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A NOTE ON PRECONTINUITY AND QUASICONTINUITY FOR MULTIFUNCTIONS

1. Introduction

in 1982, Mashhour et al. [9] introduced the notions of preopen sets and precontinuity in topological spaces. Precontinuity was also called almost-continuity in the sense of Husain [6]. Recently, Przemski [20] and the present authors [19] have independently defined the notion of precontinuity in the setting of multifunctions. In the present paper, we show that these notions are equivalent of each other and obtain several characterizations of precontinuous multifunctions. The main result of the paper is Theorem 3, which is a generalization of a theorem due to Borsík and Doboš [3].

2. Preliminaries

Let X be a topological space and A be a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be *preopen* [9] (resp. *semi-open* [8], *α-open* [11]) if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A)), A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$). The family of preopen (resp. semi-open) sets of X is denoted by $PO(X)$ (resp. $SO(X)$). By $PO(X,x)$ (resp. $SO(X,x)$), we denote the family of preopen (resp. semi-open) sets of X containing a point x of X. The complement of a preopen (resp. semi-open) set is said to be *preclosed* (resp. *semi-closed*). The intersection of preclosed (resp. semi-closed) sets containing A is called the *preclosure* [5] (resp. *semi-closure* [4]) and is denoted by $p\,\mathrm{Cl}(A)$ (resp. $s\,\mathrm{Cl}(A)$). The union of preopen sets contained in A is called the *preinterior* of A and is denoted by $p\,\mathrm{Int}(A)$.

LEMMA 1. *Let A be a subset of a topological space X and $x \in X$. The following hold for the preclosure of A:*

1) *$x \in p\,\mathrm{Cl}(A)$ if and only if $A \cap U \neq \emptyset$ for each $U \in PO(X,x)$;*
2) *A is preclosed if and only if $p\,\mathrm{Cl}(A) = A$;*
3) *$p\,\mathrm{Cl}(A) = A \cup \mathrm{Cl}(\mathrm{Int}(A))$.*

Proof. This follows from Lemmas 2.2 and 2.3 of [5] and [1, Theorem 1.5].

Throughout this paper, X and Y always mean topological spaces and $F : X \to Y$ (resp. $f : X \to Y$) presents a multivalued (resp. single valued) function. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a subset B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is

$$F^+(B) = \{x \in X : F(x) \subset B\} \quad \text{and} \quad F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

DEFINITION 1. A multifunction $F : X \to Y$ is said to be

a) *upper precontinuous* [17] if for each point x of X and each open set V of Y containing $F(x)$, there exists $U \in PO(X, x)$ such that $F^+(U) \subset V$,

b) *lower precontinuous* [17] if for each point x of X and each open set V of Y with $F(x) \cap V \neq \emptyset$, there exists $U \in PO(X, x)$ such that $U \subset F^-(V)$.

DEFINITION 2. A multifunction $F : X \to Y$ is said to be

a) *upper quasi continuous* [13] if for each $x \in X$, each open set U containing x and each open set V containing $F(x)$, there exists a nonempty open set G of X such that $G \subset U$ and $F^+(G) \subset V$,

b) *lower quasi continuous* [13] if for each $x \in X$, each open set U containing x and each open set V such that $F(x) \cap V \neq \emptyset$, there exists a nonempty open set G of X such that $G \subset U$ and $F(g) \cap V \neq \emptyset$ for every $g \in G$.

DEFINITION 3. A multifunction $F : X \to Y$ is said to be *quasi continuous* [2] if for each $x \in X$, each open set U containing x and any open sets V_1, V_2 of Y such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$, there exists a nonempty open set $G \subset U$ such that $F(g) \subset V_1$ and $F(g) \cap V_2 \neq \emptyset$ for every $g \in G$.

Remark 1. Every quasi continuous multifunction is upper and lower quasi continuous.

DEFINITION 4. A multifunction $F : X \to Y$ is said to be

a) *upper weakly continuous* [14,21] if for each point $x \in X$ and each open set V containing $F(x)$, there exists an open ser U containing x such that $F^+(U) \subset \mathrm{Cl}(V)$,

b) *lower weakly continuous* [14,21] if for each point $x \in X$ and each open set V with $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $U \subset F^-(\mathrm{Cl}(V))$.

DEFINITION 5. A multifunction $F : X \to Y$ is said to be

a) *upper α-continuous* [10] if for each point $x \in X$ and each open set V of Y containing $F(x)$, there exists an α-open set U containing x such that $F^+(U) \subset V$,

b) *lower α-continuous* [10] if for each point $x \in X$ and each open set V of Y with $F(x) \cap V \neq \emptyset$, there exists an α-open set U containing x such that $U \subset F^-(V)$.

3. Characterizations

DEFINITION 6. A multifunction $F : X \to Y$ is said to be *precontinuous* in the sense of Popa and Noiri [19] (resp. *α-continuous* [19]) if for each $x \in X$ and each open sets V_1, V_2 of Y such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$, there exists $U \in PO(X, x)$ (resp. an α-open set U containing x) such that $F(u) \subset V_1$ and $F(u) \cap V_2 \neq \emptyset$ for every $u \in U$.

DEFINITION 7. A multifunction $F : X \to Y$ is said to be *precontinuous* in the sense of Przemski [20] if for each $x \in X$ and each open sets $V_1, V_2 \subset Y$ such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$, $x \in p\operatorname{Int}(F^+(V_1) \cap F^-(V_2))$.

Remark 2. (1) A multifunction is α-continuous if and only if it is upper and lower α-continuous. (2) A precontinuous multifunction is upper and lower precontinuous.

THEOREM 1. *For a multifunction $F : X \to Y$ the following statements are equivalent*:

1) *F is precontinuous in the sense of Popa and Noiri*;

2) *F is precontinuous in the sense of Przemski*;

3) $F^+(V_1) \cap F^-(V_2) \in PO(X)$ *for any open sets* V_1, V_2 *of* Y;

4) $F^-(K_1) \cup F^+(K_2)$ *is preclosed in* X *for any closed sets* K_1, K_2 *of* Y;

5) $\operatorname{Cl}(\operatorname{Int}(F^-(B_1) \cup F^+(B_2))) \subset F^-(\operatorname{Cl}(B_1)) \cup F^+(\operatorname{Cl}(B_2))$ *for any subsets* $B_1, B_2 \subset Y$;

6) $p\operatorname{Cl}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\operatorname{Cl}(B_1)) \cup F^+(\operatorname{Cl}(B_2))$ *for any subsets* $B_1, B_2 \subset Y$;

7) $F^-(\operatorname{Int}(B_1)) \cap F^+(\operatorname{Int}(B_2))) \subset p\operatorname{Int}(F^-(B_1) \cap F^+(B_2))$ *for any subsets* $B_1, B_2 \subset Y$.

Proof. (1) $\Rightarrow$(2): Let F be precontinuous in the sense of Popa and Noiri and let V_1, V_2 be any open sets of Y such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$. Then, there exists $U \in PO(X, x)$ such that $F(u) \subset V_1$ and $F(u) \cap V_2 \neq \emptyset$ for every $u \in U$. Therefore we have $U \subset F^+(V_1) \cap F^-(V_2)$ and hence $x \in U \subset p\operatorname{Int}(F^+(V_1) \cap F^-(V_2))$.

(2)$\Rightarrow$(3): Let V_1, V_2 be any open sets of Y and $x \in F^+(V_1) \cap F^-(V_2)$. Then we have $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$ and hence $x \in p\operatorname{Int}(F^+(V_1) \cap F^-(V_2))$. Thus, we obtain $F^+(V_1) \cap F^-(V_2) \subset p\operatorname{Int}(F^+(V_1) \cap F^-(V_2))$. This shows that $F^+(V_1) \cap F^-(V_2) \in PO(X)$.

(3)$\Rightarrow$(4): This easily follows from the fact that $F^-(Y - B) = X - F^+(B)$ and $F^+(Y - B) = X - F^-(B)$ for every subset $B \subset Y$.

(4)$\Rightarrow$(5): Let B_1, B_2 be any subsets of Y. Then $F^-(\operatorname{Cl}(B_1)) \cup F^+(\operatorname{Cl}(B_2))$

is a preclosed set of Y. By Lemma 1, we obtain

$$\begin{aligned}\mathrm{Cl}(\mathrm{Int}(F^-(B_1)\cup F^+(B_2))) &\subset \mathrm{Cl}(\mathrm{Int}(F^-(\mathrm{Cl}(B_1))\cup F^+(\mathrm{Cl}(B_2))))\\ &\subset p\,\mathrm{Cl}(F^-(\mathrm{Cl}(B_1))\cup F^+(\mathrm{Cl}(B_2)))\\ &= F^-(\mathrm{Cl}(B_1))\cup F^+(\mathrm{Cl}(B_2)).\end{aligned}$$

(5)⇒(6): Let B_1, B_2 be any subsets of Y. It follows from Lemma 1 that

$$\begin{aligned}p\,\mathrm{Cl}(F^-(B_1)\cup F^+(B_2)) &= [F^-(B_1)\cup F^+(B_2)]\cup \mathrm{Cl}(\mathrm{Int}(F^-(B_1)\cup F^+(B_2)))\\ &\subset F^-(\mathrm{Cl}(B_1))\cup F^+(\mathrm{Cl}(B_2)).\end{aligned}$$

(6)⇒(7): Let B_1, B_2 be any subsets of Y. Then we have

$$\begin{aligned}X - p\,\mathrm{Int}(F^-(B_1)\cap F^+(B_2)) &= p\,\mathrm{Cl}(X - [F^-(B_1)\cap F^+(B_2)])\\ &= p\,\mathrm{Cl}[(X - F^-(B_1))\cup(X - F^+(B_2)]\\ &= p\,\mathrm{Cl}[F^+(Y - B_1)\cup F^-(Y - B_2)]\\ &\subset F^+(\mathrm{Cl}(Y - B_1))\cup F^-(\mathrm{Cl}(Y - B_2))\\ &= F^+(Y - \mathrm{Int}(B_1))\cup F^-(Y - \mathrm{Int}(B_2))\\ &= [X - F^-(\mathrm{Int}(B_1))]\cup[X - F^+(\mathrm{Int}(B_2))]\\ &= X - [F^-(\mathrm{Int}(B_1))\cap F^+(\mathrm{Int}(B_2)))].\end{aligned}$$

Therefore, we obtain

$$F^-(\mathrm{Int}(B_1))\cap F^+(\mathrm{Int}(B_2))) \subset p\,\mathrm{Int}(F^-(B_1)\cap F^+(B_2)).$$

(7)⇒(1): Let $x \in X$ and V_1, V_2 be any open sets of Y such that $F(x) \subset V_1$ and $F(x)\cap V_2 \neq \emptyset$. Then, we have $F^+(V_1)\cap F^-(V_2) \subset p\,\mathrm{Int}(F^+(V_1)\cap F^-(V_2))$. Now, put $U = F^+(V_1)\cap F^-(V_2)$. Then we obtain $U \in PO(X, x)$, $F(U) \subset V_1$ and $F(u)\cap V_2 \neq \emptyset$ for each $u \in U$.

COROLLARY 1 (Mashhour et al. [9[and Popa [15]). *For a function* $f : X \to Y$, *the following statements are equivalent*:

1) f *is precontinuous*;

2) f *is almost continuous in the sense of Husain*;

3) *for each* $x \in X$ *and each open set* V *containing* $f(x)$, *there exists* $U \in PO(X, x)$ *such that* $f(U) \subset V$;

4) $f^{-1}(K)$ *is preclosed in* X *for every closed set* K *of* Y;

5) $\mathrm{Cl}(\mathrm{Int}(f^{-1}(B))) \subset f^{-1}(\mathrm{Cl}(B))$ *for every subset* B *of* Y:

6) $p\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}(B))$ *for every subset* B *of* Y;

7) $f^{-1}(\mathrm{Int}(B)) \subset p\,\mathrm{Int}(f^{-1}(B))$ *for every subset* B *of* Y;

8) $f(p\,\mathrm{Cl}(A)) \subset \mathrm{Cl}(f(A))$ *for every subset* A *of* X.

DEFINITION 8. A subset A of a space X is said to be *α-regular* [7] if for any point $x \in A$ and any open set U of X containing x, there exists an open set G of X such that $x \in G \subset \mathrm{Cl}(G) \subset U$.

DEFINITION 9. A subset A of a space X is said to be *α-paracompact* [22] if every X-open cover of A has an X-open X-locally finite refinement which covers A.

LEMMA 2 (Kovačević [7]). *If A is an α-regular α-paracompact subset of a space X and U is an open neighborhood of A, then there exists an open set G of X such that $A \subset G \subset \mathrm{Cl}(G) \subset U$.*

LEMMA 3 (Popa [18]). *If A is an α-regular set of a space X, then for each open set U which intersects A there exists an open set U_A such that $A \cap U_A \neq \emptyset$ and $\mathrm{Cl}(U_A) \subset U$.*

For a multifunction $F : X \to Y$, a multifunction $p\,\mathrm{Cl}\,F : X \to Y$ is defined in [17] as follows: $(p\,\mathrm{Cl}\,F)(x) = p\,\mathrm{Cl}(F(x))$ for each $x \in X$.

LEMMA 4 (Noiri and Popa [12]). *Let $F : X \to Y$ be a multifunction. Then,*

1) $(p\,\mathrm{Cl}\,F)^+(V) = F^+(V)$ *for each open set V of Y if $F(x)$ is α-regular α-paracompact for each $x \in X$,*

2) $(p\,\mathrm{Cl}\,F)^-(V) = F^-(V)$ *for each open set V of Y.*

THEOREM 2. *Let $F : X \to Y$ be a multifunction such that $F(x)$ is α-regular α-paracompact for each $x \in X$. Then F is precontinuous if and only if $p\,\mathrm{Cl}\,F : X \to Y$ is precontinuous.*

Proof. *Necessity*. Suppose that F is precontinuous. Let $x \in X$ and V_1, V_2 be any open sets of Y such that $(p\,\mathrm{Cl}\,F)(x) \subset V_1$ and $(p\,\mathrm{Cl}\,F)(x) \cap V_2 \neq \emptyset$. By Lemma 4, we have $x \in (p\,\mathrm{Cl}\,F)^+(V_1) = F^+(V_1)$ and $x \in (p\,\mathrm{Cl}\,F)^-(V_2) = F^-(V_2)$ and hence $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$. Since F is precontinuous, then by Theorem 1 we obtain

$$x \in p\,\mathrm{Int}(F^+(V_1) \cap F^-(V_2))$$

and hence

$$x \in p\,\mathrm{Int}((p\,\mathrm{Cl}\,F)^+(V_1) \cap (p\,\mathrm{Cl}\,F)^-(V_2))).$$

This shows that $p\,\mathrm{Cl}\,F$ is precontinuous.

Sufficiency. Suppose that $p\,\mathrm{Cl}\,F$ is precontinuous. Let $x \in X$ and V_1, V_2 be any open sets of Y such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$. By Lemma 4, we have $x \in F^+(V_1) = (p\,\mathrm{Cl}\,F)^+(V_1)$ and $x \in F^-(V_2) = (p\,\mathrm{Cl}\,F)^-(V_2)$. By the precontinuity of $p\,\mathrm{Cl}\,F$, we obtain

$$x \in p\,\mathrm{Int}((p\,\mathrm{Cl}\,F)^+(V_1) \cap (p\,\mathrm{Cl}\,F)^-(V_2))$$

and hence

$$x \in p\,\mathrm{Int}(F^+(V_1) \cap F^-(V_2)).$$

Therefore, by Theorem 1, F is precontinuous.

4. Precontinuity and quasicontinuity

By Q_F (resp. P_F), we denote the set of all points of X at which F is quasi continuous (resp. precontinuous). If $Q_F = X$ (resp. $P_F = X$), then F is quasi continuous (resp. precontinuous).

THEOREM 3. *If $F : X \to Y$ is a multifunction such that $F(x)$ is α-regular and α-paracompact for each $x \in X$, then $P_F \cap \mathrm{Cl}(Q_F) \subset Q_F$.*

Proof. Let $x \in P_F \cap \mathrm{Cl}(Q_F)$, U be an open set of X containing x and V_1, V_2 be any open sets of Y such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$. Since $F(x)$ is α-regular and α-paracompact, by Lemma 2 there exists an open set W_1 of Y such that

$$(1) \qquad F(x) \subset W_1 \subset \mathrm{Cl}(W_1) \subset V_1.$$

Since $F(x)$ is α-regular, by Lemma 3 there exists an open set W_2 of Y such that

$$(2) \qquad F(x) \cap W_2 \neq \emptyset \quad \text{and} \quad \mathrm{Cl}(W_2) \subset V_2.$$

Since $x \in P_F$, by Remark 2 F is upper precontinuous and lower precontinuous at x. The upper precontinuity of F at x implies that there exists an open set $U_1 \subset U$ such that $x \in U_1 \subset \mathrm{Cl}(F^+(W_1))$. The lower precontinuity of F at x implies that there exists an open set $U_2 \subset U$ such that $x \in U_2 \subset \mathrm{Cl}(F^-(W_2))$. Put $U_0 = U_1 \cap U_2$, then $x \in U_0 \subset \mathrm{Cl}(F^+(W_1))$ and $x \in U_0 \subset \mathrm{Cl}(F^-(W_2))$. Since $x \in \mathrm{Cl}(Q_F)$, we have $U_0 \cap Q_F \neq \emptyset$. If $s \in U_0 \cap Q_F$, then

$$(3) \qquad s \in F^+(\mathrm{Cl}(W_1)) \quad \text{and}$$

$$(4) \qquad s \in F^-(\mathrm{Cl}(W_2)).$$

Suppose that (3) does not hold. Then, there exists $s_0 \in U_0 \cap Q_F$ such that $s_0 \in F^-(Y - \mathrm{Cl}(W_1))$. By Remark 1, F is upper quasi continuous and lower quasi continuous at s_0. The lower quasi continuity of F at s_0 implies that there exists a nonempty open set $W_1^* \subset U_1$ such that $W_1^* \subset F^-(Y - \mathrm{Cl}(W_1)) \subset F^-(Y - W_1)$ [16, Theorem 2.2]. That is in contradiction with $U_1 \subset \mathrm{Cl}(F^+(W_1))$. Next, suppose that (4) does not hold. Then there exists $s_0 \in U_0 \cap Q_F$ such that $s_0 \in F^+(Y - \mathrm{Cl}(W_2))$. The upper quasi continuity of F at s_0 implies that there exists a nonempty open set $W_2^* \subset U_2$ such that $W_2^* \subset F^+(Y - \mathrm{Cl}(W_2)) \subset F^+(Y - W_2)$. That is in contradiction with $U_2 \subset \mathrm{Cl}(F^-(W_2))$. If $s \in U_0 \cap Q_F$, it follows from (1), (2), (3) and (4) that $F(s) \subset \mathrm{Cl}(W_1) \subset V_1$ and $\emptyset \neq F(s) \cap \mathrm{Cl}(W_2) \subset F(s) \cap V_2$. Since F is quasi continuous at s, there exists a nonempty open set $G \subset U$ such that $F(G) \subset V_1$ and $F(g) \cap V_2 \neq \emptyset$ for every $g \in G$. Thus for each open set U containing x and any open sets V_1, V_2 such that $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$, there exists a nonempty open set $G \subset U$ such that $F(G) \subset V_1$

and $F(g) \cap V_2 \neq \emptyset$ for every $g \in G$. Thus F is quasi continuous at x and hence $P_F \cap \mathrm{Cl}(Q_F) \subset Q_F$.

The following two corollaries are immediate consequences of Theorem 3 and thus the proofs are omitted.

COROLLARY 2. *If a multifunction $F : X \to Y$ is precontinuous and $F(x)$ is α-regular α-paracompact for each $x \in X$, then Q_F is closed.*

COROLLARY 3 (Borsík and Doboš [3]). *If Y is a regular space and $f : X \to Y$ is precontinuous, then Q_f is closed.*

COROLLARY 4. *Let $F : X \to Y$ be a precontinuous multifunction such that $F(x)$ is α-regular α-paracompact for each $x \in X$. If $\mathrm{Cl}(Q_F) = X$, then F is continuous.*

Proof. By Corollary 2, $\mathrm{Cl}(Q_F) = Q_F = X$ and thus F is quasi continuous. Since F is precontinuous and quasi continuous, F is α-continuous [19, Theorem 5] and hence F is upper and lower α-continuous. By Remark 1, it follows from [10, Theorem 2] that F is upper weakly continuous. Since $F(x)$ is α-regular α-paracompact for each $x \in X$, it follows from [18, Theorem 1] that F is upper continuous. Similarly, F is lower weakly continuous and $F(x)$ is α-regular for each $x \in X$. Hence F is lower continuous [18, Theorem 2]. Consequently, we obtain that F is continuous.

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