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KAZIMIERZ URBANIK AND HIS RESEARCH

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The town of Krzemieniec, now in Western Ukraine, was for centuries, until World War II, a part of the Commonwealth of Poland and Lithuania. The town always played a special role in the history of Polish culture, expecially during the nineteenth century when the Polish state ceased to exist as such and its territories were partitioned among Russia, Prussia and Austria. The pride of Krzemieniec was the *Lyceum*, an educational institution of considerable prestige and tradition in Eastern Europe. It counted numerous luminaries and statesmen among its graduates. Juliusz Słowacki (1809–1849), one of the pantheon of Polish Romantic poets, was raised and educated in Krzemieniec, and so was, a century later, Mark Kac (1914–1984), a well known Polish-American mathematician.

It was in these environs that Kazimierz Urbanik was born on February 5, 1930. In due time he entered the *Lyceum* but his education at its School of Exercises was interrupted by the war. First the Soviets, then the Germans, and then, again the Soviets occupied the area, and by 1945 his family was forced to move to the town of Brzeg, 50 km south-east of Wrocław, in Lower Silesia which was just reunited with Poland as a result of the Yalta agreements. In 1948, he passed the final *matura* high school examination and matriculated at the University of Wrocław. He majored both in mathematics and physics and showed an early interest in other areas of natural sciences. At one point during his undergraduate studies he was an active participant in nine different seminars. There he met his mentors, Professors Hugo Steinhaus and Edward Marczewski (Szpilrajn) who after World War II

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transplanted the traditions of the Lwów and Warsaw Schools of Mathematics to the Polish Western Territories. The presence of those two distinguished mathematicians decisively influenced Urbanik's scholarly interests. In October 1950, as a third-year undergraduate student, he was awarded a teaching assistantship at the University where from he graduated in 1952, and where he was immediately employed as a junior faculty member.

Although throughout his career he remained interested in a broad spectrum of scientific endeavors, his focus, after a brief flirtation with topology, was now firmly in probability theory. A research seminar on this subject was then run at the Wrocław Branch of Mathematical Institute of the Polish Academy of Sciences by Marczewski and Steinhaus. Dubbed simply the "Monday, 5 o'clock seminar" by the insiders, it has continued its activities for more than fifty years. It is still alive and well today (these days it meets at 10 a.m., though) and concentrates on analytic and functional methods of probability theory. Over the last several decades it was Urbanik who directed it and was its soul and spirit. All the three authors of this biographical sketch and numerous other probabilists got their initial training there, and were beneficiaries of Urbanik's patient and forgiving mentoring style.

Urbanik's academic career was swift. In 1956, he received his Ph.D. for a dissertation on cascade processes, in 1957 obtained his docentship and was appointed Associate Professor (Docent), and three years later was promoted to the rank of Professor. In 1965, at the age of 35, he was elected to the Polish Academy of Sciences as its youngest member ever.

Throughout the years he masterly combined a steady flow of research (published in over 180 papers in a variety of areas), teaching and major administrative responsibilities. The latter were not an afterthought in his academic life. For almost thirty years (1967–1978 and 1981–1996) he guided the Institute of Mathematics of the Wrocław University as its Director, and from 1975 to 1981 he served as the President (Rektor) of Wrocław University. He formally retired from the University in the Summer of 2000. For a couple of terms he was also a Vice President of the Polish Academy of Sciences. He played key roles in developing several major projects of importance to Polish mathematics, such as the creation of the Stefan Banach Mathematical Center, which was initially an institution jointly funded by the Soviet Union, Poland, East Germany, Czechoslovakia, Hungary, Romania and Bulgaria, but located in Warsaw, Poland. It should not be overlooked that his effectiveness as a science administrator and a community leader was greatly enhanced by his prominent position within the then ruling Polish United Workers Party. However, he was never an ideological doctrinaire and kept his political views private. With his access to the highest echelons of the communist establishment, he was able to protect the mathemati-

cal community from political extremists, and many individual mathematicians from the unpleasant consequences of their "political incorrectness". Throughout the last half-century of Poland's political trials and tribulations his integrity was above reproach as he kept the respect and admiration of people from all parts of the political spectrum. It was a remarkable fact that in the nineties, after Solidarity wrestled power from the communist party, Urbanik was still elected by a popular vote to the directorship of the institute.

As a teacher Urbanik developed a large and faithful following. His delivery was crisp and velvety, and we were all mesmerized by his lectures in which deep theories unfolded effortlessly in front of our eyes without any help from notes or textbooks. He had developed original approaches to almost every subject he lectured on and we regret that most of his course offerings were never converted into published textbooks. His lectures attracted many research students to his seminars. Among the seventeen Ph.D. students who wrote their dissertations under his supervision are the first and the third authors of this article, while the second author was a Ph.D. student of the third one. Urbanik was also a popular speaker abroad, with invited visits to Berkeley, Moscow, Paris, Cambridge, New Orleans, Beijing, Göttingen, Hanoi, and Cleveland, among others. He spoke several times at the Oberwolfach Institute in Germany. In 1966, during the World Mathematical Congress in Moscow, he delivered a major invited address. Despite his retirement, he still continues to direct his Monday Seminar, teaches graduate courses and serves as the Editor-in-Chief of the journal *Probability and Mathematical Statistics* which was founded by him in 1980. Numerous awards and honors bestowed on him are listed in a separate appendix below.

Kazimierz Urbanik's most substantial research contributions, already acknowledged in Jean Dieudonné's historical analysis *A Panorama of Pure Mathematics as seen by N. Bourbaki* (cf. [a], Section B VII, pp. 223–228), were in probability and stochastic processes. He also made, however, major discoveries in other areas that included information theory, mathematical physics (including foundations of quantum mechanics), theory of universal algebras, mathematical analysis, functional analysis and topology. In this broad scope of research he was a faithful follower of his mentor, Hugo Steinhaus. In the remainder of this sketch we will attempt to describe Kazimierz Urbanik's principal lines of research dividing them into several topics as was done in the 1974 *Nauka Polska* article on Urbanik's work (cf. [g]). The numbered references refer to the complete bibliography of Urbanik's papers which is enclosed as an appendix below; letters denote other references.

(i) *Probability theory.* In the years 1956–1960 Urbanik was one of the first who investigated limit theorems for sequences of independent random elements with values in compact groups, and introduced the notion of a Gaussian measure on a locally compact abelian group . One of his fundamental and strikingly elegant results was that the existence of a Gaussian measure on a group is equivalent to the connectedness of the group. His results in this area are now a standard fare in monographs on probability theory on groups (cf. e.g., Heyer's monograph [c] and Chapters 3 and 6 in Grenander's book [b]).

While visiting Aarhus University in 1962 Urbanik learned about Kingman's work on random walks with spherical symmetry which lead him to consider a new type of convolution. In the fundamental paper [79] he introduced a formal notion of a *generalized convolution* as a binary operation on probability measures on the positive half-line satisfying five axioms, one of them being the weak law of large numbers for δ_1 measures. These axioms permit a study of generalized characteristic functions, Laplace transforms, infinitely divisible laws, stable laws, Linnik class I_0, moments, domains of attractions, and other concepts hitherto studied only for classical convolutions. In his most recent work those fundamental results are used to introduce and investigate some "generalized" special functions. Over the years Urbanik wrote almost twenty papers on that subject; in literature, generalized convolutions are now commonly referred to as "Urbanik systems". Some of the generalized convolutions are related to the theory of hypergroups. Urbanik's pioneering work in this area was followed up by numerous contributions to the subject from other mathematicians such as D. Kendall, N. Bingham, V.E. Volkovich, N. van Thu, H.Heyer, R. Jajte and Z. J. Jurek.

In 1968, Urbanik has ingeniously applied the analytical method of extreme points, and the Choquet's Theorem in particular, to find characteristic functions of many limit probability laws, including a description of the Lévy class L of selfdecomposable distributions. He also used that tool to characterize Feller's class, autoregressive systems and limit laws in non-commutative probability theory, cf. [119], [122], [141].

Four years later, in [109], he described limit laws of partial sums of random vectors normalized by matrices (linear operators). For this purpose he introduced a completely new notion of *decomposability semigroups*. These are matrix (linear operator) semigroups associated with probability measures. In numerous papers Urbanik has shown how topological and algebraic properties of those semigroups can be used to describe probability distributions; cf. [120], [123], [128], [134]. The subject, which originated independently in V. Sakovic's and M. Sharpe's dissertations, had many followers, including M. Klass, M.G. Hahn, V. Semovskii, J. Kucharczak, R. Jajte,

W. Krakowiak, B. Mincer, W.N. Hudson, Z.J. Jurek, J.A. Veeh, W. Hazod, M.M. Meerschaert, H.P. Schaefler; and again in this context, in [109] and [134], Urbanik masterfully applied Choquet's Theorem to find an analogue of the Lévy–Khintchine formula. A historical sketch of the operator-limit laws theory can be found in [e]. Chapter 3 of the latter monograph consists mostly of Urbanik's results and the book also provides a new random integral representation method which permits to circumvent the extreme points technique.

In papers [110] and [115] Urbanik introduced a classification of limit laws by introducing a countable decreasing family $L_m, m = 0, 1, 2, \ldots$, which begins with the Lévy class L of selfdecomposable distributions. This circle of ideas was picked up, extended and generalized, among others, by J. Bunge, Z. J. Jurek, K. Sato and M. Yamazato, B. Schreiber, N. van Thu. A novel identification theorem for probability distributions via moments of sums of independent random variables was obtained by Urbanik in 1993. The proof borrowed techniques from the theory of Banach algebras.

(ii) *Stochastic processes.* In one of his first papers, published in 1954, Urbanik investigated asymptotic behavior of homogenous Markov processes and, in particular, the distribution of their extreme values. He proposed a Markovian model of cosmic ray cascades, and the physical problem of forecasting the sun's activity led him to the prediction theory for stationary processes without the moment condition. He proved that in this context Orlicz spaces play a role analogous to the role of Hilbert spaces in the Wiener–Kolmogorov theory based on the covariance function. In a 1967 article [97], which was quickly followed by a joint paper with W.A. Woyczynski [98] on a similar topic, the stochastic integrability with respect to general processes with independent increments was characterized in terms of Orlicz spaces. This approach was later extended to Bartle-type stochastic integrals by J. Rosiński, and to semimartingale integrals by Kwapień and Woyczyński; cf. [g].

In 1956, Urbanik begun his systematic study of generalized stochastic processes and random fields whose sample functions are (Schwartz) distributions, and introduced local characteristics for such processes; cf. [21], [27], [30–31], [34], [38]. This work, of great importance for physics and, in particular, quantum field theory, was done contemporaneously but independently of I.M. Gelfand's investigations in the same area, and used different techniques.

More recently, in 1988 papers [157] and [165], Urbanik introduced the concept of an analytic stochastic process which was based on the Wiener–Itô decomposition of chaos. His fundamental theorem provides an isomorphism

between the class of analytic processes and the space of entire functions. This permits an application of tools from analytic function theory to random special functions. In 1992, Urbanik introduced a new analytic method for studying random functionals of transient stochastic processes, which include functionals of geometric Brownian motion, cf. [168] and [169]. The latter found applications in foundations of modern mathematical finance theory.

(iii) *Information theory and theoretical physics.* In 1957, Urbanik working together with G.S. Rubinstein, solved a problem posed by A.N. Kolmogorov, concerning the maximum value of information, [26]. His further investigations in this field were closely related to statistical physics and done in collaboration with physicist Roman S. Ingarden. In particular, using ideas of E.T. Jaynes, they proposed an original foundation for informational thermodynamics. The law of entropy increase was proved rigorously; cf. [65–66], [68–69]. In foundations of quantum mechanics, Urbanik proved a remarkable fact that commutativity of observables is equivalent to the existence of their joint distribution; cf. [67], [93].

Since 1961, Urbanik made several attempts to define information without probability theory. These efforts finally bore fruit in 1972, when he proposed new axioms for information theory based on four postulates: (1) the law of the broken choice; (2) the local character of information; (3) the indistinguishability of equivalent systems of information; (4) the law of increase of information; cf. [111], [114], [116].

(iv) *General algebras.* It was Edward Marczewski, one of Urbanik's mentors who, in 1958, initiated studies of the notion of independence in universal algebras. One of the deepest problems in that field was the characterization of those algebras whose independence has the properties of linear independence in linear spaces. During the following eight years Urbanik completely solved that problem, proving that those algebras are linear or affine spaces over appropriate fields; cf. [48], [57], [71], [75], [89]. Also, during his visit at Tulane University, New Orleans, Louisiana, in the academic year 1959–60, he made fundamental contributions to the theory of algebras with absolute values. The work contained in almost twenty papers written by Urbanik in the field of general algebras forms an essential part of George Grätzer's 1979 monograph [d]. The journal *Algebra Universalis* is now one of the main outlets for research in the area where Urbanik's work was of such fundamental importance.

(v) *Topology, measure theory and analysis.* Urbanik's first paper, written in 1953 jointly with B. Knaster, characterized zero-dimensional G_δ sets. But he did not stay in the area although, occasionally, he returned to topo-

logical issues. In [5] he proved the non-topological structure of the field of Mikusiński operators. Jointly with Paul Erdös, in [41], he proved a theorem about sets measured by multiples of irrational numbers. A collaboration with H. Fast, resulted in an extension of Titchmarsh convolution theorem, while in [64] he developed Fourier analysis on Marcinkiewicz spaces. Urbanik also solved the Hartman's problem on the existence of common extension of isomorphic images of Haar measures induced by different topologies on a given group.

The above paragraphs provide only a rough and imprecise description of Kazimierz Urbanik's opus of research. A complete listing of his publications is enclosed as an appendix. There are several characteristic features of Urbanik's style of doing mathematics in particular, and science in general. The first and foremost in our minds is the elegance of his theories and sheer power of his deductive reasoning combined with the crispness and clarity of their presentation. He never shrinks from frontal attacks on the problems he is working on and is capable to marshal considerable artillery to support his offensives. But now and then you see a totally unexpected tack in his proofs and analytic ingenuity that we, his students, all tried to emulate. There is also a persistent physical thinking behind a lot of his abstract arguments. In his work on probability, he employed powerful abstract tools, ranging from functional analysis to abstract algebras and topology, with great mastery even in situations that seemingly, at first sight, were unlikely to benefit from them. His great intuition and insight in finding the most appropriate and often eye-opening formal framework for theories he was working on has always been remarkable.

It is our considered opinion that the recognition and importance of Urbanik's multifaceted work will grow as time goes on and that the popularity of his pioneering ideas in research programs of other mathematicians and theoretical physicists will expand. The probability school he has created in Wrocław, continuing Hugo Steinhaus' traditions, has by now radiated its ideas and its style of doing mathematics to many other international research centers, and his former students spread their scholarly activities to five continents.

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List of Kazimierz Urbanik's Ph.D. Students

1962 Emanuel Strzelecki (now in Australia)
1968 Wojbor A. Woyczyński (now at Case Western Reserve University, Cleveland, Ohio, USA)
1969 Bolesław Szafnicki (now in Germany)
1971 Jerzy Gilewski (now in France)
1971 Jacek Kuiński (now at Technical University of Poznań, Poland)
1971 Marek Pieńkowski (now at Dominican Order, Kraków, Poland)
1974 Nguyen Chi Bao (now at the Embassy of Vietnam in Poland, Warsaw, Poland)
1975 Jerzy Kucharczak (now at University of Wrocław, Wrocław, Poland)
1976 Nguyen van Thu (now at Center for Natural Sciences and Technology, Hanoi, Vietnam)
1977 Purewiin Beyzżaw (now in Ułan Bator, Mongolia)
1977 Zbigniew J. Jurek (now at University of Wrocław, Wrocław, Poland)
1978 Andrzej Korzeniowski (now at University of Texas, Arlington, Texas, USA)
1979 Wiesław Krakowiak (now at University of Wrocław, Wrocław, Poland)
1981 Teresa Rajba (now at Technical University of Łódź, Łódź, Poland)
1982 Lesław Bielak (now in Germany)
1985 Bogdan Mincer (now at University of Wrocław, Wrocław, Poland)
1988 Andrzej Wiśniewski (now at University of Szczecin, Poland)

Major Administrative and Editorial Positions

Director of the Institute of Mathematics at the University of Wrocław since 1967 till 1978 and from 1981–1996.

President(Rektor) of the University of Wrocław for two terms, 1975–1978 and 1978–1981.

President of the Wrocław Branch of the Polish Academy of Sciences (PAN), 1972–1977.

Vice President of the Polish Academy of Sciences 1984–1986.

Member of the Editorial Board of the *Zeitschrift fur Wahrsheinlichkeitstheorie und Verwandte Gebiete*, 1962–1982.

Member of the Editorial Board of *Journal of Multivarite Analysis*, 1970–1980.

Member of the Editorial Board of the *Studia Mathematica*, 1967–present.

Member of the Editorial Board of the *Colloquium Mathematicum*, 1968–present.

Founder and the Editor-in-Chief of *Probability and Mathematical Statistic*, 1980–present.

Major Awards and Honors

Mazurkiewicz Prize of the Polish Mathematical Society, 1957.

Polish State Prize, Second Class, 1964.

Polish State Prize, First Class, 1973.

Award of the Polish Academy of Sciences, 1972.

The French Academy of Sciences' Palms Award, 1976.

Medal of King Leopod II awarded by the Government of Belgium, 1977.

The Alfred Jurzykowski Foundation (New York) Award, 1989.

Sierpiński Medal of the Polish Mathematical Society and Warsaw University, 1993.

Minister of Education of the Republic of Poland Award, 1994.

Doctorate *Honoris Causa* of the University of Łódź, 1995.

Doctorate *Honoris Causa* of the Technical University of Wrocław, 1995.

S. Banach Medal of the Polish Academy of Sciences, 1998.

W. Orlicz Medal of the University of Poznań, 1998.

Award of the Prime Minister of the Republic of Poland, 1998.

LIST OF RESEARCH PUBLICATIONS OF KAZIMIERZ URBANIK

1953

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1954

[2] *Sur un problème de J. F. Pàl sur les courbes continues*, Bull. Acad. Polon. Sci., Cl. III **2** (1954), 205–207.

[3] *Limit properties of homogeneous Markoff processes with a denumerable set of states*, Bull. Acad. Polon. Sci., Cl. III **2** (1954), 371–373.

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[5] *Sur la structure non-topologique du corps des opérateurs*, Studia Math. **14** (1954), 243–246.

1955

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1956

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[15] *Uwagi o równaniach procesów stochastycznych rozgałęzionych (Remarks on the equations of branching stochastic processes)*, Zeszyty Naukowe Uniwersytetu Wrocławskiego, Seria B, **1** (1956), 17–26 (English and Russian summaries).

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