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# ON THE DISTANCE BETWEEN ADJACENT ZEROES OF SOLUTIONS OF FIRST ORDER NEUTRAL DIFFERENTIAL EQUATIONS

## 1. Introduction

Consider the first order neutral delay differential equation

$$[x(t) + P(t)x(t-\tau)]' + Q(t)x(t-\sigma) = 0 \tag{1}$$

where $P, Q \in C([t_0, \infty), R^+)$, and $\tau, \sigma \in R^+$.

When $P(t) = 0$, Eq.(1) reduces to

$$x'(t) + Q(t)x(t-\sigma) = 0. \tag{2}$$

The oscillation theory of neutral differential equations has been extensively developed during the past several years. We refer to Bainov and Misher [1], Gyori and Ladas [3], and the references cited therein. But the results dealing with the distribution of zeroes of the oscillatory solution of neutral differential equation are relatively scarce. Recently, Domshlak and Stavroulakis [2] obtained estimates for the intervals length successive zeroes of solutions of Eq.(2), Liang [4] and Li [5] established estimates for the distance between adjacent zeroes of the solutions of Eq. (2). Lin [6] and Zhou [7] extend the results in [4]–[5] to Eq.(1). In this paper we establish a new estimate for the distance between adjacent zeroes of the solutions of Eq.(1). Our results improve the all results in [4]–[7].

Let $m = \max\{\tau, \sigma\}$. By a solution of Eq.(1) we mean a function $x \in C([t_x - m, \infty), R)$ , for some $t_x \geq t_0$, such that $x(t) + P(t)x(t-\tau)$ is continuously differentiable on $[t_x, \infty)$ and such that Eq.(1) is satisfied for $t \geq t_x$.

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Assume that (2) holds and let $\Phi \in C([t_0 - m, t_0], R)$ be a given initial function. Then one can easily see by the method of steps that Eq.(1) has a unique solution $x \in C([t_0 - m, \infty), R)$ such that $x(t) = \Phi(t)$ for $t_0 - m \le t \le t_0$.

## 2. Main results

First we define a sequence $\{a_i\}$ by

$$a_1 = e^{\varrho}, a_{i+1} = e^{\varrho a_i}, i = 1, 2, \dots \tag{3}$$

It is easily seen that $\{a_i\}$ is increasing for $\varrho > 0$.

Observe that when $\varrho > \frac{1}{e}$ then

$$\lim_{i \to \infty} a_i = +\infty,$$

because otherwise the sequence $\{a_i\}$ would have a finite limit $a$, such that

$$a = e^{\varrho a}.$$

Using the known inequality

$$e^x \ge ex,$$

we have

$$a = e^{\varrho a} \ge e\varrho a > a$$

which is a contradiction.

When $\frac{1}{e} < \varrho < 1$, we also define a sequence $\{b_j\}$ by

$$b_1 = \frac{2(1-\varrho)}{\varrho^2}, \quad b_{j+1} = \frac{2(1-\varrho)}{\varrho^2 + \frac{2}{b_j^2}}, \quad j = 1, 2, \dots \tag{4}$$

Observe that the sequence $\{b_j\}$ is decreasing for $\frac{1}{e} < \varrho < 1$.

For the sake of convenience, we set

$$R(t) = P(t-\sigma)\frac{Q(t)}{Q(t-\tau)}.$$

In the following, $D(x)$ denotes distance between adjacent zeros of the solution $x(t)$ of Eq.(1). Our main result is the following theorem.

THEOREM 1. *Assume that*

(A) $P \in C([t_0, \infty), [0, \infty))$, $Q \in C([t_0, \infty), (0, \infty))$, $R \in C^1([t_0, \infty), [0, \infty))$, $R'(t) \le 0$, $\sigma > \tau > 0$;

(B) *There exist $t_1 (t_1 \ge t_0)$ and positive constant $\varrho$ such that*

$$\int_{t+\tau-\sigma}^{t} \frac{Q(s)}{1 + R(s+\tau-\sigma)} ds \ge \varrho > \frac{1}{e} \quad \text{for } t \ge t_1.$$

*Then*

$$D(x) < 2\sigma + n_\varrho(\sigma - \tau), \tag{5}$$

*where*

$$n_\varrho = \begin{cases} 1, & \text{when } \varrho \geq 1, \\ \min_{i\geq 1, j\geq 1}\{i + j | a_i \geq b_j\}, & \text{when } 1/e < \varrho < 1 \end{cases}$$

*and $a_i$, $b_j$ are defined by* (3) *and* (4).

Proof. It suffices to prove that for $T_0 \geq t_1$ the solution $x(t)$ of Eq.(1) has zeros on $[T_0, T_0 + 2\sigma + n_\varrho(\sigma - \tau)]$. Otherwise, without loss of generality, we assume that $x(t)$ is positive on $[T_0, T_0 + 2\sigma + n_\varrho(\sigma - \tau)]$. Let $T_\varrho = T_0 + 2\sigma + n_\varrho(\sigma - \tau)$ and

$$z(t) = x(t) + P(t)x(t - \tau) \quad \text{for } t \geq T_0 + \tau. \tag{6}$$

We get

$$z(t) > 0 \quad \text{for } t \in [T_0 + \tau, T_\varrho] \tag{7}$$

and

$$z'(t) = -Q(t)x(t - \sigma) < 0 \quad \text{for } t \in [T_0 + \sigma, T_\varrho]. \tag{8}$$

From (1) and (6), we have

$$\begin{aligned} z'(t) &= -Q(t)x(t - \sigma) \\ &= -Q(t)[z(t - \sigma) - P(t - \sigma)x(t - \tau - \sigma)] \\ &= -Q(t)z(t - \sigma) - P(t - \sigma)\frac{Q(t)}{Q(t - \tau)}z'(t - \tau) \quad \text{for } t \geq T_0 + \sigma + \tau. \end{aligned} \tag{9}$$

That is

$$z'(t) + R(t)z'(t - \tau) + Q(t)z(t - \sigma) = 0 \quad \text{for } t \geq T_0 + \sigma + \tau. \tag{10}$$

Set

$$\omega(t) = z(t) + R(t)z(t - \tau) \quad \text{for } t \geq T_0 + 2\tau. \tag{11}$$

From (7) and (11), we have

$$\omega(t) > 0 \quad \text{for } t \in [T_0 + 2\tau, T_\varrho] \tag{12}$$

and

$$\omega'(t) = z'(t) + R'(t)z(t - \tau) + R(t)z'(t - \tau) \quad \text{for } t \geq T_0 + 2\tau. \tag{13}$$

By (10) and (13), we get

$$\omega'(t) = R'(t)z(t - \tau) - Q(t)z(t - \sigma) < 0, \quad \text{for } t \in [T_0 + \sigma + \tau, T_\varrho]). \tag{14}$$

Since $z(t)$ is decreasing in $[T_0 + \sigma, T_\varrho]$, by (11) we have

$$\omega(t) < (1 + R(t))z(t - \tau) \quad \text{for } t \in [T_0 + \sigma + \tau, T_\varrho] \tag{15}$$

and so

$$z(t-\sigma) > \frac{\omega(t+\tau-\sigma)}{1+R(t+\tau-\sigma)} \quad \text{for } t \in [T_0+2\sigma, T_\varrho]. \tag{16}$$

Substituting (16) into (14), we have

$$\omega'(t) + \frac{Q(t)}{1+R(t+\tau-\sigma)}\omega(t+\tau-\sigma) < R'(t)z(t-\tau) \leq 0 \quad \text{for } t \in [T_0+2\sigma, T_\varrho]. \tag{17}$$

Next, for convenience, we set

$$q(t) = \frac{Q(t)}{1+R(t+\tau-\sigma)}.$$

Thus, (17) implies that

$$\omega'(t) + q(t)\omega(t+\tau-\sigma) < 0 \quad \text{for } t \in [T_0+2\sigma, T_\varrho]. \tag{18}$$

We consider the following two cases:

Case 1. $\varrho \geq 1$. From (12) and (14), we have

$$\omega(t) > 0 \quad \text{for } t \in [T_0+2\tau, T_0+2\sigma+(\sigma-\tau)] \tag{19}$$

and

$$\omega'(t) < 0 \quad \text{for } t \in [T_0+\sigma+\tau, T_0+2\sigma+(\sigma-\tau)] \tag{20}$$

which implies that $\omega(t)$ is decreasing, and

$$\omega(t) > \omega(T_0+2\sigma) \quad \text{for } t \in [T_0+\sigma+\tau, T_0+2\sigma].$$

Integrating both sides of (18) from $T_0+2\sigma$ to $T_0+2\sigma+(\sigma-\tau)$, we obtain

$$\begin{aligned} \omega(T_0+2\sigma+(\sigma-\tau)) &< \omega(T_0+2\sigma) - \int_{T_0+2\sigma}^{T_0+2\sigma+(\sigma-\tau)} q(s)\omega(s+\tau-\sigma)\,ds \\ &< \omega(T_0+2\sigma)\Big\{1 - \int_{T_0+2\sigma}^{T_0+2\sigma+(\sigma-\tau)} q(s)\,ds\Big\}. \end{aligned} \tag{21}$$

By (21) and condition (B), we have

$$\omega(T_0+2\sigma+(\sigma-\tau)) < \omega(T_0+2\sigma)(1-\varrho) \leq 0$$

which contradicts with (19).

Case 2. $1/e < \varrho < 1$. Setting $n_\varrho = i^* + j^*$ and $T_1 = T_0+2\sigma+(\sigma-\tau)$, under the conditions (A) and (B), we know that

$$\int_t^{t-\tau+\sigma} q(s)\,ds \geq \varrho > \frac{1}{e} \quad \text{for } t \geq t_1.$$

Observe that $f(\lambda) = \int_t^\lambda q(s)\,ds$ is a continuous function, $f(t) = 0$ and $f(t-\tau+\sigma) \geq \varrho$ and there exists some $\lambda_t \in (t, t-\tau+\sigma)$ such that $\int_t^{\lambda_t} q(s)\,ds = \varrho$.

Integrating both sides of (18) from $t$ to $\lambda_t$, we obtain

$$\omega(t) - \omega(\lambda_t) > \int_t^{\lambda_t} q(s)\omega(s+\tau-\sigma)\,ds \tag{22}$$

$$\text{for } t \in [T_1, T_0 + 2\sigma + (i^* + j^* - 1)(\sigma - \tau)].$$

Since $t \leq s \leq t + (\sigma - \tau)$, we easily see that $T_0 + 2\sigma \leq t - (\sigma - \tau) \leq s - (\sigma - \tau) \leq t$.

Integrating both side of (18) from $s - (\sigma - \tau)$ to $t$, we get

$$\omega(s+\tau-\sigma) - \omega(t) > \int_{s+\tau-\sigma}^{t} q(u)\omega(u+\tau-\sigma)\,du.$$

From (14), it is clear that $\omega(u+\tau-\sigma)$ is decreasing in $[T_0 + 2\sigma, t]$, we have

$$\begin{aligned} \omega(s+\tau-\sigma) &> \omega(t) + \omega(t+\tau-\sigma) \int_{s+\tau-\sigma}^{t} q(u)\,du \\ &= \omega(t) + \omega(t+\tau-\sigma)\Big\{ \int_{s+\tau-\sigma}^{s} q(u)\,du - \int_t^s q(u)\,du \Big\} \\ &> \omega(t) + \omega(t+\tau-\sigma)\Big\{ \varrho - \int_t^s q(u)\,du \Big\}. \end{aligned} \tag{23}$$

From (22) and (23), we have

$$\begin{aligned} \omega(t) &> \omega(\lambda_t) + \int_t^{\lambda_t} q(s)\omega(s+\tau-\sigma)\,ds \\ &> \omega(\lambda_t) + \int_t^{\lambda_t} q(s)\Big\{\omega(t) + \omega(t+\tau-\sigma)\Big(\varrho - \int_t^s q(u)\,du\Big)\Big\}\,ds \\ &= \omega(\lambda_t) + \varrho\omega(t) + \varrho^2\omega(t+\tau-\sigma) \\ &\quad - \omega(t+\tau-\sigma) \int_t^{\lambda_t} ds \int_t^s q(s)q(u)\,du. \end{aligned} \tag{24}$$

As is well known, the identical relation

$$\int_t^{\lambda_t} ds \int_t^s q(s)q(u)\,du = \int_t^{\lambda_t} du \int_u^{\lambda_t} q(s)q(u)\,ds$$

holds. On the right-hand we may exchange the variable notation of integration $s$ and $u$ the above equality becomes

$$\int_t^{\lambda_t} ds \int_t^s q(s)q(u)\,du = \int_t^{\lambda_t} ds \int_s^{\lambda_t} q(u)q(s)\,du$$

which implies

$$\begin{aligned}\int_t^{\lambda_t} ds \int_t^s q(s)q(u)\,du &= \frac{1}{2}\int_t^{\lambda_t} ds \int_t^{\lambda_t} q(u)q(s)\,du \\ &= \frac{1}{2}\Big(\int_t^{\lambda_t} q(s)\,ds\Big)^2 = \frac{\varrho^2}{2}.\end{aligned}$$

Substituting this into (24), we have

$$\omega(t) > \omega(\lambda_t) + \varrho\omega(t) + \frac{\varrho^2}{2}\omega(t+\tau-\sigma) \tag{25}$$

and so

$$\frac{\omega(t-(\sigma-\tau))}{\omega(t)} < \frac{2(1-\varrho)}{\varrho^2} = b_1, t \in [T_1, T_0+2\sigma+(i^*+j^*-1)(\sigma-\tau)]. \tag{26}$$

When $t \in [T_1, T_0 + 2\sigma + (i^* + j^* - 2)(\sigma - \tau)]$, we easily see that $T_1 \le t \le \lambda_t \le t + \sigma - \tau \le T_0 + 2\sigma + (i^* + j^* - 1)(\sigma - \tau)$. Thus, by (26), we have

$$\omega(\lambda_t) \ge \frac{1}{b_1}\omega(\lambda_t - (\sigma - \tau)). \tag{27}$$

Since $\omega(t)$ is decreasing on $[T_0 + \sigma + \tau, T_0 + 2\sigma + (i^* + j^*)(\sigma - \tau)]$ and $T_0 + 2\sigma \le \lambda_t - (\sigma - \tau) < t < \lambda_t < T_0 + 2\sigma + (i^* + j^*)(\sigma - \tau)$, we get

$$\omega(\lambda_t) > \frac{1}{b_1}\omega(\lambda_t - (\sigma-\tau)) > \frac{1}{b_1}\omega(t) > \frac{1}{b_1^2}\omega(t-(\sigma-\tau)).$$

Substituting this into (25), we have

$$\omega(t) > \frac{1}{b_1^2}\omega(t-(\sigma-\tau)) + \varrho\omega(t) + \frac{\varrho^2}{2}\omega(t-(\sigma-\tau)).$$

Therefore

$$\frac{\omega(t-(\sigma-\tau))}{\omega(t)} < \frac{2(1-\varrho)}{\varrho^2 + \frac{2}{b_1^2}} = b_2, t \in [T_1, T_0 + 2\sigma + (i^* + j^* - 2)(\sigma - \tau)].$$

Repeating the above procedure, we obtain

$$\frac{\omega(t-(\sigma-\tau))}{\omega(t)} < \frac{2(1-\varrho)}{\varrho^2 + \frac{1}{b_{j^*-1}^2}} = b_{j^*}, t \in [T_1, T_0 + 2\sigma + i^*(\sigma - \tau)]. \tag{28}$$

Setting $t = T_0 + 2\sigma + i^*(\sigma - \tau)$ in (28), we get

$$\frac{\omega(T_0 + 2\sigma + (i^* - 1)(\sigma - \tau))}{\omega(T_0 + 2\sigma + i^*(\sigma - \tau))} < b_{j^*}. \tag{29}$$

On the other hand, from (14) we know that $\omega(t)$ is decreasing in $[T_0 + \sigma + \tau, T_\varrho]$, hence

$$\frac{\omega(t - (\sigma - \tau))}{\omega(t)} > 1 \quad \text{for } t \in [T_0 + 2\sigma, T_\varrho]. \tag{30}$$

When $t \in [T_0 + 2\sigma + (\sigma - \tau), T_\varrho]$, dividing (18) by $\omega(t)$, and integrating from $t - (\sigma - \tau)$ to $t$, we get

$$\ln\left(\frac{\omega(t)}{\omega(t - (\sigma - \tau))}\right) + \int_{t-(\sigma-\tau)}^{t} q(s)\frac{\omega(s - (\sigma - \tau))}{\omega(s)}\, ds < 0.$$

By using (30) and (B), we have

$$\ln\left(\frac{\omega(t - (\sigma - \tau))}{\omega(t)}\right) > \int_{t-(\sigma-\tau)}^{t} q(s)\frac{\omega(s - (\sigma - \tau))}{\omega(s)}\, ds > \varrho.$$

It follows that

$$\frac{\omega(t - (\sigma - \tau))}{\omega(t)} > e^{\varrho} = a_1 \quad \text{for } t \in [T_0 + 2\sigma + (\sigma - \tau), T_\varrho]. \tag{31}$$

Repeating the above procedure, we get

$$\frac{\omega(t - (\sigma - \tau))}{\omega(t)} > e^{\varrho a_{i^*-1}} = a_{i^*} \quad \text{for } t \in [T_0 + 2\sigma + i^*(\sigma - \tau), T_\varrho]. \tag{32}$$

Setting $t = T_0 + 2\sigma + i^*(\sigma - \tau)$ in (32), we have

$$\frac{\omega(T_0 + 2\sigma + (i^* - 1)(\sigma - \tau))}{\omega(T_0 + 2\sigma + i^*(\sigma - \tau))} > a_{i^*}. \tag{33}$$

From (29) and (33), we obtain

$$a_{i^*} < b_{j^*} \tag{34}$$

which contradicts (5) and completes the proof of Theorem 1.

THEOREM 2. *Assume that*

(C) $P \in C([t_0, \infty), [0, \infty))$, $Q \in C([t_0, \infty), (0, \infty))$, $R'(t) \le a$, $a > 0$ *and* $\sigma > \tau > 0$;

(D) *There exist* $t_1 (t_1 \geq t_0)$ *and positive constant* $\varrho$ *such that*

$$\int_{t+\tau-\sigma}^{t} \frac{Q(s)}{1+a}\, ds \geq \varrho > \frac{1}{e} \quad \textit{for } t \geq t_1.$$

*Then* (5) *holds.*

Proof. Let $x(t)$ be a solution of Eq.(1), without loss of generality, we assume that $x(t)$ is positive in $[T_0, T_\varrho]$, $T_0 \geq t_1$, $T_\varrho = T_0 + 2\sigma + n_\varrho(\sigma - \tau)$. Let

$$z(t) = x(t) + P(t)x(t-\tau) \quad \text{for } t \geq T_0 + \tau. \tag{35}$$

Then

$$z(t) > 0 \quad \text{for } t \in [T_0 + \tau, T_\varrho] \tag{36}$$

and

$$z'(t) = -Q(t)x(t-\sigma) < 0 \quad \text{for } t \in [T_0 + \sigma, T_\varrho]. \tag{37}$$

From (1) and (35), we have

$$z'(t) + R(t)z'(t-\tau) + Q(t)z(t-\sigma) = 0 \quad \text{for } t \geq T_0 + \sigma + \tau. \tag{38}$$

By (C) and (38), we have

$$z'(t) + az'(t-\tau) + Q(t)z(t-\sigma) \leq 0 \quad \text{for } t \geq T_0 + \sigma + \tau. \tag{39}$$

Set

$$\omega(t) = z(t) + az(t-\tau) \quad \text{for } t \geq T_0 + 2\tau. \tag{40}$$

From (36) and (40), we get

$$\omega(t) > 0 \quad \text{for } t \in [T_0 + 2\tau, T_\varrho] \tag{41}$$

and

$$\omega'(t) = z'(t) + az'(t-\tau) < 0 \quad \text{for } t \in [T_0 + \sigma + \tau, T_\varrho]. \tag{42}$$

By (39) and (42), we get

$$\omega'(t) \leq -Q(t)z(t-\sigma) < 0 \quad \text{for } t \in [T_0 + \sigma + \tau, T_\varrho]. \tag{43}$$

Since $z(t)$ is decreasing in $[T_0 + \sigma, T_\varrho]$, by (40) we have

$$\omega(t) < (1+a)z(t-\tau) \quad \text{for } t \in [T_0 + \sigma + \tau, T_\varrho] \tag{44}$$

and so

$$z(t-\sigma) > \frac{\omega(t+\tau-\sigma)}{1+a} \quad \text{for } t \in [T_0 + 2\sigma, T_\varrho]. \tag{45}$$

Substituting (45) into (43), we have

$$\omega'(t) + \frac{Q(t)}{1+a}\omega(t+\tau-\sigma) < 0, \quad \text{for } t \in [T_0 + 2\sigma, T_\varrho].$$

By using a similar method as in the proof of Theorem 1, we can derive a contradiction. The proof is complete.

When $P(t) = p$, $Q(t) = q$ are constants, we get

COROLLARY 1. *Assume that*

(A′) $p \geq 0, q > 0, \sigma > \tau > 0$;

(B′) $\frac{q(\sigma-\tau)}{1+p} = \varrho > \frac{1}{e}$.

*Then* (5) *holds.*

COROLLARY 2. *Assume that*

(a) $P(t) = 0$, $Q \in C([t_0, \infty), [0, \infty))$, $\sigma > 0$;
(b) *There exist* $t_1 (t_1 \geq t_0)$ *and positive constant* $\varrho$ *such that*

$$\int_{t-\tau}^{t} Q(s)\,ds \geq \varrho > \frac{1}{e} \quad \textit{for } t \geq t_1$$

*and* $x(t)$ *is a solution of Eq.(2) on* $[t_x, \infty)$, $t_x \geq t_1$. *Then*

$$D(x) < (2 + n_\varrho)\sigma,$$

*where*

$$n_\varrho = \begin{cases} 1, & \textit{when } \varrho \geq 1 \\ \min_{i\geq 1, j\geq 1}\{i + j | a_i \geq b_j\}, & \textit{when } 1/e < \varrho < 1 \end{cases} \tag{46}$$

*and* $a_i$, $b_j$ *are defined by* (3) *and* (4).

REMARK. Theorem 1 and Theorem 2 improve and extend all results in [4]–[7].

EXAMPLE 1. Consider the delay differential equation

$$x'(t) + x(t - 0.4) = 0, \tag{47}$$

here $Q(t) = 1$. We have $\varrho = \sigma = 0.4$ and $a_1 = 1.491$, $a_2 = 1.816, \ldots, a_{10} = 4.387$, $a_{11} = 5.784$, $a_{12} = 10.111, \ldots$; $b_1 = 7.500$, $b_2 = 6.136$, $b_3 = 5.631$, $b_4 = 5.379, \ldots$.

We find

$$a_i < 5 < b_j, 1 \leq i \leq 10, j \geq 1; a_{11} > b_j, j \geq 3; a_{12} > b_j, j \geq 1;$$

Hence, by Corollary 2, we have $n_\varrho = 12 + 1 = 13$ and $D(x) < 15 \times 0.4$. This improves the result of [4]: $D(x) < 28 \times 0.4$.

EXAMPLE 2. Consider the neutral differential equation

$$[x(t) + x(t - 0.45)]' + 2x(t - 1) = 0, \tag{48}$$

here $p = 1$, $q = 2$, and $\tau = 0.45$, $\sigma = 1$. We have $\varrho = \frac{2(1-0.45)}{1+1} = 0.55$ and $a_1 = 1.733$, $a_2 = 2.594$, $a_3 = 4.165$, $a_4 = 9.884, \ldots$; $b_1 = 2.975$, $b_2 = 1.703$, $b_3 = 0.907, \ldots$.

We find

$$a_1 > b_j, j \geq 2; a_2 > b_j, j \geq 2; a_3 > b_j, j \geq 1.$$

Hence, by Corollary 1, we have $n_\varrho = 1 + 2 = 3$ and $D(x) < 2 \times 1 + 3 \times (1 - 0.45) = 3.65$.

## References

[1] D. D. Bainov and D. P. Mishev, *Oscillation theory for neutral differential equations with delay*, Hilger, Bristol, 1991.

[2] Y. Domshlak and I. P. Stavroulakis, *Oscillations of first order delay differential equations in a critical state*, Applicable Analysis, 61 (1996), 359–371.

[3] I. Gyori and G. Ladas, *Oscillation theory of differential equation with applications*, Oxford Univ. Press, London/New York, 1991.

[4] F. X. Liang, *The distribution of solutions of first-order delay differential equations*, J. Math. Anal. Appl., 186 (1994), 383–392.

[5] B. T. Li, *An estimate for distance between delay adjacent zeros of solutions first order differential equations*, Acta. Math. Appl. Sinica., 13(4) (1990), 467–472.

[6] S. Z. Lin, *An estimate for distance between adjacent zeroes of solutions of first order differential equations*, Acta. Math. Appl. Sinica., 17(3) (1994), 458–461.

[7] Y. Zhou, *An estimate for distance between adjacent zeroes of solutions of neutral equation*, Chinese Quart. J. Math., 11(4) (1996), 1–6.

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