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ON THE SUPPORT THEOREM FOR STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS

Dedicated to Professor Janina Wolska-Bochenek

1. Introduction

The theorem on the support of a measure generated by the solution to a functional stochastic differential equation is examined. We consider a model similar to that in the papers of Dawidowicz and Twardowska [4] and of Twardowska [18], [19]. This note is a relatively simple consequence of the approximation theorem of Wong-Zakai type for the above equations from [18], [19] and of the support theorem of Millet and Sanz-Solé (see [13]). However, we restrict ourselves to spaces of continuous functions instead of spaces of Hölder functions since the generalization to Hölder functions is straightforward on the base of the quoted papers.

There are some papers dealing with the support of probability measures connected with stochastic differential equations in finite dimension; see e.g. Stroock and Varadhan [16], [17], Ikeda and Watanabe [8] for finite multidimensional stochastic differential equations. See also the paper of Aida, Kusuoka and Stroock [1], which uses a sequence of non-absolutely continuous transformations of a probability space. A characterization of the support in the finite-dimensional case on the space of Hölder-continuous functions was given by Ben Arous and Gradinaru in [3], Bally, Millet and Sanz-Solé in [2], Millet and Sanz-Solé in [13], [14]. The support of diffusion processes considered on manifolds was examined by Kunita in [9]. For stochastic differential equations driven by finite multidimensional continuous semimartingales, support theorems were given by Mackevičius in [11], [12] as well as by Gyöngy in [5], [6], Gyöngy and Pröhle in [7].

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It is well known that support theorems are important for the characterization of invariant sets and, consequently, for ergodic theorems.

2. Definitions and notations

Let $t \in [0,T]$ and let $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ be a complete probability space with $\mathcal{F}_t = (\mathcal{F}_t)_{t\in[0,T]}$ an increasing family of sub-σ-algebras of the σ-algebra $\mathcal{F}$. We put $J = [-r, 0]$ and we introduce the metric spaces $\mathcal{C}_- = C(J, \mathbb{R}^d), \mathcal{C}_1 = C([-r,T], \mathbb{R}^d)$ and $\mathcal{C}_2^0 = C([-r,T], \mathbb{R}^m) = \widetilde{\Omega}$ of continuous functions. The spaces $\mathcal{C}_-$, $\mathcal{C}_1$ and $\mathcal{C}_2^0$ are endowed with the usual norms of uniform convergence. Here d is the dimension of the state space and m is the dimension of the Wiener process; in the space $\mathcal{C}_2^0$ all functions are equal to zero at zero. Below we denote by $\mathcal{X}$ any of the above spaces.

Let $\mathcal{B}(\mathcal{X})$ denote the topological σ-algebra of the space $\mathcal{X}$. It is obvious that it is identical with the σ-algebra generated by the family of all Borel cylinder sets in $\mathcal{X}$. So we construct the Wiener space $(\mathcal{C}_2^0, \mathcal{B}(\mathcal{C}_2^0), P^w)$, where P^w is the Wiener measure ([8], Chapter I). The coordinate process $B(t,w) = w(t)$, $w \in \mathcal{C}_2^0$, is the m-dimensional Wiener process.

The smallest Borel algebra that contains $\mathcal{B}_1, \mathcal{B}_2, \ldots$ is denoted by $\mathcal{B}_1 \cup \mathcal{B}_2 \cup \ldots$; $\mathcal{B}_{u,v}(X)$ denotes the smallest Borel σ-algebra for which a given stochastic process $X(t)$ is measurable for every $t \in [u,v]$ and $\mathcal{B}_{u,v}(dB)$ denotes the smallest Borel algebra for which $B(s) - B(t)$ is measurable for every (t,s) with $u \le t \le s \le$v.

Let $B^n(t,w) = w_n(t)$ be the following piecewise linear $\mathcal{F}_t$-adapted approximation of $B(t,w) = w(t)$:

$$B^{n,p}(t,w) = w^p\left(\frac{k}{2^n}\right) + 2^n\left(t - \frac{k}{2^n}\right)\left(w^p\left(\frac{k+1}{2}\right) - w^p\left(\frac{k}{2^n}\right)\right) \tag{2.1}$$

for each $p = 1, \ldots, m$ and $(k+1)T/2^n \le t < (k+2)T/2^n$ for $k = 0, 1, \ldots$ $\ldots, 2^n - 1$.

For the stochastic process $X(t,w)$ and for a fixed $t \in [0,T]$ we define

$$X_t(\theta, w) = X(t+\theta, w), \qquad \theta \in J;$$

therefore $X_t(\cdot, w)$ denotes the segment of the trajectory $X(\cdot, w)$ on $[-r, t]$.

3. Description of the model

Now we consider $\widetilde{\Omega} = \mathcal{C}_2^0$. Let X be a continuous stochastic process $X(t,w) : [-r,T] \times \widetilde{\Omega} \to \mathbb{R}^d$, that is, $X : \widetilde{\Omega} \to \mathcal{X} = \mathcal{C}_1$.

We take some fixed initial constant stochastic processes $X^i(0+\theta, w) = X_0^i(w) = X_0^{n,i}(w) = Y_0^i(w)$ for $\theta \in J$, $i = 1, \ldots, d$. We also consider operators $b : \mathcal{C}_- \to \mathbb{R}^d$, $\sigma : \mathcal{C}_- \to L(\mathbb{R}^m, \mathbb{R}^d)$ (where $L(\mathbb{R}^m, \mathbb{R}^d)$ is the Banach space of linear functions from $\mathbb{R}^m$ to $\mathbb{R}^d$ with uniform operator norm $|\cdot|_L$).

We consider the following stochastic differential equation with delayed argument:

$$(3.1)\quad X^i(t,w) = X_0^i(w) + \int_0^t b^i(X_s(\cdot,w))\,ds + \sum_{p=1}^m \int_0^t \sigma^{ip}(X_s(\cdot,w))\,dw^p(s)$$

for $i = 1, \ldots, d$.

Let P_X be the probability law of the solution $X = \{X(t)\}$, $t \in [0,T]$, to equation (3.1). Let $\mathcal{H}$ be the Cameron–Martin space associated with the Brownian motion, that is, the space of functions $h : [0,T] \to \mathbb{R}^m$ which are absolutely continuous and whose derivative $\dot{h}$ belongs to $L^2([0,T], \mathbb{R}^m)$. Let

$$(3.2)\qquad \mathcal{S}_{\mathcal{H}} = \{h \in C_0^2 : h \in \mathcal{H},\ h(0) = 0\}.$$

We consider for given $h \in \mathcal{S}_{\mathcal{H}}$ and $x = X_0(w) \in \mathbb{R}^d$ the equation

$$(3.3)\qquad \xi^i(t) = X_0^i + \int_0^t b^i(\xi_s(\cdot))\,ds + \sum_{p=1}^m \int_0^t \sigma^{ip}(\xi_s(\cdot))\dot{h}^p(s)\,ds - \frac{1}{2}\sum_{p=1}^m \sum_{j=1}^d \int_0^t \widetilde{D}_j\sigma^{ip}(\xi_s(\cdot))\sigma^{jp}(\xi_s(\cdot))\,ds$$

for every $i = 1, \ldots, d$. Further, $D\sigma^{ip}$ is the Fréchet derivative from $\mathcal{C}_-$ to $L(\mathcal{C}_-, \mathbb{R})$. From the Riesz Theorem (see Rudin [15]) it follows that there exists a family of measures $\mu = \mu_g^{ipj}$ with bounded variation such that

$$D\sigma^{ip}(g)(\Phi) = \sum_{j=1}^d \int_{-r}^0 \Phi_j(v)\mu_g^{ipj}(dv)$$

is a directional derivative, for any $\Phi, g \in \mathcal{C}_-$. The measure μ has the following decomposition:

$$\mu(A) = \mu(A \cap [-r,0)) + \mu(A \cap \{0\}) = \widetilde{\mu}(A) + \mu(\{0\})\delta_0(A),$$

where δ_0 is the Dirac measure, $A \in \mathcal{B}([-r,0))$. We denote the value $\mu_g^{ipj}(\{0\})$ by $\widetilde{D}_j\sigma^{ip}(g)$, that is, $\widetilde{D}_j\sigma^{ip}(\xi_s(\cdot,w)) = \mu_{s,w,\xi}^{ipj}(\{0\})$, where $\mu_{s,v,\xi}^{ipj} = \mu_{\xi_s(\cdot,w)}^{ipj}$.

Let

$$(3.4)\qquad S_1 = \operatorname{supp} P_X \quad \text{in } G = \mathcal{C}_1,$$

$$(3.5)\qquad S_2 = \overline{\{\xi = \xi(x,h) : h \in \mathcal{S}_{\mathcal{H}}\}} \quad \text{(closure in } G).$$

Denote by T_n^h the map $T_n^h(w) = w - B^n + \mathrm{h}$ and observe that by the Girsanov theorem the measures $P \circ (T_n^h)^{-1}$ are absolutely continuous with respect to P.

Let us introduce the following conditions:

($\widetilde{A}1$) The initial stochastic process X_0 is $\mathcal{F}_0$ – measurable and $P(|X_0(w)| < \infty) = 1$, where $|X_0(w)| = \sum_{j=1}^{d} |X_0^i(w)|$, $\mathcal{B}_{-r,0}(X_0)$ is independent of $\mathcal{B}_{0,T}(B)$.

($\widetilde{A}2$) For every $\varphi, \psi \in \mathcal{C}_-$ the following Lipschitz condition is satisfied:

$$|b(\varphi) - b(\psi)|^2 + |\sigma(\varphi) - \sigma(\psi)|_L^2 \leq L^1 \int_{-r}^{0} |\varphi(\theta) - \psi(\theta)|^2 \, dK(\theta) + \\ + L^2 |\varphi(0) - \psi(0)|^2,$$

where $K(\theta)$ is a certain bounded measure on J, and L^1, L^2 are some constants.

($\widetilde{A}3$) For every $\varphi \in \mathcal{C}_-$ the following growth condition is satisfied:

$$|b(\varphi)|^2 + |\sigma(\varphi)|_L^2 \leq L^1 \int_{-r}^{0} (1 + \varphi^2(\theta)) \, dK(\theta) + L^2(1 + \varphi^2(0)),$$

where $\varphi^2(0) = \sum_{i=1}^{d} \varphi_i^2(0)$.

($\widetilde{A}4$) $b^i, \sigma^{ip} \in C_b^1(\mathcal{C}_-)$, for every $i = 1, \ldots, d$, $p = 1, \ldots, m$, where C_b^1 denotes the space of bounded mappings with continuous bounded first derivative, and the first derivatives of σ^{ip} satisfy the Lipschitz condition.

Notice that conditions ($\widetilde{A}1$)–($\widetilde{A}4$) ensure the existence and uniqueness of solutions for equations considered in this paper (compare [18]).

Let $(E, \|\cdot\|)$ be a separable Banach space. Here $(E, \|\cdot\|) = (\mathcal{C}_1, \sup|\cdot|)$. Further we consider the following equations for $i = 1, \ldots, d$:

$$X^{n,i}(t, w) = X_0^{n,i}(w) + \int_0^t b^i(X_s^n(\cdot, w)) \, ds + \\ + \sum_{p=1}^{m} \int_0^t \sigma^{ip}(X_s^n(\cdot, w)) \, dw^p(s) - \sum_{p=1}^{m} \int_0^t \sigma^{ip}(X_s^n(\cdot, w)) \dot{B}^{n,p}(s, w) \, ds + \\ + \sum_{p=1}^{m} \int_0^t \sigma^{ip}(X_s^n(\cdot, w)) \dot{h}^p(s) \, ds, \tag{3.6^n}$$

$$(3.7^n) \qquad \xi^{n,i}(t,w) = X_0^{n,i}(w) + \int_0^t b^i(\xi_s^n(\cdot,w))\,ds + $$
$$+ \sum_{p=1}^m \int_0^t \sigma^{ip}(\xi_s^n(\cdot,w))\dot{B}^{n,p}(s)\,ds -$$
$$-\frac{1}{2}\sum_{p=1}^m \sum_{j=1}^d \int_0^t \widetilde{D}_j\sigma^{ip}(\xi_s^n(\cdot,w))\sigma^{jp}(\xi_s^n(\cdot,w))\,ds.$$

Both processes X^n and ξ^n are particular cases of the stochastic process $Y^n = (Y^{n,1},\dots,Y^{n,d})$ which components are the solutions to the stochastic differential equations:

$$(3.8^n) \qquad Y^{n,i}(t,w) = Y_0^{n,i}(w) + \sum_{p=1}^m \int_0^t \widehat{F}^{ip}(Y_s^n(\cdot,w))\,dw^p(s) +$$
$$+ \int_0^t \widehat{B}^i(Y_s^n(\cdot,w))\,ds + \sum_{p=1}^m \int_0^t \widehat{G}^{ip}(Y_s^n(\cdot,w))\dot{B}^{n,p}(s,w)\,ds +$$
$$+ \sum_{p=1}^m \int_0^t \widehat{H}^{ip}(Y_s^n(\cdot,w))\dot{h}^p(s)\,ds,$$

where the coefficients $\widehat{F}, \widehat{G}, \widehat{H}$ and $\widehat{B}$ satisfy by our assumptions the condition:

(C) the functions $\widehat{F}, \widehat{G}, \widehat{H} : \mathcal{C}_- \to L(\mathbb{R}^m, \mathbb{R}^d)$ are globally Lipschitz, $\widehat{G}$ is of class C^1 with bounded partial derivatives and the first derivatives satisfying the Lipschitz condition, $\widehat{B} : \mathcal{C}_- \to \mathbb{R}^d$ is globally Lipschitz.

Given the coefficients $\widehat{F}, \widehat{G}, \widehat{H}$ and $\widehat{B}$, let the process Z has the components Z^i being the solutions to the stochastic differential equations, for $i = 1,\dots,d$:

$$(3.9) \quad Z^i(t,w) = X_0^i(w) + \sum_{p=1}^m \int_0^t (\widehat{F}^{ip}(Z_s(\cdot,w)) + \widehat{G}^{ip}(Z_s(\cdot,w)))\,dw^p(s) +$$
$$+ \sum_{p=1}^m \int_0^t \widehat{H}^{ip}(Z_s(\cdot,w))\dot{h}^p(s)\,ds + \int_0^t \widehat{B}^i(Z_s(\cdot,w))\,ds +$$
$$+ \sum_{p=1}^m \sum_{j=1}^d \int_0^t \widetilde{D}_j\widehat{G}^{ip}(Z_s(\cdot,w))(\widehat{F}^{jp}(Z_s(\cdot,w)) + \frac{1}{2}\widehat{G}^{jp}(Z_s(\cdot,w)))\,ds.$$

Remark 3.1. Observe that if $\widehat{F} = 0$, $\widehat{G} = \sigma$, $\widehat{H} = 0$, $\widehat{B} = b - \frac{1}{2}\widetilde{D}_j\sigma \cdot \sigma$, then we obtain

$$Y^n(t) - Z(t) = \xi^n(t) - X(t), \tag{3.10}$$

while if $\widehat{F} = \sigma$, $\widehat{G} = -\sigma$, $\widehat{H} = \sigma$ and $\widehat{B} = b$ we obtain

$$Y^n(t) - Z(t) = X^n(t) - \xi(t). \tag{3.11}$$

4. Approximation theorem of Wong–Zakai type

In [18], [19] the following is proved:

THEOREM 4.1. *Let conditions* $(\widetilde{A}1)$-$(\widetilde{A}4)$ *be satisfied. Let* $B^n(t,w)$ *be the approximation of the type* (2.1) *of the Wiener process. Assume that* X^n *and* ξ *are the solutions to* (3.6^n) *and* (3.3), *respectively, and that* ξ^n *and* X *are the solutions to* (3.7^n) *and* (3.3), *respectively, with a constant initial stochastic process. Then, for every* $T > 0$,

$$\lim_{n\to\infty} \sup_{0\le t\le T} E[|X^n(t,w) - \xi(t,w)|^2] = 0, \tag{4.1}$$

$$\lim_{n\to\infty} \sup_{0\le t\le T} E[|\xi^n(t,w) - X(t,w)|^2] = 0. \tag{4.2}$$

Remark 4.1. Instead of the interval $J = (-\infty, 0]$ in [18], [19] we can consider $J = [-r, 0]$, $r > 0$, and the proof is analogous to that of Theorem 2.4.1 in [18]. But, instead of considering $X^i(t_i^n + s) - X^i(t_{i-1}^n + s)$ on the whole interval of definition like in [18], [19], we observe that

$$X^i(t_i^n + s) - X^i(t_{i-1}^n + s) = \\ = \begin{cases} X_0^i(t_i^n + s) - X_0^i(t_{i-1}^n + s) & \text{for } t_i^n + s \le 0, \\ X_0^i(0) - X_0^i(t_{i-1}^n + s) + \int\limits_0^{t_i^n + s} b^i(X_u(\cdot))\,du + \\ \quad + \sum_{p=1}^m \int\limits_0^{t_i^n + s} \sigma^{ij}(X_u(\cdot))\,dw^p(u) & \text{for } t_{i-1}^n + s \le 0 \le t_i^n + s, \\ \int\limits_{t_{i-1}^n + s}^{t_i^n + s} b^i(X_u(\cdot))\,du + \sum_{p=1}^m \int\limits_{t_{i-1}^n + s}^{t_i^n + s} \sigma^{ij}(u, X_u(\cdot))\,dw^p(u) & \text{for } t_{i-1}^n + s > 0, \end{cases}$$

and we can estimate each part separately by expressions converging to zero to obtain the proof of Theorem 4.1.

5. Auxiliary lemmas

LEMMA 5.1. *Let $\widehat{F}$, $\widehat{G}$, $\widehat{H}$ and $\widehat{B}$ be bounded coefficients satisfying condition* (C) *and let Y^n be the solution to* (3.8^n). *Then for given $p \in [1, \infty)$ there exists a constant C_1 such that for every $s, t \in [0, T]$*

$$\sup_n E(|Y^n(t) - Y^n(s)|^{2p}) \le C_1 |t - s|^p. \tag{5.1}$$

Proof. The proof is analogous to that of Proposition 3.1 in [13].

Remark 5.1. The proofs of Lemmas 5.1 and 5.4 are not different from those of analogous lemmas in [13] because in [13] the authors do not use the form of the equation but only the form of the stochastic differential.

LEMMA 5.2. *Let $\widehat{F}$, $\widehat{G}$, $\widehat{H}$ and $\widehat{B}$ be bounded coefficients satisfying condition* (C) *and let $Z(t)$ be the solution to* (3.9). *Then for given $p \in [1, \infty)$ there exists a constant C_2 such that for every $s, t \in [0, T]$*

$$E(|Z(t) - Z(s)|^{2p}) \le C_2 |t - s|^p. \tag{5.2}$$

Proof. For the proof of this property see e.g. Lemma 4.11 of the book of Liptser and Shiryaev [10].

LEMMA 5.3. *Let $Y_n(t) = Y^n(t) - Z(t)$. Then for any $\varepsilon > 0$ we have*

$$\lim_{n\to\infty} P\Big(\sup_{0\le i\le 2^n} \Big| Y_n\Big(\frac{i}{2^n}\Big)\Big| > \varepsilon \Big) = 0. \tag{5.3}$$

Proof. Using the Wong–Zakai type Theorem 4.1 and Remark 3.1, i.e. (3.11), we conclude from the Chebyshev inequality that

$$P\Big(\sup_{0\le i\le 2^n} \Big| Y_n\Big(\frac{i}{2^n}\Big)\Big| > \varepsilon \Big) \le \frac{E\big(\sup_{0\le i\le 2^n} |Y_n(\frac{i}{2^n})|\big)^2}{\varepsilon^2} =$$

$$= \frac{E(\sup_{0\le i\le 2^n} |Y^n(\frac{i}{2^n}) - Z(\frac{i}{2^n})|)^2}{\varepsilon^2} \le \frac{E(\sup_t |Y^n(t) - Z(t)|)^2}{\varepsilon^2} \to 0$$

as $n \to \infty$, which completes the proof.

As a natural consequence of Lemma 5.3 we formulate

LEMMA 5.4. *Let the sequence $Y_n(t) = Y^n(t) - Z(t)$ satisfy* (5.1)–(5.3). *Then for every $\varepsilon > 0$ we get*

$$\lim_{n\to\infty} P(\|Y_n\|_{C_1} > \varepsilon) = 0. \tag{5.4}$$

We have

LEMMA 5.5. *Let $F : \Omega \to E$ be a measurable map.*

(i) *Let $\zeta_1 : \mathcal{H} \to E$ be a measurable map, and let $H_n : \Omega \to \mathcal{H}$ be a sequence of random variables such that for any $\varepsilon > 0$,*

$$\lim_n P(\|F(\omega) - \zeta_1(H_n(\omega))\| > \varepsilon) = 0. \tag{5.5}$$

Then

$$\mathrm{supp}(P \circ F^{-1}) \subset \overline{\zeta_1(\mathcal{H})}. \tag{5.6}$$

(ii) *Fix a map $\zeta_2 : \mathcal{H} \to E$ and for given $h \in \mathcal{H}$ consider a sequence of measurable transformations $T_n^h : \Omega \to \Omega$ such that $P \circ (T_n^h)^{-1} \ll P$, and for any $\varepsilon > 0$,*

$$\limsup_n P(\|F(T_n^h(\omega)) - \zeta_2(h)\| < \varepsilon) > 0. \tag{5.7}$$

Then

$$\mathrm{supp}(P \circ F^{-1}) \supset \overline{\zeta_2(\mathcal{H}}. \tag{5.8}$$

Proof. (i) Let $U \cap \zeta_1(\mathcal{H}) = \emptyset$, where U is an open set in E. It is sufficient to show that $(P \circ F^{-1})(U) = 0$, that is, $P(F(\omega) \in U) = 0$. For a contradiction, suppose that $P(\{\omega : F(\omega) \in U\}) > 0$. Then there exists a ball $K(z, \varepsilon) \subset U$ such that $P(\{\omega : F(\omega) \in K(z, \varepsilon)\}) > 0$.Hence, there exists $\varepsilon' < \varepsilon$ such that $P(\{\omega : F(\omega) \in K(z, \varepsilon')\}) > 0$. Define $\varepsilon'' = \varepsilon - \varepsilon' > 0$. From (5.5) it follows that $\lim_n P(\|F(\omega) - \zeta_1(H_n(\omega))\| > \varepsilon'') = 0$. But on the other hand,

$$P(\|F(\omega) - \zeta_1(H_n(\omega))\| > \varepsilon'') \geq P(\{\omega : F(\omega) \in K(z, \varepsilon')\}) > 0,$$

which contradicts our hypothesis.

(ii) Let U be an open set in E. We need to show that if $P(F(\omega) \in U) = 0$ then $U \cap \zeta_2(\mathcal{H}) = \emptyset$. This, by our assumptions, means that for every n we have $P(F(T_n^h(\omega)) \in U) = 0$ for some $h \in \mathcal{H}$. For a contradiction, suppose that $\zeta_2(h) \in U, h \in \mathcal{H}$. Then there exists a ball $K(\zeta_2(h), \varepsilon) \subset U$ such that $P(\|F(T_n^h(\omega)) - \zeta_2(h)\| < \varepsilon) = 0$, which contradicts our hypothesis (5.7) and completes the proof.

6. The support theorem

We shall prove the following

THEOREM 6.1. *Let σ and b be functions satisfying conditions $(\widetilde{A}1)$–$(\widetilde{A}4)$ and $X(t)$ be the solution to equation (3.1). Let S_1 and S_2 be given by (3.4) and (3.5), respectively. Then $S_1 = S_2$.*

Proof. We have proved in Lemma 5.4 that $\lim_{n\to\infty} P(\|Y_n\|_{C_1} > \varepsilon) = 0$, where $Y_n(t) = Y^n(t) - Z(t)$. Using (3.10) and (3.11) we obtain the following particular cases of (5.4):

$$\lim_{n\to\infty} P(\|\xi^n - X\|_{C_1} > \varepsilon) = 0 \tag{6.1}$$

and

$$\lim_{n\to\infty} P(\|X^n - \xi\|_{C_1} > \varepsilon) = 0. \tag{6.2}$$

Now we denote by $F(w)$ the solution to equation (3.1) and by $\zeta_1(h) = \zeta_2(h)$ we denote the solution to equation (3.3) for given $h \in \mathcal{H}$. We also set $H_n(w) = B^n$, $T_n^h(w) = w - B^n$+h. From (6.1) and (6.2) we immediately obtain (5.5) and (5.7) for this notation. From Lemma 5.5 we obtain (5.6) and (5.8). From this we conclude that $S_1 = S_2$, which completes the proof.

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