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ON ALMOST-OPENNESS AND SPACES INVERTNESS VIA DENSITY

1. Introduction

In 1943, Hewitt [1] presented the concepts of MI-spaces, resolvability spaces and SI-spaces; these types of spaces are obtained via one of the important properties of topological spaces which called „density". Several topological properties of these types have been investigated. But, in 1966, Bourbarki [2] had given another type of spaces which is established depend on the density property which is known as a submaximal space. Also, the hyperconnected space has been defined by Steen and Seebach [3] in 1978, by the concept of dense sets. Recently, the author in [4] defined the almost-open space by using the concept of almost-openness which depends on the dense sets. Many properties of all previous concepts and some correlated spaces have been studied in [5]–[8].

Therefore, this paper is devoted to these types of spaces which are mentioned before. The first part contains the complete introduction to the subject of study which is presented throughout this paper. While, the fundamental notations and basic preliminaries which are necessarily used in this work will be given in the second section. In the third one, the general framework between the spaces which under the subject of disscution here have been constructed. This framework gives the relationships and shows the common properties among more than one type of these spaces. But, several characterizations of the previous spaces with their related sets and functions are established in the forth section. Depending on the framework in the third part, some suggested conditions have been found under which

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the equivalence between the above mentioned spaces are satisfied, in the fifth section of this paper. All other new properties concerning the spaces via density and other notions are investigated throughout the final part of this work.

2. Useful notations and basic preliminaries

Throughout the present paper, all notations will be defined with respect to the topological space (X, τ), whenever such spaces are needed it will be explicitly stated. Also, all topological spaces used here will not include any separation properties which are assumed unless they are otherwise needed in which case they will be given. In (X, τ), the closure, the interior and the derived set of any $W \in P(X)$ will be denoted by $cl(W)$, $int(W)$ and $d(W)$, respectively. Recall that, W is said to be dense, codense, nowhere dense and dense-in-itself if $\mathrm{cl}(W) = X$, $\mathrm{int}(W) = \emptyset$, $\mathrm{int}(\mathrm{cl}(W)) = \emptyset$ and $W = d(W)$, respectively. Dense-in-itself of any $W \subseteq X$, equivalently that W does not have any isolated points. $D(X, \tau)$ and $C(X, \tau)$ will denote the class of all dense and codense sets of (X, τ), respectively.

In (X, τ), any $W \subseteq X$ is said to be almost-open [9] if $W \subseteq \mathrm{int}(\mathrm{cl}\, W)$ and $AO(X, \tau)$ means the collection of almost-open sets in (X, τ). For any space (X, τ) let τ_A be the smallest topology on X containing $AO(X, \tau)$. While the topology $\tau^\alpha = AO(X, \tau) \cap SO(X, \tau)$, where $W \in SO(X, \tau)$, iff W is semi-open [10], i.e., $W \subseteq \mathrm{cl}(\mathrm{int}\, W)$. And thus for any space (X, τ), $\tau \subseteq \tau^\alpha \subseteq AO(X, \tau) \subseteq \tau_A$, and it is also known that $AO(X, \tau^\alpha) = AO(X, \tau)$.

A space (X, τ) is an MI-space [1] if it is dense-in-itself and has the property that each dense subset is open. But, (X, τ) is resolvable [1] if it is the disjoint union of two dense subsets. A space which is not resolvable is called irresolvable. A subset of X is resolvable (iiresolvable) if it is resolvable (irresolvable) as a subspace. A space is hereditarily irresolvable if each of its nonempty subsets is irresolvable. In the same paper [1], the concept of SI-spaces was defined as follows: an SI-space is the space which is dense-in-itself and hereditarily irresolvable. (X, τ) is submaximal [2] if each of its dense subsets are open. Also, (X, τ) is called hyperconnected [3] if each nonempty open set is dense. A space (X, τ) is an almost-open space [4] if $\tau = \tau_A$.

3. Connections between spaces via density

This article is devoted to study the relationships between the spaces defined depending on the known types of dense sets and mentioned previously.

By the meaning of these spaces, we can construct the following implication which shows the common properties among more than one type of these spaces.

$$\begin{array}{ccccc} \text{Submaximal spaces} & \longrightarrow & \text{Hereditarily irresolvable spaces} & \longrightarrow & \text{Irresolvable spaces} \\ \uparrow & & \uparrow & & \not\updownarrow \\ \text{MI-spaces} & \longrightarrow & \text{SI-spaces} & \not\leftrightarrow & \text{Resolvable spaces.} \end{array}$$

Also, it is clear that: (i) MI-spaces are not resolvable and therefore MI-spaces stand for the "maximally irresolvable" property.

(ii) Resolvability is independent of each of SI-spaces and irresolvability.

(iii) Hyperconnectedness is dual with respect to submaximality.

Note that we cannot demand the converse of the relations in the previous implication as the following examples illustrate.

EXAMPLE 1. For any fixed element (say x) of an infinite set X, with a topology $\tau = \{X, \emptyset, \{x\}, X - \{x\}\}$. One can show that a space (X, τ) is irresolvable but not hereditarily irresolvable.

EXAMPLE 2. If $X = \{(0,0) \cup \{(0, \frac{1}{n}) \cup (\frac{1}{m}, \frac{1}{n}) : m,\ n \in \mathbb{N}\}\}$, with a topology τ, its openness takes the form $(\frac{1}{m}, \frac{1}{n})$. The basic neighbourhood of $(0, \frac{1}{n})$ containing $(0, \frac{1}{n})$ with at most a finite number of points, differrent than the same point, must be deleted. Also, a neighbourhood of the origin point $(0,0)$ contains neighbourhoods of all but finitely many points $(0, \frac{1}{n})$. Hence (X, τ) is hereditarily irresolvable but fails to be submaximal.

Remark 1. Since each MI-space and SI-space is dense-in-itself and submaximal, hereditarily irresolvable, respectively, then the following fact must be verified: if the condition dense-in-itself is add to the space in Example 2, the fact that MI-space implies SI-space can not be reversed.

4. Characterizations of spaces via density

Some equivalent definitions of each type of spaces which are under the discussion throughout this paper will be investigated in this section. In the beginning, one of the basic and useful concepts which is an ideal must be given.

$I(C)$ denotes the ideal generated by $C(X, \tau)$, and therefore the following two obvious results are obtained which are very useful to investigate the resolvability equivalent nextly.

LEMMA 1. *For any $W \subseteq X$ in (X, τ), $W \in I(C)$ iff W is a finite union of codense sets.*

Proposition 1. *In* (X, τ) *the following statements are equivalent for any* $W \subseteq X$:

(i) $W \in I(C)$;
(ii) $\operatorname{int} W$ *is resolvable*;
(iii) $\operatorname{int}(\operatorname{cl} W)$ *is resolvable.*

As a consequence of two previous results, an immediate equivalent is given the following theorem.

Theorem 1. *An open set* U *in a space* (X, τ) *is resolvable iff* $U \in I(C)$; *and therefore,* (X, τ) *is resolvable iff* X *is a finite union of elements of* $I(C)$.

Also, in [4] the author gives the following property which is useful throughout the sequel.

Lemma 2 [4]. *Each dense set in any space is almost-open.*

Proposition 2. *The topology* τ_A *is discrete if a space* (X, τ) *is resolvable.*

Proof. Let be any $x \in X$ and $D, D^* \in D(X, \tau)$ such that $D \cap D^* = \emptyset$. Then $D \cup \{x\}, D^* \cup \{x\} \in D(X, \tau)$ and therefore they are almost-open sets in (X, τ) (see Lemma 2). Hence $(D \cup \{x\}) \cap (D^* \cup \{x\}) = \{x\} \in \tau_A$ and this completes the proof.

Theorem 2. *A connected space* (X, τ) *is resolvable iff* τ_A *is discrete.*

Proof. One direction follows by Proposition 2, previously. While the converse is established by using the hypothesis and Lemma 2 of [11].

Another important result related to the resolvability of (X, τ) due to Hewitt [1] will be stated as a useful fact to establish an obvious resolvability equivalent to other results which will be presented next.

Proposition 3 [1]. *Any space* (X, τ) *can be represented uniquely as a disjoint union* $X = F \cup G$ *where* F *is closed and resolvable and* G *is open and hereditarily irresolvable. This is called the "Hewitt-representation"* [1].

Theorem 3. *Let* X *be the Hewitt-representation of* (X, τ), *then*:

(i) (X, τ) *is resolvable iff* G *is an empty set.*
(ii) (X, τ) *is hereditarily irresolvable iff* F *is empty.*

Proposition 4 [5]. *For a space* (X, τ) *the followings are equivalent*:

(i) (X, τ) *contains an open, dense and hereditarily irresolvable subspace.*
(ii) *Every open ultrafilter on* X *is a base for an ultrafilter on* X.
(iii) *Every nonempty open set is irresolvable.*
(iv) *For each dense subset* D *of* (X, τ) $\operatorname{int} D$ *is dense.*
(v) *For every* $W \subseteq X$, *if* $\operatorname{int} W = \emptyset$, *then* W *is nowhere dense.*

THEOREM 4. *In* (X,τ) *the next statements are equivalent*:

(i) $\tau = \tau^{\alpha} = AO(X,\tau) = \tau_A \subseteq SO(X,\tau)$.

(ii) (X,τ) *is almost-open, contains an open, dense and hereditarily irresolvable subspace* $D \subseteq X$.

Proof. (i)→(ii): The almost-openness of (X,τ) follows by the fact that $\tau = \tau_A$. Let $D \in D(X,\tau)$, then $D \in AO(X,\tau)$, since τ_A is a topology generated by a class $AO(X,\tau)$ as a subbase. This means that $D \in \tau_A$ and therefore $D \in SO(X,\tau)$ which gives $X = clD \subseteq \mathrm{cl}(\int D)$. Hence, $\int D \in D(X,\tau)$ and by Proposition 4, the statement (ii) will be verified.

(ii)→(i): Since (X,τ) is almost-open, then the equalities in (i) must be satisfied. To show the equality also, let $W \in \tau_A$. Then, by Lemma 2 in [4] $W = U \cap D$, where $U \in \tau$ and $D \in D(X,\tau)$ which leads to $\mathrm{int}\, D \in D(X,\tau)$ (see (iv) of previous proposition). Therefore, $\mathrm{cl}(\mathrm{int}\, W) = \mathrm{cl}(U \cap \mathrm{int}\, D) = \mathrm{cl}\, U$, but $W \subseteq U \subseteq \mathrm{cl}\, U = \mathrm{cl}(\mathrm{int}\, W)$. Hence the result.

Consequently, one characterization of almost-openness will be obtained nextly, which has an obvious proof.

THEOREM 5. (X,τ) *is almost-open iff* $\tau = AO(X,\tau)$.

Here, $I_{cd}(X,\tau)$ denotes the ideal of closed and discrete subsets of a space (X,τ). But $I_n(X,\tau)$ means the ideal generating the class of nowhere dense sets with respect to (X,τ) and $\tau(I_n)$ is the extension topology defined by an ideal $I_n(X,\tau)$. These notions help to characterize submaximality as the following straightforward results.

THEOREM 6. *For a space* (X,τ) *the following statements are satisfied*:

(i) (X,τ) *is submaximal iff* $C(X,\tau) \subseteq I_{cd}(X,\tau)$.

(ii) $(X,\tau(I_n))$ *is submaximal iff* $C(X,\tau) = I_n(X,\tau)$.

THEOREM 7. *Let* (X,τ) *be an almost-open space, then the following statements are equivalent*:

(i) (X,τ) *is submaximal.*

(ii) (X,τ_A) *is submaximal.*

(iii) $\tau = \tau_A = AO(X,\tau) \subseteq SO(X,\tau)$.

(iv) $W \subseteq X$ *is nowhere dense if it is codense.*

(v) *There exists an open, dense and hereditarily irresolvable subspace* $D \subseteq X$ *and* $\tau = \tau_A$.

Proof. (i)↔(ii): Established by the almost-openness of (X,τ). While the equivalent of (i) with each of other statements follows from Lemma 2 in [5], Proposition 4 and Theorem 4 above.

THEOREM 8. *If (X,τ) is an almost-open space, then the following statements are equivalent:*

(i) *(X,τ) is hyperconnected.*
(ii) *$D(X,\tau)$ coincides with the class of nonempty sets of τ_A.*
(iii) *Any $W \in P(X)$ or its complement is dense.*
(iv) *$D(X,\tau)$ contains the nonempty class of $SO(X,\tau)$.*

Proof. (i)$\leftrightarrow$(ii): Established directly by the meaning of an almost-openness and applying Lemma 2 in [4].

(i)$\leftrightarrow$(iii): Letting $intW \neq \emptyset$, then $W \in D(X,\tau)$ for $X = \mathrm{cl}(\mathrm{int}\, W) \subseteq \mathrm{cl}\, W$. But if not, this gives $\mathrm{cl}(X-W) = X - \mathrm{int}\, W = X$. Therefore $X - W \in D(X,\tau)$.

(iii)$\rightarrow$(i): For each $\emptyset \neq U \in \tau, X \neq X - U = cl(X-U)$. Hence $X - U \notin D(X,\tau)$, by (iii), U must be a dense set.

(i)$\leftrightarrow$(iv): Since each $\emptyset \neq W \in SO(X,\tau)$, this gives $\emptyset \neq \mathrm{int}\, W \in D(X,\tau)$. This shows one direction, while the other follows from $\tau \subseteq SO(X,\tau)$.

5. Basic properties of spaces via density

The benefits of spaces have been apparent by studying their properties. Moreover, this study shows the common results and the extension ones among more than one type of the different spaces. So, this article will contain several properties of spaces constructed via dense sets and other related ones.

Hewitt in [1] showed that: "open subsets of a resolvable space are resolvable". This fact will be strengthened as follows.

THEOREM 9. *If (X,τ) is resolvable and $W \in SO(X,\tau)$, then W is resolvable.*

Proof. Since $W \in SO(X,\tau)$, i.e., $W \subseteq \mathrm{cl}(\mathrm{int}\, W)$ and (X,τ) is resolvable. Then $\mathrm{int}\, W$ is resolvable and $W - \mathrm{int}\, W$ is nowhere dense in $(W, \tau/W)$. Thus, if $D \cup D^*$ is a disjoint union of dense subsets of $(\mathrm{int}\, W)$, then $[D \cup (W - \mathrm{int}\, W)]$ and D^* are disjoint and also are dense in W. Hence W is resolvable.

THEOREM 10. *The union of a disjoint family of open resolvable sets in any space is resolvable.*

Proof. Let a space (X,τ) and W be the union of a disjoint resolvable family $\{U_i : \emptyset \neq U_i \in \tau,\ i \in I\}$. Then for each $i \in I$, there exist disjoint sets G_i, H_i which are dense in U_i such that $U_i = G_i \cup H_i$. If $G = \bigcup_{i\in I} G_i$, $H = \bigcup_{i\in I} H_i$, then G, H are disjoint and dense in their union W, and this verifies the result.

One can show that: "any subspace of a hereditarily irresolvable space is hereditarily irresolvable". The condition under which this result must be reversible will be established by the following theorem.

THEOREM 11. *(X,τ) is hereditarily irresolvable, if X can be expressed as a disjoint union of Y and $Y^* \in \tau$ and both $(Y,\tau/Y)$ and $(Y^*,\tau/Y^*)$ are hereditarily irresolvable.*

Proof. Let us assume $\emptyset \neq W \subseteq X$ and $(W,\tau/W)$ is resolvable. Then there exist disjoint, dense in W subsets D and D^* with $W = D \cup D^*$. Suppose that $D \cap Y^* \neq \emptyset$ and $D^* \cap Y^* \neq \emptyset$. Then since Y^* is open in $X, (D \cap Y^*)$ and $(D^* \cap Y^*)$ are disjoint and dense in $(W \cap Y^*)$. For, if $x \in D^* \cap Y^*$ and V is open with $x \in V$, since D is dense in $W, V \cap W \cap D \neq \emptyset$. If U is open in X and $x \in U$ then, for $V = U \cap Y^*$, $V \in \tau$ and $x \in V$ so that $U \cap W \cap (D \cap Y^*) \neq \emptyset$. Thus, $D \cap Y^*$ and similarly $D^* \cap Y^*$ are dense in $W \in Y^*$ and disjoint. Thus, $W \cap Y^*$ is a resolvable subspace of Y^* which contradicts Y^* being hereditarily irresolvable. Apparently, either $D \cap Y^* = \emptyset$ or $D^* \cap Y^* = \emptyset$. But in either case $W \cap Y$ contains a dense set in W. Thus, $cl_W(W \cap Y) = W \subseteq W \cap Y \subseteq Y$, since Y is closed. Thus W is a resolvable subspace of Y which cannot be, since Y is hereditarily irresolvable. This final contradiction proves that (X,τ) is hereditarily irresolvable.

THEOREM 12. *(X,τ) is irresolvable if $\{\operatorname{int} D : D \in D(X,\tau)\}$ is a filter-base on X.*

Proof. Let (X,τ) be resolvable, then $X = D \cup D^*$ where $D, D^* \in D(X,\tau)$ and $D \cap D^* = \emptyset$. This means that both of $(\operatorname{int} D)$ and $(\operatorname{int} D^*)$ are empty. This contradicts the hypothesis. Hence the result.

THEOREM 13. *Each semi-open subspace of a submaximal space is submaximal.*

Proof. Let (X,τ) be submaximal and $W \in SO(X,\tau)$. Then $\tau = \tau^\alpha$ and there is an open, dense, hereditarily irresolvable subset $D \subseteq X$. If W is nonempty, then $D \cap \operatorname{int} W$ is a dense, open, hereditarily irresolvable subspace of $(W,\tau/W)$, and also $\mathrm{cl}_W(D \cap \operatorname{int} W) = W \cap \mathrm{cl}(D \cap \operatorname{int} W) = W \cap \mathrm{cl}(\operatorname{int} W) = W$, because $W \in SO(X,\tau)$ and so $\tau/W = \tau^\alpha/W = (\tau/W)^\alpha$. Hence $(W,\tau/W)$ is submaximal.

6. Spaces via density and some functions

Recall that a bijection $f : (X,\tau) \to (Y,\sigma)$ is a semihomeomorphism [11] and α-homeomorphism [12], if both f and f^{-1} preserve semiopen sets and α-sets, respectively. Any property transmitted by semihomeomorphisms is called semitopological [11].

PROPOSITION 5. *Let $f : (X,\tau) \to (Y,\sigma)$ be a bijection. Then f is a semihomeomorphism iff f is an α-homeomorphism.*

By the above result, a property P is semitopological if and only if P is an α-topological property [13] which is defined as follows: an α-topological property is any topological property shared by all numbers of the α-class when possessed by any one member of the α-class. In particular, it is any topological property possessed by both (X,τ) and (X,τ^α) when possessed by either of [14].

Also, for any topology τ on X, the semiregularization of τ [15] is the topology τ_s having for a basis the set of regular open subsets of (X,τ). The semiregular class of τ is the set $[\tau]_s$ of all topologies on X having the same semiregularization as τ. A topological property P is a semiregular property if it is shared by all members of $[\tau]_s$ when possessed by any one member. This is equivalent to saying that (X,τ) and (X,τ_s) both have P whenever either does.

It is clearly shown that: "spaces (X,τ) and (X,τ^α) share the same family of dense subsets", also resolvability is one of the α-topological and hence semitopological properties. This illustrates our belief that generally the best way to demonstrate that a property P is semitopological is to show that it is α-topological. Also, clearly semiregular properties are α-topological [16].

Therefore, the following example shows that semitopological properties are not semiregular.

EXAMPLE 3. Let (X,τ) be the two-point Sierpiński space. Then (X,τ) is not resolvable whereas the indiscrete semiregularization (X,τ_s) is resolvable.

Recall that any function $f : (X,\tau) \to (Y,\sigma)$ is called dual almost-continuous [4] A-function [9] if for each $W \in AO(Y,\sigma)$, $f^{-1}(W)$ is open, or almost-open, respectively.

One can show that: "for the usual space of real numbers $(\mathbb{R},\mathcal{U})$, every non-constant function $f : (\mathbb{R},\mathcal{U}) \to (\mathbb{R},\mathcal{U})$ is not dual almost-continuous". This fact turns the attention to offer the following obvious consequence.

PROPOSITION 6. *If (X,τ) is connected and (Y,σ) is resolvable then $f : (X,\tau) \to (Y,\sigma)$ is dual almost-continuous iff f is a constant function.*

Some other equivalent definitions of dual almost-continuity will be given nextly via some previously mentioned spaces.

THEOREM 14. *If (Y,σ) is resolvable, the following statements are equivalent:*

(i) *$f : (X,\tau) \to (Y,\sigma)$ is dual almost-continuous.*

(ii) *$f : (X,\tau) \to (Y,\mathbb{D})$ is continuous, ($\mathbb{D}$ is the discrete topology).*

(iii) $f^{-1}(y)$ *is clopen (closed and open) for each* $y \in Y$.
(iv) $f^{-1}(B)$ *is clopen for each* $B \subseteq Y$.

Proof. Since a function $f : (X, \tau) \to (Y, \sigma)$ is dual almost-continuous iff $f : (X, \tau) \to (Y, \sigma_A)$ is continuous, and by using Proposition 2, we get (i)$\leftrightarrow$(ii). While the other equivalents are established immediately.

COROLLARY 1. *If* (X, τ) *is dense-in-itself and* (Y, σ) *is a nonempty resolvable space, then there is no injection* $f : (X, \tau) \to (Y, \sigma)$ *dual almost-continuous.*

PROPOSITION 7. *For any submaximal space* (X, τ) *the class* $AO(X, \tau)$ *coincides with its topology* τ.

Proof. Let $W \in AO(X, \tau)$, then, by Lemma 2 in [4], W can be expressed as the intersection of $U \in \tau$ with $D \in D(X, \tau)$ and the submaximality of (X, τ) gives that D is open and so $U \cap D \in \tau$. Hence, $AO(X, \tau) \subseteq \tau$, whereas the other inclusion follows immediately, which established the result.

THEOREM 15. *The following statements hold for any* $f : (X, \tau) \to (Y, \sigma)$.

(i) *A-function of* f *and dual almost-continuity of it are equivalent if* (X, τ) *is a submaximal space.*

(ii) *Dual almost-continuity coincides with continuity if* (Y, σ) *is an almost-open space.*

(iii) *Continuity, A-function and dual almost-continuity are equivalent if* (X, τ) *is submaximal and* (X, σ) *is almost-open.*

Proof. It follows directly by applying Proposition 7 above and the meaning of an almost-openness of any space.

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