DEMONSTRATIO MATHEMATICA
Vol. XXXII No 4 1999

Wojciech Jabłoński

STABILITY OF THE PEXIDER-TYPE HOMOGENEOUS EQUATION

Abstract. Let f be a function defined on a subset U ($\mathbb{R}_0 U \subset U$) of a real linear space X with the values in a sequentially complete locally convex linear topological Hausdorff space Y. We show that if there exist a bounded subset $V \subset Y$, a non-empty subset $A \subset \mathbb{R}_0$, $\delta : A \to [0, \infty)$ and $\phi : A \to \mathbb{R}_0$, $K : U \to [0, \infty)$ such that $\phi(\alpha)^{-1} f(\alpha x) - f(x) \in \delta(\alpha) K(x) V$ for all $\alpha \in A$, $x \in U$, then, under certain assumptions on A and K, there is a unique $\bar{\phi}$-homogeneous mapping $F : U \to Y$ such that the difference $F(x) - f(x)$ is suitably bounded on U. Next we investigate the stability of the Pexider-type homogeneous equation.

Introduction

J. Tabor proved in [6] that every mapping from a real vector space X into a normed space Y satisfying the inequality

$$(1) \qquad \|\alpha^{-1} f(\alpha x) - f(x)\| \leq \varepsilon \quad \text{for } \alpha \in \mathbb{R} \setminus \{0\}, x \in X,$$

where $\varepsilon \geq 0$ is given, is homogeneous. Z. Kominek and J. Matkowski investigated in [4] the condition

$$(2) \qquad \alpha^{-1} f(\alpha x) - f(x) \in V, \qquad \alpha \in A, x \in S,$$

for the mapping f from a cone $S \subset X$ into a sequentially complete locally convex linear topological Hausdorff space and a subset $A \subset (1, \infty)$. Their result has next been generalized in [5]. J. Schwaiger has examined the condition

$$(3) \qquad f(\alpha x) - \phi(\alpha) f(x) \in V(\alpha) \quad \text{for } \alpha \in A, x \in X,$$

where

— G is a semigroup with unit acting on the non-empty set X;

1991 *Mathematics Subject Classification*: Primary 39 B72.
Key words and phrases: homogeneous function, linear topological space, Hausdorff space, locally convex space, sequentially complete space.

— Y is a sequentially complete locally convex linear topological Hausdorff space Y over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$;
— $A \subset G$ generates G as a semigroup;
— $\phi : G \to \mathbb{K}$ is a function;
— $V : G \to \mathcal{B}(Y)$ is a mapping from G into the set $\mathcal{B}(Y)$ of bounded subsets of Y.

It is proved there that if the functions f and ϕ satisfy (3) and $f(X)$ is unbounded, then ϕ is a multiplicative function and there is a function $F : X \to Y$ satisfying

$$F(\alpha x) = \phi(\alpha)F(x) \quad \text{for } \alpha \in G, x \in X,$$

(we say then that F is *ϕ-homogeneous*) and such that the difference $F - f$ is suitably bounded on X.

In [1] and [7] it has been investigated the inequality

$$\|f(\alpha x) - \alpha^v f(x)\| \leq g(\alpha, x), \qquad \alpha \in \mathbb{R} \setminus \{0\}, x \in X,$$

with some functions g mapping $\mathbb{R} \times X$ into $\mathbb{R}$. Moreover S. Czerwik studied in [2] the following Pexider-type stability condition

$$\|f(\alpha x) - \phi(\alpha)g(x)\| \leq g(\alpha, x), \qquad \alpha \in \mathbb{R}, x \in X.$$

In this paper we investigate some kind of stability of the homogeneity and stability of the Pexider-type homogeneous equation.

Main result

Throughout this paper the letters $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{R}_0$ stand for positive integers, non-negative integers, reals, non-negative reals, and reals different from zero, respectively. From now on X stands for a real linear space and Y– for a sequentially complete locally convex linear topological Hausdorff space. The sequentially closure of V will be denoted by $\operatorname{seq\,cl} V$. By $\operatorname{conv} V$ we will denote the convex hull of V. The subset $V \subset Y$ is said to be bounded if for each neighbourhood W of zero there exists an $r \in \mathbb{R}_0$ such that $r\,V \subset W$. By $\langle A \rangle$ we denote a multiplicative group generated by the set $A \subset \mathbb{R}_0$.

It is known that for $V \subset Y$ and $0 \leq \alpha \leq \beta$, $\alpha V \subset \beta \operatorname{conv}(V \cup \{0\})$ holds. Moreover, if V is symmetric with respect to the origin, then $\alpha V \subset \beta \operatorname{conv} V$.

We have the following

THEOREM 1. *Let $\emptyset \neq U \subset X$ be such that $\mathbb{R}_0 U \subset U$. Assume that the function $K : U \to \mathbb{R}_+$ satisfies*

$$K(\alpha x) \leq |\alpha|^p K(x) \quad \textit{for } \alpha \in \mathbb{R}_0, x \in U, \tag{4}$$

with certain $p \in \mathbb{R} \setminus \{1\}$. Fix $A \subset \mathbb{R}_0$ and let $\delta : A \to \mathbb{R}_+$, $\phi : A \to \mathbb{R}_0$ be given mappings. Let $V \subset Y$ be a bounded set and let $f : U \to Y$ satisfy

$$\phi(\alpha)^{-1} f(\alpha x) - f(x) \in \delta(\alpha)K(x)V \quad \textit{for all } \alpha \in A,\ x \in U. \tag{5}$$

If $A \supset A_0 := \{\alpha \in A : |\phi(\alpha)| \neq |\alpha|^p\} \neq \emptyset$, *then there exists a unique function* $F : U \to Y$ *satisfying*

$$F(\alpha x) = \phi(\alpha)F(x) \quad \textit{for } \alpha \in A_0, x \in U,$$

and such that

$$F(x) - f(x) \in cK(x)\operatorname{seq cl\,conv}(V \cup (-V)), \qquad x \in U,$$

where

$$c := \min\left(\inf_{\alpha\in A_0^1} \frac{\delta(\alpha)\,|\phi(\alpha)|}{|\phi(\alpha)| - |\alpha|^p}, \inf_{\alpha\in A_0^2} \frac{\delta(\alpha)\,|\phi(\alpha)|}{|\alpha|^p - |\phi(\alpha)|}\right),$$

$$A_0^1 := \{\alpha \in A : |\alpha|^p < |\phi(\alpha)|\}, A_0^2 := \{\alpha \in A : |\alpha|^p > |\phi(\alpha)|\}.$$

Furthermore, if $F \neq 0$, *then there is a unique multiplicative function* $\tilde{\phi} : \langle A_0 \rangle \to \mathbb{R}_0$ *such that* F *is* $\tilde{\phi}$*-homogeneous. If, moreover,* $0 \in U$ *and* $\phi(\alpha_0) \neq 1$ *for some* $\alpha_0 \in A_0$, *then there exists a unique multiplicative function* $\bar{\phi} : \langle A_0 \rangle \cup \{0\} \to \mathbb{R}$, *such that* F *is* $\bar{\phi}$*-homogeneous.*

Proof. We give only the main idea of the proof, which follows as that one of Theorem 1 from [3]. Fix an $\alpha \in A_0$. Consider two cases:

1) $|\alpha|^p < |\phi(\alpha)|$,
2) $|\alpha|^p > |\phi(\alpha)|$.

In the first case one can show that $\{\phi(\alpha)^{-n} f(\alpha^n x) : n \in \mathbb{N}\}$ is a Cauchy sequence. Hence we may define the function $F_\alpha : U \to Y$,

$$F_\alpha(x) := \lim_{n\to\infty} \phi(\alpha)^{-n} f(\alpha^n x)$$

which satisfies

$$\begin{aligned} F_\alpha(\alpha x) &= \lim_{n\to\infty} \phi(\alpha)^{-n} f(\alpha^{n+1} x) \\ &= \phi(\alpha) \lim_{n\to\infty} \phi(\alpha)^{-(n+1)} f(\alpha^{n+1} x) = \phi(\alpha) F_\alpha(x). \end{aligned} \tag{6}$$

Moreover

$$F_\alpha(x) - f(x) \in \frac{\delta(\alpha)K(x)|\phi(\alpha)|}{|\phi(\alpha)| - |\alpha|^p} \operatorname{seq cl\,conv}(V \cup (-V)), \qquad x \in U. \tag{7}$$

Now consider the second case. Replacing in (5) x by $\alpha^{-1}x$ we get

$$\phi(\alpha)^{-1} f(x) - f(\alpha^{-1}x) \in \delta(\alpha)K(\alpha^{-1}x)V,$$

and so, using (4), we get

$$\phi(\alpha) f(\alpha^{-1}x) - f(x) \in \delta(\alpha)|\phi(\alpha)||\alpha|^{-p} K(x) \operatorname{conv}(V \cup (-V)). \tag{8}$$

Similary as above we prove that $\{\phi(\alpha)^n f(\alpha^{-n}x) : n \in \mathbb{N}\}$ is a Cauchy sequence. The function $F_\alpha : U \to Y$ given by

$$F_\alpha(x) := \lim_{n\to\infty} \phi(\alpha)^n f(\alpha^{-n}x)$$

is well defined and satisfies

$$(9) \qquad F_\alpha(\alpha x) = \phi(\alpha)F_\alpha(x) \quad \text{for } x \in U.$$

Moreover we have

$$(10) \quad F_\alpha(x) - f(x) \in \frac{\delta(\alpha)K(x)|\phi(\alpha)|}{|\alpha|^p - |\phi(\alpha)|} \operatorname{seq\,cl\,conv}(V \cup (-V)), \qquad x \in U.$$

Let us notice that from (6) and (9) it follows that

$$F_\alpha(\alpha^n x) = \phi(\alpha)^n F_\alpha(x) \quad \text{for } \alpha \in A_0, x \in U, n \in \mathbb{Z}.$$

Using this fact and considering suitable cases (cf. [3]) one can prove that $F_\alpha = F_\beta$ for α, $\beta \in A_0$. Whence we may define

$$F := F_\beta, \qquad \beta \in A_0,$$

which satisfies

$$(11) \qquad F(\alpha x) = \phi(\alpha)F(x) \quad \text{for all } \alpha \in A_0, x \in U.$$

Moreover, by (7) and (10) we get

$$F(x) - f(x) \in c\,K(x) \operatorname{seq\,cl\,conv}(V \cup (-V)), \qquad x \in U,$$

where c is given as in the statement of our Theorem. The simply verification that F is unique can be concluded as in [3] and we left it to the reader.

Now let us assume that F obtained above is different from 0, that is there is an $x_0 \in U$ with $F(x_0) \neq 0$. From (11) we have that

$$F(\alpha^{-1}x) = \phi(\alpha)^{-1}F(x) \qquad \text{for all } \alpha \in A_0, x \in U.$$

Consequently, for $\alpha = \lambda_1 \cdot \ldots \cdot \lambda_n \mu_1^{-1} \cdot \ldots \cdot \mu_m^{-1} \in \langle A_0 \rangle$, we obtain

$$(12) \quad F(\alpha x) = \phi(\lambda_1) \cdot \ldots \cdot \phi(\lambda_n)\, \phi(\mu_1)^{-1} \cdot \ldots \cdot \phi(\mu_m)^{-1} F(x) \quad \text{for } x \in U.$$

Define a function $\tilde{\phi} : \langle A_0 \rangle \to \mathbb{R}_0$ as follows. For $\alpha = \lambda_1 \cdot \ldots \cdot \lambda_n \mu_1^{-1} \cdot \ldots \cdot \mu_m^{-1} \in \langle A_0 \rangle$, we take

$$\tilde{\phi}(\alpha) := \phi(\lambda_1) \cdot \ldots \cdot \phi(\lambda_n)\, \phi(\mu_1)^{-1} \cdot \ldots \cdot \phi(\mu_m)^{-1}.$$

We show that $\tilde{\phi}$ is well defined. Take $\lambda_1 \cdot \ldots \cdot \lambda_n \mu_1^{-1} \cdot \ldots \cdot \mu_m^{-1} = \eta_1 \cdot \ldots \cdot \eta_p \nu_1^{-1} \cdot \ldots \cdot \nu_q^{-1} \in \langle A_0 \rangle$. Then we have

$$\begin{aligned}
&\phi(\lambda_1) \cdot \ldots \cdot \phi(\lambda_n)\, \phi(\mu_1)^{-1} \cdot \ldots \cdot \phi(\mu_m)^{-1} F(x_0) \\
&= F\left(\lambda_1 \cdot \ldots \cdot \lambda_n \mu_1^{-1} \cdot \ldots \cdot \mu_m^{-1} x_0\right) = F\left(\eta_1 \cdot \ldots \cdot \eta_p \nu_1^{-1} \cdot \ldots \cdot \nu_q^{-1} x_0\right) \\
&= \phi(\eta_1) \cdot \ldots \cdot \phi(\eta_p)\, \phi(\nu_1)^{-1} \cdot \ldots \cdot \phi(\nu_q)^{-1} F(x_0).
\end{aligned}$$

Since $F(x_0) \neq 0$, so we have

$$\begin{aligned}
&\phi(\lambda_1) \cdot \ldots \cdot \phi(\lambda_n)\, \phi(\mu_1)^{-1} \cdot \ldots \cdot \phi(\mu_m)^{-1} \\
&\qquad = \phi(\eta_1) \cdot \ldots \cdot \phi(\eta_p)\, \phi(\nu_1)^{-1} \cdot \ldots \cdot \phi(\nu_q)^{-1}.
\end{aligned}$$

Therefore $\tilde{\phi}$ is well defined. Moreover, by (12), we obtain

$$F(\alpha x) = \tilde{\phi}(\alpha)F(x) \qquad \text{for } \alpha \in \langle A_0 \rangle, x \in U. \tag{13}$$

For $\alpha, \beta \in \langle A_0 \rangle$ we have

$$\tilde{\phi}(\alpha\beta)F(x_0) = F(\alpha\beta x_0) = \tilde{\phi}(\alpha)F(\beta x_0) = \tilde{\phi}(\alpha)\tilde{\phi}(\beta)F(x_0),$$

o $\tilde{\phi}(\alpha\beta) = \tilde{\phi}(\alpha)\tilde{\phi}(\beta)$ and $\tilde{\phi}$ is a multiplicative function. The uniquenes we obtain from the construction of $\tilde{\phi}$. From (13) we get that F is $\tilde{\phi}$–homogeneous.

Next assume that F obtained above is different to 0, $0 \in U$ and $\phi(\alpha_0) \neq 1$ for some $\alpha_0 \in A_0$. From (13) we get

$$F(0) = F(\alpha_0 0) = \tilde{\phi}(\alpha_0)F(0),$$

so $F(0) = 0$. By (13) we may uniquelly extend $\tilde{\phi}$ as follows

$$\bar{\phi}(\alpha) = \begin{cases} \tilde{\phi}(\alpha) & \text{for } \alpha \in \langle A_0 \rangle \\ 0 & \text{for } \alpha = 0, \end{cases}$$

and then $\bar{\phi} : \langle A_0 \rangle \cup \{0\} \to \mathbb{R}$ is a unique multiplicative function such that F is $\bar{\phi}$–homogeneous. ■

REMARK 1. The similar example as in [3] shows that the estimation obtained in the assertion of above Theorem is the best one.

Stability of Pexider-type homogeneous equation

In this section we consider, for given functions $f, g : U \to Y$, the following stability condition

$$\phi(\alpha)^{-1} f(\alpha x) - g(x) \in \delta(\alpha)K(x)V, \qquad \alpha \in A, x \in U, \tag{14}$$

with a certain function $\phi : A \to \mathbb{R}_0$.

We prove

THEOREM 2. *Let $\emptyset \neq U \subset X$ be such that $\mathbb{R}_0 U \subset U$. Assume that the function $K : U \to \mathbb{R}_+$ satisfies (4). Let $\{1\} \subset A \subset \mathbb{R}_0$ and let $\delta : A \to \mathbb{R}_+$, $\phi : A \to \mathbb{R}_0$ be given mappings. Assume that $\phi(1) = 1$. Let $V \subset Y$ be a bounded set and let functions $f, g : U \to Y$ satisfy (14).*

If $A \supset A_0 := \{\alpha \in A : |\phi(\alpha)| \neq |\alpha|^p\} \neq \emptyset$, then there exists a unique function $F : U \to Y$ satisfying

$$F(\alpha x) = \phi(\alpha)F(x) \qquad \text{for } \alpha \in A_0, x \in U,$$

and such that

$$F(x) - f(x) \in c_1 K(x) \operatorname{seq cl conv}(V \cup (-V)), \qquad x \in U,$$

$$F(x) - g(x) \in c_2 K(x) \operatorname{seq cl conv}(V \cup (-V)), \qquad x \in U,$$

where

$$c_1 := \min\left(\inf_{\alpha\in A_0^1}\frac{(\delta(\alpha)+\delta(1))\,|\phi(\alpha)|}{|\phi(\alpha)|-|\alpha|^p},\ \inf_{\alpha\in A_0^2}\frac{(\delta(\alpha)+\delta(1))\,|\phi(\alpha)|}{|\alpha|^p-|\phi(\alpha)|}\right),$$

$$c_2 := \min\left(\inf_{\alpha\in A_0^1}\frac{\delta(\alpha)\,|\phi(\alpha)|+\delta(1)|\alpha|^p}{|\phi(\alpha)|-|\alpha|^p},\ \inf_{\alpha\in A_0^2}\frac{\delta(\alpha)\,|\phi(\alpha)|+\delta(1)|\alpha|^p}{|\alpha|^p-|\phi(\alpha)|}\right),$$

and the sets A_0^1, A_0^2 are given as in Theorem 1.

Furthermore, if $F \neq 0$, then there is a unique multiplicative function $\tilde{\phi} : \langle A_0\rangle \to \mathbb{R}_0$ such that F is $\tilde{\phi}$-homogeneous. If, moreover, $0 \in U$ and $\phi(\alpha_0) \neq 1$ for some $\alpha_0 \in A_0$, then there exists a unique multiplicative function $\bar{\phi} : \langle A_0\rangle \cup \{0\} \to \mathbb{R}$, such that F is $\bar{\phi}$-homogeneous.

Proof. From (14) we have

$$\begin{aligned}\phi(\alpha)^{-1}f(\alpha x) - f(x) &= \phi(\alpha)^{-1}f(\alpha x) - g(x) + g(x) - f(x)\\ &\in (\delta(\alpha)+\delta(1))\,K(x)\,\mathrm{conv}\,(V\cup(-V)),\end{aligned}$$

and similarly

$$\begin{aligned}\phi(\alpha)^{-1}g(\alpha x) - g(x) &= \phi(\alpha)^{-1}g(\alpha x) - \phi(\alpha)^{-1}f(\alpha x) + \phi(\alpha)^{-1}f(\alpha x) - g(x)\\ &\in |\phi(\alpha)|^{-1}\,\delta(1)K(\alpha x)\,(-V) + \delta(\alpha)K(x)V\\ &\subset \left(\delta(1)|\alpha|^p\,|\phi(\alpha)|^{-1} + \delta(\alpha)\right)K(x)\,\mathrm{conv}\,(V\cup(-V)).\end{aligned}$$

By Theorem 1 there exist functions $F, G : U \to Y$ and real constants c_1, c_2 such that

$$F(\alpha x) = \phi(\alpha)F(x) \quad \text{for } \alpha\in A_0, x\in U,$$

$$G(\alpha x) = \phi(\alpha)G(x) \quad \text{for } \alpha\in A_0, x\in U,$$

and

$$F(x) - f(x) \in c_1\,K(x)\,\mathrm{seq\,cl\,conv}\,(V\cup(-V)), \qquad x\in U,$$

$$G(x) - g(x) \in c_2\,K(x)\,\mathrm{seq\,cl\,conv}\,(V\cup(-V)), \qquad x\in U.$$

We are going to show that $F(x) = G(x)$ for each $x \in U$. Let us consider two cases:

1) there exists an $\alpha \in A_0$ such that $|\alpha|^p < |\phi(\alpha)|$. Then

$$F(x) = \lim_{n\to\infty}\phi(\alpha)^{-n}f(\alpha^n x),\ G(x) = \lim_{n\to\infty}\phi(\alpha)^{-n}g(\alpha^n x)$$

and from (14) we get

$$\begin{aligned}\phi(\alpha)^{-n}f(\alpha^n x) - \phi(\alpha)^{-n}g(\alpha^n x) &\in |\phi(\alpha)|^{-n}\delta(1)K(\alpha^n x)\,\mathrm{conv}(V\cup(-V))\\ &\subset (|\alpha|^p|\phi(\alpha)|^{-1})^n\delta(1)K(x)\,\mathrm{conv}(V\cup(-V)).\end{aligned}$$

Consequently

$$F(x) - G(x) = \lim_{n\to\infty} \phi(\alpha)^{-n} f(\alpha^n x) - \lim_{n\to\infty} \phi(\alpha)^{-n} g(\alpha^n x)$$
$$= \lim_{n\to\infty} (\phi(\alpha)^{-n} f(\alpha^n x) - \phi(\alpha)^{-n} g(\alpha^n x)) = 0;$$

2) there exists an $\alpha \in A_0$ such that $|\alpha|^p > |\phi(\alpha)|$. Then

$$F(x) = \lim_{n\to\infty} \phi(\alpha)^n f(\alpha^{-n} x), G(x) = \lim_{n\to\infty} \phi(\alpha)^n g(\alpha^{-n} x)$$

and from (14) we obtain

$$\phi(\alpha)^n f(\alpha^{-n}x) - \phi(\alpha)^n g(\alpha^{-n}x) \in |\phi(\alpha)|^n \delta(1) K(\alpha^{-n}x) \operatorname{conv}(V \cup (-V))$$
$$\subset (|\phi(\alpha)||\alpha|^p)^n \delta(1) K(\alpha x) \operatorname{conv}(V \cup (-V)),$$

so we have

$$F(x) - G(x) = \lim_{n\to\infty} \phi(\alpha)^n f(\alpha^{-n} x) - \lim_{n\to\infty} \phi(\alpha)^n g(\alpha^{-n} x)$$
$$= \lim_{n\to\infty} (\phi(\alpha)^n f(\alpha^{-n} x) - \phi(\alpha)^n g(\alpha^{-n} x)) = 0.$$

The last statement of our theorem we obtain similarly as in Theorem 1. ∎

Stability in Banach spaces

The conditions (5) and (14) may be considered, in a special case, for a normed space Y. We take $V = \{y \in Y : \|y\| \leq 1\}$. Then (5) and (14) may be rewriten as

(15) $$\|\phi(\alpha)^{-1} f(\alpha x) - f(x)\| \leq \delta(\alpha) K(x), \qquad \alpha \in A, x \in U,$$

or

(16) $$\|\phi(\alpha)^{-1} f(\alpha x) - g(x)\| \leq \delta(\alpha) K(x), \qquad \alpha \in A, x \in U,$$

respectively.

From Theorems 1 and 2 we obtain

THEOREM 3. *Let X, U, A, δ, K and A_0 be as in Theorem 1. Let Y be a Banach space, $\langle A_0 \rangle = \mathbb{R}_0$ and let $\phi : A \to \mathbb{R}_0$ be a given mapping.*

(i) *If a function $f : U \to Y$ satisfies (15), then there exists a unique function $F : U \to Y$ such that*

$$F(\alpha x) = \phi(\alpha) F(x) \quad \text{for } \alpha \in A_0, x \in U,$$

and

$$\|F(x) - f(x)\| \leq cK(x), \qquad x \in U$$

where c is defined as in Theorem 1. If $F \neq 0$, $0 \in U$ and $\phi(\alpha_0) \neq 1$ for some $\alpha_0 \in A_0$, then there exists a unique multiplicative function $\bar{\phi} : \mathbb{R} \to \mathbb{R}$ such that F is $\bar{\phi}$- homogeneous.

(ii) *If* $\{1\} \subset A_0$, $\phi(1) = 1$ *and functions* $f, g : U \to Y$ *satisfy* (16), *then there exists a unique function* $F : U \to Y$ *such that*

$$F(\alpha x) = \phi(\alpha) F(x) \quad \text{for } \alpha \in A_0, x \in U,$$

and

$$\|F(x) - f(x)\| \leq c_1 K(x), \qquad x \in U,$$

$$\|F(x) - g(x)\| \leq c_2 K(x), \qquad x \in U,$$

where c_1, c_2 *are defined as in Theorem* 2. *If* $F \neq 0$, $0 \in U$ *and* $\phi(\alpha_0) \neq 1$ *for some* $\alpha_0 \in A_0$, *then there exists a unique multiplicative function* $\bar{\phi} : \mathbb{R} \to \mathbb{R}$ *such that* F *is* $\bar{\phi}$- *homogeneous.*

References

[1] S. Czerwik, *On the stability of the homogeneous mapping*, G. R. Math. Rep. Acad. Sci. Canada 14 (6), (1992), 268–272.

[2] S. Czerwik, *Some results on the stability of the homogeneous functions*, (manuscript).

[3] W. Jabłoński, *On the stability of the homogeneous equation*, Publicationes Math. 55/1-2 (1999), 33–45.

[4] Z. Kominek and J. Matkowski, *On stability of the homogeneity condition*, Results Math. 27 (1995), 370–380.

[5] J. Schwaiger, *The Functional Equation of Homogeneity and its Stability Properties*, Sitzungsber. Abt. II (1996) 205, 3–12.

[6] J. Tabor, *On aproximately linear mappings*, Th. M. Rassias and J. Tabor, eds., Stability of Mappings of Hyers-Ulam Type, Hadronic Press, Palm Harbor, Florida 34682-1577, U.S.A., (1994), 157–163.

[7] Jacek Tabor, Józef Tabor, *Homogeneity is superstable*, Publ. Math. 45/1-2(1994), 123–130.

DEPARTMENT OF MATHEMATICS
SILESIAN UNIVERSITY
Bankowa 14
40-007 KATOWICE, POLAND
E-mail: wj@ux2.math.us.edu.pl

DEPARTMENT OF MATHEMATICS
PEDAGOGICAL UNIVERSITY
Rejtana 16 A
35-310 RZESZÓW, POLAND
E-mail: wojciech@atena.univ.rzeszow.pl

Received November 9, 1998.