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A NEW CLASS OF FOURIER BESSEL-JACOBI SERIES OF MEIJER'S G-FUNCTION

1. Introduction

In this paper, we introduce a new class of Fourier Bessel-Jacobi series of Meijer's G-function ([3], pp. 206–222), and present a Fourier Bessel-Jacobi series of the class.

The following formulae are required in the proofs.

The integral ([2], p. 37, (2.2)]):

$$(1.1)\quad \int_0^\infty x^{\rho-1}J_\nu(x)K_\nu(x)G^{m,n}_{p,q}\left[zx^{4\delta}\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right]dx$$
$$= \frac{2^{3/2\delta-5/2}\delta^{\rho-3/2}}{(2\pi)^{\delta-1/2}}G^{m,n+3\delta}_{p+4\delta,q}\left[\frac{z(2\delta)^{4\delta}}{2^{-2\delta}}\middle|\begin{matrix}\Delta(2\delta,1-\frac{\rho}{2}),\Delta(\delta,1-\frac{\rho}{4}-\frac{\nu}{2}),\\ a_p,\Delta(\delta,1-\frac{\rho}{4}+\frac{\nu}{2}),\\ b_q\end{matrix}\right],$$

where δ is a positive integer, $p+q<2(m+n)$, $|\arg z|<(m+n-1/2p-1/2q)\pi$, $\mathrm{Re}(\sigma+2\nu+4\delta b_j)>0$ $(j=1,\dots,m)$.

The integral ([1], p. 177,(2.1)):

$$(1.2)\quad \int_{-1}^1(1-y)^\sigma(1+y)^\beta P_u^{(\alpha,\beta)}(y)G^{m,n}_{p,q}\left[z(1-y)^\lambda\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right]dy$$
$$=\frac{2^{\beta+\sigma+1}\Gamma(\beta+u+1)}{\lambda^{\beta+1}u!}$$
$$\times G^{m+\lambda,n+\lambda}_{p+2\lambda,q+2\lambda}\left[z2^\lambda\middle|\begin{matrix}\Delta(\lambda,-\sigma),a_p,\Delta(\lambda,\alpha-\sigma)\\ \Delta(\lambda,\alpha-\sigma+u),b_q,\Delta(\lambda,-1-\beta-\sigma-u)\end{matrix}\right],$$

where λ is a positive integer, $2(m+n)>p+q$, $|\arg z|<(m+n-1/2p-1/2q)\pi$, $\mathrm{Re}\,\beta>-1$, $\mathrm{Re}(\rho+\lambda b_j)>-1$ $(j=1,\dots,m)$.

The orthogonality property of the Jacobi polynomials ([4], p. 285, (5), and (9)):

$$(1.3)\quad \int_{-1}^{1} (1-x)^{\alpha}(1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\beta)}(x)\,dx$$

$$= \begin{cases} 0, & \text{if } m \neq n, \\ \dfrac{2^{\alpha+\beta+1}\Gamma(\alpha+n+1)\Gamma(\beta+n+1)}{n!(\alpha+\beta+2n+1)\Gamma(\alpha+\beta+n+1)}, & \text{if } m = n; \end{cases}$$

where $\operatorname{Re}\alpha > -1$, $\operatorname{Re}\beta > -1$.

The orthogonality property of the Bessel Functions ([5], p. 291, (6))

$$(1.4)\quad \int_{0}^{\infty} x^{-1} J_{\nu+2n+1}(x) J_{\nu+2m+1}(x)\,dx$$

$$= \begin{cases} 0, & \text{if } m \neq n, \\ (4n+2\nu+2)^{-1}, & \text{if } m = n,\ \operatorname{Re}\nu + m + n > -1. \end{cases}$$

2. Fourier Bessel-Jacobi series

The Fourier Bessel-Jacobi series to be established is

$$(2.1)\quad x^{\rho}(1-y)^{\sigma} K_{\nu+2v+1}(x) G_{p,q}^{m,n}\left[zx^{4\delta}(1-y)^{\lambda} \middle| \begin{matrix} a_p \\ b_q \end{matrix} \right]$$

$$= \frac{2^{\sigma+3/2\delta-3/2}\delta^{\rho-3/2}}{(2\pi)^{\delta-1/2}\lambda^{\beta+1}}$$

$$\times \sum_{r=0}^{\infty}\sum_{t=0}^{\infty} \frac{(\nu+2r+1)(\alpha+\beta+2t+1)\Gamma(\alpha+\beta+t+1)}{\Gamma(\alpha+t+1)} J_{\nu+2r+1}(x) P_t^{(\alpha,\beta)}(y)$$

$$\times G_{p+4\delta+2\lambda,q+2\lambda}^{m+\lambda,n+3\delta+\lambda}\left[\frac{z(2\delta)^{4\delta}2^{\lambda}}{2^{-2\delta}} \middle| \begin{matrix} \Delta(2\delta, 1-\frac{\rho}{2}), \Delta(\delta, \frac{1}{2}-\frac{\rho}{4}-\frac{\nu}{2}-\mathrm{r}), \Delta(\lambda, -\alpha-\sigma), \\ \mathrm{a_p}, \Delta(\delta, \frac{3}{2}-\frac{\rho}{4}+\frac{\nu}{2}+\mathrm{r}), \Delta(\lambda, -\sigma) \\ \Delta(\lambda, \mathrm{t}-\sigma), \mathrm{b_q}, \Delta(\lambda, -1-\alpha-\beta-\sigma-\mathrm{t}) \end{matrix} \right],$$

valid under the conditions of (1.1), (1.2), (1.3) and (1.4).

$$(2.2)\quad f(x,y) = x^{\rho}(1-y)^{\sigma} K_{\nu+2v+1}(x) G_{p,q}^{m,n}\left[zx^{4\delta}(1-y)^{\lambda} \middle| \begin{matrix} a_p \\ b_q \end{matrix} \right]$$

$$= \sum_{r=0}^{\infty}\sum_{t=0}^{\infty} C_{r,t} J_{\nu+2r+1}(x) P_t^{(\alpha,\beta)}(y).$$

Equation (2.2) is valid, since $f(x,y)$ is continuous and of bounded variation in the region $0 < x < \infty$, $-1 < y < 1$.

Multiplying both sides of (2.2) by $(1-y)^{\alpha}(1+y)^{\beta}P_u^{(\alpha,\beta)}(y)$, integrating with respect to y from -1 to 1, and using (1.2) and (1.3). Then multiplying both sides of the resulting expression by $x^{-1}J_{\nu+2v+1}(x)$, integrating with respect to x from 0 to ∞ and using (1.1) and (1.4), the value of $C_{r,t}$ is obtained. Substituting this value of $C_{r,t}$ in (2.2), the Fourier Bessel-Jacobi series (2.1) is obtained.

Note: On applying the same procedure as above, we can establish three other forms of two-dimensional expansions of this class with the help of alternative forms of (1.1) and (1.2).

Since on specializing the parameters Meijer's G-function yields almost all special functions appearing in applied mathematics and physical sciences. Therefore, the result presented in this paper is of a general character and hence may encompass several cases of interest.

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