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OSCILLATION FOR SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS WITH FUNCTIONAL ARGUMENTS

Abstract. Sufficient conditions are established for the oscillations of systems of partial differential equations of the form (1).

1. Introduction

Recently, the oscillation problem for the partial functional differential equation has been studied by many authors. We refer the reader to [1]–[3] for parabolic equations and to [4]–[7] for hyperbolic equations. But only [8], [9] studied the oscillation of systems of partial functional differential equations.

In this paper, we study the oscillation of systems of partial differential equations with functional arguments of the form

$$(1)\quad \frac{\partial}{\partial t}\Big(p(t)\frac{\partial}{\partial t}u_i(x,t)\Big) = a_i(t)\Delta u_i(x,t) + \sum_{k=1}^{s} a_{ik}(t)\Delta u_i(x,\rho_k(t)) - q_i(x,t)u_i(x,t) - \sum_{j=1}^{m}\sum_{h=1}^{l} q_{ijh}(x,t)u_j(x,\sigma_h(t)),$$
$$(x,t)\in\Omega\times[0,\infty)\equiv G,\quad i=1,2,\dots,m,$$

where Ω is a bounded domain in R^n with a piecewise smooth boundary $\partial\Omega$, and $\Delta u_i(x,t)=\sum_{r=1}^{n}\frac{\partial^2 u_i(x,t)}{\partial x_r^2}$, $i=1,2,\dots,m$.

Key words and phrases: oscillation, system, partial differential equation, functional argument.

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Suppose that the following conditions hold:

(A1) $p \in C^1([0,\infty);[0,\infty)), \lim_{t\to\infty}\int_{t_0}^{t}\frac{1}{p(s)}ds = +\infty, t_0 > 0$;

(A2) $q_i \in C(\overline{G};[0,\infty)), q_i(t) = \min_{x\in\overline{\Omega}} q_i(x,t), q(t) = \min_{1\le i\le m} q_i(t)$, $i \in I_m = \{1,2,\ldots,m\}$;

(A3) $q_{ijh} \in C(\overline{G};R)$, $q_{iih}(x,t) > 0$, $q_{iih}(t) = \min_{x\in\overline{\Omega}} q_{iih}(x,t)$, and

$$\overline{q}_{ijh}(t) = \max_{x\in\overline{\Omega}} |q_{ijh}(x,t)|, Q_h(t) = \min_{1\le i\le m}\left\{q_{iih}(t) - \sum_{j=1,j\ne i}^{m} \overline{q}_{jih}(t)\right\} \ge 0,$$

$$i,j \in I_m, h \in I_l = \{1,2,\ldots,l\};$$

(A4) $a_i, a_{ik} \in C([0,\infty);[0,\infty))$, $i \in I_m$, $k \in I_s = \{1,2,\ldots,s\}$;

(A5) $\sigma_j, \rho_k \in C([0,\infty);R)$, $\sigma_j(t) \le t$, $\rho_k(t) \le t$, the functions σ_j, ρ_k are nondecreasing and $\lim_{t\to\infty}\sigma_j(t) = \lim_{t\to\infty}\rho_k(t) = \infty, j \in I_m, k \in I_s$.

We consider two kinds of boundary conditions:

$$\frac{\partial u_i(x,t)}{\partial N} + g_i(x,t)u_i(x,t) = 0, (x,t) \in \partial\Omega \times [0,\infty), \quad i \in I_m, \tag{2}$$

where N is the unit exterior normal vector to $\partial\Omega$ and $g_i(x,t)$ is a nonnegative continuous function on $\partial\Omega \times [0,\infty), i \in I_m$, and

$$u_i(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,\infty), \quad i \in I_m. \tag{3}$$

DEFINITION 1.1. The vector function $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ is said to be a solution of the problem (1), (2) (or (1), (3)) if it satisfies (1) in $G = \Omega \times [0,\infty)$ and boundary condition (2) (or (3)).

DEFINITION 1.2. A nontrivial component $u_i(x,t)$ of the vector function $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ is said to oscillate in $\Omega\times[\mu_0,\infty)$ if for each $\mu > \mu_0$ there is a point $(x_0,t_0) \in \Omega\times[\mu,\infty)$ such that $u_i(x_0,t_0) = 0$.

DEFINITION 1.3. The vector solution $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ of the problem (1), (2) (or (1), (3)) is said to be oscillatory in the domain $G = \Omega \times [0,\infty)$ if at least one of its nontrivial components is oscillatory in G. Otherewise, the vector solution $u(x,t)$ is said to be nonoscillatory.

We note that a particular case of system (1) with $p(t) = 1, l = 1$ has been studied in [8].

2. Oscillation of the problem (1), (2)

THEOREM 2.1. *If there exists some $h_0 \in I_l$ such that $\sigma'_{h_0}(t) \ge 0$, and*

$$\int_{t_0}^{\infty} Q_{h_0}(t)dt = \infty, \qquad t_0 > 0, \tag{4}$$

then every solution $u(x,t)$ of problem (1), (2) is oscillatory in G.

Proof. Suppose to the contrary that there is a nonoscillatory solution $u(x,t) = \{u_1(x,t), u_2(x,t), \dots, u_m(x,t)\}^T$ of the problem (1), (2). We assume that $|u_i(x,t)| > 0$ for $t \geq t_0 \geq 0$, $i \in I_m$. Let $\delta_i = \operatorname{sgn} u_i(x,t)$, $Z_i(x,t) = \delta_i u_i(x,t)$, then $Z_i(x,t) > 0, (x,t) \in \Omega \times [t_0, \infty)$, $i \in I_m$. From (A5) there exists a number $t_1 \geq t_0$ such that $Z_i(x,t) > 0$, $Z_i(x, \rho_k(t)) > 0$ and $Z_i(x, \sigma_h(t)) > 0$ in $\Omega \times [t_1, \infty)$, $i \in I_m$, $k \in I_s$.

Integrating (1) with respect to x over the domain Ω, we have

$$\begin{aligned}(5)\quad &\frac{d}{dt}\left(p(t)\frac{d}{dt}\int_\Omega u_i(x,t)dx\right)\\ &\quad = a_i(t)\int_\Omega \Delta u_i(x,t)dx + \sum_{k=1}^{s} a_{ik}(t)\int_\Omega \Delta u_i(x,\rho_k(t))dx\\ &\quad - \int_\Omega q_i(x,t)u_i(x,t)dx - \sum_{j=1}^{m}\sum_{h=1}^{l}\int_\Omega q_{ijh}(x,t)u_j(x,\sigma_h(t))dx,\\ &\qquad\qquad t \geq t_1,\ i \in I_m.\end{aligned}$$

Therefore,

$$\begin{aligned}(6)\quad &\frac{d}{dt}\left(p(t)\frac{d}{dt}\int_\Omega Z_i(x,t)dx\right)\\ &= a_i(t)\int_\Omega \Delta Z_i(x,t)dx + \sum_{k=1}^{s} a_{ik}(t)\int_\Omega \Delta Z_i(x,\rho_k(t))dx - \int_\Omega q_i(x,t)Z_i(x,t)dx\\ &- \frac{\delta_j}{\delta_i}\sum_{j=1}^{m}\sum_{h=1}^{l}\int_\Omega q_{ijh}(x,t)Z_j(x,\sigma_h(t))dx, \qquad t \geq t_1, i \in I_m.\end{aligned}$$

From Green's formula and boundary condition (2), it follows that

$$(7)\quad \int_\Omega \Delta Z_i(x,t)dx = \int_{\partial\Omega} \frac{\partial Z_i(x,t)}{\partial N}dS = -\int_{\partial\Omega} g_i(x,t)Z_i(x,t)dS \leq 0,$$

$$\begin{aligned}(8)\quad &\int_\Omega \Delta Z_i(x,\rho_k(t))dx = \int_{\partial\Omega} \frac{\partial Z_i(x,\rho_k(t))}{\partial N}dS\\ &= -\int_{\partial\Omega} g_i(x,\rho_k(t))Z_i(x,\rho_k(t))dS \leq 0, \quad t \geq t_1, \quad i \in I_m, \quad k \in I_s,\end{aligned}$$

where dS is the surface element on $\partial\Omega$.

Combining (6)–(8), we get

$$\frac{d}{dt}\left(p(t)\frac{d}{dt}\int_\Omega Z_i(x,t)dx\right) \tag{9}$$
$$\leq -q_i(t)\int_\Omega Z_i(x,t)dx - \sum_{h=1}^{l} q_{iih}(t)\int_\Omega Z_i(x,\sigma_h(t))dx$$
$$+\sum_{h=1}^{l}\sum_{j=1,j\neq i}^{m} \overline{q}_{ijh}(t)\int_\Omega Z_j(x,\sigma_h(t))dx, \quad t\geq t_1, i\in I_m.$$

Set $V_i(t) = \int_\Omega Z_i(x,t)dx$, $t \geq t_1$, $i \in I_m$. From (9) we have

$$[p(t)V_i'(t)]' + q_i(t)V_i(t) + \sum_{h=1}^{l}[q_{iih}(t)V_i(\sigma_h(t)) \tag{10}$$
$$- \sum_{j=1,j\neq i}^{m} \overline{q}_{ijh}(t)V_j(\sigma_h(t))] \leq 0, \quad t \geq t_1,\ i\in I_m.$$

Let $V(t) = \sum_{i=1}^{m} V_i(t)$, $t \geq t_1$. Then, from (10) we have

$$[p(t)V'(t)]' + q(t)V(t) \tag{11}$$
$$+\sum_{h=1}^{l}\left\{\sum_{i=1}^{m}[q_{iih}(t)V_i(\sigma_h(t)) - \sum_{j=1,j\neq i}^{m}\overline{q}_{ijh}(t)V_j(\sigma_h(t))]\right\} \leq 0,\ t\geq t_1.$$

Noting that

$$\sum_{i=1}^{m}[q_{iih}(t)V_i(\sigma_h(t)) - \sum_{j=1,j\neq i}^{m}\overline{q}_{ijh}(t)V_j(\sigma_h(t))]$$
$$= [q_{11h}(t)V_1(\sigma_h(t)) - \sum_{j=1,j\neq 1}^{m}\overline{q}_{1jh}(t)V_j(\sigma_h(t))]$$
$$+ [q_{22h}(t)V_2(\sigma_h(t)) - \sum_{j=1,j\neq 2}^{m}\overline{q}_{2jh}(t)V_j(\sigma_h(t))] + \cdots\cdots$$
$$+ [q_{mmh}(t)V_m(\sigma_h(t)) - \sum_{j=1,j\neq m}^{m}\overline{q}_{mjh}(t)V_j(\sigma_h(t))]$$
$$= [q_{11h}(t) - \sum_{j=1,j\neq 1}^{m}\overline{q}_{j1h}(t)]V_1(\sigma_h(t))$$

$$+ [q_{22h}(t) - \sum_{j=1, j\neq 2}^{m} \overline{q}_{j2h}(t)] V_2(\sigma_h(t)) + \ldots\ldots$$

$$+ [q_{mmh}(t) - \sum_{j=1, j\neq m}^{m} \overline{q}_{jmh}(t)] V_m(\sigma_h(t))$$

$$\geq \min_{1\leq i\leq m} \left\{ q_{iih}(t) - \sum_{j=1, j\neq i}^{m} \overline{q}_{jih}(t) \right\} \sum_{i=1}^{m} V_i(\sigma_h(t))$$

$$= Q_h(t) V(\sigma_h(t)), \quad t \geq t_1, \quad h \in I_l.$$

From (11) we get

$$(12) \qquad [p(t)V'(t)]' + q(t)V(t) + \sum_{h=1}^{l} Q_h(t)V(\sigma_h(t)) \leq 0, \quad t \geq t_1.$$

The inequality (12) shows that $[p(t)V'(t)]' < 0$ for $t \geq t_1$. Hence $p(t)V'(t)$ is a decreasing function in the interval $[t_1, \infty)$. We can claim that $V'(t) > 0$ for $t \geq t_1$. In fact, if $V'(t) \leq 0$ for $t \geq t_1$, then there exists a $T > t_1$ such that $p(T)V'(T) < 0$. This implies that

$$V'(t) \leq \frac{p(T)V'(T)}{p(t)} \text{ for } t \geq T.$$

Hence

$$V(t) - V(T) \leq p(T)V'(T) \int_T^t \frac{ds}{p(s)}, \quad t \geq T.$$

Therefore,

$$\lim_{t\to\infty} V(t) = -\infty,$$

which contradicts the fact that $V(t) = \sum_{i=1}^{m} V_i(t) > 0$.

From (12) we obtain that there exists some $h_0 \in I_l$ such that

$$(13) \qquad [p(t)V'(t)]' + Q_{h_0}(t)V(\sigma_{h_0}(t)) \leq 0, \quad t \geq t_1.$$

Integrating the inequality (13), we have

$$(14) \qquad p(t)V'(t) - p(t_1)V'(t_1) + \int_{t_1}^{t} Q_{h_0}(s)V(\sigma_{h_0}(s))ds \leq 0, \quad t \geq t_1.$$

Then we obtain

$$(15) \qquad \int_{t_1}^{t} Q_{h_0}(s)V(\sigma_{h_0}(s))ds \leq -p(t)V'(t) + p(t_1)V'(t_1), \quad t \geq t_1.$$

Hence

$$(16)\quad \int_{t_1}^{t} Q_{h_0}(s)ds$$

$$\leq \frac{1}{V(\sigma_{h_0}(t_1))}[-p(t)V'(t) + p(t_1)V'(t_1)] \leq \frac{p(t_1)V'(t_1)}{V(\sigma_{h_0}(t_1))}, \quad t \geq t_1,$$

which contradicts the condition (4).

This completes the proof of Theorem 2.1.

THEOREM 2.2. *If*

$$(17)\qquad \int^{\infty} q(t)dt = \infty$$

then every solution $u(x,t)$ *of the problem* (1), (2) *oscillates in* G.

Proof. As in the proof of Theorem 2.1, we obtain (12). Therefore,

$$(18)\qquad [p(t)V'(t)]' + q(t)V(t) \leq 0, \quad t \geq t_1.$$

The remainder of the proof is similar to that of Theorem 2.1 and we omit it.

DEFINITION 2.1. The solution $V(t)$ of the differential inequality (12) is called eventually positive if there exists a number $\mu \geq t_1$ such that $V(t) > 0$ holds for all $t \geq \mu$.

COROLLARY 2.1. *If the inequality* (12) *has no eventually positive solution, then every solution* $u(x,t)$ *of the problem* (1), (2) *is oscillatory in* G.

3. Oscillation of the problem (1), (3)

The following fact will be used. The smallest eigenvalue α_0 of the Dirichlet problem

$$\begin{cases} \Delta\omega(x) + \alpha\omega(x) = 0 \text{ in } \Omega, \\ \omega(x) = 0 \text{ on } \partial\Omega, \end{cases}$$

where α is a constant, is positive and the corresponding eigenfunction $\varphi(x)$ is positive in Ω.

THEOREM 3.1. *If all assumptions of Theorem* 2.1 *hold, then every solution of the problem* (1), (3) *is oscillatory in* G.

Proof. Suppose to the contrary that there is a nonoscillatory solution $u(x,t) = \{u_1(x,t), u_2(x,t), \ldots, u_m(x,t)\}^T$ of the problem (1), (3). We assume that $|u_i(x,t)| > 0$ for $t \geq t_0 \geq 0$, $i \in I_m$. Let $\delta_i = \operatorname{sgn} u_i(x,t)$, $Z_i(x,t) = \delta_i u_i(x,t)$, then $Z_i(x,t) > 0$, $(x,t) \in \Omega \times [t_0,\infty)$, $i \in I_m$. From (A5) there exists a number $t_1 \geq t_0$ such that $Z_i(x,t) > 0, Z_i(x,\rho_k(t)) > 0$ and $Z_i(x,\sigma_h(t)) > 0$ in $\Omega \times [t_1,\infty)$, $i \in I_m$, $k \in I_s$.

Multiplying both sides of (1) by $\varphi(x) > 0$ and integrating with respect to x over the domain Ω, we have

$$
\begin{aligned}
(19)\quad & \frac{d}{dt}(p(t)\frac{d}{dt}\int_\Omega u_i(x,t)\varphi(x)dx) \\
&= a_i(t)\int_\Omega \Delta u_i(x,t)\varphi(x)dx \\
&\quad + \sum_{k=1}^{s} a_{ik}(t)\int_\Omega \Delta u_i(x,\rho_k(t))\varphi(x)dx - \int_\Omega q_i(x,t)u_i(x,t)\varphi(x)dx \\
&\quad - \sum_{j=1}^{m}\sum_{h=1}^{l}\int_\Omega q_{ijh}(x,t)u_j(x,\sigma_h(t))\varphi(x)dx, \quad t \geq t_1,\ i \in I_m.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
(20)\quad & \frac{d}{dt}\left(p(t)\frac{d}{dt}\int_\Omega Z_i(x,t)\varphi(x)dx\right) \\
&= a_i(t)\int_\Omega \Delta Z_i(x,t)\varphi(x)dx \\
&\quad + \sum_{k=1}^{s} a_{ik}(t)\int_\Omega \Delta Z_i(x,\rho_k(t))\varphi(x)dx - \int_\Omega q_i(x,t)Z_i(x,t)\varphi(x)dx \\
&\quad - \frac{\delta_j}{\delta_i}\sum_{j=1}^{m}\sum_{h=1}^{l}\int_\Omega q_{ijh}(x,t)Z_j(x,\sigma_h(t))\varphi(x)dx, \quad t \geq t_1,\ i \in I_m.
\end{aligned}
$$

Green's formula and boundary condition (3) yield

$$
(21)\quad \int_\Omega \Delta Z_i(x,t)\varphi(x)dx = \int_\Omega Z_i(x,t)\Delta\varphi(x)dx = -\alpha_0\int_\Omega Z_i(x,t)\varphi(x)dx \leq 0
$$

and

$$
\begin{aligned}
(22)\quad & \int_\Omega \Delta Z_i(x,\rho_k(t))\varphi(x)dx = \int_\Omega Z_i(x,\rho_k(t))\Delta\varphi(x)dx \\
&\qquad = -\alpha_0\int_\Omega Z_i(x,\rho_k(t))\varphi(x)dx \leq 0, \quad t \geq t_1,\ i \in I_m,\ k \in I_s.
\end{aligned}
$$

Then, we have

$$\begin{aligned}(23)\quad & \frac{d}{dt}\left(p(t)\frac{d}{dt}\int_\Omega Z_i(x,t)\varphi(x)dx\right)\\ &\le -q_i(t)\int_\Omega Z_i(x,t)\varphi(x)dx\\ &\quad -\sum_{h=1}^{l} q_{iih}(t)\int_\Omega Z_i(x,\sigma_h(t))\varphi(x)dx\\ &\quad +\sum_{h=1}^{l}\sum_{j=1,j\ne i}^{m}\overline{q}_{ijh}(t)\int_\Omega Z_j(x,\sigma_h(t))\varphi(x)dx, \qquad t\ge t_1,\ i\in I_m.\end{aligned}$$

Set $V_i(t)=\int_\Omega Z_i(x,t)\varphi(x)dx,\ t\ge t_1, i\in I_m$. Then, from (23) we have

$$\begin{aligned}(24)\quad & [p(t)V_i'(t)]' + q_i(t)V_i(t) + \sum_{h=1}^{l}[q_{iih}(t)V_i(\sigma_h(t))\\ &\qquad - \sum_{j=1,j\ne i}^{m}\overline{q}_{ijh}(t)V_j(\sigma_h(t))] \le 0, \qquad t\ge t_1,\ i\in I_m.\end{aligned}$$

Let $V(t)=\sum_{i=1}^m V_i(t), t\ge t_1$. From (24) we have

$$\begin{aligned}(25)\quad & [p(t)V'(t)]' + q(t)V(t)\\ &+\sum_{h=1}^{l}\left\{\sum_{i=1}^{m}[q_{iih}(t)V_i(\sigma_h(t)) - \sum_{j=1,j\ne i}^{m}\overline{q}_{ijh}(t)V_j(\sigma_h(t))]\right\}\le 0, \qquad t\ge t_1.\end{aligned}$$

As in the proof of Theorem 2.1, from (25) we obtain

$$(26)\qquad [p(t)V'(t)]' + q(t)V(t) + \sum_{h=1}^{l} Q_h(t)V(\sigma_h(t)) \le 0, \qquad t\ge t_1.$$

The remainder of the proof is similar to that of Theorem 2.1 and we omit it.

COROLLARY 3.1. *If the differential inequality* (26) *has no eventually positive solution, then every solution* $u(x,t)$ *of the problem* (1), (3) *oscillates in* G.

It is not difficult to see that the following theorem is true.

THEOREM 3.2. *If the assumption of Theorem* 2.2 *hold, then every solution* $u(x,t)$ *of the problem* (1), (3) *is oscillatory in* G.

4. Examples

EXAMPLE 4.1. Consider the system of two partial differential equations

$$(27)\quad \begin{cases} \frac{\partial}{\partial t}\left(t\frac{\partial}{\partial t}u_1(x,t)\right) = t\Delta u_1(x,t) + (2+t)\Delta u_1(x,t-\frac{3\pi}{2}) - u_1(x,t) \\ -3u_1(x,t-\pi) - u_2(x,t-\pi) - (2+t)u_1(x,t-\frac{\pi}{2}) - 2u_2(x,t-\frac{\pi}{2}), \\ \frac{\partial}{\partial t}\left(t\frac{\partial}{\partial t}u_2(x,t)\right) = t\Delta u_2(x,t) + \Delta u_2(x,t-\frac{3\pi}{2}) - 3u_2(x,t) \\ -u_1(x,t-\pi) - 2u_2(x,t-\pi) - u_1(x,t-\frac{\pi}{2}) - 3u_2(x,t-\frac{\pi}{2}), \\ \qquad (x,t) \in (0,\pi)\times[0,\infty), \end{cases}$$

with boundary conditions

$$(28)\qquad \frac{\partial}{\partial x}u_i(0,t) = \frac{\partial}{\partial x}u_i(\pi,t) = 0, \quad t \geq 0, \quad i = 1,2.$$

Here $n = 1$, $m = 2$, $s = 1$, $l = 2$, $p(t) = t$, $a_1(t) = t$, $a_{11}(t) = 2+t$, $\rho_1(t) = t - \frac{3\pi}{2}$, $q_1(x,t) = 1$, $q_{111}(x,t) = 3$, $q_{121}(t) = 1$, $\sigma_1(t) = t-\pi$, $q_{112}(x,t) = 2+t$, $q_{122}(x,t) = 2$, $\sigma_2(t) = t - \frac{\pi}{2}$, $a_2(t) = t$, $a_{21}(t) = 1$, $q_2(x,t) = 3$, $q_{211}(x,t) = 1$, $q_{221}(x,t) = 2$, $q_{212}(x,t) = 1$, $q_{222}(x,t) = 3$. It is easy to see that $Q_1(t) = 1, Q_2(t) = 1$, and

$$\lim_{t\to\infty}\int_{t_0}^{t}\frac{1}{p(s)}ds = \lim_{t\to\infty}\int_{t_0}^{t}\frac{1}{s}ds = +\infty,$$

$$\sigma_1'(t) = (t-\pi)' = 1 \geq 0, \qquad \int_{t_0}^{\infty} Q_1(t)dt = \int_{t_0}^{\infty} dt = \infty, \qquad t_0 > 0.$$

Hence all conditions of Theorem 2.1 are fulfilled. Then every solution of the problem (27), (28) oscillates in $(0,\pi)\times[0,\infty)$. In fact, $u_1(x,t) = \cos x \sin t, u_2(x,t) = \cos x \cos t$ is such a solution.

EXAMPLE 4.2. Consider the system of two partial differential equations

$$(29)\quad \begin{cases} \frac{\partial}{\partial t}\left(e^{-t}\frac{\partial}{\partial t}u_1(x,t)\right) = e^{-t}\Delta u_1(x,t) + 2e^{-t}\Delta u_1(x,t-\frac{3\pi}{2}) - 4u_1(x,t) \\ -2u_1(x,t-\pi) - u_2(x,t-\pi) - (1+e^{-t})u_1(x,t-\frac{\pi}{2}) - 2u_2(x,t-\frac{\pi}{2}), \\ \frac{\partial}{\partial t}\left(e^{-t}\frac{\partial}{\partial t}u_2(x,t)\right) = (2+\frac{1}{2}e^{-t})\Delta u_2(x,t) + (4+e^{-t})\Delta u_2(x,t-\frac{3\pi}{2}) \\ -\frac{1}{6}e^{-t}u_2(x,t) - u_1(x,t-\pi) - 2u_2(x,t-\pi) \\ -\frac{1}{3}e^{-t}u_1(x,t-\frac{\pi}{2}) - 3u_2(x,t-\frac{\pi}{2}), \qquad (x,t)\in(0,\pi)\times[0,\infty), \end{cases}$$

with boundary conditions

$$(30)\qquad u_i(0,t) = u_i(\pi,t) = 0, \quad t \geq 0,\ i = 1,2.$$

It is easy to see that all conditions of Theorem 3.1 are fulfilled. Then every solution of the problem (29), (30) oscillates in $(0,\pi)\times[0,\infty)$. In fact, $u_1(x,t)=\sin x\cos t, u_2(x,t)=\sin x\sin t$ is such a solution.

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