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T. Tracy

## THE SCIENTIFIC AND DIDACTIC WORK OF PROFESSOR TADEUSZ TRACZYK

Professor Tadeusz Traczyk was born on November 14, 1921, at Roznieszew, near Warka (Poland). In 1950 he has received a M.Sc. in Mathematics from the University of Warsaw. Before he had worked as a teacher of mathematics at elementary and high schools. In 1950 he has started his didactic and scientific career at Technical University of Warsaw by accepting the position of assistant at the Chair of Mathematics at the Faculty of Chemistry. In 1959 he obtained his Ph.D. in Mathematics from the Institute of Mathematics of Polish Academy of Sciences writing a thesis on Boolean algebras under the direction of Professor Roman Sikorski. In 1963 he obtained the degree of Privatdozent in Mathematics from the University of Wrocław – his thesis were on the analogies between Boolean and Post algebras. In 1964 he was promoted to the position of dozent at the Technical University of Warsaw and in 1971 he obtained the title of Professor of Mathematics. During his professorship he has been a vice-dean at the Faculty of Chemistry, the dean of the Studium of Basic Technical Problems and the chairman of the Semester on Algebra at the Banach International Mathematical Center. He has also been a visiting professor at the University of Science and Technology in Kumasi (Ghana), and a visiting professor at Al-Fatah University in Tripoli (Libya). He has visited the McMaster University, University of Manitoba, University of Calgary, University of Toronto, Simon Fraser University, Illinois University at Chicago Circle, West Virginia University, University of Technology in Helsinki, Université de Liege, Technische Hochschule Darmstadt, Universität Kaiserslautern, Gesamthochschule Kassel, University in Berlin, University in Halle, Technical University of Vienna, Zagrzeb University, Comenius University Bratislava and Technical University in Prague. In Warsaw, he has been for many years the Head of the Chair of Mathematics at the Faculty of Chemistry and later the Head of the Department of Algebra and the Department of Algebraical Methods at the Institute of Mathematics at the Faculty of Technical Physics and Applied Mathematics of Technical University of Warsaw. In 1991 he retired, but

has been further working part-time and taking active part in the Seminar of Algebra.

Professor Tadeusz Traczyk has been the creator of the school of Algebra at the Technical University of Warsaw. His scientific interest were stimulated by Professors Roman Sikorski and Edward Marczewski and concerned the theory of Boolean and Post algebras, measure theory and topology, the theory of structural numbers and the theory of BCK algebras, lattice theory and the theory of partially ordered sets and the foundations of computer science. He has published almost 50 research papers in these fields. His first papers were dealing with Boolean and Post algebras. He has solved an open problem on the homomorphisms not induced by point mappings and on weak and strong distributivity of Boolean algebras and he has made a fundamental contribution to the theory of Post algebras by formulating a simple set of axioms, proving important theorems on representation and independence in Post algebras. Later he has generalized the theory of Post algebras to the so-called,  $P_0$ -lattices. He has generalized the Loomis-Sikorski theorem and investigated the structure of prime filters in generalized Post algebras. He investigated minimal extensions and free products of  $m$ -distributive Boolean algebras and the  $m$ -amalgamation property. He also studied the generalized Łukasiewicz algebras. The results of Professor Tadeusz Traczyk on Boolean algebras has been included in the monographs in these fields and he has written an encyclopedic chapter on the theory of Post algebras in application to computer sciences. He has been recognized as a leading international expert in these fields. Professor Traczyk has also written interesting papers on developable sets and almost limit points and on the approximation of mappings by Baire mappings. His second field of interest has been connected with the theory of structural numbers. He has given a strict algebraic formulation to the rings of structural numbers invented by S. Bellert in connection with his interest in the theory of electrical nets and proved a representation theorem. Together with A. Romanowska and other authors, he has developed the theory of BCK algebras investigating the structure of commutative BCK algebras, subdirectly irreducible BCK algebras and their varieties, embedding commutative BCK algebras in distributive lattices, and investigating simple commutative BCK chains. Professor Tadeusz Traczyk has made important contribution to the theory of orthoposets and partially ordered sets admitting a full set of atoms, proving a representation theorem and investigating numerical representations of such posets, Boolean algebras and Post algebras. He has used algebraic methods to investigate stochastic informational systems and query language of Boolean informational systems. Together with Professor Tadeusz Kaczorek he investigated partial derivatives of Vandermonde determinant and their applications to the synthesis of linear control

systems – they have found sufficient conditions for the existence of unique solution for the synthesis of many-dimensional linear stationar systems with delay. It is impossible to present in a short review all important results of Professor Traczyk, so we finish with this our review of his scientific work.

Professor Tadeusz Traczyk has stimulated many papers in algebra among his collaborators and students. He has published some papers with coauthors, among others with R. Sikorski, E. Marczewski, M. Mączyński, W. Marek, Z. Burakowski, T. Kaczorek, S. Bellert, W. Zarębski, J. Klukowski, B. Konikowska, A. Romanowska, W. Cornish, T. Sturm, J. Varlet and R. Godowski (in chronological order), just proving his distinguished ability to collaborate with others forming research teams. He has directed 12 doctoral thesis. Among his doctoral students, 3 have obtained dozent degree and 2 have obtained the title of professor. For many years he has directed the Seminar on Algebra (later with A. Romanowska and M. Mączyński), which has had many participants from Opole, Kielce, Białystok, Wrocław, Siedlce, Łódź and of course Warsaw, as well as from Libya, Algeria, Czechoslovakia, Austria, Germany, Finland and Italy. Professor Traczyk is still active in the seminar and stimulates research papers with new ideas.

Besides his scientific work, Professor Traczyk has conducted a very active didactive service at the Technical University of Warsaw and the Medical Academy. He has lectures mathematical subjects at the Faculty of Chemistry, Faculty of Chemical Engineering, Faculty of Technical Physics and Applied Mathematics and Faculty of Farmacy. He is author or coauthor of 8 textbooks, among them the first monograph in the Polish language on Boolean algebras, the textbooks on applied mathematics in chemical engineering and chemistry, and the basic textbooks on mathematics for technical universities. All his textbooks are still used and highly valued by students.

Professor Tadeusz Traczyk has achieved a very important and high position in the field of algebra. His opinion and advice is highly valued by the members of the Scientific Council of Institute of Mathematics. In recognition of his merits, he has been elected President of the Warsaw Chapter of the Polish Mathematical Society.

Since 1977 Professor Traczyk is a member of the Editorial Board of *Demonstratio Mathematica*. He has influenced many papers published in the journal acting as referee or editor. Owing to him, the algebraic side of the journal is well developed and attracts many papers from the country and abroad.

The Editorial Board of *Demonstratio Mathematica* is pleased to publish the present volume dedicated to Professor Tadeusz Traczyk on the occasion of his seventies anniversary of his birthday. The volume includes the papers dedicated to him by his disciples and followers and other authors who have

been influenced by Professor Traczyk's work or who wanted to express their admiration to his achievements. Some of them have been presented at the conference on algebra held in Jachranka 8–13 June 1993. To present Professor Traczyk's scientific work in more details, we are including a list of his scientific publication. We are also including a list of his didactic publication. We hope that by publishing this volume we will pay a tribute to Professor Traczyk's achievements. We also hope this volume will revive the interest in the classical theory of Boolean and Post algebras, where there are still many unsolved problems. The collection of papers published in this volume may be also useful for researchers who would like to get acquainted with the actual scientific activity in the fields connected with domains initiated by Professor Traczyk.

The Editorial Board of *Demonstratio Mathematica* would like to wish Professor Traczyk all the best on the occasion of his seventies birthday. We wish Professor Traczyk many further active years of scientific and didactic activity and we hope that *Demonstratio Mathematica* will enjoy Professor Traczyk's collaboration and expertise also in future, which will contribute to keeping the scientific standard and international recognition of our journal at a high level.

Maciej Mączyński  
Editor-in-Chief

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