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AN INTEGRAL ANALOGUE OF A THEOREM OF LEINDLER

Dedicated to Professor Janina Wolska-Bochenek

1. Preliminaries

Let L_{loc} be the set of all measurable (complex-valued) functions Lebesgue-integrable on every finite subinterval of $R = (-\infty, \infty)$. Denote by L^∞ the set of those functions $f \in L_{\mathrm{loc}}$ for which

$$\|f\| = \operatorname*{ess\,sup}_{-\infty<t<\infty} |f(t)| < \infty.$$

Write $f \in W^\alpha$, $1 \le \alpha < \infty$, if $f \in L_{\mathrm{loc}}$ and

$$\int_{-\infty}^{\infty} \left(\frac{|f(t)|}{1+|t|}\right)^\alpha dt < \infty.$$

If $f \in L_{\mathrm{loc}}$ and if

$$\|f\|_2 = \left\{ \int_{-\infty}^{\infty} |f(t)|^2 dt \right\}^{1/2} < \infty,$$

the Fourier transform $\widehat{f}$ is defined by

$$\widehat{f}(v) = \underset{\varrho\to\infty}{\overset{(2)}{\mathrm{l.i.m.}}} \frac{1}{\sqrt{2\pi}} \int_{-\varrho}^{\varrho} f(u) e^{-iuv} du \quad \text{for a.e. } v \in R.$$

In this case, the Parseval formula

$$\|\widehat{f}\|_2 = \|f\|_2 \tag{1}$$

holds (see [1, p. 209]).

Let E_σ, $0 \le \sigma < \infty$, be the class of all entire functions of exponential type, of order σ at most. Considering a function $f \in L_{\mathrm{loc}}$, introduce the quantity

$$A_\sigma(f) = \inf_{F\in E_\sigma} \|f - F\|$$

called the best approximation of f by entire functions of class E_σ. Obviously, $A_\sigma(f)$ is non-negative and, sometimes, equals infinity. Moreover, $A_\sigma(f) \geq A_\tau(f)$ for $\tau > \sigma \geq 0$.

Take into account the entire function D_λ, $0 \leq \lambda < \infty$, given by

$$D_\lambda(z) = \frac{\sin \lambda z}{z} \quad \text{for } z \neq 0.$$

Suppose that $f \in L_{\text{loc}}$ and write

$$S_\lambda[f](z) = \frac{1}{\pi} \int\limits_{\to -\infty}^{\to +\infty} f(u) D_\lambda(z-u)\,du,$$

whenever the improper Lebesgue integral on the right-hand side exists. This is the so-called Dirichlet (singular) integral of f at a point $z = x + iy$ $(x, y \in R)$.

Put

$$V_{n,m}^{\{p\}}[f](x) = \left\{ \frac{1}{m} \int\limits_{n-m}^{n} |S_\lambda[f](x) - f(x)|^p d\lambda \right\}^{1/p}$$

for positive numbers m, n, p $(m \leq n)$ and $x \in R$. Let, by convention,

$$V_{n,m}(x) = V_{n,m}[f](x) = V_{n,m}^{\{1\}}[f](x)$$

and

$$\tau_{n,m}(x) = \tau_{n,m}[f](x) = m\{V_{n+m,m}(x) - V_{n,m}(x)\}.$$

It was proved in [3, Section 4] that under the assumptions $f \in W^1 \cap W^2$, $A_\sigma(f) < \infty$ for $\sigma > 0$ and $0 < m < n < \infty$, the estimate

$$\|V_{n,m}^{\{p\}}[f]\| \leq K(p) A_{n-m}(f) \log \frac{2n}{m} \tag{2}$$

holds with a certain constant $K(p)$ depending only on p. Now, in case $p = 1$, an other result will be presented. It corresponds to the theorem on strong approximation by Fourier series formulated in [2, p. 29].

The symbols K, K_1, K_2 used below will mean positive absolute constants not necessarily the same at each occurrence.

2. Basic estimates

Consider a function f from $W^1 \cap W^2$, with $A_\sigma(f) < \infty$ for $\sigma > 0$.

LEMMA 1. *Suppose that $0 < m < n < \infty$, $m \leq q < \infty$. Then $S_\lambda[f](x)$ for $x \in R$ is continuous in $\lambda \geq 0$ and*

$$\sup_{x\in R}\left\{\frac{1}{m}\int_{n-m}^{n}|S_{\lambda+q}[f](x)-S_\lambda[f](x)|d\lambda\right\}\le KA_{n-m}(f)\log\frac{2q}{m}. \tag{3}$$

Proof. Since $f\in W^1$, the Dirichlet integral $S_\lambda[f](x)$ is continuous in $\lambda\ge 0$ (see Remark in [3, Sec. 2]).

Given any $\varepsilon>0$, let F be a function of class E_{n-m} such that

$$\|f-F\|<A_{n-m}(f)+\varepsilon.$$

Evidently $F\in W^2$ and, by Lemma 1 of [3, Sec. 3],

$$S_\lambda[f-F](x)=S_\lambda[f](x)-F(x)\quad\text{for } \lambda>n-m \text{ and } x\in R. \tag{4}$$

In view of (4),

$$\begin{aligned}
&\int_{n-m}^{n}|S_{\lambda+q}[f](x)-S_\lambda[f](x)|d\lambda\\
&=\int_{n-m}^{n}|S_{\lambda+q}[f-F](x)-S_\lambda[f-F](x)|d\lambda\\
&=\int_{n-m}^{n}\left|\frac{1}{\pi}\lim_{\varrho\to\infty}\int_{-\varrho}^{\varrho}\{f(x-t)-F(x-t)\}\{D_{\lambda+q}(t)-D_\lambda(t)\}dt\right|d\lambda\\
&=\int_{n-m}^{n}\left|\frac{1}{2\pi}\lim_{\varrho\to\infty}\int_{-\varrho}^{\varrho}\{f(x-t)-F(x-t)+f(x+t)-F(x+t)\}\right.\\
&\left.\times\{D_{\lambda+q}(t)-D_\lambda(t)\}dt\right|d\lambda.
\end{aligned}$$

Putting

$$g_x(t)=\begin{cases}\frac{f(x-t)-F(x-t)+f(x+t)-F(x+t)}{t} & \text{if } |t|\ge\frac{1}{m},\\ 0\quad\text{otherwise,}\end{cases}$$

we obtain

$$\begin{aligned}
&\int_{n-m}^{n}|S_{\lambda+q}[f](x)-S_\lambda[f](x)|d\lambda\\
&\le\int_{n-m}^{n}\left\{\frac{1}{\pi}\left(\int_0^{1/q}+\int_{1/q}^{1/m}\right)|f(x-t)-F(x-t)+f(x+t)-F(x+t)|\right.\\
&\left.\times\left|\frac{\sin(\lambda+q)t-\sin\lambda t}{t}\right|dt\right\}d\lambda\\
&\quad+\int_{n-m}^{n}\left|\frac{1}{2\pi}\lim_{\varrho\to\infty}\int_{-\varrho}^{\varrho}g_x(t)\{\sin(\lambda+q)t-\sin\lambda t\}dt\right|d\lambda
\end{aligned}$$

$$=: I_1 + I_2 + I_3.$$

Clearly,

$$I_1 \le \frac{2}{\pi} \int\limits_{n-m}^{n} \left\{ (A_{n-m}(f) + \varepsilon) \int\limits_{0}^{1/q} \left| \frac{\sin(\lambda+q)t - \sin\lambda t}{t} \right| dt \right\} d\lambda$$

$$\le \frac{2}{\pi}(A_{n-m}(f) + \varepsilon) \int\limits_{n-m}^{n} \left\{ \int\limits_{0}^{1/q} q\,dt \right\} d\lambda = \frac{2}{\pi}(A_{n-m}(f) + \varepsilon) m.$$

Analogously,

$$I_2 \le \frac{2}{\pi} \int\limits_{n-m}^{n} \left\{ (A_{n-m}(f) + \varepsilon) \int\limits_{1/q}^{1/m} \frac{2}{t} dt \right\} d\lambda = \frac{4}{\pi}(A_{n-m}(f) + \varepsilon) m \log \frac{q}{m}.$$

By the Cauchy-Schwarz inequality and the Parseval formula (1),

$$\frac{1}{m} I_3 \le \left\{ \frac{1}{m} \int\limits_{n-m}^{n} \left| \frac{1}{2\pi} \lim_{\varrho\to\infty} \int\limits_{-\varrho}^{\varrho} g_x(t)(\sin(\lambda+q)t - \sin\lambda t)\, dt \right|^2 d\lambda \right\}^{1/2}$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \frac{1}{m} \int\limits_{n-m}^{n} |\widehat{g}_x(\lambda+q) - \widehat{g}_x(\lambda)|^2 d\lambda \right\}^{1/2}$$

$$\le \sqrt{\frac{2}{\pi m}} \left\{ \int\limits_{-\infty}^{\infty} |g_x(t)|^2 dt \right\}^{1/2} = \sqrt{2}\sqrt{\frac{2}{\pi m}} \left\{ \int\limits_{1/m}^{\infty} |g_x(t)|^2 dt \right\}^{1/2}$$

$$\le 4\sqrt{\frac{1}{\pi m}} (A_{n-m}(f) + \varepsilon) \left\{ \int\limits_{1/m}^{\infty} \frac{1}{t^2}\, dt \right\}^{1/2}.$$

Consequently,

$$I_3 \le 4\sqrt{1/\pi}(A_{n-m}(f) + \varepsilon) m.$$

Summing up we get at once the desired assertion (3).

LEMMA 2. *Let* $0 < m \le n < \infty$ *and let* $0 < 2\mu \le m$. *Then*

$$\|\tau_{n,m}[f] - \tau_{n-\mu,m-\mu}[f]\| \le K\mu A_{n-\mu}(f) \log \frac{m}{\mu}. \tag{5}$$

Proof. Clearly, for a.e. $x \in R$,

$$\tau_{n,m}(x) - \tau_{n-\mu,m-\mu}(x) = \Big(\int\limits_{n}^{n+m} - \int\limits_{n-\mu}^{n} - \int\limits_{n-\mu}^{n+m-2\mu} \Big) |S_\lambda[f](x) - f(x)| d\lambda$$

$$= \Big(\int\limits_{n+m-2\mu}^{n+m} - \int\limits_{n-\mu}^{n} - \int\limits_{n-\mu}^{n} \Big) |S_\lambda[f](x) - f(x)| d\lambda.$$

By Lemma 1,

$$\Big|\Big(\int\limits_{n+m-2\mu}^{n+m-\mu} - \int\limits_{n-\mu}^{n}\Big)|S_\lambda[f](x) - f(x)|d\lambda\Big| = \mu|V_{n+m-\mu,\mu}(x) - V_{n,\mu}(x)|$$

$$= \Big|\int\limits_{n-\mu}^{n} (|S_{\lambda+m-\mu}[f](x) - f(x)| - |S_\lambda[f](x) - f(x)|)d\lambda\Big|$$

$$\leq \mu\cdot\frac{1}{\mu}\int\limits_{n-\mu}^{n} |S_{\lambda+m-\mu}[f](x) - S_\lambda[f](x)|d\lambda$$

$$\leq \mu K A_{n-\mu}(f)\log\frac{2(m-\mu)}{\mu},$$

$$\Big|\Big(\int\limits_{n+m-\mu}^{n+m} - \int\limits_{n-\mu}^{n}\Big)|S_\lambda[f](x) - f(x)|d\lambda\Big|$$

$$\leq \int\limits_{n-\mu}^{n} |S_{\lambda+m}[f](x) - S_\lambda[f](x)|d\lambda$$

$$\leq \mu K A_{n-\mu}(f)\log\frac{2m}{\mu}.$$

Hence

$$\|\tau_{n,m} - \tau_{n-\mu,m-\mu}\| \leq K\mu A_{n-\mu}(f)\Big\{\log\frac{2(m-\mu)}{\mu} + \log\frac{2m}{\mu}\Big\},$$

and the estimate (5) is established.

PROPOSITION . *If* $0 < m < n < \infty$ *then*

$$\|\tau_{n,m}[f]\| \leq K\sum_{\nu=0}^{[m]} A_{n-m+\nu}(f). \tag{6}$$

P r o o f. In case $0 < m < 2$ Lemma 1 gives

$$\|\tau_{n,m}[f]\| = m\|V_{n+m,m} - V_{n,m}\|$$

$$= \Big\|\int\limits_{n}^{n+m} |S_\lambda[f] - f|d\lambda - \int\limits_{n-m}^{n} |S_\lambda[f] - f|d\lambda\Big\|$$

$$\leq \Big\|\int\limits_{n-m}^{n} |S_{\lambda+m}[f] - S_\lambda[f]|d\lambda\Big\|$$

$$\leq KmA_{n-m}(f)\log\frac{2m}{m} \leq K_1A_{n-m}(f).$$

Therefore, we may suppose that $m \geq 2$.

Following Stechkin ([2, p. 33]) take into account the positive sequence defined by

$$m_0 = m, \quad m_s = m_{s-1} - [m_{s-1}/2] \; (s = 1, 2, \ldots).$$

Evidently, there exists a smallest index $t \geq 1$ such that $1 \leq m_t < 2$ and

$$m = m_0 > m_1 > \ldots > m_t \geq 1, \quad m_{t-1} - m_t = 1.$$

Since

$$\tau_{n,m} = \sum_{s=1}^{t} \{\tau_{n-m+m_{s-1},m_{s-1}} - \tau_{n-m+m_s,m_s}\} + \tau_{n-m+m_t,m_t},$$

we have

$$\|\tau_{n,m}\| \leq \sum_{s=1}^{t} \|\tau_{n-m+m_{s-1},m_{s-1}} - \tau_{n-m+m_s,m_s}\| + \|\tau_{n-m+m_t,m_t}\|.$$

In view of Lemma 2,

$$\|\tau_{n-m+m_{s-1},m_{s-1}} - \tau_{n-m+m_s,m_s}\| \leq K\mu A_{n-m+m_{s-1}-\mu}(f) \log \frac{m_{s-1}}{\mu}$$

where $\mu = m_{s-1} - m_s > m_{s-1}/4$ $(s = 1, \ldots, t)$.

By Lemma 1,

$$\|\tau_{n-m+m_t,m_t}\| = m_t \|V_{n-m+2m_t,m_t} - V_{n-m+m_t,m_t}\|$$
$$\leq 2 \left\| \frac{1}{m_t} \int_{n-m}^{n-m+m_t} |S_{\lambda+m_t}[f] - S_\lambda[f]| d\lambda \right\| \leq K A_{n-m}(f) \log 2.$$

Hence

$$\|\tau_{n,m}\| \leq K_1 \Big\{ \sum_{s=1}^{t} (m_{s-1} - m_s) A_{n-m+m_s}(f) + A_{n-m}(f) \Big\}$$
$$\leq K_1 \Big\{ 4 \sum_{s=1}^{t-1} (m_s - m_{s+1}) A_{n-m+m_s}(f) + A_{n-m+m_t} + A_{n-m}(f) \Big\},$$

i.e.

$$\|\tau_{n,m}\| \leq K_2 \Big\{ \sum_{s=1}^{t-1} \int_{m_{s+1}}^{m_s} A_{n-m+\nu}(f) d\nu + A_{n-m}(f) \Big\}$$
$$= K_2 \Big\{ \int_{m_t}^{m_1} A_{n-m+\nu}(f) d\nu + A_{n-m}(f) \Big\}.$$

Further,

$$\int_{m_t}^{m_1} A_{n-m+\nu}(f)d\nu \le \int_{1}^{m-1} A_{n-m+\nu}(f)d\nu \le \sum_{k=1}^{[m]-1} \int_{k}^{k+1} A_{n-m+\nu}(f)d\nu.$$

Consequently,

$$\|\tau_{n,m}\| \le K_2 \sum_{k=0}^{[m]-1} A_{n-m+k}(f),$$

which leads to (6), immediately.

3. Main result

Suppose that $f \in W^1 \cap W^2$, $A_\sigma(f) < \infty$ for $\sigma > 0$, $\lim_{\sigma\to\infty} A_\sigma(f) = 0$.

THEOREM . *If* $0 < m < n < \infty$ *then*

$$\|V_{n,m}[f]\| \le K \sum_{\nu=0}^{[n]} \frac{A_{n-m+\nu}(f)}{m+\nu}. \tag{7}$$

Proof. When $0 < n < 1$,

$$\left\| \int_{n-m}^{n} |S_\lambda[f] - f| d\lambda \right\| \le K_1 A_{n-m}(f).$$

Indeed, given any $\varepsilon > 0$ there is an entire function $F \in E_{n-m}$ such that

$$\|f - F\| < A_{n-m}(f) + \varepsilon.$$

Thus, by Lemma 1 and Lemma 2′ of [3, Sec. 3],

$$\begin{aligned}
&\int_{n-m}^{n} |S_\lambda[f](x) - f(x)| d\lambda \\
&= \int_{n-m}^{n} |S_\lambda[f-F](x) + F(x) - f(x)| d\lambda \\
&< Km\{A_{n-m}(f) + \varepsilon\} \log \frac{2n}{m} \\
&\quad + m|F(x) - f(x)| \quad \text{for a.e. } x \in R.
\end{aligned}$$

Consequently,

$$\left\| \int_{n-m}^{n} |S_\lambda[f] - f| d\lambda \right\| \le KmA_{n-m}(f) \log \frac{2}{m} + A_{n-m}(f) \le K_1 A_{n-m}(f),$$

and the estimate (7) follows.

In case $n \geq 1$ we construct, similarly to [2, p. 34], a finite sequence $\{n_k\}$ of positive numbers $n_0 = n, n_1 > n_0, \ldots$ Namely, assuming that $n_s < 2n$, we define n_{s+1} as follows. Denote by ν_s the smallest non-negative integer such that

$$A_{n_s-m+\nu_s}(f) \leq \frac{1}{2}A_{n_s-m}(f) \tag{8}$$

and set

$$n_{s+1} = \begin{cases} n_s + m & \text{if } \nu_s \leq m, \\ n_s + \nu_s & \text{if } m < \nu_s < 2n + m - n_s, \\ 2n + m & \text{if } \nu_s \geq 2n + m - n_s. \end{cases}$$

When $n_{s+1} < 2n$ we continue the procedure; in case $n_{s+1} \geq 2n$ we stop the construction and define $t := s + 1$.

Clearly,

$$n = n_0 < n_1 < \ldots < n_t, \quad 2n \leq n_t \leq 2n + m \tag{9}$$

and $n_{s+1} - n_s \geq m$ for $s = 0, 1, \ldots, t-1$. By (8),

$$A_{n_{s+1}-m}(f) \leq \frac{1}{2}A_{n_s-m}(f) \quad \text{for } s = 0, 1, \ldots, t-2. \tag{10}$$

Moreover, if $n_{s+1} - n_s > m$ $(0 \leq s \leq t-1)$ then

$$\frac{1}{2}A_{n_s-m}(f) \leq A_{n_{s+1}-m-1}(f) \tag{11}$$

(see [2, p. 35]).

Since

$$V_{n,m} = \sum_{s=0}^{t-1}\{V_{n_s,m} - V_{n_{s+1},m}\} + V_{n_t,m},$$

we have

$$\|V_{n,m}\| \leq \sum_{s=0}^{t-1}\|V_{n_s,m} - V_{n_{s+1},m}\| + \|V_{n_t,m}\|.$$

In view of (2) and (9),

$$\begin{aligned} \|V_{n_t,m}\| &\leq KA_{n_t-m}(f)\log\frac{2n_t}{m} \leq KA_{2n-m}(f)\log\frac{4n+2m}{m} \\ &\leq 2KA_{2n-m}(f)\log\frac{2[n]+m+2}{m} \leq 2KA_{2n-m}(f)\sum_{\nu=0}^{2[n]+1}\frac{1}{m+\nu} \\ &\leq 4KA_{n-m+[n]}(f)\sum_{\nu=0}^{[n]}\frac{1}{m+\nu} \leq 4K\sum_{\nu=0}^{[n]}\frac{A_{n-m+\nu}(f)}{m+\nu}. \end{aligned}$$

Putting $\lambda_s = [n_{s+1} - n_s]$, we observe that

$$(12) \quad \|V_{n_{s+1},m} - V_{n_s,m}\| \le K \sum_{\nu=0}^{\lambda_s} \frac{A_{n_s-m+\nu}(f)}{m+\nu} \quad \text{for } s = 0, 1, \dots, t-1.$$

Indeed, if $n_{s+1} - n_s = m$ then

$$V_{n_{s+1},m} - V_{n_s,m} = V_{n_s+m,m} - V_{n_s,m} = \frac{1}{m}\tau_{n_s,m}.$$

Hence, by Proposition,

$$\|V_{n_{s+1},m} - V_{n_s,m}\| \le \frac{K}{m} \sum_{\nu=0}^{[m]} A_{n_s-m+\nu}(f) \le 2K \sum_{\nu=0}^{[m]} \frac{A_{n_s-m+\nu}(f)}{m+\nu}.$$

In case $n_{s+1} - n_s > m$, Lemma 1 and (11) yield

$$\begin{aligned} \|V_{n_{s+1},m} - V_{n_s,m}\| &\le K A_{n_s-m}(f) \log \frac{2(n_{s+1} - n_s)}{m} \\ &\le K A_{n_s-m}(f) \frac{2}{m} \quad \text{if } \lambda_s = 0, \end{aligned}$$

and

$$\begin{aligned} \|V_{n_{s+1},m} - V_{n_s,m}\| &\le 4K A_{n_{s+1}-m-1}(f) \sum_{\nu=0}^{\lambda_s} \frac{1}{m+\nu} \\ &\le 8K \sum_{\nu=0}^{\lambda_s-1} \frac{A_{n_s-m+\nu}(f)}{m+\nu} \quad \text{if } \lambda_s \ge 1. \end{aligned}$$

Thus (12) holds.

In view of (12),

$$\sum_{s=0}^{t-1} \|V_{n_{s+1},m} - V_{n_s,m}\| \le K \sum_{s=0}^{t-1} \sum_{\nu=0}^{\lambda_s} \frac{A_{n_s-m+\nu}(f)}{m+\nu} =: K\Sigma_2.$$

Since $\lambda_s \le n_{s+1} - n_s \le 2n + m - n = n + m$ for $s = 0, 1, \dots, t-1$, we get

$$\Sigma_2 \le \sum_{\nu=0}^{[n+m]} \frac{1}{m+\nu} \sum_{s:\lambda_s \ge \nu} A_{n_s-m+\nu}(f).$$

Let $p = \min\{s : \lambda_s \ge \nu\}$. Then

$$\sum_{s:\lambda_s \ge \nu} A_{n_s-m+\nu}(f) = A_{n_p-m+\nu}(f) + \sum_{s \ge p+1:\lambda_s \ge \nu} A_{n_s-m+\nu}(f)$$

and, by (10),

$$\sum_{s\geq p+1:\lambda_s\geq\nu} A_{n_s-m+\nu}(f) \leq \sum_{s\geq p+1} A_{n_s-m}(f) \leq 2A_{n_{p+1}-m}(f)$$
$$\leq 2A_{n_p+\nu-m}(f) \leq 2A_{n-m+\nu}(f).$$

Consequently,

$$\Sigma_2 \leq \sum_{\nu=0}^{[n+m]} \frac{1}{m+\nu}\cdot 3A_{n-m+\nu}(f)$$
$$\leq 3\Big(\sum_{\nu=0}^{[n]} + \sum_{\nu=[n]+1}^{[2n]}\Big)\frac{A_{n-m+\nu}(f)}{m+\nu}$$
$$\leq 6\sum_{\nu=0}^{[n]}\frac{A_{n-m+\nu}(f)}{m+\nu}.$$

Thus

$$\|V_{n,m}\| \leq K_1\sum_{\nu=0}^{[n]}\frac{A_{n-m+\nu}(f)}{m+\nu} + K_2\sum_{\nu=0}^{[n]}\frac{A_{n-m+\nu}(f)}{m+\nu},$$

and the proof of (7) is complete.

Remark. In case $A_0(f) < \infty$, Theorem also holds when $0 < m = n < \infty$.

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