is a Markov modulated Poisson process, i.e. $S(t) = X_0 + X_1 + \ldots + X_{N(t)}$, where subsequent claim sizes X_1, $X_2, \ldots$ are iid r.v's, $X_0 = 0$, $\{N(t), t \geq 0\}$ is a counting process with random intensity which is a function of an unobserved Markovian environment state process. Depending on its distribution law, the mean premium income may not cover mean losses in which case there is a need to stop the process so as to recalculate the

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premium. In the classical continuous time model one assumes that $G(t) = ct$, $c > 0$, and that the number of subsequent claims till time t, $t \geq 0$, $N(t)$ is modelled by a renewal process. Ferenstein and Sierociński (1997) [3] considered the optimal stopping time problem of the classical risk process in which the insurer goal is to stop the process before the ruin time and to maximize the mean reward function depending on the current state of the risk process.

In this paper we consider a modified version of the classical discrete time risk process which models random unobserved change in claim size distribution. Distribution law of subsequent claim sizes is like in the disorder problem investigated in Chow et al. (1971) [2], Shiryayev (1978) [6], Bojdecki (1979) [1]. Subsequent claim sizes are assumed to be nonnegative iid r.v's till an unobserved random time. At that time the common distribution law of claim sizes changes to another one which is unfavarouble for the insurer since its mean excess the premium. In Section 2 we give precise model of the risk process and state the problem. In Section 3 we find general form of optimal stopping rule and derive the Bellman equation for the optimal mean reward. Explicit form of optimal stopping time is found in Section 4 where we formulate additional assumptions on the claim sizes distributions under which the monotone case is fulfilled. Example with truncated exponential losses is presented.

2. The model

Let $(\Omega, \mathcal{F}, P)$ be some fixed probability space on which all considered random variables are determined. Suppose that

$$U_n = u + cn - \sum_{i=0}^{n} X_i, \tag{2.1}$$

$n \in \mathbb{N} = \{0, 1, \ldots\}$, $X_0 = 0$, $u > 0$, $c > 0$, is a discrete time risk process. We assume that the distribution of the claim sequence $X_1, X_2, \ldots$ depends on the unobserved random time θ similarly as in the disorder problem considered by Shiryaev (1978) [6] and generalized by Bojdecki (1979) [1]. More precisely, let θ, X_1, $X_2, \ldots$ have the distribution such that

$$p_0 = P(\theta = 0) = \pi,$$
$$p_n = P(\theta = n) = (1 - \pi)(1 - p)^{n-1} p, \ n \geq 1,$$

where $\pi \in [0, 1]$, $p \in (0, 1]$ are fixed and known and

$$X_n(\omega) = \begin{cases} X_n^0(\omega), & \text{if } \ \theta(\omega) > n \\ X_n^1(\omega), & \text{if } \ \theta(\omega) \leq n \end{cases},$$

where nonnegative r. v's $X_1^0, X_2^0, \ldots$ have the density f_0, nonnegative r. v's $X_1^1, X_2^1, \ldots$ have the density f_1 and f_0, f_1 are different functions with supports in $\mathbb{R}_+ \cup \{0\}$. Moreover, the random variables θ, $X_1^0, X_2^0, \ldots, X_1^1, X_2^1, \ldots$ are independent.

As we observe subsequent losses, the σ-field of events observed at the nth moment is $\mathcal{F}_n = \sigma(X_1, \ldots, X_n) = \sigma(U_1, \ldots, U_n)$, $n \geq 1$, and $\mathcal{F}_0 = \{\emptyset, \Omega\}$. Let $\mathcal{T}$ and $\overline{\mathcal{T}}$ be sets of stopping rules and Markov times, respectively, adapted to the filtration $\mathbb{F} = \{\mathcal{F}_n : n \in \mathbb{N}\}$. Hence $\mathcal{T} \subset \overline{\mathcal{T}}$, since for any stopping time τ we have $P(\tau < \infty) = 1$ while Markov times admit infinite values. Let us denote, for $u \in (-\infty, \infty)$ and $\pi \in [0,1]$ optimal mean rewards in $\mathcal{T}$ and $\overline{\mathcal{T}}$, as

$$s(u,\pi) = \sup_{\tau \in \mathcal{T}} \mathbb{E}(U_\tau), \quad \overline{s}(u,\pi) = \sup_{\tau \in \overline{\mathcal{T}}} \mathbb{E}(U_\tau).$$

In what follows we will characterize the above mean rewards and we will find an optimal stopping time $\tau_0 \in \mathcal{T}$, i.e.

$$s(u,\pi) = \mathbb{E}(U_{\tau_0}).$$

3. Solution of the problem

Let, for any moment n, π_n denote the conditional probability that the change in distribution of claim sizes has occurred not later than at n, given observations till that moment, i.e.

$$\pi_n = P(\theta \leq n \mid \mathcal{F}_n).$$

Then, using Bayes' formula we obtain (Shiryaev (1978) [6])

$$\pi_{n+1} = \frac{\pi_n f_1(X_{n+1}) + (1-\pi_n) p f_1(X_{n+1})}{\pi_n f_1(X_{n+1}) + (1-\pi_n) p f_1(X_{n+1}) + (1-\pi_n)(1-p) f_0(X_{n+1})},$$

a.s., which may be rewritten in more convenient form

$$\pi_{n+1} = \frac{1}{1 + h(\pi_n)\lambda(X_{n+1})}, \tag{3.1}$$

where the functions h and λ are defined as follows

$$h(\pi) = \frac{(1-\pi)(1-p)}{\pi + (1-\pi)p}, \quad \lambda(x) = \frac{f_0(x)}{f_1(x)}.$$

Now, let us note that the sequence $\{(U_n, \pi_n), n \in \mathbb{N}\}$, where $(U_0, \pi_0) = (u, \pi)$, forms a homogenous Markov chain adapted to the filtration $\mathbb{F}$ since (3.1) is satisfied and the conditional distribution of X_{n+1} given $\mathcal{F}_n$ has the density

$$f_{\pi_n} = \pi_n f_1 + (1-\pi_n) p f_1 + (1-\pi_n)(1-p) f_0. \tag{3.2}$$

Hence our optimal stopping time problem has been reduced to the problem of optimal stopping of the homogenous Markov chain $\{(U_n, \pi_n) : n \in \mathbb{N}\}$ with the reward function $g(u, \pi) = u$, $u \in (-\infty, \infty)$, $\pi \in [0,1]$. Thus, we may apply a rich range of results as presented in Chow, Robbins and Siegmund (1971) [2] and Shiryaev (1978) [6].

In what follows, we will use the denotations

$$\mu_i = \int_{-\infty}^{+\infty} x f_i(x)\, dx, \quad i = 0, 1. \tag{3.3}$$
$$\mu_p = p\mu_1 + (1-p)\mu_0.$$

To prove theorems on existence and form of optimal stopping times we need below Lemmas 1-4.

LEMMA 1. *(a) If $\mu_1 > c$, then $\mathbb{E}(U_n) \to -\infty$ as $n \to \infty$.*
(b) If $\mu_1 < c$, then $\mathbb{E}(U_n) \to \infty$ as $n \to \infty$.
(c) If $\mu_1 = c$, then $\mathbb{E}(U_n) \to u - (\mu_0 - \mu_1)(E\theta - 1)$ as $n \to \infty$.

Proof. Let $\mathbb{I}_{(A)}$ be an indicator function of the set A. Then,

$$\mathbb{E}(U_n) = u + nc - \mathbb{E}\Big(\mathbb{I}_{\{\theta>n\}} \sum_{j=0}^{n} X_j^0 + \sum_{i=0}^{n} \mathbb{I}_{\{\theta=i\}}\Big(\sum_{j=0}^{i-1} X_j^0 + \sum_{j=i}^{n} X_j^1\Big)\Big) =$$
$$= u + nc - \Big(\mu_0 n \sum_{i=n+1}^{\infty} p_i + \sum_{i=0}^{n} p_i(\mu_0(i-1) + \mu_1(n-i+1))\Big) =$$
$$= u + n(c-\mu_1) + \mu_1 n \sum_{i=n+1}^{\infty} p_i - (\mu_0 - \mu_1)\Big(\sum_{i=0}^{n} i p_i - \sum_{i=0}^{n} p_i\Big) - \mu_0 n \sum_{i=n+1}^{\infty} p_i.$$

Now let us note that

$$n \sum_{i=n+1}^{\infty} p_i = n(1-\pi)(1-p)^n \text{ tends to 0 as } n \to \infty.$$

Hence, $\lim_{n\to\infty} \mathbb{E}(U_n) = c - \mu_1$ which completes the proof of (a) and (b). The case (c) is obvious. ■

According to Lemma 1 the most interesting case is $\mu_1 > c \geq \mu_0$ which means that the premium was established correctly for the distribution f_0 and then after the change into f_1 the situation is unfavorable for the insurer and there is an urgent need to recalculate the premium.

The following lemma is the simplified version of Theorem 4.13 in Chow, Robbins and Siegmund (1971) [2].

LEMMA 2. *Let $Y_1, Y_2, \ldots$ be a sequence of i.i.d. random variables with $\mathbb{E}(Y_1) = 0$ and $\mathbb{E}\left(Y_1^+\right)^2 < \infty$. Then, for any $a > 0$,*

$$\mathbb{E}\Big(\sup_{n\in\mathbb{N}}\Big(\sum_{i=1}^{n} Y_i - na\Big)^+\Big) < \infty.$$

LEMMA 3. *Assume that $\mu_1 > c$ and $\mathbb{E}(X_i^1)^2 < \infty$. Then,*

$$\mathbb{E}(\sup_{n\in\mathbb{N}} U_n^+) < \infty.$$

Proof. Let us denote $S_n = X_1 + \ldots + X_n$. We need to prove the following

$$\mathbb{E}(\sup_{n\in\mathbb{N}}(nc - S_n)^+) < \infty.$$

Let us rewrite

$$\mathbb{E}(\sup_{n\in\mathbb{N}}(nc - S_n)^+) = \sum_{i=0}^{\infty}\mathbb{E}(\mathbb{I}_{\{\theta=i\}}\mathbb{E}(\sup_{n\in\mathbb{N}}(nc - S_n)^+ \mid \theta = i)).$$

Now let us note that

$$\mathbb{E}(\sup_{n\in\mathbb{N}}(nc - S_n)^+ \mid \theta = i) =$$

$$\mathbb{E}(\max\{\max_{n<i}(nc - S_n)^+, \sup_{n\geq i}(nc - S_n)^+\} \mid \theta = i) \leq$$

$$\mathbb{E}(\max_{n<i}(nc - S_n)^+ \mid \theta = i) + \mathbb{E}(\sup_{n\geq i}(nc - S_n)^+ \mid \theta = i).$$

The last inequality is the consequence of the obvious inequality holding for any two nonnegative random variables X, Y, say, i.e.

$$\mathbb{E}(\max\{X, Y\}) \leq \mathbb{E}(X) + \mathbb{E}(Y).$$

We have

$$\mathbb{E}(\max_{n<i}(nc - S_n)^+ \mid \theta = i) \leq \mathbb{E}(\max_{n<i}(nc + S_n) \mid \theta = i) \leq$$

$$\mathbb{E}\Big(\sum_{n=0}^{i-1}(nc + S_n) \mid \theta = i\Big) = \sum_{n=0}^{i-1}\mathbb{E}(nc + S_n \mid \theta = i) =$$

$$\sum_{n=0}^{i-1}(nc + n\mu_0) = (c + \mu_0)\frac{i(i-1)}{2} < \infty.$$

Note that by our assumptions on X_1, $X_2, \ldots$ we have

$$\mathbb{E}(\sup_{n\geq i}(nc - S_n)^+|\theta = i) =$$

$$= \mathbb{E}(\sup_{n\geq i}(nc - \sum_{k=0}^{i-1} X_k - \sum_{k=i}^{n} X_k)^+|\theta = i) =$$

$$= \mathbb{E}\Big(\sup_{n\geq i}\Big((i-1)c - \sum_{k=0}^{i-1} X_k^0 + (n-i+1)c - \sum_{k=i}^{n} X_i^1\Big)^+\Big) \leq$$

$$\leq \mathbb{E}_0((i-1)c - S_{i-1})^+ + \mathbb{E}_1(\sup_{n\in\mathbb{N}}(nc - S_n)^+),$$

where the expectations on the right hand side of the last inequality are calculated with respect to the densities f_0 and f_1, respectively.

It is easy to see that

$$\mathbb{E}_0((i-1)c - S_{i-1})^+ \leq (i-1)(\mu_0 + c) < \infty.$$

In order to see that $\mathbb{E}_1(\sup_{n\in\mathbb{N}}(nc - S_n)^+) < \infty$ we use Lemma 2 since

$$\mathbb{E}_1(\sup_{n\in\mathbb{N}}(nc - S_n)^+) = \mathbb{E}(\sup_{n\in\mathbb{N}}(\tilde{S}_n - na)^+),$$

where we have introduced $\tilde{S}_n = \sum_{i=1}^{n} Y_i$, $Y_i = \mu_1 - X_i^1, i = 1, 2, \ldots, a = \mu_1 - c$. $Y_1, Y_2, \ldots$ are i.i.d. r. v's, $\mathbb{E}(Y_1) = 0$, $\mathbb{E}(Y_1^+)^2 < \infty$ and $a > 0$.

So finally we have

$$\mathbb{E}(\sup_{n\in\mathbb{N}}(nc - S_n)^+) \leq$$

$$\leq \sum_{i=0}^{\infty} \mathbb{E}(\mathbb{I}_{\{\theta=i\}}\Big((c+\mu_0)\frac{i(i-1)}{2} + (i-1)(c+\mu_0) + \mathbb{E}_1(\sup_{n\in\mathbb{N}}(nc - S_n)^+)\Big)\Big) \leq$$

$$\leq K(\mathbb{E}(\theta^2) + \mathbb{E}(\theta)) + \mathbb{E}_1(\sup_{n\in\mathbb{N}}(nc - S_n)^+) < \infty$$

where K is some positive constant. ∎

LEMMA 4. *If* $\mu_1 > c$, *then* $\lim_{n\to\infty} U_n = -\infty$.

Proof.

$$U_n = \sum_{k=0}^{\infty} \mathbb{I}_{\{\theta=k\}} U_n = \sum_{k=0}^{\infty} \mathbb{I}_{\{\theta=k\}}\Big(u + nc - \sum_{i=0}^{(k\wedge n)-1} X_i - \sum_{i=(k\wedge n)}^{n} X_i\Big) =$$

$$= \sum_{k=0}^{\infty} \mathbb{I}_{\{\theta=k\}}\Big(u - \sum_{i=1}^{(k\wedge n)-1} X_i\Big) + \sum_{k=0}^{\infty} \mathbb{I}_{\{\theta=k\}}\Big(nc - \sum_{i=(k\wedge n)}^{n} X_i\Big).$$

Now let us analyze the limit behavior of U_n as $n \to \infty$ on $\{\theta = k\}$, for any fixed k. Then, for n sufficiently large, $n > k$, we have

$$U_n \mathbb{I}_{\{\theta=k\}} = \Big(u - \sum_{i=1}^{k-1} X_i\Big)\mathbb{I}_{\{\theta=k\}} + n\left(c - \frac{\sum_{i=k}^{n} X_i}{n-k+1} \cdot \frac{n-k+1}{n}\right)\mathbb{I}_{\{\theta=k\}}.$$

On the right hand side of the above the first term is finite and the second one tends to $-\infty$, since by the strong law of large numbers $\frac{\sum_{i=k}^{n} X_i}{n-k+1} \to \mu_1$ *a.s.*, which completes the proof. ∎

THEOREM 1. *Suppose that* $\mu_1 > c$ *and* $\mathbb{E}(X_i^1)^2 < \infty$. *Then,*

a) $s(u,\pi) = \bar{s}(u,\pi)$,

b) $s(u,\pi) = \max\{u, \mathbb{E}(s(U_1,\pi_1)|U_0 = u, \pi_1 = \pi)\}$, *where*

$$\mathbb{E}(s(U_1,\pi_1)|U_0 = u, \pi_1 = \pi) = \int_0^\infty s\Big(u + c - x, \frac{1}{1 + h(\pi)\lambda(x)}\Big) f_\pi(x)dx,$$

c) $\tau_0 = \inf\{n \in \mathbb{N} : U_n = s(U_n, \pi_n)\}$ *is an optimal stopping time.*

P r o o f. The cases *a*) and *b*) follow directly from Theorem 7 in Shiryaev (1978) [6] since $\{(U_n, \pi_n) : n \in \mathbb{N}\}$ is a homogenous Markov chain and by Lemma 3 the condition $\mathbb{E}(\sup_n (g(U_n, \pi_n))^+) < \infty$, where $g(u,\pi) = u$ is satisfied. Similarly, the case *c*) is a direct consequence of Theorem 8 in Shiryaev (1978) [6] because Lemmas 3 and 4 assure the appropriate assumptions. ∎

4. The monotone case

In the so called monotone case under some assumptions one may efficiently find the form of an optimal stopping time.

Let $A_n = \{\mathbb{E}(U_{n+1} - U_n|\mathcal{F}_n) \le 0\}$, $n \in \mathbb{N}$.

We say that we are in the monotone case if

$$\text{(M)} \qquad A_0 \subseteq A_1 \subseteq \ldots; \quad \bigcup_{n=0}^{\infty} A_n = \Omega.$$

LEMMA 5. *Let us assume that* $\mu_1 > \mu_0$ *and the following inequality is fulfilled*

$$\text{(4.1)} \qquad \frac{p}{1-p} \ge \sup_x \left(\frac{f_0(x)}{f_1(x)}\right) - 1.$$

Then, the condition (M) is satisfied.

P r o o f. Note that

$$\mathbb{E}(U_{n+1} - U_n|\mathcal{F}_n) = c - \mathbb{E}(X_{n+1}|\mathcal{F}_n)$$

and, using the formulas (3.2) and (3.3), we have

$$\mathbb{E}(X_{n+1}|\mathcal{F}_n) = \mu_1\pi_n + \mu_p(1 - \pi_n).$$

Hence, in order to show that $A_n \subseteq A_{n+1}$ it is enough to prove that the inequality

$$\mu_1\pi_n + \mu_p(1-\pi_n) \geq c$$

implies

$$\mu_1\pi_{n+1} + \mu_p(1-\pi_{n+1}) \geq c,$$

which is obvious if the sequence $\{\pi_n, n \geq 1\}$ is not decreasing. The latter is true by the assumption (4.1). ∎

THEOREM 2. *Assume that the condition (M) is satisfied and $X_i^j < K < \infty$ a.s., for $i \geq 1$, $j = 0, 1$ and some fixed K. Then, the stopping time*

$$\sigma = \inf\{n \in \mathbb{N} : \mathbb{E}(U_{n+1} - U_n | \mathcal{F}_n) \leq 0\} = \inf\{n \in \mathbb{N} : \pi_n \geq \alpha\}, \tag{4.2}$$

where $\alpha = \dfrac{c - \mu_p}{\mu_1 - \mu_p}$, *is optimal in the set* $\mathcal{C} = \{\tau \in \mathcal{T} : \mathbb{E}(\tau) < \infty\}$.

Proof. First, we will show that $\sigma \in \mathcal{C}$. Put

$$\tilde{p} = P(\pi_k < \alpha | \pi_{k-1} < \alpha).$$

It is obvious that $\tilde{p} < 1$. From the homogenity of the Markov chain $\{(U_n, \pi_n) : n \in \mathbb{N}\}$ we have

$$\begin{aligned}\mathbb{E}(\sigma) &= \sum_{k=1}^{\infty} kP(\pi_1 < \alpha, \ldots, \pi_{k-1} < \alpha, \pi_k \geq \alpha) = \\ &= \sum_{k=1}^{\infty} k \prod_{j=2}^{k-1} P(\pi_j < \alpha | \pi_{j-1} < \alpha) \cdot P(\pi_1 < \alpha) \cdot P(\pi_k \geq \alpha) = \\ &= \sum_{k=1}^{\infty} k \cdot \tilde{p}^{k-2} \cdot P(\pi_1 < \alpha) \cdot P(\pi_k \geq \alpha) < \infty.\end{aligned}$$

Let us note that for $\tau \in \mathcal{C}$ we have

$$\int_{\{\tau > n\}} U_n^- \leq \int_{\{\tau > n\}} Kn \leq \int_{\{\tau > n\}} K\tau \to 0 \tag{4.3}$$

as $n \to \infty$. Moreover,

$$\liminf_{n \in \mathbb{N}} \int_{\{\sigma > n\}} U_n^+ = 0 \tag{4.4}$$

because $\mathbb{E}(\sup_{n \in \mathbb{N}}(nc - S_n)^+) < \infty$.

Now, in the monotone case, (4.3) and (4.4) are assumptions of Theorem 3.3 in Chow, Robbins and Siegmund (1971) [2]. Hence, σ is optimal in $\mathcal{C}$. ∎

EXAMPLE 1. Let X_i^j, $j = 0, 1$, have exponential distribution truncated at K, i.e. with the density

$$f_j(x) = \begin{cases} \lambda_j e^{-\lambda_j x}, & \text{if } 0 < x < K \\ e^{-\lambda_j K}, & \text{if } x = K \\ 0, & \textit{otherwise} \end{cases}$$

with respect to the measure

$$\mu(A) = \nu(A \cap (0, K)) + \mathbb{I}_A(K), \ A \in \mathcal{B}(\mathbb{R}),$$

where $\nu(\cdot)$ is the Lesbegue measure.

Let us note that $\mu_j = \dfrac{1 - e^{-\lambda_j K}}{\lambda_j}$. It is easy to show that $\mu_0 < \mu_1$ iff one of the below cases (a)-(c) is satisfied:

(a) $\lambda_0 < \lambda_1 < \lambda^*$,

(b) $\lambda_0 < \lambda^* < \lambda_1$,

(c) $\lambda^* < \lambda_1 < \lambda_0$,

where λ^* is the unique positive solution of the equation $K\lambda + 1 = e^{K\lambda}$.

Let us note that in the cases (a) and (b) $\sup_x \dfrac{f_0(x)}{f_1(x)} = \dfrac{\lambda_0}{\lambda_1}$, while in the case (c) it is equal to $e^{-K(\lambda_0 - \lambda_1)}$. Hence, from Lemma 5, the condition (M) is satisfied in the cases (a) and (b) for $p \geq 1 - \dfrac{\lambda_1}{\lambda_0}$ and in the case (c) for $p \geq 1 - e^{-K(\lambda_0 - \lambda_1)}$. From Theorem 2 we know that σ defined by (4.2):

$$\sigma = \inf \left\{ n \in \mathbb{N} : \pi_n \geq \frac{c - p\dfrac{1 - e^{-K\lambda_1}}{\lambda_1} - (1-p)\dfrac{1 - e^{-K\lambda_0}}{\lambda_0}}{(1-p)\left(\dfrac{1 - e^{-K\lambda_1}}{\lambda_1} - \dfrac{1 - e^{-K\lambda_0}}{\lambda_0}\right)} \right\}$$

is an optimal stopping time in the class of stopping times with finite expectations.

References

[1] T. Bojdecki, *Probability maximizing approach to optimal stopping and its application to a disorder problem*, Stochastics, 3 (1979), 61–71.

[2] Y. S. Chow, H. Robbins and D. Siegmund, *Great Expectations: The Theory of Optimal Stopping*, Houghton Mifflin, Boston, 1971.

[3] E. Z. Ferenstein and A. Sierocinski, *Optimal stopping of a risk process*, Appl. Math., 24, 3 (1997), 335–342.

[4] U. Jensen, *Monotone stopping rules for stochastic processes in a semimartingale representation with applications*, Optimization, 20, 6 (1989), 837–852.

[5] U. Jensen, *An optimal stopping problem in risk theory*, Scand. Actuarial J., 2 (1997), 149–159.

[6] A. N. Shiryaev, *Optimal Stopping Rules*, Springer-Verlag, New York, 1978.

[7] A. Schottl, *Optimal stopping of a risk reserve process with interest and cost rates*, J. Appl. Prob., 35 (1998), 115–123.

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