DEMONSTRATIO MATHEMATICA
Vol. XXXII No 3 1999

R. A. Mahmoud

ON PREIRRESOLUTE MULTIVALUED FUNCTIONS

In this paper, the author establishes some new characterizations of the preirresolute multifunctions due to Popa et al. [11]. Also, other properties have been presented and some results in [11] are improved.

1. Introduction

Recently, Popa et al. [11] defined both upper and lower-preirresolute multifunctions as a generalization of preirresoluteness of single-one due to Reilly and Vamanamurthy [13], in 1985. Several of their characterizations and properties have been studied in [11]. Therefore, this paper is devoted to present other new characterizations of preirresolute-multifunctions. Moreover, some basic topological properties have been investigated and known results in [11], we have succeeded to strengthen them. The relationship between preirresolute-multifunctions with preclosed graphs have been discussed.

2. Preliminaries

The topological spaces or simply spaces which will be used here are (X,τ) and $Y,\sigma)$ without any separation axioms and whenever such properties are needed they will be explicity assumed. By $f:(X,\tau)\rightarrow(Y,\sigma)$ and $F:(X,\tau)\rightarrow(Y,\sigma)$ we will represent a single and a multivalued function or simply a function and a multifunction, respectively. For a multifunction $F:(X,\tau)\rightarrow(Y,\sigma)$, the upper and lower inverse of a subset B of Y are denoted by $F^+(B)$ and $F^-(B)$ respectively, that is, $F^+(B)=\{x\in X : F(x)\subseteq B\}$ and $F^-(B)=\{x\in X : F(x)\cap B\neq\phi\}$. If W is subset of X, the closure and the interior of W with respect to τ are

Key words and phrases. Upper-preirresolute, lower-preirresolute and preirresolute multifunctions, preclosed graphness of multifunctions.
1991 AMS Classification Codes: 54C8 and 54C60.

denoted by τ-cl(W) and τ-int(W), respectively. $W \subseteq X$ is said to be preopen [6] if $W \subset \tau$-int$(\tau$-cl$(W))$. While, the semi-openness consept was given by $W \subseteq \tau$-cl$(\tau$-int$(W))$ [5]. The family of all preopen (resp. semi-open) sets in (X,τ) will be denoted as usual by $PO(X,\tau)$ (resp. $SO(X,\tau)$). But for any point $p \in X$, $PO(X,p) = \{W \in PO(X,\tau) : p \in W\}$. The class $PO(X,p)$ and all of its super sets is the usual preneighbourhood system of p and will be denoted by pre-$N(p)$. A complement of a preopen set is preclosed [6], equivalently, τ-cl$(\tau$-int$(W)) \subseteq W$. Therefore [3] the preclosure operator of any $W \subseteq X$ is denoted by precl(W), and it is the intersection of all preclosed sets containes W. The authors in [13] denoted the class τ^{α}, with respect to (X,τ) which contains the sets having both properties of semi- and preopenness. In (X,τ), $W \subseteq X$ is called α-set, if $W \subseteq \tau$-int$(\tau$-cl$(\tau$-int$(W))$ as in [9]. A function $f : (X,\tau) \to (Y,\sigma)$ is preirresolute [13] if the inverse image of each preopen in (Y,σ) is preopen in (X,τ). Preirresoluteness of f is equivalent with $f(\text{precl}(A)) \subseteq \text{precl}(f(A))$, for each $A \subseteq X$ and $\text{precl}(f^{-1}(B)) \subseteq f^{-1}(\text{precl}(B))$ for each $B \subseteq Y drop(X)$. Following Popa [11] et al. a multifunction $F : (X,\tau) \to (Y,\sigma)$ is said to be upper-preirresolute (resp. lower-preirresolute) at a point $p \in X$ if for each $W \in PO(Y,\sigma)$ such that $F(p) \subseteq W$ (resp. $F(p) \cap W \neq \phi$ there exists $H \in PO(X,p)$ such that for each $h \in H$, $F(h) \subseteq W$ (resp. $F(h) \cap W \neq \phi$). If F is upper-preirresolute (resp. lower-preirresolute) at all points of its domain, then it is called upper-preirresolute (resp. lower-preirresolute). But $F : (X,\tau) \to (Y,\sigma)$ is strongly lower (resp. strongly upper) [10] semicontinuous, if for any $B \subseteq Y$, $F^{-}(B) \in \tau$ (resp. $F^{+}(B) \in \tau$) A space (X,τ) is said to be strongly compact [7] if every preopen cover of X admits a finite subcover. While, in (X,τ) any $W \subseteq X$ is called α-paracompact [14] if every of its open cover has an X-open X-locally finite retinement which covers W. Therefore, $F : (X,\tau) \to (Y,\sigma)$ is punctually α-paracompact [12] (punctually $\alpha - p$-regular [11]) if for each $p \in X$, $F(p)$ is α-paracompact ($\alpha - p$-regular). A subset W of a space (X,τ) is $\alpha - p$-regular, if for any $p \in W$ and any preopen set H containing p, there exists $U \in \tau$ such that $p \in U \subseteq \tau$-cl$(U) \subseteq H$ [11].

3. Characterizations

Several characterizations of upper and lower-preirresoluteness have been given in [11] and we show a bit more.

Definition 1. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is called preirresolute at a point $p \in X$ if for each pair $W_i \in PO(Y,\sigma)$, $i = 1,2$ such that $F(p) \subseteq W_1$ and $F(p) \cap W_2 \neq \phi$, there exists $H \in PO(X,p)$ with $F(H) \subseteq W_1$, such that for any $h \in H$, $F(h) \cap W_2 \neq \phi$.

Therefore, $F : (X,\tau) \to (Y,\sigma)$ is preirresolute if it has the above property at any point $p \in X$.

PROPOSITION 1. *Any preirresolute multifunction $F : (X,\tau) \to (Y,\sigma)$ at any $p \in X$ is both upper and lower-preirresolute at the same point.*

THEOREM 1. *For a multifunction $F : (X,\tau) \to (Y,\sigma)$, the following statements are equivalent:*

(i) *F is preirresolute at any $p \in X$.*

(ii) *For any $W_1, W_2 \in PO(Y,\sigma)$ such that $F(p) \subseteq W_1$ and $F(p) \cap W_2 \neq \phi$, we have $p \in \tau\text{-int}(\tau\text{-cl}(F^+(W_1) \cap F^-(W_2)))$.*

(iii) *For each $W_1, W_2 \in PO(Y,\sigma)$ having $F(p) \subseteq W_1$, $F(p) \cap W_2 \neq \phi$ and for any $U \in \tau(p)$, there exists $\phi \neq G \in \tau$ with $G \subseteq U$, $F(G) \subseteq W_1$ and $F(G) \cap W_2 \neq \phi$ for each $g \in G$.*

Proof. (i)→(ii): Take any $W_i \in PO(Y,\sigma)$, $i = 1,2$ with $F(p) \subseteq W_1$ and $F(p) \cap W_2 \neq \phi$. By hypothesis, there exists $H \in PO(Y,p)$ such that $F(H) \subseteq W_1$ and $F(h) \cap W_2 \neq \phi$ for each $h \in H$. Thus, $p \in H \subseteq F^+(W_1)$ and $p \in H \subseteq F(W_2) \neq \phi$. Hence $p \in H \subseteq F^+(W_1) \cap F^-(W_2)$. Preopenness of H implies that $p \in \tau\text{-int}(\tau\text{-cl}(H)) \subseteq \tau\text{-int}(\tau\text{-cl}(F^+(G_1) \cap F^-(G_2)))$.

(ii)→(iii): Assume $W_1, W_2 \in PO(Y,\sigma)$ be such that $F(p) \subseteq W_1$ and $F(p) \cap W_2 \neq \phi$. Notice that (ii) gives $p \in \tau\text{-int}(\tau\text{-cl}(F^-(W_1) \cap F^-(W_2)))$. Letting also $U \in \tau(p)$ we have $\phi \neq U \cap \tau\text{-int}(F^+(W_1) \cap F^-(W_2)) \subseteq U \cap (\tau\text{-int}(F^+(W_1)) \cap \tau\text{-int}(F^-(W_2))) = G$ which is open and contains in U with $G \subseteq \tau\text{-int}(F^+(W_1))$ and also $G \subseteq \tau\text{-int}(F^-(W_2)) \subseteq F^-(W_2)$. Therefore, $F(G) \subseteq W_1$ and $F(g) \cap W_2 \neq \phi$ for each $g \in G$. (iii)→(i): Immediately, since $\tau(p) \subseteq PO(X,p)$.

LEMMA 1 [1]. *In a space (X,τ),* $\text{precl}(A) = A \cup \tau\text{-cl}(\tau\text{-int}(A))$, *for any $A \subseteq X$.*

Although the mentioned statements (i)–(iv) in the next theorem are studied in [11] separately in both upper and lower-preirresoluteness, they will be investigated here, together with others more general. Namely we have:

THEOREM 2. *The following statements are equivalent for $F : (X,\tau) \to (Y,\sigma)$:*

(i) *F is preirresolute.*

(ii) *For any pair $W_1, W_2 \in PO(Y,\sigma)$, $F^+(W_1) \cap F^-(W_2) \in PO(X,\tau)$.*

(iii) *For each preclosed sets $S_1, S_2 \subseteq Y$, $F^-(S_1) \cup F^+(S_2)$ is preclosed.*

(iv) *For every $B_i \in P(Y)$, $i = 1,2$. $\tau\text{-cl}(\tau\text{-int}(F^-(B_1) \cup (B_2))) \subseteq F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2))$.*

(v) $\text{precl}(F^-(B_1) \cup F^+(B_2)) \subseteq F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2))$, *for any $B_1, B_2 \subseteq Y$.*

(vi) $F^-(\text{preint}(B_1)) \cap F^+(\text{preint}(B_2)) \subseteq \text{preint}(F^-(B_1) \cap F^+(B_2))$ *for each* $B_i \in P(Y)$, $i = 1, 2$.

(vii) *For any* $p \in X$ *and each* $N \in$ pre-$N(F(p))$, *then for every* $W \in PO(Y, \sigma)$ *such that* $W \cap F(p) \neq \phi$, $(F^+(N) \cap F^-(W)) \in$ pre-$N(p)$.

(viii) *For each* $p \in X$ *and any* $N \in$ pre-$N(F(p))$, *then for any* $W \in PO(Y, \sigma)$ *such that* $W \cap F(p) \neq \phi$. *There is* $N^* \in$ pre-$N(p)$ *with* $F(N^*) \subseteq N$ *and* $F(n) \cap W \neq \phi$ *for each* $n \in N*$.

Proof. (i)→(ii) For any $W_1, W_2 \in PO(Y, \sigma)$ and let $p \in F^+(W1) \cap F^-(W_2)$. Thus $F(p) \subseteq W_1$ and $F(p) \cap W_2 \neq \phi$. Since F is preirresolute then Theorem 1 gives, $p \in \tau\text{-int}(\tau\,\text{cl}(F^+(W_1) \cap F^-(W_2)))$. Hence (ii) follows directly.
(ii)→(iii): Follows immediately.

(iii)→(iv): Let B_1, B_2 be any subsets of Y. Then $\text{precl}(B_i)$, $i = 1, 2$ are preclosed. By (iii), $F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2))$ is also preclosed in (X, τ), i.e. $\tau\,\text{cl}(\tau\text{-int}(F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2)))) \subseteq F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2))$. Since $F^+(B_2) \subseteq F^+(\text{precl}(-B_2)$ and $F^-(B_1) \subseteq F^-(\text{precl}(B_1))$ so $\tau\text{-cl}(\tau\text{-int}(F^-(B_1) \cup F^+(B_2))) \subseteq \tau\text{-cl}(\tau\text{-int}(F^-(\text{precl}(B_1)) \cup F^+(\text{precl}(B_2)))) \subseteq F^-(\text{precl}(B_1) \cup F^+(\text{precl}(B_2)$.

(iv)→(v): By the meaning of preclosure of any set as the above lemma shows.

(v)→(vi): From the relation between preclosure (resp. upper) with preinterior (resp. lower) of any set (resp. multifunction).

(vi)→(vii): Let $p \in X$ and $N \in$ pre-$N(F(p))$. Then there exists $W^* \in PO(Y, \sigma)$ such that $F(p) \subseteq W^* \subseteq N$. Since, W is also preopen with $F(p) \cap W \neq \phi$, therefore $p \in F^+(W^*) \cap F^-(W^*) = F^+(\text{preint}(W^*) \cap F^-(\text{preint}(W)) \subseteq \text{preint}(F^+(W^*) \cap F^-(W)) \subseteq \text{preint}\, F^+(N) \cap F^-(W)) \subseteq F^+(N) \cap F^-(W)$. Hence the conclusion follows.

(vii)→(viii): Immediately by taking $N^* = F^+(N) \cap F^-(W)$.

(viii)→(i): It is evidently from the hypothesis. This completes the proof.

THEOREM 3. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$, *the following statements are equivalent*:

(i) *F is upper (resp. lower) preirresolute.*

(ii) $F^+(W)$ *(resp.* $F^-(W)$*)* $\in PO(X, \tau)$, *for each* $W \in PO(Y, \sigma)$.

(iii) *For any* $p \in X$ *and each* $W \in PO(Y, \sigma)$ *such that* $F(p) \subseteq W$ *(resp.* $F(p) \cap W \neq \phi$*), there exists* $H \in PO(X, p)$ *having* $F(H) \subseteq W$ *(resp.* $F(h) \cap W \neq \phi$ *for every* $h \in H$*).*

(iv) $F^-(W)$ *(resp.* $F^+(W)$*) is preclosed, for each* $W \in PO(Y, \sigma)$.

(v) $\tau\text{-cl}(\tau\text{-int}(F^-(W))) \subseteq F^-(W)$ *(resp.* $(\tau\text{-cl}(\tau\text{-int}(F^+(W))) \subseteq F^+(W))$, *for each* $W \in PO(Y, \sigma)$.

Proof. Here, the proof will be given with respect to the lower-preirresolute (the upper one follows similarily).

(i)→(ii), (ii)→(iii), and (iii)→(iv) have been investigated in [11].

(iv)↔(v): Follows directly by preclosedness of $F^+(W)$.

(v)→(i): Take any $p \in X$ and any $W \in PO(Y,\sigma)$ such that $F(p) \cap W \neq \phi$. By hypothesis, $F^-(W)$ is a preopen set containing p. Taking $H = F^-(W)$, the result follows directly and this complete the proof.

THEOREM 4. *The following statements are equivalent for a multifunction* $F : (X,\tau) \to (Y,\sigma)$.

(i) *F is lower preirresolute.*

(ii) $F(\tau\text{-cl}(\tau\text{-int}(H))) \subseteq F(H)$, *for each* $H \in PO(X,\tau)$.

(iii) $F(\text{precl}(H)) \subseteq F(H)$, *for each* $H \in PO(X,\tau)$.

Proof. (i)↔(ii): By the equivallence of (i) and (v) of the above theorem and by considering $W = F(H)$.

(ii)→(iii): Since $\text{precl}(H) = H \cup (\tau\text{-cl}(\tau\text{-int}(H)))$ [1], hence the required hold.

(iii)→(i): Take $p \in X$ and any $W \in PO(Y,\sigma)$ having $F(p) \cap W \neq \phi$. Then $p \in F^-(W)$. By hypothesis, $F(\text{precl}(F^+(Y-W)) \subseteq F(F^+(Y-W)) \subseteq Y - W$. Therefore, $\text{precl}(F^+(Y-W)) \subseteq F^+(Y-W)$. This shows that $(F^+(Y-W))$ is preclosed set in (X,τ) what in turn implies that $F^-(W) \in PO(X,\sigma)$. Putting $H = F^-(W)$, $H \in PO(X,p)$ and $F(h) \cap W \neq \phi$ for every $h \in H$. Hence F is lower preirresolute.

Now we shall present two characterizations of upper and lower-pre-irresoluteness via the preclosure multifunction which is denoted by $(\text{precl}\, F)$: $(X,\tau) \to (Y,\sigma)$ for any multifunction $F : (X,\tau) \to (Y,\sigma)$ and defined as $(\text{precl}\, F)(X) = \text{precl}(F(x))$, for each $x \in X$ [11].

Two next Theorems are formulated in [11], and we offer their proofs.

LEMMA 2 [11]. *Let* $F : (X,\tau) \to (Y,\sigma)$ *be a punctually α-paracompact and punctually $\alpha - p$-regular-multifunction. Then,* $(\text{precl}\, F)^+(W) = F^+(W)$ *for every* $W \in PO(X,\sigma)$.

THEOREM 5. *If* $F : (X,\tau) \to (Y,\sigma)$ *is both punctually α-paracompact and punctually $\alpha - p$-regular-multifunction. Then it is upper-preirresolute iff* $(\text{precl}\, F) : (X,\tau) \to (Y,\sigma)$ *is so.*

Proof. Take any $p \in X$ and $(\text{precl}\, F)(p) \subseteq W \in PO(Y,\sigma)$. By Lemma 2. $p \in F^+(W)$ and the upper-preirresoluteness of F means that, there is $H \in PO(X,p)$ with $F(H) \subseteq W$. But for any $h \in H$ which is both α-paracompact and $\alpha - p$-regular, Lemma (3.2) in [11] gives that there exists $V \in \sigma$ having $\overline{F}(h) \subseteq V \subseteq \sigma\text{-cl}(V) \subseteq W$. Hence for any $h \in H(\text{precl}\, F)(h) \subseteq \text{precl}(V) \subseteq \sigma\text{-cl}(V) \subseteq W$. So, $(\text{precl}\, F)(V) \subseteq W$. Thus $(\text{precl}\, F)$ is upper-preirresolute. Conversely, let $p \in X$ and $W \in PO(X,\tau)$ be such that $F(p) \subseteq W$ i.e. $p \in F^+(W) = (\text{precl}\, F)^+(W)$, as in the above lemma shows.

Hence $(\mathrm{precl}\, F)(p) \subseteq W$. By hypothesis, there exists $H \in PO(X,p)$ with $(\mathrm{precl}\, F)(H) \subseteq W$ and so $F(H) \subseteq W$. Therefore, F is upper-preirresolute.

LEMMA 3 [11]. *For a multifunction $F : (X,\tau) \to (Y,\sigma)$, $(\mathrm{precl}\, F)^-(W) = F^-(W)$, for any $W \in PO(Y,\sigma)$.*

THEOREM 6. *A multifunction $F : (X,\tau) \to (Y,\sigma)$ is lower-preirresolute iff $(\mathrm{precl}\, F) : (X,\tau) \to (Y,\sigma)$ is so.*

Proof. Necessity. Let $p \in X$ and $W \in PO(Y,\sigma)$ be such that $(\mathrm{precl}\, F)(p) \cap W \neq \phi$ i.e. $p \in (\mathrm{precl}\, F)^-(W)$. Lemma 3 shows that $p \in F^-(W)$ and this means that $F(p) \cap W \neq \phi$. Lower-preirresoluteness of F implies that there exists $H \in PO(X,p)$ having $F(h) \cap W \neq \phi$, for every $h \in H$. So, $(\mathrm{precl}\, F)(h) \cap W \neq \phi$ for every $h \in H$. Hence, $(\mathrm{precl}\, F)$ is lower-preirresolute. Sufficiency. Take any $p \in X$ and $W \in PO(Y,\sigma)$ with $F(p) \cap W \neq \phi$, therefore $p \in (\mathrm{precl}\, F)^-(W)$ (see above lemma), i.e. $(\mathrm{precl}\, F)(p) \cap W \neq \phi$. By hypothesis of $(\mathrm{precl}\, F)$ there exists $H \in PO(X,p)$ such that for every $h \in H$, $(\mathrm{precl}(H)) \cap W \neq \phi$ for every $h \in H$. Hence F is a lower-preirresolute.

4. Other properties

THEOREM 7. *The composition of two lower (resp. upper) preirresolute multifunctions is so as well.*

Proof. Let $F : (X,\tau) \to (Y,\sigma)$ and $F^*(Y,\sigma) \to (Z,\delta)$ be two lower-preirresolute multifunctions and $W \in PO(Z,\delta)$. Observe that $(F^* \circ F)(W) = F(F^*(W)) \in PO(X,\tau)$ (see Theorem (3.4) in [11]). Hence, $(F^* \circ F)$ is lower-preirresolute. The upper-one is satisfies by a similar argument.

These equivalencies throughout the previous theorem help to establish the following connection between three types of multifunctions.

THEOREM 8. *An upper-preirresolute multifunction which is strongly lower-semicontinuous is preirresolute.*

Proof. Let $F : (X,\tau) \to (Y,\sigma)$ be multifunction with hypothesis above, $p \in X$ and $W_1, W_2 \in PO(Y,\sigma)$ with $p \in F^+(W_1)$. Upper-preirresoluteness of F means that there exists $H \in PO(X,p)$ and $F(H) \subseteq W_1$, such that $p \in H \subseteq \tau\text{-int}(\tau\text{-cl}(H)) \subseteq \tau\text{-int}(\tau\text{-cl}(F^+(W_1)))$. As was chosen arbitrary in $F^+(W_1)$, then it follows that $F^+(W_1) \subseteq \tau\text{-int}(\tau\text{-cl}(F^+(W_1)))$ i.e. $F^+(W_1) \in PO(X,\tau)$. Since F is strongly lower-semicontinuous, then $F^-(W_2) \in \tau$ and therefore $F^+(W_1) \cap F^-(W_2)$ is preopen in (X,τ). Hence F is preirresolute.

The next results may be considered as a strengthening one direction of Theorems (3.7) and (3.8) in [11].

LEMMA 4 [8]. *If $W \in PO(X,\tau)$ and $V \in SO(X,\tau)$, then $W \cap V \in PO(V,\tau/V)$.*

THEOREM 9. *If $\{H_i : i \in I\}$ is a semi-open cover of X and a multifunction $F : (X,\tau) \to (Y,\sigma)$ is upper (resp. lower) preirresolute. Then for each $i \in I$ the restriction multifunctions $F/H_i : (H_i, \tau/H_i) \to (Y,\sigma)$ are also upper (resp. lower) preirresolute.*

Proof. Let $p \in H_i \in SO(X,\tau_i)$ for any $i \in I$ and let $W \in PO(Y,\sigma)$ be such that $(F/H_i)(p) \subseteq W$. The upper preirresoluteness of F implies that there exists $G \in PO(X,p)$ such that $F(G) \subseteq W$. Put $H = G \cap H_i \in PO(H_i,p)$ (see above lemma). Then $(F/H_i)(H) = F(H) \subseteq W$ and hence for each $i \in I$, F/H_i is upper-preirresolute. While the proof of the second part follows analogously as in the first one, taking in the consideration the meaning of the lower-preirresoluteness.

The following theorem is an improvement in the second direction of Theorems (3.7) and (3.8) in [11].

LEMMA 5 [2]. *In (X,τ), if $V \subseteq G \in PO(X,\tau)$ such that $V \in PO(G,\tau/G)$. Then $V \in PO(X,\tau)$.*

THEOREM 10. *Assume that $X = \bigcup_{i\in I} G_i$, $G_i \in PO(X,\tau)$ for each $i \in I$ and a multifunction $F : (X,\tau) \to (Y,\sigma)$ is such that the restriction $F/G_i : (G_i, \tau/G_i) \to (Y,\sigma)$, $i \in I$ are upper (resp. lower) preirresolute. Then F is upper (resp. lower) preirresolute.*

Proof. We will present the proof for the lower-case, while for the upper-one it follows similarily. Let $p \in X$ and $W \in PO(X,\tau)$ be such that $F(p) \cap W \neq \phi$. Then there is $i \in I$, such that $p \in G_i$. Since $F(p) = (F/G_i)(p)$, this gives $(F/G_i)(p) \cap W \neq \phi$. By lower-preirresoluteness of F/G_i, there exists $H \in PO(G_i,p)$ having $(F/G_i)(h) \cap W \neq \phi$ for each $h \in H$. The above lemma shows that $H \in PO(X,\tau)$ and so $F(h) \cap W \neq \phi$ for every $h \in H$. Hence F is lower-preirresolute.

As a consequence of Theorems 9 and 10 the following equivalence will be established.

THEOREM 11. *Assume that $\{W_i : i \in I\}$ is a cover of X by a α-sets. Then a multifunction $F : (X,\tau) \to (Y,\sigma)$ is upper (resp. lower) preirresolute iff the restriction multifunction $F/W_1 : (W_i, \tau/W_i) \to (Y,\sigma)$ is, for each $i \in I$ also upper (resp. lower) preirresolute.*

Proof. It follows by above Theorems 9 and 10 and the fact that $\tau^\alpha = SO(X,\tau) \cap PO(X,\tau)$ due to Reilly and Vamanamurthy [13].

LEMMA 6 [4]. *Let H be an open subset of a space (X,τ) and let $S \subseteq H$. Then $S \in PO(X,\tau)$ iff $S \in PO(H,\tau/H)$.*

The next result is considered as a corollary of the above theorem and its proof is clear by the previously Lemma 6.

COROLLARY 1. *If* $\{\bigcup_i : i \in I\}$ *is an open cover of* X. *A multifunction* $F : (X, \tau) \to (Y, \sigma)$ *is upper (resp. lower) preirresolute iff for each* $i \in I$ $F/\bigcup_i : (\bigcup_i, \tau/\bigcup_i) \to (Y, \sigma)$ *is upper (resp. lower) preirresolute.*

References

[1] D. Andrijevic, *On the topology generated by preopen sets*, Math. Vesnik, 39 (1987), 367–376.

[2] R. H. Atia, S. N. El-Deeb and I. A. Hasanein, *A note on strong compactness and S-closedness*, Mat. Vesnik 6 (19) (34) (1982), 23–28.

[3] S. N. El-Deeb and I. A. Hasanein, A. S. Mashhour and T. Noiri, *On P-regular spaces*, Bull. Math. Soc. Sci. Math. R. S. Romanie 3 (1982).

[4] M. Ganster, *Preopen sets and resolvabl spaces*, Kyungpook. Math. J. 24 (2) (1987), 135–143.

[5] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly 70 (1963), 36–41.

[6] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt 53 (1982), 47–53.

[7] A. S. Mashhour, M. E. Abd El-Monsef, I. A. Hasanein and T. Noiri, *Strongly compact spaces*, Delta J. Sci. 8(1) (1984), 30–46.

[8] A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, *A note on semi-continuity and precontinuity*, Indian J. Pure and Appl. Math. 13 (10) (1982).

[9] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math. 15 (1965), 961–970.

[10] V. Popa, *Strongly continuous multifunctions* (Romanian), Bull. Saint. Tehn. Inst. Politehn. "Traian Vuai" Timisoara 27 (41) (1982), 5–7.

[11] V. Popa, Y. Küçük and T. Noiri, *On upper and lower preirresolute multifunctions*, To appear in Pure and Appl. Math. Vol. (46).

[12] V. Popa, T. Noiri, *Some properties of irresolute multifunctions*, Matemat. Vesnik 43 (1991), 11–17.

[13] I. L. Reilly, M. K. Vamanamurthy, *On α-continuity in topological spaces*, Acta Math. Hung. 45 (1–2) (1985), 27–32.

[14] J. D. Wine, *Locally paracompact spaces*, Glasnik Math. 10 (30) (1975), 351–357.

DEPARTAMENT OF MATHEMATICS
FACULTY OF SCIENCE
MENOUFIA UNIVERSITY, EGYPT

Received December 29, 1997; revised version March 8, 1999.